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Single Valued Neutrosophic Soft Approach to Rough Sets, Theory and Application

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Abstract. This paper aims to introduce a single valued neutrosophic soft approach to rough sets based on neutrosophic right minimal structure. Some of its properties are deduced and proved. A comparison between traditional rough model and suggested model, by using their properties is concluded to show that Pawlak's approach to rough sets can be viewed as a special case of single valued neutrosophic soft approach to rough sets. Some of rough concepts are redefined and then some properties of these concepts are deduced, proved and illustrated by several examples. Finally, suggested model is applied in a decision making problem, supported with an algorithm.

Keywords: Neutrosophic set, soft set, rough set approximations, neutrosophic soft set, single valued neutrosophic soft set.

1 Introduction

Set theory is a basic branch of a classical mathematics, which requires that all input data must be precise, but almost, real life problems in biology, engineering, economics, environmental science, social science, medical science and many other fields, involve imprecise data. In 1965, L.A. Zadeh [1] introduced the concept of fuzzy logic which extends classical logic by assigning a membership function ranging in degree between 0 and 1 to variables. As a generalization of fuzzy logic, F. Smarandache in 1995, initiated a neutrosophic logic which introduces a new component called indeterminacy and carries more information than fuzzy logic. In it, each proposition is estimated to have three components: the percentage of truth (t %), the percentage of indeterminacy (i %) and the percentage of falsity (f %), his work was published in [2]. From scientific or engineering point of view, neutrosophic set's operators need to be specified. Otherwise, it will be difficult to apply in the real applications. Therefore, Wang et al.[3] defined a single valued neutrosophic set and various properties of it. This thinking is further extended to many applications in decision making problems such as [4, 5].

Rough set theory, proposed by Z. Pawlak [6], is an effective tool in solving many real life problems, based on imprecise data, as it does not need any additional data to discover a knowledge hidden in uncertain data. Recently, many papers have been appeared to development rough set model and then apply it in many real life applications such as [7-11]. In 1999, D. Molodtsov [12], suggested a soft set model. By using it, he created an information system from a collected data. This model has been successfully used in the decision making problems and it has been modified in many papers such as [13-17]. In 2011, F. Feng et al.[18] introduced a soft rough set model and proved its properties. E.A. Marei generalized this model in [19]. In 2013, P.K. Maji [20] introduced neutrosophic soft set, which can be viewed as a new path of thinking to engineers, mathematicians, computer scientists and many others in various tests. In 2014, Broumi et al. [21] introuduced the concept of rough neutrosophic sets. It is generalized and applied in many papers such as [22-31]. In 2015, E.A. Marei [32] introduced the notion of neutrosophic soft rough sets and its modification.

This paper aims to introduce a new approach to soft rough sets based on the neutrosophic logic, named single valued neutrosophic soft (VNS in short) rough set approximations. Properties of VNS-lower and VNS-upper approximations are included along with supported proofs and illustrated examples. A comparison between traditional rough and single valued neutrosophic soft rough approaches is concluded to show that Pawlak's approach to rough sets can be viewed as a special case of single valued neutrosophic soft approach to rough sets. This paper delves into single valued neutrosophic soft rough set by defining some concepts on it as a generalization of rough concepts. Single valued neutrosophic soft rough concepts (NR-concepts in short) include NR-definability, NR-membership function, NR-membership relations, NR-inclusion relations and NR-equality relations. Properties of these concepts are deduced, proved and illustrated by several examples. Finally, suggested model is applied in a decision making problem, supported with an algorithm.

2 Preliminaries

In this section, we recall some definitions and properties regarding rough set approximations, neutrosophic set, soft set and neutrosophic soft set required in this paper.

Definition 2.1 [6] Lower, upper and boundary approximations of a subset $X \subseteq U$, with respect to an equivalence relation, are defined as

$$\underline{E}(X)=\cup\{[x]_E:[x]_E\subseteq, X\}, \overline{E}(X)=\cup\{[x]_E:[x]_E\cap X\neq\phi\},$$

$$BND_E(X)=\overline{E}(X)-\underline{E}(X), where$$

$$[x]_E=\{x' \in U: E(x)=E(x')\}.$$

Definition 2.2 [6] Pawlak determined the degree of crispness of any subset $X \subseteq U$ by a mathematical tool, named the accuracy measure of it, which is defined as

$$\alpha_E(X)=\overline{E}(X)/\underline{E}(X),\overline{E}(X)\neq\phi.$$

Obviously, $0\le\alpha_E(X)\le 1$. If $\underline{E}(X)=\overline{E}(X)$, then X is crisp (exact) set, with respect to E, otherwise X is rough set.

Properties of Pawlak's approximations are listed in the following proposition.

Proposition 2.1 [6] Let (U, E) be a Pawlak proximation space and let $X, Y \subseteq U$. Then,

(a) $\underline{E}(X)\subseteq X\subseteq\overline{E}(X)$.

(b) $\underline{E}(\phi)=\phi=\overline{E}(\phi)$ and $\underline{E}(U)=U=\overline{E}(U)$.

(c) $\overline{E}(X\cup Y)=\overline{E}(X)\cup\overline{E}(Y)$.

(d) $\underline{E}(X\cap Y)=\underline{E}(X)\cap\underline{E}(Y)$.

(e) $X\subseteq Y$, then $\underline{E}(X)\subseteq\underline{E}(Y)$ and $\overline{E}(X)\subseteq\overline{E}(Y)$.

(f) $\underline{E}(X\cup Y)\supseteq\underline{E}(X)\cup\underline{E}(Y)$.

(g) $\overline{E}(X\cap Y)\subseteq\overline{E}(X)\cap\overline{E}(Y)$.

(h) $\underline{E}(X^c)=[\overline{E}(X)]^c$, X^c is the complement of X.

(i) $\overline{E}(X^c)=[\underline{E}(X)]^c$.

(j) $\underline{E}(\underline{E}(X))=\overline{E}(\underline{E}(X))=\underline{E}(X)$.

(k) $\overline{E}(\overline{E}(X))=\underline{E}(\overline{E}(X))=\overline{E}(X)$.

Definition 2.3 [33] An information system is a quadruple $IS=(U,A,V,f)$, where U is a non-empty finite set of objects, A is a non-empty finite set of attributes, $V=\cup\{V_e, e\in A\}$, V_e is the value set of attribute e, $f: U\times A\rightarrow V$ is called an information (knowledge) function.

Definition 2.4 [12] Let U be an initial universe set, E be a set of parameters, $A\subseteq E$ and let $P(U)$ denotes the power set of U. Then, a pair $S=(F,A)$ is called a soft set over U, where F is a mapping given by $F: A\rightarrow P(U)$. In other words, a soft set over U is a parameterized family of subsets of U. For $e\in A, F(e)$ may be considered as the set of e-approximate elements of S.

Definition 2.5 [2] A neutrosophic set A on the universe of discourse U is defined as

$$A=\{\langle x,T_A(x),I_A(x),F_A(x)\rangle : x\in U\}, where$$

$$^{-}0\le T_A(x)+I_A(x)+F_A(x)\le 3^{+}, and T, I, F\rightarrow \rfloor^{-}0,1^{+}\lfloor$$

Definition 2.6 [20] Let U be an initial universe set and E be a set of parameters. Consider $A\subseteq E$, and let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F,A) is termed to be the neutrosophic soft set over U, where F is a mapping given by $F: A\rightarrow P(U)$.

Definition 2.7 [3] Let X be a space of points (objects), with a generic element in X denoted by x. A single valued neutrosophic set A in X is characterized by truth-embership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A. For each point x in X, $T_A(X), I_A(X), F_A(X)\in[0,1]$. When X is continuous, a single valued neutrosophic set A can be written as $A=\int_X (T(x),I(x),F(x))/x, x\in X$. When X is discrete, A can be written as $A=\Sigma_{i=1}^{n}(T(x_i),I(x_i),F(x_i))/x_i, x_i\in X$.

3 Single valued neutrosophic soft rough set approximations

In this section, we give a definition of a single valued neutrosophic soft (VNS in short) set. VNS-lower and VNS-upper approximations are introduced and their properties are deduced, proved and illustrated by many counter examples.

Definition 3.1 Let U be an initial universe set and E be a set of parameters. Consider $A\subseteq E$, and let $P(U)$ denotes the set of all single valued neutrosophic sets of U. The collection (G,A) is termed to be VNS set over U, where G is a mapping given by $G: A\rightarrow P(U)$.

For more illustration the meaning of VNS set, we consider the following example

Example 3.1 Let U be a set of cars under consideration and E is the set of parameters (or qualities). Each parameter is a neutrosophic word. Consider E = {elegant, trustworthy, sporty, comfortable, modern}. In this case, to define a VNS means to point out elegant cars, trustworthy cars and so on. Suppose that, there are five cars in the universe U, given by $U=\{h_1,h_2,h_3,h_4,h_5\}$ and the set of parameters $A=\{e_1,e_2,e_3,e_4\}$, where $A\subseteq E$ and each e_i is a specific criterion for cars: e_1 stands for elegant, e_2 stands for trustworthy, e_3 stands for sporty and e_4 stands for comfortable.

A VNS set can be represented in a tabular form as shown in Table 1. In this table, the entries are c_{ij} corresponding to the car h_i and the parameter e_j, where c_{ij} = (true membership value of h_i, indeterminacy-membership value of h_i, falsity membership value of h_i) in $G(e_i)$.

U	e_1	e_2	e_3	e_4
h_1	(.6, .6, .2)	(.8, .4, .3)	(.7, .4, .3)	(.8, .6, .4)
h_2	(.4, .6, .6)	(.6, .2, .4)	(.6, .4, .3)	(.7, .6, .6)
h_3	(.6, .4, .2)	(.8, .1, .3)	(.7, .2, .5)	(.7, .6, .4)
h_4	(.6, .3, .3)	(.8, .2, .2)	(.5, .2, .6)	(.7, .5, .6)
h_5	(.8, .2, .3)	(.8, .3, .2)	(.7, .3, .4)	(.9, .5, .7)

Table1: Tabular representation of (G, A) of Example 3.1.

Definition 3.2 Let (G, A) be a VNS set on a universe U. For any element $h \in U$, a neutrosophic right neighborhood, with respect to $e \in A$ is defined as follows

$$h_e = \{h_i \in U : T_e(h_i) \geq T_e(h), I_e(h_i) \geq I_e(h), F_e(h_i) \leq F_e(h)\}.$$

Definition 3.3 Let (G,A) be a VNS set on U. Neutrosophic right minimal structure is defined as follows

$$\zeta = \{U, \phi, h_e : h \in U, e \in A\}$$

Illustration of Definitions 3.2 and 3.3 is introduced in the following example

Example 3.2 According Example 3.1, we can deduce the following results: $h_{1e_1} = h_{1e_2} = h_{1e_3} = h_{1e_4} = \{h_1\}$, $h_{2e_1} = h_{2e_3} = \{h_1, h_2\}$, $h_{2e_2} = \{h_1, h_2, h_4, h_5\}$, $h_{2e_4} = \{h_1, h_2, h_3\}$, $h_{3e_1} = h_{3e_4} = \{h_1, h_3\}$, $h_{3e_2} = \{h_1, h_3, h_4, h_5\}$, $h_{3e_3} = \{h_1, h_3, h_5\}$, $h_{4e_1} = \{h_1, h_3, h_4\}$, $h_{4e_2} = \{h_4, h_5\}$, $h_{4e_3} = U$, $h_{4e_4} = \{h_1, h_2, h_3, h_4\}$, $h_{5e_1} = h_{5e_2} = h_{5e_4} = \{h_5\}$, $h_{5e_3} = \{h_1, h_5\}$.

It follows that,

$$\zeta = \{\{h_1\}, \{h_5\}, \{h_1, h_2\}, \{h_1, h_3\}, \{h_1, h_5\}, \{h_4, h_5\}, \{h_1, h_2, h_3\}, \{h_1, h_3, h_4\}, \{h_1, h_3, h_5\}, \{h_1, h_2, h_3, h_4\}, \{h_1, h_2, h_4, h_5\}, \{h_1, h_3, h_4, h_5\}, U, \phi\}$$

Proposition 3.1 Let (G, A) be a VNS set on a universe U, ξ is the family of all neutrosophic right neighborhoods on it, and let

$$R_e : U \to \xi, R_e(h) = h_e$$

Then,

(a) R_e is reflexive relation.

(b) R_e is transitive relation.

(c) R_e may be not symmetric relation.

Proof Let $\langle h_1, T_e(h_1), I_e(h_1), F_e(h_1)\rangle$, $\langle h_2, T_e(h_2), I_e(h_2), F_e(h_2)\rangle$ and $\langle h_3, T_e(h_3), I_e(h_3), F_e(h_3)\rangle \in G(A)$. Then,

(a) Obviously, $T_e(h_1) = T_e(h_1)$, $I_e(h_1) = I_e(h_1)$ and $F_e(h_1) = F_e(h_1)$. For every $e \in A$, $h_1 \in h_{1e}$. Then $h_1 R_e h_1$ and then R_e is reflexive relation.

(b) Let $h_1 R_e h_2$ and $h_2 R_e h_3$, then $h_2 \in h_{1e}$ and $h_3 \in h_{2e}$. Hence, $T_e(h_2) \geq T_e(h_1)$, $I_e(h_2) \geq I_e(h_1)$, $F_e(h_2) \leq F_e(h_1)$, $T_e(h_3) \geq T_e(h_2)$, $I_e(h_3) \geq I_e(h_2)$ and $F_e(h_3) \leq F_e(h_2)$. Consequently, we have $T_e(h_3) \geq T_e(h_1)$, $I_e(h_3) \geq I_e(h_1)$ and $F_e(h_3) \leq F_e(h_1)$. It follows that, $h_3 \in h_{1e}$. Then $h_1 R_e h_3$ and then R_e is transitive relation.

The following example proves (c) of Proposition 3.1.

Example 3.3 From Example 3.2, we have, $h_{1e_1} = \{h_1\}$ and $h_{3e_1} = \{h_1, h_3\}$. Hence, $(h_2, h_1) \in R_{e_1}$ but $(h_1, h_3) \notin R_{e_1}$. Then, R_e isn't symmetric relation.

Definition 3.4 Let (G,A) be a VNS set on U, and let ζ be a neutrosophic right minimal structure on it. Then, VNS-lower and VNS-upper approximations of any subset X based on ζ, respectively, are

$$S_* X = \cup\{Y \in \zeta : Y \subseteq X\},$$
$$S^* X = \cap\{Y \in \zeta : Y \supseteq X\}.$$

Remark 3.1 For any considered set X in a VNS set (G,A), the sets

$$P_{NR} X = S_* X, \quad N_{NR} X = [S^* X]^c,$$
$$b_{NR} X = S^* X - P_{NR} X$$

are called single valued neutrosophic positive, single valued neutrosophic negative and single valued neutrosophic boundary regions of a considered set X, respectively. The real meaning of single valued neutrosophic positive of X is the set of all elements which are surely belonging to X, single valued neutrosophic negative of X is the set of all elements which are surely not belonging to X and single valued neutrosophic boundary of X is the elements of X which are not determined by (G,A). Consequently, the single valued neutrosophic boundary region of any considered set is the initial problem of any real life application.

VNS rough set approximations properties are introduced in the following proposition.

Proposition 3.2 Let (G,A) be a VNS set on U, and let $X, Z \subseteq U$. Then the following properties hold

(a) $S_*X \subseteq X \subseteq S^*X$.

(b) $S_*\varnothing = S^*\varnothing = \varnothing$.

(c) $S_*U = S^*U = U$.

(d) $X \subseteq Z \Rightarrow S_*X \subseteq S_*Z$.

(e) $X \subseteq Z \implies S^*X \subseteq S^*Z$.

(f) $S_*(X \cap Z) \subseteq S_*X \cap S_*Z$.

(g) $S_*(X \cup Z) \supseteq S_*X \cup S_*Z$.

(h) $S^*(X \cap Z) \subseteq S^*X \cap S^*Z$

(i) $S^*(X \cup Z) \supseteq S^*X \cup S^*Z$.

Proof

(a) From Definition 3.3, obviously, we can deduce that,

$S_*X \subseteq X \subseteq S^*X$.

(b) From Definition 3.4, we can deduce that $S_*\phi = \phi$ and $S^*\phi = \cap\{Y \in \zeta : Y \supseteq \phi\} = \phi$.

(c) From Property (a), we have $U \subseteq S^*U$ but U is the universe set, then $S^*U \subseteq U$. Also, from Definition 3.4, we have $S_*U = \cup\{Y \in \xi : Y \subseteq U\}$, but $U \in \xi$. Then, $S_*U = U$

(d) Let $X \subseteq Z$ and $h \in S_*X$, then there exists $Y \in \xi$ such that $h \in Y \subseteq X$. But $X \subseteq Z$, then $h \in Y \subseteq Z$. Hence, $h \in S_*Z$. Consequently $S_*X \subseteq S_*Z.$

(e) Let $X \subseteq Z$ and $h \notin S^*Z$. But $S^*Z = \cap\{Y \in \xi : Y \supseteq$. $h \notin Y$ and $Y \supseteq Z$ such that $U \in \xi$ there exists Then. $Z\}$ But $X \subseteq Z$, then $Y \supseteq X$ and $h \notin Y$. Hence $h \notin S^*Z$. Thus $S^*X \subseteq S^*Z$.

(f) Let $h \in S_*(X \cap Z) = \cup\{Y \in \xi : Y \subseteq X \cap Z\}$. So, there exists $Y \in \xi$ such that, $h \in Y \subseteq X \cap Z$, then $h \in Y \subseteq X$ and $h \in Y \subseteq Z$. Consequently, $h \in S_*X$ and $h \in S_*Z$, then $h \in S_*X \cap S_*Z$. Thus $S_*(X \cap Z) \subseteq S_*X \cap S_*Z.$

(g) Let $h \notin S_*(X \cup Z) = \cup\{Y \in \xi : Y \subseteq X \cup Z\}$. So, for all $Y \in \xi, h \in Y$, we have $Y \not\subset X \cup Z$, then $Y \not\subset X$ and $Y \not\subset Z$. Consequently, $h \notin S_*X$ and $h \notin S_*Z$. So $h \notin S_*X \cup S_*Z$. Thus $S_*(X \cup Z) \supseteq S_*X \cup S_*Z$.

(h) Let $h \notin S^*X \cap S^*Z$. Then, $h \notin S^*X$ or $h \notin S^*Z$ and then there exists $Y \in \xi$ such that $Y \supseteq X, h \notin Y$ or $Y \supseteq X$, $h \notin Y$. Consequently $h \notin S^*(X \cap Z)$. Thus $S^*(X \cap Z) \subseteq S^*X \cap S^*Z$.

(i) Let $h \notin S^*(X \cup Z)$. But $S^*(X \cup Z) = = \cap\{Y \in \xi : Y \supseteq X \cup Z\}$. Then, there exists $Y \in \xi$ such that $Y \supseteq X \cup Z$ and $h \notin Y$. Then, $Y \supseteq X$, $h \notin Y$ and $Y \supseteq Z$, $h \notin Y$. It follows that, $h \notin S^*X \cup S^*Z$. Thus $S^*(X \cup Z) \supseteq S^*X \cup S^*Z$.

The following example illustrates that the converse of Property (a) doesn't hold
Example 3.4 From Example 3.1, if $X = \{h_3\}$, then $S_*X =$. $X \neq S^*X$ and $S_*X \neq X$ Hence. $S^*X = \{h_1, h_3\}$ and ϕ

The following example illustrates that the converse of Property (d) doesn't hold
Example 3.5 From Example 3.1, if $X = \{h_2\}$ and $Z = \{h_1, h_2\}$, then $S_*X = \phi$, $S_*Z = \{h_1, h_2\}$. Thus $S_*X \neq S_*Z$.

The following example illustrates that the converse of Property (e) doesn't hold

Example 3.6 From Example 3.1, if $X = \{h_5\}$ and $Z = \{h_2, h_5\}$, then, $S^*X = \{h_5\}$ and $S^*Z = \{h_1, h_2, h_4, h_5\}$. Hence, $S^*X \neq S^*Z$.

The following example illustrates that the converse of Property (f) doesn't hold

Example 3.7 From Example 3.1, If $X = \{h_1, h_3, h_4\}$ and $Z = \{h_1, h_4, h_5\}$, then $S_*X = \{h_1, h_3, h_4\}$, $S_*Z = \{h_1,$ $S_*(X \cap Z) \neq S_*X$ Hence. $S_*(X \cap Z) = \{h_1\}$ and $h_4, h_5\}$ $. \cap S_*Z$

The following example illustrates that the converse of Property (g) doesn't hold

Example 3.8 From Example 3.1, if $X = \{h_1\}$ and $Z = \{h_2\}$ then $S_*X = \{h_1\}$, $S_*Z = \phi$ and $S_*(X \cup Z) = \{h_1, h_2\}$. Hence $S_*(X \cup Z) \neq S_*X \cup S_*Z$.

The following example illustrates that the converse of Property (h) doesn't hold

Example 3.9 From Example 3.1, if $X = \{h_1, h_2, h_4\}$ and $Z = \{h_1, h_2, h_5\}$ then $S^*X = \{h_1, h_2, h_4\}$, $S^*Z = \{h_1, h_2, h_4, h_5\}$ and $S^*(X \cap Z) = \{h_1, h_2\}$. Hence $S^*(X \cap Z) \neq S^*X \cap S^*Z$

The following example illustrates that the converse of Property (i) doesn't hold

Example 3.10 From Example 3.1, if $X = \{h_2, h_3\}$ and

$Z=\{h_5\}$ then $S^*X=\{h_1,h_2,h_3\}$, $S^*Z=\{h_5\}$ and $S^*(X\cup Z)=U$. Hence $S^*(X\cup Z)\neq S^*X\cup S^*Z$.

Proposition 3.3 Let (G,A) be a neutrosophic soft set on a unverse U , and let $X,Z\subseteq U$. Then the following properties hold.

(a) $S_*S_*X=S_*X$

(b) $S^*S^*X=S^*X$

(c) $S_*S^*X\subseteq S^*X$

(d) $S^*S_*X\supseteq S_*X$

Proof

(a) Let $W=S_*X$ and $h\in W=\cup\{Y\in\zeta:Y\subseteq X\}$. Then, for some $e\in A$, we have $h\in Y\subseteq W.$ So $h\in S_*W$. Hence $W\subseteq S_*W$. Thus, $S_*W\subseteq S_*S_*W$. Also, from Property (a) of Proposition 3.2, we have $S_*X\subseteq X$ and by using Property (d) of Proposition 3.2, we get $S_*S_*X\subseteq S_*X$. Consequently. $S_*X=S_*S_*X$

(b) Let $W=S^*X$ and $h\notin W$, from Definition 3.4, we have $W=\cap\{Y\in\xi:Y\supseteq X\}$. Then there exists $Y\in\xi$, such that $Y\supseteq X$ and $h\notin Y$. Hence, there exists $Y\in\xi$, such that $Y\supseteq W$ and $h\notin Y$, it follows that $h\notin S^*W$. Consequently $W\supseteq S^*W$. Also, by using Property (a) of Proposition 3.2, we have $W\subseteq S^*W$. Thus $S^*S^*W=S^*W$

Properties (c) and (d) can be proved directly from Proposition 3.2.

The following example illustrates that the converse of Property (c) doesn't hold.
Example 3.11 From Example 3.1, if $X=\{h_4\}$. Then $S^*X=\{h_4\}$ and $S_*S^*X=\phi$. Hence, $S_*S^*X\neq S^*X$.

The following example illustrates that the converse of Property (c) doesn't hold.
Example 3.12 From Example 3.1, if $X=\{h_1,h_2,h_5\}$, then $S_*X=\{h_1,h_2,h_5\}$ and $S^*S_*X=\{h_1,h_2,h_4,h_5\}$. Hence $S^*S_*X\neq S_*X$

Proposition 3.4 Let (G,A) be a VNS set on U and let $X,Z\subseteq U$. Then

$$S_*(X-Z)\subseteq S_*X-S_*Z$$

Proof
Let $h\in S_*(X-Z)=\cup\{Y\in\xi:Y\subseteq(X-Z)\}$. So, there exists $Y\in\xi$ such that $h\in Y\subseteq(X-Z)$, then $h\in Y\subseteq X$ and $h\in Y\not\subset Z$. Consequently, $h\in S_*X$ and $h\notin S_*Z$, then $h\in S_*X-S_*Z$. Therefore $S_*(X-Z)\subseteq S_*X-S_*Z$.

The following example illustrates that the converse of Proposition 3.4 doesn't hold.
Example 3.13 From Example 3.1, if $X=\{h_1,h_3,h_5\}$ and $Z=\{h_1,h_5\}$, then $S_*X=\{h_1,h_3,h_5\}$, $S_*Z=\{h_1,h_5\}$, $S_*(X-Z)=\phi$ and $S_*X-S_*Z=\{h_3\}$. Hence, $S_*(X-Z)\neq S_*X-S_*Z$

Proposition 3.5 Let (G,A) be a VNS set on U and let $X,Z\subseteq U$. Then the following properties don't hold

(a) $S_*X^c=[S^*X]^c$

(b) $S^*X^c=[S_*X]^c$

(c) $S^*(X-Z)=S^*X-S^*Z$

The following example proves Properties (a) and (b) of Proposition 3.5.
Example 3.14 From Example 3.1, if $X=\{h_1\}$. Then, $S_*X=S^*X=\{h_1\}$, $S_*X^c=\{h_4,h_5\}$ and $S^*X^c=U.$ Thus $S_*X^c\neq[S^*X]^c$ and $S^*X^c\neq[S_*X]^c$

The following example proves Property (c) of Proposition 3.5.
Example 3.15 From Example 3.1, if $X=\{h_1,h_2\}$ and $Z=\{h_1\}$. Then $S^*X=\{h_1,h_2\},S^*Z=\{h_1\}$, $S^*(X-Z)=\{h_1,h_2\}$. Hence $S^*(X-Z)\neq S^*X-S^*Z.$

Remark 3.2 A comparison between traditional rough and single valued neutrosophic soft rough approaches, by using their properties, is concluded in Table 2, as follows

4 Single valued neutrosophic soft rough concepts

In this section, some of single valued neutrosophic soft rough concepts (NR-concepts in short) are defined as a generalization of traditional rough concepts.

Definition 4.1 Let (G,A) be a VNS set on U . A subset $X\subseteq U$ is called

(a) NR-definable (NR-exact) set if $S_*X=S^*X=X$

(b) Internally NR-definable set if $S_*X=X$ and $S^*X\neq X$

(c) Externally NR-definable set if $S_*X\neq X$ and $S^*X=X$

(d) NR-rough set if $S_*X\neq X$ and $S^*X\neq X$

The following example illustrates Definition 4.1.
Example 4.1 From Example 3.1, we can deduce that $\{h_1\}$, $\{h_5\},\{h_1,h_2\},\{h_1,h_3\},\{h_1,h_5\},\{h_4,h_5\}$, $\{h_1,h_2,h_3\},\{h_1,h_3,h_4\},\{h_1,h_3,h_5\},\{h_1,h_4,h_5\},\{h_1,h_2,h_3,h_4\}$, $\{h_1,h_2,h_4,h_5\},\{h_1,h_3,h_4,h_5\}$ are NR-definable sets, $\{h_1,h_2,h_5\},\{h_1,h_2,h_3,h_5\}$ are internally NR-definable sets, $\{h_4\},\{h_1,h_4\},\{h_1,h_2,h_4\}$ are externally NR-definable sets and the rest of proper subsets of U are

NR-rough sets.

We can determine the degree of single valued neutrosophic soft-crispness (exactness) of any subset $X \subseteq U$ by using NR-accuracy measure, denoted by $C_* X$, which is defined as follows

Definition 4.2 Let (G,A) be a VNS on U, and let $X \subseteq U$. Then

$$C_* X = S_* X / S^* X, X \neq \phi$$

Remark 4.1 Let (G,A) be a VNS on U. A subset $X \subseteq U$ is NR-definable (NR-exact) if and only if $C_* X = 1$.

Definition 4.3 Let (G,A) be a VNS on U and let $X \subseteq U$, $x \notin X$. NR-membership function of an element x to a set X denoted by $\mu_X x$ is defined as follows:
$\mu_X x = | x_A \cap X | / | x_A |$, where $x_A = \cap\{x_e : e \in A\}$ and x_e is a neutrosophic right neighborhood, defined in Definition 3.2.

Proposition 4.1 Let (G,A) be a VNS on U, $X \subseteq U$ and let $\mu_X x$ be the membership function defined in Definition 4.3. Then

$$\mu_X x \in [0,1]$$

Proof

Where $\phi \subseteq x_A \cap X \subseteq x_A$ then $0 \le |x_A \cap X| \le |x_A|$ and then $0 \le \mu_X x \le 1$.

Proposition 4.2 Let (G,A) be a VNS on U and let $X \subseteq U$, then

$$\mu_X x = 1 \Rightarrow x \in X$$

Proof

Let $\mu_X x = 1$, then $|x_A \cap X| = |x_A|$. Consequantly $x_A \subseteq X$. From Proposition 3.1, we have R_e is a reflexive relation for all $e \in A$. Hence $x \in x_e \forall e \in A$. It follows that $x \in x_A$. Thus $x \in X$

The following example illustrates that the converse of Proposition 4.2 doesn't hold.

Example 4.2 From Example 3.2, we get $h_{3A} = \{h_1, h_3\}$. If $X = \{h_2, h_3, h_5\}$, then $\mu_X h_3 = 1/2$. Although $h_3 \in X$

Proposition 4.3 Let (G,A) be a VNS on U and let $X, Z \subseteq U$. If $X \subseteq Z$, then the following properties hold

(a) $\mu_X x \le \mu_Z x$

(b) $\mu_{S_* X} x \le \mu_{S_* Z} x$

(c) $\mu_{S^* X} x \le \mu_{S^* Z} x$

Proof

(a) Where $X \subseteq U$, for any $x \subseteq U$ we can deduce that $\mu_X x \le \mu_Z x$. Thus $|x_A \cap X| \le |x_A \cap Z|$ then $\subseteq x_A \cap Z$, $x_A \cap X$

We get the proof of Properties (b) and (c) of Proposition 4.3, directly from property (a) of Proposition 4.3 and properties (d) and (e) of Proposition 3.2.

Traditional rough properties	VNS rough properties
$\overline{E}(X \cup Z) = \overline{E}X \cup \overline{E}Z$	$S^*(X \cup Z) \supseteq S^* X \cup S^* Z$
$\underline{E}(X \cap Y) = \underline{E}(X) \cap \underline{E}(Y)$	$S_*(X \cap Z) \subseteq S_* X \cap S_* Z$
$\underline{E}(\overline{E}(X)) = \overline{E}(X)$	$S_* S^* X \subseteq S^* X$
$\overline{E}(\underline{E}(X)) = \underline{E}(X)$	$S^* S_* X \supseteq S_* X$
$\underline{E}(X^c) = [\overline{E}(X)]^c$	$S_* X^c \neq [S^* X]^c$
$\overline{E}(X^c) = [\underline{E}(X)]^c$	$S^* X^c \neq [S_* X]^c$

Table 2: Comparison between traditional, VNS rough

Proposition 4.4 Let (G,A) be a VNS on U and let $X \subseteq U$, then the following properties hold

(a) $\mu_{S_* X} x \le \mu_X x$

(b) $\mu_X x \le \mu_{S^* X} x$

(c) $\mu_{S_* X} x \le \mu_{S^* X} x$

Proof can be obtained directly from Propositions 3.2 and property (a) of Proposition 4.3.

Definition 4.4 Let (G,A) be a VNS set on U, and let $x \in U$, $X \subseteq U$. NR-membership relations, denoted by $\in_*$ and $\in^*$ are defined as follows

$$x \in_* X \text{ if } x \in S_* X \text{ and } x \in^* X \text{ if } x \in S^* X$$

Proposition 4.5 Let (G,A) be a VNS set on U, and let $x \in U$, $X \subseteq U$. Then

(a) $x \in_* X \Rightarrow x \in X$

(b) $x \notin^* X \Rightarrow x \notin X$

Proof

(a) Let $x \in_* X$, hence by using Definition 4.4, we get $x \in S_* X$.
 But from Proposition 3.2, we have $S_* X \subseteq X$, then $x \in X$.

(b) Let $x \in X$, according to Proposition 3.2, we have $X \subseteq S^* X$, then $x \in S^* X$, by using Definition 4.4, we can deduce that $x \in^* X$.
 Consequently $x \notin^* X \Rightarrow x \notin X$.

The following example illustrates that the converse of Proposition 4.5 doesn't hold.

Example 4.3 From Example 3.1, if $X = \{h_2, h_5\}$, then $S_* X = \{h_5\}$ and $S^* X = \{h_1, h_2, h_4, h_5\}$. Hence, $h_2 \notin_* X$, although $h_2 \in X$ and $h_4 \notin X$, although $h_4 \in^* X$.

Proposition 4.6 Let (G,A) be a VNS on U and let $X \subseteq U$. Then the following properties hold

(a) $x \in_* X \Rightarrow \mu_X x = 1$

(b) $\mu_X x = 1 \Rightarrow x \in^* X$

Proof can be obtained directly from Definition 4.4 and Propositions 4.2 and 4.5.

The following example illustrates that the converse of property (a) does not hold.

Example 4.4 From Example 3.1, if $X = \{h_1, h_4\}$ then $S_* X = \{h_1\}$ and $h_{4A} = \{h_4\}$, it follows that $\mu_X h_4 = 1$. Although $h_4 \notin_* X$

The following example illustrates that the converse of property (b) does not hold.

Example 4.5 From Example 3.1, if $X = \{h_2\}$, then $S^* X = \{h_1, h_2\}$ and $h_{2A} = \{h_1, h_2\}$, it follows that $h_2 \in^* X$, although $\mu_X h_2 \neq 1$

Proposition 4.7 Let (G,A) be a VNS on U and let $X \subseteq U$. Then

(a) $\mu_X x = 0 \Rightarrow x \notin X$

(b) $\mu_X x = 0 \Rightarrow x \notin_* X$

Proof is straightforward and therefore is omitted.

The following example illustrates that the converse of property (a), does not hold.

Example 4.6 From Example 3.1, if $X = \{h_1, h_3, h_4\}$ and from Example 3.2, we get $h_{2A} = \{h_1, h_2\}$, then $\mu_X h_2 \neq 0$, although $h_2 \notin X$

The following example illustrates that the converse of property (b), does not hold.

Example 4.7 From Example 3.1, if $X = \{h_1, h_4, h_5\}$, then $S_* X = \{h_1, h_4, h_5\}$, from Example 3.2, we get $h_{2A} = \{h_1, h_2\}$, it follows that $\mu_X h_2 \neq 0$, although $h_2 \notin_* X$

Proposition 4.8 Let (*G*,*A*) be a VNS on *U* and let *X*⊆*U*. The following property does not hold

$$\mu_X x = 0 \Rightarrow x \notin^* X$$

The following example proves Proposition 4.8.

Example 4.8 From Example 3.1, if $X = \{h_2\}$ then $S^* X = \{h_1, h_2\}$, from Example 3.2, we get $h_{1A} = \{h_1\}$, it follows that $h_1 \in^* X$, although $\mu_X h_1 = 0$

Definition 4.5 Let (*G*,*A*) be a VNS on U and let $X, Z \subseteq U$. NR-inclusion relations, denoted by $\subset_*$ and $\subset^*$ which are defined as follows

$$X \subset_* Z \text{ If } \quad S_* X \subseteq S_* Z$$

$$X \subset^* Z \text{ If } \quad S^* X \subseteq S^* Z$$

Proposition 4.9 Let (G,A) be a VNS on U and let $X, Z \subseteq U$. Then

$$X \subseteq Z \Rightarrow X \subset_* Z \wedge X \subset^* Z$$

Proof comes directly From Proposition 3.2.

The following example illustrates that, the converse of Proposition 4.9 doesn't hold.

Example 4.9 In Example 3.1, if $X = \{h_1, h_4\}$ and $Z = \{h_1, h_2, h_5\}$, then $S_* X = \{h_1\}$, $S_* Z = \{h_1, h_2, h_5\}$, $S^* X = \{h_1, h_4\}$ and $S^* Z = \{h_1, h_2, h_4, h_5\}$. Hence, $X \subset_* Z$ and $X \subset^* Z$. Although $X \not\subset Z$

From Definition 4.5 and Proposition 4.3, the following remarks can be deduced

Remark 4.2 Let (*G*,*A*) be a VNS on U and let $X, Z \subseteq U$. If $X \subset_* Z$, then the following properties hold

(a) $\mu_{S_* X} x \leq \mu_{S_* Z} x$

(b) $\mu_{S_* X} x \leq \mu_Z x$

(c) $\mu_{S_* X} x \leq \mu_{S^* Z} x$

Remark 4.3 Let (*G*,*A*) be a VNS on U and let $X, Z \subseteq U$. If $X \subset^* Z$, then the following properties hold

(a) $\mu_{S^* X} x \leq \mu_{S^* Z} x$

(b) $\mu_X x \leq \mu_{S^* Z} x$

(c) $\mu_{S_* X} x \leq \mu_{S^* Z} x$

Definition 4.6 Let (G,A) be a VNS on U and let $X, Z \subseteq U$. NR-equality relations are defined as follows

$$X =_* Z \text{ If } \quad S_* X = S_* Z$$

$$X =^* Z \text{ If } \quad S^* X = S^* Z$$

$$\text{If } X =_*^* Z \quad X =_* Z \wedge X =^* Z$$

The following example illustrates Definition 4.6.

Example 4.10 According to Example 3.1. Let $A = \{e_1\}$, then $\xi = \{U, \phi, \{h_1\}, \{h_5\}, \{h_1, h_2\}, \{h_1, h_3\}, \{h_1, h_3, h_4\}\}$. If $X_1 = \{h_2\}$, $X_2 = \{h_3\}, X_3 = \{h_1, h_2\}, X_4 = \{h_2, h_3\}$ and $X_5 = \{h_2, h_4\}$, then $S_* X_1 = S_* X_2 = \phi$, $S^* X_1 = S^* X_3 = \{h_1, h_2\}$, $S_* X_4 = S_* X_5 = \phi$ and $S^* X_4 = S^* X_5 = U$. Consequently $X_1 =_* X_2$, $X_1 =^* X_3$ and $X_4 =_*^* X_5$

Proposition 4.10 Let (*G*,*A*) be a VNS set on U and let $X, Z \subseteq U$. Then

(a) $X =_* S_* X$

(b) $X =^* S^* X$

(c) $X = Z \Rightarrow X =_*^* Z$

(d) $X \subseteq Z, Z =_* \phi \Rightarrow X =_* \phi$

(e) $X \subseteq Z, X =_* U \Rightarrow Z = U$

(f) $X \subseteq Z, Z =^* \phi \Rightarrow X = \phi$

(g) $X \subseteq Z, X =^* U \Rightarrow Z =^* U$

Proof. From Definition 4.6 and Propositions 3.2 and 3.3 we get the proof, directly.

From Definition 4.6 and Proposition 4.3, the following remarks can be deduced

Remark 4.4 Let (*G*,*A*) be a VNS on U and let $X, Z \subseteq U$. If $X =_* Z$, then the following properties hold

(a) $\mu_{S_*X} x = \mu_{S_*Z} x$

(b) $\mu_{S_*X} x \leq \mu_Z x$

(c) $\mu_{S_*X} x \leq \mu_{S^*Z} x$

Remark 4.5 Let (*G*,*A*) be a VNS on U and let $X, Z \subseteq U$. If $X =^* Z$, then the following properties hold

(a) $\mu_{S^*X} x \leq \mu_{S^*Z} x$

(b) $\mu_X x \leq \mu_{S^*Z} x$

(c) $\mu_{S_*X} x \leq \mu_{S^*Z} x$

The following remark is introduced to show that Pawlak's approach to rough sets can be viewed as a special case of proposed model.

Remark 4.6 Let (*G*,*A*) be a VNS on U and let $X, Z \subseteq U$. If we consider the following case

$$(\text{ If } T_e(h_i) \geq 0.5, \text{ then } e(h) = 1, \text{ otherwise } e(h) = 0)$$

and the neutrosophic right neighborhood of an element *h* is replaced by the following equivalence class

$$[h]_e = \{h_i \in U \ : e(h_i) = e(h), e \in A\}.$$

Then VNS-lower and VNS-upper approximations will be traditional Pawlak's approximations. It follows that NR-concepts will be Pawlak's concepts. Therefor Pawlak's approach to rough sets can be viewed as a special case of suggested single valued neutrosophic soft approach to rough sets.

5 A decision making problem

In this section, suggested single valued neutrosophic soft rough model is applied in a decision making problem. We consider the problem to select the most suitable car which a person X is going to choose from n cars ($h_1, h_2, \ldots, h_n$) by using m parameters ($e_1, e_2, .., e_m$). Since these data are not crisp but neutrosophic, the selection is not straightforward. Hence our problem in this section is to select the most suitable car with the choice parameters of the person *X*. To solve this problem, we need the following definitions

Definition 5.1 Let (*G*,*A*) be a VNS set on $U = \{h_1, h_2, \ldots, h_n\}$ as the objects and $A = \{e_1, e_2, .., e_m\}$ is the set of parameters. The value matrix is a matrix whose rows are labeled by the objects, its columns are labeled by the parameters and the entries C_{ij} are calculated by

$$C_{ij} = (T_{ej}(h_i) + I_{ej}(h_i) - F_{ej}(h_i)), \quad 1 \leq i \leq n, 1 \leq j \leq m$$

Definition 5.2 Let (*G*,*A*) be a VNS set on $U = \{h_1, h_2, \ldots, h_n\}$, where $A = \{e_1, e_2, .., e_m\}$. The score of an object h_j is defined as follows

$$S(h_i) = \Sigma_{j=1}^{m} C_{ij}$$

Remark 5.1 Let (*G*, *A*) be a VNS set on U and $A = \{e_1, e_2,$ then is the set of parameters. $.., e_m\}$

(a) $-1 \leq C_{ij} \leq 2, \quad 1 \leq i \leq n, 1 \leq j \leq m$

(b) $-m \leq S(h_i) \leq 2m, \quad h_i \in U$

The real meaning of $C_* A$ is the degree of crispness of A. Hence, if $C_* A = 1$, then A is *NR*-definable set. It means that the collected data are sufficient to determine the set A. Also, from the meaning of the neutrosophic right neighborhood, we can deduce the most suitable choice by using the following algorithm.

Algorithm

1. Input VNS set (*G*,*A*)
2. Compute the accuracy measures of all singleton sets
3. Consider the objects of *NR*-definable singleton sets
4. Compute the value matrix of the considered objects
5. Compute the score of all considered objects in a tabular form
6. Find the maximum score of the considered objects
7. If there are more than one object has the maximum scare, then any object of them could be the suitable choice
8. If there is no *NR*-definable singleton set, then we consider the objects of all *NR*-definable sets consisting two elements and then repeat steps (4-7), else, consider the objects of all *NR*-definable sets consisting three elements and then repeat steps (4-7),and so on...

For illustration the previous technique, the following example is introduced.

Example 5.1 According to Example 3.1, we can create Tables 3, as follows

Singleton sets	$\{h_1\}$	$\{h_2\}$	$\{h_3\}$	$\{h_4\}$	$\{h_5\}$
$C_* X$	1	0	0	0	1

Table 3: Accuracy measures of all singleton sets.

Hence $C_*\{h_1\} = C_*\{h_5\} = 1$. It follows that h_1 and h_5 are the *NR*-definable singleton sets. Consequently h_1 and h_5 are concidered objects. Therefore Table 4 can be created as follows

Object	e_1	e_2	e_3	e_4
h_1	(.6,.6,.2)	(.8,.4,.3)	(.7,.4,.3)	(.8,.6,.4)
h_5	(.8,.2,.3)	(.8,.3,.2)	(.7,.3,.4)	(.9,.5,.7)

Table 4: Tabular representation of considered objects.

The value matrix of considered objects can be viewed as Table 5.

Object	e_1	e_2	e_3	e_4
h_1	1	0.9	0.8	1
h_5	0.7	0.9	0.6	0.7

Table 5: Value matrix of considered objects.

Finally, the scores of considered objects are concluded in Table 6, as follows

Object	Score of the object
h_1	3.7
h_5	2.9

Table 6: The scores of considered objects.

Clearly, the maximum score is 3.7, which is scored by the car h_1. Hence, our decision in this case study is that a car h_1 is the most suitable car for a person X, under his choice parameters. Also, the second suitable car for him is a car h_5.

Obviously, the selection is dependent on the choice parameters of the buyer. Consequently, the most suitable car for a person X need not be suitable car for another person Y.

Conclusion

This paper introduces the notion of single valued neutrosophic soft rough set approximations by using a new neighborhood named neutrosophic right neighborhood. Suggested model is more realistic than the other traditional models, as each proposition is estimated to have three components: the percentage of truth, the percentage of indeterminacy and the percentage of falsity. Several properties of single valued neutrosophic soft rough sets have been defined and propositions and illustrative examples have been presented. It has been shown that Pawlak's approach to rough sets can be viewed as a special case of single valued neutrosophic soft approach to rough sets. Finally, proposed model is applied in a decision making problem, supported with algorithm.

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