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Natural or Violent Motion? Galileo's Conjectures on the Fall of Heavy Bodies

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1. Introduction

According to Westfall, "At its inception seventeenth-century mechanics denied the distinction of natural and violent motion. All motion was to be treated as one, subject all to the same laws"[1]. Paolo Rossi also stressed that the rise of the new physics, in the 17th century, required overthrowing and forsaking (among other things) the Aristotelian distinction between natural and violent motions[2].

According to Aristotelian physics, there was a fundamental distinction between natural and violent motion[3]. When the cause of the motion was internal to the moving body, that motion was regarded as natural. Violent motion was supposed to have an external efficient cause. It should stop as soon as this external cause ceased its action. The fall of a heavy body was believed to have an internal cause – the very nature of the body – but the motion of a projectile was supposed to be accidental and a violent one, produced by an external agent and maintained by the continuous action of the air around it.

On the other side, there was a clear distinction between the natural motions of heavenly bodies, and those belonging to terrestrial (sublunary) bodies[4]. The natural motion of celestial bodies was supposed to be circular (they move around the Earth), and the natural motion of terrestrial bodies is straight (they fall or rise in a radial direction, relative to the Earth). Upwards straight natural motion was associated to fire and air, downwards straight natural motion was ascribed to earth and water, and circular natural motion around the centre of the universe was ascribed to the ether.

Those distinctions disappeared from physics during the 17th century. Descartes and Newton do not address those problems. However, in Galileo's thought those distinctions were still present, and they did play a relevant role in his physics. As Alexandre Koyré remarked, "Modern science did not spring perfect and complete – as Athena from the head of Zeus – from the brains of Galileo and Descartes"[5].

Some central arguments presented by Galileo in the *Dialogues concerning the two chief world systems* depended on the characterisation of circular motion as "natural". Considerations concerning natural or violent motion, circular and straight motions, provided a trap that led Galileo astray to ideas that seem extremely odd, from the point of view of classical mechanics. This article will show how Galileo struggled with those Aristotelian concepts when he tried to understand the nature of the motion of falling bodies.

2. Copernicus' theory and natural motions

Let us try to understand the problem addressed by Galileo.

In the early 17th century, if one accepted Copernicus' theory, it was necessary to admit that the Earth is a planet, of the same nature as Mars or Jupiter. This entailed a rejection of *part* of the old theory of motions, because Aristotle's theory required that the heavens were made of a different kind of matter, endowed with natural circular motions, while all earthly matter was endowed with natural motion in a straight line (either downwards, or upwards).

Now, in the early 17th century it seemed that all heavenly bodies did have circular motions (either around the Earth or around the Sun)[6]. If that is not due to their special kind of nature, why do they move in a way that seems completely different from the motion of a stone or other earthly bodies?

Newton's answer is well known: All bodies have mass and inertia, and they move in straight lines, with uniform motion, unless they are acted by external forces. Circular motion is accelerated motion, of essentially the same nature as the accelerated motion of a falling stone. Planets are acted by gravitational forces that continuously deviate their motions and produce curved (elliptical) paths. The same kind of force that produces those curved trajectories also produces the fall of stones and other phenomena.

Well, that was *not* Galileo's understanding. Galileo did not arrive to the idea of inertia *in a straight line*, as Descartes and Newton. Also, he did not even imagine the possibility of forces between the planets and the Sun, or between the Moon and the Earth. He had to frame a different view, and so he did, keeping the Aristotelian concept of natural motion.

The motion of each planet is periodical, and regular, as recorded since Antiquity. No astronomer believed that they could be slowing down or accelerating. Except for theological reasons, those motions seemed *eternal*. Now, one of the distinctions between natural and violent motions, for Aristotle, was that the former ones did not wear away, while the later did wear away, and could not be eternal. As celestial motions seemed eternal, one should accept that they were *natural* motions. Galileo accepted this conclusion, and tried to fit other kinds of motion to this central concept.

It seems that Galileo got his main ideas on this subject from Copernicus. According to the *De revolutionibus*, all round bodies – irrespective of their material – have a natural motion of rotation around their respective centres. This is the main natural motion, and the only kind of motion that can occur when all bodies are in their natural places. However, when a body is outside its natural place, it can have another kind of natural motion, in a straight line, toward its proper position.

Of course, this was a deep change of Aristotle's theory. The table in the next page presents some points of disagreement between Aristotle's and Copernicus' views concerning natural motions. Why did Copernicus change Aristotle's theory in that way? It was necessary for Copernicus to ascribe to the Earth a *natural* motion of rotation because in his theory this rotation was to be regular and eternal. And Copernicus chose to associate natural rotation with a round form:

> *"[...] let us hold as certain that the Earth is held together between its two poles and terminates in a spherical surface. Why therefore should we hesitate any longer to grant to it the movement which accords naturally with its form [...]?"* (De revolutionibus I.8, p. 6v; On the revolutions, p. 519)[7].

ARISTOTLE	COPERNICUS
(1) Only the heavenly ether has a natural motion of rotation. The centre of this motion is the centre of the universe.	(1') Any round body, whatever its substance, has a natural motion of rotation. The centre of this motion is the centre of the body.
(2) When the sublunary substances (earth, water, air, fire) are outside their natural places, they have a natural motion in straight line toward their natural place. When they arrive to their natural place, they attain a natural rest.	(2') When any substance is outside its natural place, it has a natural straight motion toward its natural place. When it arrives to its natural place, it acquires a natural motion of rotation.
(3) Heavy bodies fall toward the Earth because they move toward their natural place (the centre of the universe), and the Earth is at the centre of the universe.	(3') Terrestrial matter falls toward the Earth because it has a natural appetency to unite to the similar matter that composes the body of the Earth. Something similar to this applies to the other planets.
(4) Each body can only have a single natural motion. When a heavy body falls, it has a simple natural motion in a straight line toward the centre of the universe.	(4') When a heavy body falls, it has two motions: a straight motion toward the centre of the Earth, and a circular motion around the centre of the Earth.

Tabela

Besides that, assuming that this rotation was natural allowed him to answer a classical objection ascribed to Ptolemy: If the Earth has a very fast motion of rotation around its axis, why aren't terrestrial bodies thrown away from its surface?

> "*But if someone opines that the Earth revolves, he will also say that the movement is natural and not violent. Now things which are according to nature produce effects contrary to those which are violent. For things to which force of violence is applied get broken up and are unable to subsist for a long time. But things which are caused by nature are in a right condition and are kept in their best organisation.*" (De revolutionibus I.8, p. 5r; On the revolutions, p. 518).

In the case of terrestrial bodies, it was necessary for Copernicus to explain their downwards (or upwards) motion without taking into account any tendency of matter toward the centre of the universe, since the Earth is supposed to move through space.

> "*I myself think that gravity or heaviness is nothing except a certain natural appetency implanted in the parts by the divine*

> *providence of the universal Artisan, in order that they should unite with one another in their oneness and wholeness and come together in the form of a globe. It is believable that this effect is present in the sun, moon, and the other bright planets and that through its efficacy they remain in the spherical figure in which they are visible, though they nevertheless accomplish their circular movements in many different ways"* (De revolutionibus I.9, p. 7v; On the revolutions, p. 521).

But if heavy bodies had only a simple radial motion toward the centre of the Earth, they would not follow the rotation of the Earth, and their fall, as observed by terrestrial inhabitants, would not follow a straight line. In order to account for the apparent straight fall of a stone, it was necessary to assume that it has a twofold movement: both downwards and around the Earth's axis. Why do heavy bodies have this second, rotational motion? Because they have the same nature as the Earth.

> *"But we must confess that in comparison with the world the movement of falling and of rising bodies is twofold and is in general compounded of the rectilinear and the circular. As regards things which move downward on account of their weight because they have very much earth in them, doubtless their parts possess the same nature as the whole [...]"* (De revolutionibus I.8, p. 6v-r; On the revolutions, p. 519).

However, according to Aristotle, a simple body can only have a simple movement – either rotation, or motion in a straight line. How could a heavy body have two *natural* motions at the same time? Copernicus changed Aristotle's theory, denying that a simple body *always* has a simple motion: that would happen *only when it is in its natural place*. When it is moving toward its natural place, it will have two simultaneous natural motions (circular and straight motions):

> *"Accordingly, as they say, a simple body possesses a simple movement – this is first verified in the case of circular movement – as long as the simple body remain in its natural place. In this place, in fact, its movement is none other than the circular, which remains entirely in itself, as though at rest. Rectilinear movement, however, is added to those bodies which journey away from their natural place or are shoved out of it or are outside it somehow. But nothing is more repugnant to the order of the whole and to the form of the world than for anything to be outside of its place. Therefore rectilinear movement belongs only to bodies which are not in the right condition and are not perfectly conformed to their nature – when they are separated from their whole and abandon its unity"* (De revolutionibus I.8, p. 6v; On the revolutions, p. 520).

Notice that Copernicus is not completely clear about the cause of the natural circular motion. If the Earth turns around its centre because it is a sphere, then a lead ball would also turn around *its own* centre, not around the centre of the Earth. On the other side, if all terrestrial matter has a natural circular motion around a specific centre, how is it possible that this centre moves itself around the Sun?

Within Aristotle's theory, each kind of matter has one and only one peculiar kind of natural motion. If we accept that the Earth is a planet, it has at least *two* different motions: rotation around its axis and rotation around the Sun. Are all those motions *natural*, or not? If they are all natural ones, how can a body have more than one natural circular motion, around different centres? If they are not all natural, which one is natural? Why do they all seem regular and eternal?

Copernicus seemingly assumed that all the Earth's motions are natural. It is likely that he still thought that the motion of the planets was due to real transparent spheres[8], and in that case it was possible to ascribe different motions to different spheres – each one revolving around its own centre with a constant speed.

3. Galileo and the natural motion of heavenly bodies

Much of the use Galileo made of the idea of natural motion in his book *Dialogue concerning the two chief world systems* follows from Copernicus' concepts[9]. In the first day of the *Dialogue*, he introduced a difference between circular and rectilinear motions, arguing that motion in a straight line is unsuitable for an ordered universe:

> *"Salviati – This principle being established then*[10]*, it may be immediately concluded that if all integral bodies in the world are by nature movable, it is impossible that their motions should be straight, or anything else but circular; and the reason is very plain and obvious. For whatever moves straight changes place and, continuing to move, goes ever farther from its starting point and from every place through which it successively passes. If that were the motion which naturally suited it, then at the beginning it was not in its proper place. So then the parts of the world were not disposed in order; and in that case, it is impossible that it should be their nature to change place, and consequently to move in a straight line"* (Dialogue, p. 19; "Straight motion cannot exist in a well-ordered universe")[11].

At another place (second day), Galileo argued that only circular motion can be eternal, and that only eternal motion can be natural[12]:

> *"Salviati – But if that which is forced cannot be eternal, then by the converse that which cannot be eternal cannot be natural; but there is no way for the earth's downward motion to be eternal, and so much the less can it be natural, nor can any motion be natural to it which cannot be eternal to it. But if we make the earth circularly movable, this can be eternal to it and to its parts, and therefore natural"* (Dialogue, p. 134; "What is forced cannot be eternal, and what cannot be eternal cannot be natural.").

The idea of natural circular motions was so strong in Galileo's thought, that he never accepted the possibility of elliptical orbits. In the *Dialogue* he always describes the movement of planets and satellites as *uniform circular motions*, never taking into account epicycles, eccentric circles or equants (*Dialogue*, book 3, pp. 320-325; p. 342; p. 345; pp. 390-391; p. 396).

In the case of the Earth, Galileo is confronted with the difficulty of ascribing two or more natural motions to the same body. Galileo did not accept any more the theory of transparent spheres, and therefore he could not solve this problem in the same way as Copernicus. Galileo seemingly thought that the Earth moved through space unattached to anything, turning at the same time around its axis and around the Sun. How could a single body have two different natural motions? That is an objection presented by Simplicio in the third day of the *Dialogue*. Galileo did not deny that the Earth had several different natural motions, but replied:

> "*Salviati – Then as to the annual and diurnal movements, these, being made in the same direction, are quite compatible, in the same way that if we were to let a ball run down a steep surface, it would, in descending spontaneously along that, turn upon itself*" (Dialogue, p. 389; "Annual motion and diurnal motion compatible in the earth.").

A few pages later, Galileo compared the Earth with a magnet, and argued that a lodestone has three different natural motions – therefore, it was not impossible that the Earth also had several different natural motions:

> "*Sagredo – What I wanted to bring up for consideration was precisely the lodestone, to which three movements are sensibly seen to belong naturally: One toward the center of the earth as a heavy object; a second is the horizontal circular motion by which it restores and conserves its axis in a direction of certain parts of the universe; and third is this one discovered by Gilbert, of dipping its axis in the meridian plane toward the surface of the earth, in greater or less degree proportionate to its distance from the equator (where it remains parallel to the axis of the earth). Besides these three, it is perhaps not improbable that it may have a fourth motion of turning about its own axis, whenever it is balanced and suspended in air or some other fluid and yielding medium and all external and accidental impediments are taken away; Gilbert himself also shows his approval of this idea. So you see, Simplicio, how shaky Aristotle's axiom is*" (Dialogue, p. 411; "Three diverse natural motions of lodestone.").

So, Galileo does not deny the very *concept* of natural motion: he uses it, but denies that a single body can only have a single natural motion.

According to Galileo, although the natural motion of the parts of the world must be circular, straight motion could also have a place in the universe:

> "*Salviati – But someone might say nevertheless that although a straight line (and consequently the motion along it) can be extended in infinitum (that is to say, is unending), still nature has, so to speak, arbitrarily assigned to it some terminus, and has given her natural bodies natural instincts to move toward that. And I shall reply that this might perhaps be fabled to have occurred in primordial chaos, where vague substances wandered confusedly in disorder, to regulate which nature would very properly have used straight motions. By means of these, just as well-arranged bodies would become disordered in moving, so those which were previously badly disposed might be arranged in order. But after their optimum distribution and arrangement it is impossible that there should remain in them natural*

> *inclinations to move any more in straight motions, from which nothing would now follow but their removal from their proper and natural places; which is to say, their disordering.*
> *We may therefore say that straight motion serves to transport materials for the construction of a work; but this, once constructed, is to rest immovable – or, if movable, is to move only circularly"* (Dialogue, pp. 19-20: "Straight motion possible in primordial chaos."; "Straight motion suitable for arranging badly disposed bodies.").

A few pages later, Galileo returned to this subject, and maintained that only circular motions are suitable for the maintenance of the order of the universe (*Dialogue*, pp. 31-32).

> *"Salviati – I therefore conclude that only circular motion can naturally suit bodies which are integral parts of the universe as constituted in the best arrangement, and that the most which can be said for straight motion is that it is assigned by nature to its bodies (and their parts) whenever these are to be found outside their proper places, arranged badly, and are therefore in need of being restored to their natural state by the shortest path"* (Dialogue, p. 32: "Straight motion assigned to natural bodies to restore them to perfect order when they are disordered."; "Only rest and circular motion suitable to maintain order.").

However, we might ask how bodies that have attained their natural place obtain a natural motion of rotation. Is this motion due to an intrinsic cause? Galileo didn't think so.

4. The cause of the motion of the planets

It was between those two points of the *Dialogue* that Galileo introduced for the first time his idea of circular inertia. When a body descends along an inclined plane, its speed increases; when it rises, its speed decreases. If it moves along a surface that neither goes up nor down, and if there is no other cause that interferes with its motion (friction, etc.), it will move forever with constant speed. This motion would be a circular motion around the Earth – that is, all bodies tend to keep an uniform speed when they turn around the centre of the Earth.

> *"Salviati – (...) motion in a horizontal line which is tilted neither up nor down is circular motion about the centre; therefore circular motion is never acquired naturally without straight motion to precede it; but, being one acquired, it will continue perpetually with uniform velocity"* (Dialogue, p. 28: "Circular motion can never be acquired naturally without straight motion preceding it."; "Circular motion perpetually uniform.").

The same laws apply to terrestrial and celestial bodies. Therefore, Galileo tried to explain the circular motion of the planets as due to a fall toward the Sun, followed by a change of direction.

> *"Salviati – We may therefore say that straight motion serves to transport materials for the construction of a work; but this, once constructed, is to rest immovable – or, if movable, is to move only circularly. Unless we wish to say with Plato that these world bodies, after their creation and the establishment of the whole, were for a*

> *certain time set in straight motion by their Maker. Then later, reaching certain definite places, they were set in rotation one by one, passing from straight to circular motion, and have ever since been preserved and maintained in this. A sublime concept, and worthy indeed of Plato, which I remember having heard discussed by our friend, the Lincean Academician"* (Dialogue, p. 20; "According to Plato, worlds were moved first in straight and afterward in circular motions.").

> *"Salviati – (...) let us suppose God to have created the planet Jupiter, for example, upon which He had determined to confer such-and-such a velocity, to be kept perpetually uniform forever after. We may say with Plato that at the beginning He gave it a straight and accelerated motion; and later, when it had arrived at that degree of velocity, converted its straight motion into motion whose speed thereafter was naturally uniform"* (Dialogue, p. 21; "Nature makes the body move in a straight line to induce in it any given speed."; "Uniform speed agrees with circular motion.").

So, according to Galileo, the circular motion of the planets is not due to some intrinsic cause, but is acquired in a straight, accelerated motion. At another place he explained that this accelerated motion was directed toward the centre of the system, and that all planets started their motions from the same place:

> *"Salviati – And here I wish to add one particular observation of our Academic friend which is quite remarkable. Let us suppose that among the decrees of the divine Architect was the thought of creating in the universe those globes which we behold continually revolving, and of establishing a center of their rotations in which the sun was located immovably. Next, suppose all the said globes to have been created in the same place, and there assigned tendencies of motion, descending toward the center[13] until they had acquired those degrees of velocity which originally seemed good to the Divine mind. These velocities being acquired, we lastly suppose that the globes were set in rotation, each retaining in its orbit its predetermined velocity. Now, at what altitude and distance from the sun would have been the place where the said globes were first created, and could they all have been created in the same place?"* (Dialogue, p. 29; "Sizes of orbits and speeds of planetary motions accord in ratio with descent from the same place.").

Galileo's answer is affirmative. According to him, it is possible to compute "from the proportions of the two velocities of Jupiter and Saturn and from the distance between their orbits, and from the natural ratio of acceleration of natural motion, at what altitude and distance from the center of their revolutions must have been the place from which they originally departed" (*Dialogue*, p. 29). Of course, Galileo did not anticipate Newton's law of gravitation, and he could only assume that this accelerated motion of the planets obeyed the same law as the fall of stones toward the Earth – a strange assumption, anyway. Afterwards, using this result, he stated that it was possible to compute the speeds of Mars, the Earth, Venus and Mercury, assuming that they had all started their accelerated motions toward their respective orbits from the same point, and that "the size of [their] orbits and the velocities of [their] motions agree so closely with those given by the computations that the matter is truly wonderful" (*Dialogue*, p. 29).

It seems likely that Galileo never computed those velocities and distances – the advertised success of his theory was a bluff. Father Marin Mersenne, however, read the *Dialogue* and checked Galileo's statements, publishing his results in 1635[14]. According to the data used by Mersenne[15], the medium speed of Saturn was about 1730 feet per second, and that of Jupiter was about 2336 feet per second. Taking into account the measured acceleration of bodies falling toward the Earth, Mersenne computed that those speeds would be acquired if they had fallen with accelerated motion during 72 seconds (Saturn) and 97 seconds (Jupiter), respectively, and that they had fallen 62,393 feet or about 4 leagues[16] (Saturn) and 113,751 feet or about 7 ½ leagues (Jupiter) – very small distances, from the astronomical point of view. Now, the mean radius of Saturn's orbit was supposed to be about 17,070,000 leagues and that of Jupiter about 9,280,000 leagues. The computed heights of fall are incompatible with the distance between the orbits. Even taking into account the least distance between Saturn and the Sun and the largest distance between Jupiter and the Sun, it is impossible to conciliate the sizes of the orbits with such small heights of fall. Of course, when one extends the computation to the other planets, the disagreement is also too large to be accounted by any secondary cause. In the case of the Earth, for instance, its speed would have been acquired in a fall of just 50 leagues, which is negligible even when compared to its own diameter (1145 leagues, according to Mersenne). It is impossible that all planets fell from the same place toward their orbits.

5. The rotation of the earth and the motion of terrestrial bodies

When Galileo discussed the arguments against the motion of the Earth, in the second day of the *Dialogue*, he tried to solve a few problems using the concept of natural motion. This appears in a conspicuous way in the famous analysis of the tower experiment. According to Galileo (and Copernicus) a stone falling from a tower has two simultaneous motions: one straight motion downward, and a circular motion accompanying the rotation of the Earth. Why does the stone keep its horizontal motion, when it falls? Nowadays we would appeal to inertia. In one of his arguments, however, Galileo invoked natural motions:

> *"But the diurnal rotation is being taken as the terrestrial globe's own and natural motion, and hence that of all its parts, as a thing indelibly impressed upon them by nature. Therefore the rock at the top of the tower has as its primary tendency a revolution about the center of the whole in twenty-four hours, and it eternally exercises this natural propensity no matter where it is placed"* (Dialogue, p. 142).

At another place, after arguing that a stone left from the top of the mast of a moving ship would accompany the motion of the ship, Galileo added:

> *"Now if in this example no difference whatever appears, what is it that you claim to see in the stone falling from the top of the tower, where the rotational movement is not adventitious and accidental to the stone, but natural and eternal, and here the air as punctiliously follows the motion of the earth as the tower does that of the terrestrial globe?"* (Dialogue, p. 154).

At some places, the rotational motion of the earthly bodies is described in a way that cannot be distinguished from the Newtonian concept of inertia. At other places, however, it becomes clear that Galileo's concept is completely different:

> "*Salviati – Copernicus will say that this natural tendency of elemental bodies to follow the terrestrial motion has a limited sphere, outside of which such a natural tendency ceases*" (Dialogue, p. 238; "Tendency of elemental bodies to follow the earth has a limited sphere.").

Also, when describing the westward winds which he presented as effects produced by the rotation of the Earth, Galileo stated that the air has no tendency to follow the motion of the Earth (*Dialogue*, pp. 437-439).

When Galileo discussed the argument of the cannon ball shot vertically upwards, he also claimed:

> "*Sagredo – Keeping up with the earth is the primordial and eternal motion ineradicably and inseparably participated in by this ball as a terrestrial object, which it has by its nature and will possess forever*" (Dialogue, pp. 177-178).

At those places, therefore, the natural motion of rotation of the Earth and of terrestrial bodies provided a central argument for the defence of Galileo's views.

6. Is the fall of heavy bodies a natural motion?

We have already seen that Galileo attributed a prominent role to natural motions of *rotation*, and ascribed to straight motions the role of restoring natural bodies to their proper places. Between those straight movements one should include the fall of heavy bodies toward the Earth. Now, is this kind of movement really a *natural* motion? The answer is "yes", according to Galileo. He presented an account of gravity similar to the one introduced by Copernicus:

> "*Salviati – Second, if it should be said that the parts of the earth do not move so as to go toward the center of the universe, but so as to unite with the whole earth (and that consequently they have a natural tendency toward the center of the terrestrial globe, by which tendency they coöperate to form and preserve it) [...]*
> "*Now, just as all the parts of the earth mutually coöperate to form its whole, from which it flows that they have equal tendencies to come together in order to unite in the best possible way and adapt themselves by taking a spherical shape, why may we not believe that the sun, moon, and other world bodies are also round in shape merely by a concordant instinct and natural tendency of all their component parts? If at any time one of these parts were forcibly separated from the whole, is it not reasonable to believe that it would return spontaneously and by natural tendency?*" (Dialogue, p. 33-34; "Natural tendency of the parts of all world globes is toward their centers.").

Galileo added that the straight natural motion of the parts toward the whole is accelerated:

> "*Sagredo – Furthermore, if a greater or lesser velocity can alter the simplicity of motion, no simple body would ever move*

with a simple motion, since in all natural straight motions the velocity is ever increasing and consequently always changing in simplicity [...]" (Dialogue, p. 17).

"Salviati – Every body constituted in a state of rest but naturally capable of motion will move when set at liberty only if it has a natural tendency toward some particular place; for if it were indifferent to all places it would remain at rest, having no more cause to move one way than another. Having such a tendency, it naturally follows that in its motion it will be continually accelerating" (Dialogue, p. 20).

"Sagredo – Then you believe that a stone, leaving its state of rest and entering into its natural motion toward the center of the earth, passes through every degree of slowness less than any degree of speed? Salviati – I believe it; indeed I am confident of it; confident in the certainty that I can make you also equally convinced of it" (Dialogue, pp. 21-22).

When a stone is released from the top a tower, therefore, it has two different *natural* motions at the same time: a rotation following the diurnal motion of the Earth, and a straight motion toward the centre of the Earth. As we have already seen, Copernicus had already proposed this view, which strongly conflicted with Aristotle's physics. At several points of the *Dialogue* Galileo tried to argue that it was not absurd to conceive two simultaneous natural motions.

"Simplicio – For to expect the rock to go grazing the tower if that were carried along by the earth would be requiring the rock to have two natural motions; that is, a straight one toward the center, and a circular one about the center, which is impossible.
"Salviati – So Aristotle's defence consists in its being impossible, or at least in his having considered it impossible, that the rock might move with a motion mixed of straight and circular. [...] Nevertheless this does not excuse Aristotle, not only because if he did have this idea he ought to have said so, it being such an important point in the argument, but also, and more so, because it cannot be said either that such an effect is impossible or that Aristotle considered it impossible" (Dialogue, pp. 140-141).

"Salviati – [...] in the first place it is obvious that these two motions (I mean the circular around the center and the straight motion toward the center) are not contraries, nor are they destructive of one another, nor incompatible" (Dialogue, p. 149).

Several years later, while writing the *Discorsi*, Galileo did not bother much about this issue and simply drew out the consequences of the combination of the two movements. However, it does seem that when Galileo wrote the *Dialogue* he did worry about the possibility of two simultaneous natural motions – especially in the case of the odd combination of circular diurnal rotation with straight fall of heavy bodies. Indeed, he tried to *eliminate* natural straight motion and to include free fall among *circular* natural movements, as will be shown below.

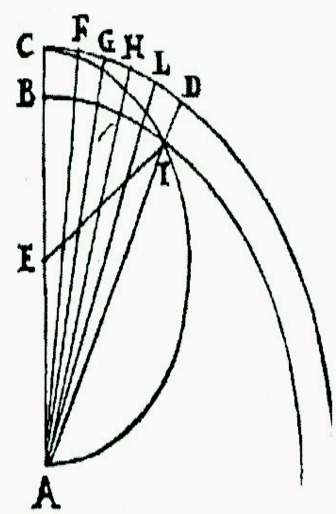
7. Falling to the centre of the earth

Let us suppose that a hole was bore in the Earth, from one point on its surface to the opposite side, passing through the centre. What would happen if we dropped a stone from one of the ends of such a tunnel? Galileo addressed that problem, in the second day of the *Dialogue*. That kind of question had already occurred to several people before Galileo. Nicoló Tartaglia, for instance, thought that the body should fall to the centre of the Earth, increasing its velocity, and then it would continue to move, going up the second half of the diameter, with decreasing speed, until it reached the opposite end of the tunnel. Galileo agreed, but he asked something more. He attempted to find out the motion of the falling stone, taking into account, in his analysis, an important feature: the Earth's rotation.

Galileo accepted the rotation of the Earth, but nevertheless he believed that the stone would fall through the hole to the very centre of the Earth[17], with accelerated motion. How could both motions be conciliated? Galileo's solution was not incorporated into classical mechanics, but it was extremely clever. He assumed that the falling body would have a compound motion: it would maintain a *constant angular velocity* around the axis of the Earth[18], keeping therefore in pace with it; and at the same time would fall with increasing velocity through the hole. However, Galileo did not use his well-known free fall law, in this case. Instead of that, Galileo introduced a completely new hypothesis: he assumed that the composition of the falling motion with the motion of rotation around the Earth could produce a semicircular trajectory, ending at the very centre of the Earth.

This is an odd idea, from the point of view of our post-Galilean physics. It is easier to understand Galileo's proposal from the drawing he used to illustrate his concept (Fig. 1). When the stone is dropped in the well, its radial velocity increases slowly, beginning from rest. Its initial motion should be, therefore, tangential to the circumference of the Earth. Afterwards, the downward speed of the stone should be ever increasing. On the other side, Galileo assumed that the motion should end at the centre of the Earth. A semicircular trajectory obeyed those conditions, and besides that it had other advantages that were perceived by Galileo (*Dialogue*, pp. 164-167). Let us adopt Galileo's hypothesis, and see what follows.

First of all, the semicircular trajectory leads to an initial radial motion that obeys the law of falling bodies: the distance between the stone and the starting point would be proportional to the square of time, for small times. Let us prove it, using present day mathematical techniques.



[FIGURE 1]

Suppose the falling body moves along the semicircle and that its angular displacement, relative to its initial position, is ϕ. How much did this body descend along the well? This radial distance will be equal to the difference between the radius of the Earth (R = OA) and the distance OB between the centre of the Earth and the semicircle. It is straightforward to see that AB is perpendicular to OB and that OB = R.cos ϕ. For small angles, we can compute the cosines using the first terms of the series:

$$\cos \phi = 1 - \phi^2/2 + \phi^4/6 - \ldots$$

A good approximation can be obtained using terms up to the second power of the angle:

$$OB \cong R.(1 - \phi^2/2)$$

Therefore, the stone fell a distance $h \cong R - OB = R.\phi^2/2$. Now, the angle ϕ is proportional to time, since the Earth has an uniform rotation and the falling body accompanies the rotation of the Earth. Therefore, making $\phi=\omega.t$, we obtain:

$$h \cong R.(\omega.t)^2/2 = (R.\omega^2/2).t^2 = k.t^2$$

Therefore, in the case of small angles (corresponding to small times), the vertical distance travelled by the falling body will be proportional to the square of time, in agreement with the Galilean law for falling bodies.

Notice, however, that the agreement is not exact. Motion along the semicircle is not rigorously compatible with a fall proportional to the square of times for large angles. Galileo probably noticed that, because he entertained some doubts concerning this circular motion: "But that the descent of heavy bodies does take place in exactly this way, I will not at present declare; I shall only say that if the line described by a falling body is not exactly this, it is very near to it" (Dialogue, p. 167). If the falling body follows the law of uniform accelerated motion *exactly*, the motion cannot be circular, and vice-versa. Which alternative should one choose? It seems that Galileo preferred the hypothesis of circular motion, because it would harmoniously fit with his concept of natural uniform rotation, because at other places he stated that the motion was "probably" circular (*Dialogue*, p. 242, p. 264).

The hypothetical semicircular motion had other curious properties that Galileo presented in the *Dialogue*:

1) The falling body "really moves in nothing other than a simple circular motion, just as when it rested on the tower it moved with a simple circular motion" (*Dialogue*, p. 166; "Body falling from the top of a tower moves along the circumference of a circle.").

2) The motion along the semicircle is uniform; that is, its angular velocity is constant – amounting to exactly twice the angular speed of the Earth. The linear speed along the semicircle is therefore also uniform. It is exactly equal to the tangential speed of a body resting on the surface of the Earth (*Dialogue*, p. 166; "It moves neither more nor less than if it had stayed on top.").

3) Relative to the universe (but not relative to us), the motion of a falling body "is never accelerated at all, but is always equable and uniform" (*Dialogue*, p. 166; "It moves equably and is not accelerated").

Galileo regarded those properties as extremely significative, since they could suggest that the acceleration observed in the case of falling bodies was a mere appearance: as a matter of fact, the falling body keeps an uniform motion, with the very same speed it had when it was "still" at the surface of the Earth. Accelerated falling motion is therefore only an illusion we have because we cannot observe the motion as it really is.

Galileo's interpretation allowed him to introduce the following idea: every natural motion is circular and uniform. There are no natural straight motions. There is no difference between natural motions of terrestrial and celestial bodies – they have the same nature and all of them move in the same way – in circles.

> "Sagredo – Well, Salviati, there is another remarkable thing which I have just been reflecting about. It is that, according to these considerations, straight motion goes entirely out the window and nature never makes any use of it at all. Even that use which you granted to it at the beginning, of restoring to their places such integral, natural bodies as were separated from the whole and badly disorganized, is now taken away and assigned to circular motion" *(Dialogue, p. 167; "Straight motion seems entirely excluded from nature.").*

That was the strongest feature of Galileo's argument: it eliminated the need of two different simultaneous natural motions in a falling body, reducing free fall to uniform rotation. Koyré once remarked that this was a strange and "non Galilean" statement[19]. This might be indeed a strange statement, and it certainly is not Newtonian, but it *is* Galileo's. Instead of conflicting with Galileo's main ideas, it provides a clue to his deeper thoughts on motion.

8. Difficulties of Galileo's hypothesis

The transformation of free fall into circular motion solved one problem, but was not devoid of difficulties. Mersenne soon pointed them out, in his book *Harmonie universelle*[20].

According to Galileo's hypothesis, the stone falling from the top of the tower will follow a circular trajectory and will reach the centre of the Earth after completing the semicircle (180 degrees), exactly at the same time when the tower and the Earth complete a 90 degrees rotation. Now, this would always take 6 hours, whatever the height of the tower. If the tower had a negligible height, the time of fall would be 6 hours. If the height of the tower were equal to the radius of the Earth, the stone would also reach the centre of the Earth in 6 hours. In the second case, however, the acceleration of the stone would be twice that of the former case, of course. If Galileo's hypothesis were correct, the acceleration of gravity should *increase* proportionally to the distance from the centre of the Earth.

Did Galileo ever conjecture that the acceleration of gravity could depend on the height of the falling body? It does not seem so. Indeed, he even assumed that the acceleration of gravity was constant when he computed the time of fall of a stone *from the sphere of the Moon to the Earth*[21].

Of course, one could assume that the acceleration of gravity does indeed depend on the distance from the Earth. Mersenne remarked that it was impossible to confirm or to reject that hypothesis by experiments made at different places, because the difference of acceleration would be negligible.

If the stone falling from the surface of the Earth is to arrive to the centre of the Earth in exactly 6 hours, and if the radius of the Earth is known, it is possible to compute the expected acceleration of gravity from the hypothesis of circular fall. Mersenne used an ingenious geometrical argument. In 3 seconds of time, the Earth turns an angle equal to 45". Mersenne computed the distance between the circumference of the Earth and the circular trajectory of the falling body corresponding to this angle of 45", and concluded that the stone should fall only 4 inches and 11 lines in this time[22] – a distance much smaller than expected, since

Mersenne had observed that a heavy body fell 108 feet in 3 seconds[23]. Conversely, supposing that the acceleration of the body corresponds to a fall of 108 feet in 3 seconds, Mersenne computed that it would require only 48,5 seconds to reach the centre of the Earth, instead of 6 hours. Galileo's hypothesis was, therefore, unacceptable. Mersenne remarked:

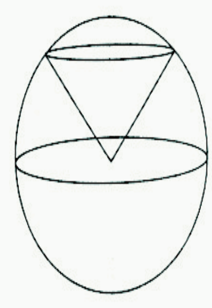
> "*It is easy to conclude from all this discourse that Galileo contented himself with a proportion of fall that seemed to him to accord with the appearances, and that, thinking mainly on the beautiful agreements and consequences that he deduced from it, he did not fathom into this subject, because it is impossible to believe that such a man could have mistaken himself if he had examined with more attention the fall of heavy bodies, according to experiments that he had made himself*"[24].

Mersenne tried to correct Galileo's analysis. He assumed that the downward motion of the stone would have the known acceleration of gravity, and added to this radial motion the assumed rotation around the centre of the Earth. He arrived at a curve that was clearly completely different from a semicircle[25]. Mersenne wrote a letter to Fermat on this subject, and Fermat agreed that Galileo was wrong[26]. In 1637, Fermat wrote to P. de Carcavi on this subject, and Carcavi wrote to Galileo, who replied that he had proposed the semicircular trajectory only as a joke. We agree, however, with Koyré, who found it difficult to believe that Galileo's proposal was a mere joke[27]. Galileo's hypothesis was wrong, but it was intended to solve an important conceptual problem, and fitted nicely with his views on natural motion. Notice also that Galileo's hypothesis was the starting point of Ismael Bouillad's work, who in 1639 proposed the exclusion of all natural straight motions, reducing them to combinations of uniform circular motions[28].

It is remarkable, however, that even in a *qualitative* way Galileo's hypothesis did not solve the problem it was intended to answer. Indeed, the motion of the falling body could only be circular (or close to a circle, taking into account Galileo's doubts) if the motion had place in the equatorial plane. If the hole through the Earth went from one pole to the other, the rotation of the Earth could not interfere with the downward movement, and the motion would follow a straight line.

What would happen if the motion started at any other place? According to Galileo, its motion relative to the Earth would be always a straight radial motion toward the centre, but relative to the universe we should also take into account the uniform rotation around the Earth's axis. The resultant motion would correspond to a curved line on the surface of a cone, since the falling body would have an accelerated downwards motion along a radius which turns around the axis of the Earth.

As a matter of fact, Galileo did not analyse those cases, but an equivalent situation had already been introduced in Johann Georg Locher's 1614 thesis[29], discussed at length by Galileo in the second day of the *Dialogue* (Dialogue, pp. 218 ff.). Locher discussed what would happen if a heavy body fell



[FIGURA 2]

from the height of the Moon to the Earth, according to the Copernican theory, and tried to show that the consequences would be absurd. In the *Dialogue*, Galileo puts the description of Locher's ideas in Simplicio's mouth:

> "*Simplicio – Now if by Divine power, or by means of some angel, a very large cannon ball were miraculously transported there [to the orbit of the Moon] and placed vertically over us and released, it is indeed a most incredible thing (in his view and mine) that during its descent it should keep itself always in our vertical line, continuing to turn with the earth about its center for so many days, describing at the equator a spiral line in the plane of the great circle, and at all other latitudes spiral lines about cones, and falling at the poles in a simple straight line*" (Dialogue, p. 219).

After a long digression in which Galileo discussed the time of fall of a stone from the height of the Moon (using a constant gravity acceleration) he returned again to the initial point, and agreed that the cannon ball would move, according to Copernicus' theory, exactly as described, but denied that those consequences of the theory were absurd (Dialogue, p. 243). Well, granted that they are not absurd, they nevertheless counteract Galileo's attempt to dismiss straight natural motion and reduce all natural motions to circular ones.

9. Conflict between the concepts of *impetus* and natural motion

Not every motion was natural, for Galileo. A stone thrown in the air has a violent motion that obeys to different laws[30]. Following the anti-Aristotelian tradition[31], Galileo applied to those movements the *impetus* concept[32]. When a stone is thrown, it receives an *impetus* to move in a straight line, even if it is thrown away after undergoing a circular motion: "The impressed impetus, I say, is undoubtedly in a straight line" (Dialogue, p. 191; "Motion impressed by the thrower is along a straight line only.")[33].

> "*Salviati – Up to this point you knew all by yourself that the circular motion of the projector impresses an impetus upon the projectile to move, when they separate, along the straight line tangent to the circle of motion at the point of separation, and that continuing with this motion, it travels ever farther from the thrower*" (Dialogue, p. 193).

In Aristotle's physics, the motion of a projectile was maintained by an external cause: it was carried by the air around the body. The *impetus*, however, was something acquired by the moving body and it was *inside* the projectile. In this sense, projectile motion became similar to natural motion, because in both cases the cause was internal[34].

> "*Salviati – You call that principle external, preternatural, and constrained which moves heavy projectiles upward, but perhaps it is no less internal and natural than that which moves them downward. It may perhaps be called external and constrained while the movable body is joined to its mover; but once separated, what external thing remains as the mover of an arrow or a ball? It must be admitted that the force that takes this on high is no less internal than that which moves it down. Thus I consider the upward motion of heavy bodies due to received*

> *impetus to be just as natural as their downward motion dependent upon gravity"* (Dialogue, p. 235; "The force which conducts heavy bodies thrown upward is no less natural than the heaviness which moves them downward".).

There was also, of course, a difference: in the case of projectile motion the cause was accidental (the same body could acquire an *impetus* in any direction with any speed); in the case of natural motion the cause was the very nature of the body. Besides that, the *impetus* could explain why a heavy body tends to keep its motion, but cannot explain how it came to acquire its motion; on the other hand, gravity explained why a heavy body began to move and increased its speed as it moved[35].

In the case of circular motion, there was another relevant difference: if one regarded the rotating motion of the Earth and terrestrial bodies as *natural*, it was not necessary to add any explanation for the motion being circular; on the other side, if the motion of terrestrial bodies was regarded as due to *impetus*, it was necessary to explain why terrestrial bodies do not escape from the Earth, moving along straight tangential lines. At this point, we notice that Galileo and Copernicus replied in different ways to the argument of extrusion of terrestrial bodies by rotation. As shown above, Copernicus denied that the rotation of the Earth could project terrestrial bodies to the space, because this rotational motion was *natural*. Galileo did also suppose that the rotation was natural, but nevertheless thought that the argument should be replied in a different way. He took into account the tangential *impetus* of terrestrial bodies and attempted to show that gravity will bend this tangential motion and will keep terrestrial bodies on the surface of the Earth (Dialogue, pp. 188-203)[36].

A conflict between the concepts of *impetus* and natural rotation arises when Galileo discusses whether a falling body will follow the motion of the Earth. According to the concept of natural motion, every terrestrial body has a natural tendency to turn around the axis of the Earth in 24 hours. Therefore, a falling body will always keep the same *angular* velocity as the Earth, and will follow its motion *exactly*. In that case, when a stone is released from the top a tower, it will have a perfectly straight vertical motion as seen by men on the Earth – that is, the rotation of the Earth will produce no observable effect. On the other hand, let us think about the *impetus* of the stone before it is released. Its speed at the top of the tower is greater than the speed of the bodies at the foot of the tower[37]. Therefore, the falling body should fall slightly *ahead of the tower* (that is, toward the east). Its motion would not be exactly vertical, and the rotation of the Earth would produce a small but observable effect[38].

Which of those conflicting theories does Galileo apply? Well, he uses *both*. As we have already seen, Galileo assumed that a heavy body falling to the very centre of the Earth would follow a straight line exactly, relative to the Earth, and moving at a constant angular speed around the axis of the Earth, relative to the universe. In that argument, he did not use the concept of *impetus*. However, at another place, he reached a different conclusion:

> *"Simplicio – [...] it seems to me a remarkable thing in any case that in coming from the moon's orbit, distant by such a huge interval, the ball should have a natural tendency to keep itself always over the same point of the earth which it stood over at its departure, rather than to fall behind in such a very long way.*

> *"Salviati – The effect might be remarkable or it might be not at all remarkable, but natural and ordinary, depending upon what had gone on before. If, in agreement with the supposition made by the author, the ball had possessed the twenty-four-hour circular motion while it remained in the moon's orbit, together with the earth and everything else contained within that orbit, then that same force which made it go around before it descended would continue to make it do so during its descent too. And far from failing to follow the motion of the earth and necessarily falling behind, it would even go ahead of it, seeing that in its approach toward the earth the rotational motion would have to be made in ever smaller circles, so that if the same speed were conserved in it which it had within the orbit, it ought to run ahead of the whirling of the earth, as I said"* (Dialogue, p. 233).

Which argument did Galileo *prefer*? It seems that he favoured the concept of natural circular motion, because at several points of the *Dialogue* he stated that the rotation of the Earth produces no observable effect.

10. Final comments

In the *Dialogue concerning the two chief world systems*, Galileo maintained the concept of natural motion[39]. Galileo's concept was similar to Copernicus' and widely different from Aristotle's theory. He clung to the idea of perfect circular natural motions and went as far as transforming the free fall of a heavy body in a circular movement. This step was part of his attempt at unifying celestial and terrestrial mechanics – we should not believe, in an anachronical way, that there was any *a priori* reason to reject the concept of natural motion as a condition for this unification. Galileo's "circular inertia" was the terrestrial equivalent to the uniform motion of rotation of the heavenly bodies, not an improved version of Buridan's *impetus*.

The preservation of the concept of natural motion led to conceptual tensions and to several inconsistencies in Galileo's work – some of them being clearly perceived by contemporaneous thinkers, such as Mersenne. Only after the abandonment of this concept, by Descartes and later philosophers, it became possible to build a completely new mechanics.

What is, after all, the ultimate cause of motion in the universe? That was the fundamental philosophical doubt that motivated the concept of natural motion. Did Descartes and Newton reach the final solution of this question? No. Classical mechanics simply *dismissed* this question, adopting the equivalence between motion and rest and stating that it is not necessary to explain the permanence of the states of motion or rest. However, the old philosophical doubt remained as a ghost in the background, and that was the reason why Newton still referred to the *vis inertiae* or *vis insita* as the cause of motion.

From the philosophical point of view, the concept of natural motion – a simple motion produced by an internal essential cause – was highly attractive. However, it sometimes happens that a very nice philosophical concept may block the development of science, instead of assisting its progress. The development of scientific thought sometimes requires the abandonment of former problems and concepts – not because they are obviously inadequate, but because they bring a series of questions that nobody is able to answer.

There are several others similar episodes in 17th century science.

From Aristotle onwards, most philosophers agreed that a void space was impossible in nature, because it could not be conceived (because it was impossible to *think* in a coherent way about it). Torricelli, Pascal, Boyle and other adepts of the vacuum, did not answer the classical arguments concerning the impossibility of void space[40]. The classical arguments were simply left aside – researchers *assumed* that a vacuum was possible and turned their attention to experimental investigations and other issues.

In a similar fashion, from Aristotle onwards most philosophers agreed that direct action at a distance was impossible, because it could not be conceived. Descartes, Newton, Huygens, Leibniz and other philosophers tried to envisage a hidden mechanism that could explain gravity, but they were not successful[41]. In the 18th century, the most common attitude was simply to ignore all philosophical problems concerning direct action at a distance, and to use Newton's law of gravitation without bothering about its hidden explanation.

Can motion itself be conceived? Is there a good answer to Zeno's arguments? A steady flow of new attempts to solve Zeno's paradoxes is a nice evidence that his arguments have not been answered. If we had to solve Zeno's paradoxes before developing a science of motion, perhaps we should give up all hope of building a mechanics.

Galileo should not be regarded anachronically as a man in possession of Newtonian mechanics. Galileo was a man of his time, trying to build a new science but, at the same time, entrapped by some old Aristotelian concepts.

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1 Richard S. Westfall. "Circular motion in seventeenth-century mechanics", in *Isis*. **63** (1972). pp. 184-189. at p. 189.

2 Paolo Rossi. *Il pensiero di Galileo Galilei*. Torino. Loescher. 1968. pp. ix-xv.

3 E. J. Dijksterhuis. *The mechanization of the world picture*. Oxford. Clarendon Press. 1961. pp. 20-36.

4 According to Aristotle. the region of the universe below the sphere of the Moon would be built of Empedocles' four principles – earth. fire. water and air. The heavenly world. from the sphere of the Moon onward. would be made of a fifth element. the ether.

5 Alexandre Koyré. *Études d'histoire de la pensée scientifique*. Paris. Gallimard. 1973. p. 196.

6 Kepler. of course. admitted elliptical orbits. but Galileo never accepted that idea.

7 All citations of Copernicus' *De revolutionibus* are taken from Wallis' English translation: Nicolaus Copernicus. *On the revolutions of the heavenly spheres*. translated by Charles Glenn Wallis. Chicago. Encyclopaedia Britannica. 1952 (Great books of the western world 16). We have added. however. the page of the original 1543 Latin edition. for easier comparison.

8 This is a disputed interpretation. According to Noel Swerdlow. Copernicus' "orbs" meant real. material spheres that turned around their centres: Noel Swerdlow. "Pseudoxia

copernicana". *Archives Internationales d'Histoire des Sciences*. **26** (1976). pp. 105-158. This is the only interpretation that agrees with the concept of one natural motion of rotation for each round body. Edward Rosen, however, did not accept this interpretation: Edward Rosen. "Reply to Swerdlow". *Archives Internationales d'Histoire des Sciences*. **26** (1976). pp. 301-304.

[9] This article will only analyse Galileo's *Dialogue concerning the two chief world systems*. The evolution of Galileo's thought on natural motions, as displayed in his other works, will not be dealt here.

[10] The principle that the world is a most perfect and orderly body: *Dialogue*, p. 19: "The author assumes the universe to be perfectly ordered".

[11] All citations of Galileo's *Dialogue* are reproduced from Drake's translation: Galileo Galilei. *Dialogue concerning the two chief world systems – Ptolemaic and Copernican*, translated by Stillman Drake. 2nd ed., Berkeley. University of California. 1967. The sentences after the page number are the marginal titles used by Galileo.

[12] Of course, this concept does not agree with Aristotle's theory of motion. It seems that Galileo drew this concept from Giovanni Benedetti: "The upward or downward straight motion of natural bodies is not *primo* and *per se* natural, because natural motion is perpetual or unending, and it can only be circular, and a part together with its whole can only have the natural motion belonging to the whole" – Benedetti, *apud* Alexandre Koyré, *op. cit.* (*Études d'histoire*), p. 151.

[13] Here Galileo means, of course, the centre of the solar system, that is, the Sun.

[14] Marin Mersenne. *Harmonie universelle contenant la théorie et la pratique de la musique*. Paris, Centre National de la Recherche Scientifique. 1975. 3 vols. at vol. 1. book 2. proposition 6. pp. 103-7.

[15] Mersenne's data are different from current data, but this does not affect his general conclusions.

[16] One league was equivalent to 15.000 royal French feet.

[17] Everyone accepted this belief, at that time. It was first challenged by Hooke, and Newton was the first to understand that the stone could never reach to the centre of the Earth, because it has an initial angular momentum.

[18] Of course, this concept is incompatible with Newton's mechanics, and one might ask whether Galileo did really suppose that a falling body should keep a constant angular speed around the axis of the Earth. Doubtless he did, because that assumption is required by the mathematical consequences drawn by Galileo.

[19] Alexandre Koyré, *Chute des corps et mouvement de la terre*, Paris, Vrin, 1973, p. 26, footnote 29.

[20] Koyré discussed at length Galileo's argument and Mersenne's criticism: Alexandre Koyré, *op. cit.* (*Chute des corps*), pp. 21-45.

[21] In his computation, Galileo arrived to a time of fall equal to 3 hours, 22 minutes and 4 seconds (*Dialogue*, pp. 223-224). It seems that he did not notice that this result was incompatible with his former hypothesis.

[22] In modern notation, as has been shown, this distance could be computed as h $R.\phi^2/2$. Substituting the current value of the radius of the Earth, one obtains 10.6 cm, in good agreement with Mersenne's result.

[23] Marin Mersenne, *op. cit.*, vol. 1, p. 96. Mersenne's measurements corresponded to an acceleration of gravity of about 7.2 m/s^2. Although that is not the value accepted nowadays, Mersenne's conclusion is valid.

[24] Marin Mersenne, *op. cit.*, vol. 1, p. 96.

[25] Marin Mersenne, *op. cit.*, vol. 1, pp. 96-98.

[26] Alexandre Koyré, *op. cit.* (*Chute des corps*), p. 30, pp. 45-48.

27 Alexandre Koyré, *op. cit.* (*Chute des corps*), p. 30, p. 49.

[28] Alexandre Koyré, *op. cit.* (*Chute des corps*), pp. 56-66.

[29] For an account of Locher's work, consult Alexandre Koyré, *op. cit.*, pp. 14-21.

[30] Sometimes Galileo conjectured that there might be *three* types of motion: natural, violent and a third or intermediate kind. Violent motion, in this case, would be motion directly opposite to natural motion (for instance, the upward motion of a stone). The horizontal motion of the same stone, however, would be neither natural nor violent. According to Wallace, similar ideas can be found in the works of several authors read by Galileo: William A. Wallace, *Prelude to Galileo – essays on medieval and sixteenth-century sources of Galileo's thought*. Dordrecht, D. Reidel, 1981, pp. 313-314.

[31] Moody described Buridan's theory of *impetus* in Ernest A. Moody, "Laws of motion in medieval physics", pp. 220-234 in: Robert M. Palter, *Toward modern science*. New York, Dutton, 1969. See also E. J. Dijksterhuis, *op. cit.*, pp. 179-185. A translation of Buridan's arguments can be found in Marshall Clagett, *The science of mechanics in the Middle Ages*. Madison, The University of Wisconsin Press, 1959, pp. 532-540.

[32] There are several studies concerning the evolution of Galileo's thought on *impetus*. See, for instance: Alexandre Koyré, *Études galiléennes*. Paris, Hermann, 1966, pp. 60-79.

[33] Giovanni Benedetti, in the 16th century, had already clearly stated that the stone leaving the sling would receive an impetus to move along the tangent straight line (E. J. Dijksterhuis, *op. cit.*, p. 270). Benedetti's argument is presented in Alexandre Koyré, op. cit. (Études d'histoire), pp. 148-150.

[34] According to Wallace, in the 16th century Domenico de Soto had already stated that the *impetus* is a motive power completely analogous to the gravity of a falling body: William A. Wallace, *op. cit.*, p. 117.

[35] According to Galileo, "the ball, in descending, is always gaining greater impetus and velocity" (*Dialogue*, p. 22). A similar argument had already been presented by Jean Buridan: "Because the motion becomes faster, the impetus also becomes greater and stronger, and consequently the body is moved by its natural gravity in combination with the greater impetus, and consequently it will move faster again" (E. J. Dijksterhuis, *op. cit.*, pp. 182-184). Giovanni Benedetti also proposed a similar explanation, in the 16th century (E. J. Dijksterhuis, *op. cit.*, pp. 269-270).

[36] By the way: Galileo's argument was mistaken, as shown in Roberto de Andrade Martins, "Galileo e a rotação da Terra", *Caderno Catarinense de Ensino de Física* 11 (1994), pp. 196-211.

[37] In current notation: supposing that the tower is at the equator, the tangential speed of the stone at the top of the tower will be $(R+h)\omega$, where R is the radius of the Earth, h is the height of the tower and ω is the angular speed of the Earth; the speed of bodies at the surface of the Earth will be $R\omega$.

[38] According to Newtonian physics, a falling body must indeed deviate to the east, but the effect should be computed assuming the law of conservation of angular momentum, and not the conservation of the tangential speed.

[39] At some places, Galileo perceived that the distinction between natural and violent motion or internal and external causes was not altogether clear (*Dialogue*, pp. 234-235), but he did not abandon the concept.

[40] Roberto de Andrade Martins, "Em busca do nada: considerações sobre os argumentos a favor e contra o vácuo", in *Trans/Form/Ação*, 16 (1993), pp. 7-27; Roberto de Andrade Martins, "O vácuo e a pressão atmosférica, da antigüidade a Pascal", in *Cadernos de História e Filosofia da Ciência*, [2nd. series] 1 (1989), pp. 9-48.

[41] Roberto de Andrade Martins, "Descartes e a impossibilidade de ações à distância", pp. 79-126 in Samuel Fuks (org.), *Descartes 400 anos: um legado científico e filosófico*. Rio de Janeiro, Relume Dumará, 1997.