Non-citable preprint. Forthcoming in the *Oxford Handbook for the Philosophy of Logic*.

The Practice-Based Approach to the Philosophy of Logic

Ben Martin (University of Padova)

Abstract. Philosophers of logic are particularly interested in understanding the aims, epistemology, and methodology of logic. This raises the question of how the philosophy of logic should go about these enquires. According to the practice-based approach, the most reliable method we have to investigate the methodology and epistemology of a research field is by considering in detail the activities of its practitioners. This holds just as true for logic as it does for the recognised empirical and abstract sciences. If we wish to systematically understand the aims and epistemology of logic, we are best placed achieving this by looking at what logicians do, rather than reflecting upon the nature of logic itself. In this entry, we outline the motivations for a practice-based approach towards the philosophy of logic and highlight its advantages over more "top-down" approaches, which attempt to infer conclusions about the nature of logic and its methodology on the basis of traditional assumptions about knowledge, metaphysics, or logic itself. We end by addressing several prominent concerns raised against the practice-based approach.

Keywords: Practice-based approach · Logical Practice · Methodology of Logic · Epistemology of Logic · Philosophy of Logic

1. Introduction

Just as the recognised natural, social and mathematical sciences are social constructs, built out of the coordinated decisions and actions of their practitioners, so is the field of logic. Research fields are not themselves natural kinds. They are not timeless, socially independent objects whose essence can be discovered through rumination on the concepts of SCIENCE, MATHEMATICS, or LOGIC.

Recognising these facts has an impact upon how we ought to go about investigating these research areas. Gone are the hopes of understanding the subject matter and epistemic properties of biology or logic by *a priori* reflection. Doing so would simply serve to detach us from the reality of the fields as they currently exist. Rather, if we want to understand a field's research goals, methodological procedures, and epistemology, we are best placed looking at the activities and decisions of its practitioners.

In this respect, understanding logic or the sciences is no different to understanding other important social activities. If we want to understand carpentry, we are best placed observing the activities of a master carpenter. If our aim is to appreciate the purposes and procedures of the law, we must consider the activities of legislatures and the judiciary. The same is true of the sciences and logic. Failing to recognise these facts can lead to highly idealized accounts of the fields based upon traditional philosophical presumptions which end up at best neglecting important methodological features of the field, and at worst deeming the field as it's currently practiced disciplinarily inappropriate.

This, in essence, is the motivation behind the practice-based approach to the philosophy of logic: to base our conclusions about the aims, methodology, and epistemology of a field not upon traditional philosophical assumptions, but rather upon the activities of its practitioners. As such, the practice-based approach is a meta-methodological position about how to go about doing the *philosophy of* logic if we want to best understand the field of logic.[1]

Recently, motivated by the successes of a broadly practice-based approach towards the philosophy of science and mathematics, there has been a noticeable increase in the use of a similar approach to the philosophy of logic. This is particularly the case when it comes to answering epistemological and methodological questions about logic, where the approach has produced a range of results on topics, including: the various *aims* of logic (Commandeur 2022; Martin 2022), what constitutes evidence for logics (Martin 2021b; Tajer 2022), the mechanisms for theory-choice in logic (Martin 2024; Martin & Hjortland 2022; Peregrin & Svoboda 2017), the norms for formalization in

[1] Importantly, it is not a thesis about how we ought to go about doing *logic*. Though, as we'll point out in Section 5, resultant findings from the use of the practice-based approach can have repercussions for how practitioners of logic ought to go about their business, just as results from the philosophy of biology can sometimes impact biology itself, and jurisprudence can inform the practice of law.

logic (Dutilh Novaes 2012; Peregrin & Svoboda 2017), whether and how logics provide extra-systemic explanations of target systems (Martin 2021a; Payette & Wyatt 2018), and the form that disagreements between logicians take, and how they are rationally resolved (Martin 2021c).

Yet, despite this increased use of the approach within the philosophy of logic and its apparent successes, there is still a noticeable underappreciation of the underlying rationale for the approach, and some misapprehension over its commitments. This requires addressing. If the approach is to be successful in rectifying the perceived shortcomings within the philosophy of logic that motivate it, it will not be enough to simply have tangible successes when it comes to answering particular philosophical questions. Instead, a concerted effort must be made to engage with philosophers of logic who have up to this point been more reticent than colleagues in the philosophy of science and mathematics to embrace a practice-based approach.

Our goal in this entry, therefore, is not so much to recapitulate recent findings using the practice-based approach, which can in the main be found in other entries in this handbook. Rather it will be to focus on the general justification for the approach to the philosophy of logic underlying these findings, how it differs from traditional philosophical approaches to the philosophy of logic, and some of the challenges the approach faces.

We begin, in Section 2, with the development of the practice-based approach in the philosophies of science and mathematics. Given that the practice-based approach in the philosophy of logic takes its inspiration from these, it makes sense to start there. Section 3 then highlights how many of the concerns raised over work in the philosophy of science and mathematics apply equally to research in the philosophy of logic, while Section 4 explains how a practice-based approach help us avoid these pitfalls. Finally, in Section 5 we address several common concerns with, and misconceptions about, the practice-based approach. Some will naturally be concerned that emphasising the socially constructed features of a *research area* commits us to the field's subject matters themselves being social constructions, or that we lose any ability whatsoever to distinguish between the brute fact of how a research area operates and how it *ought to* operate, thereby reducing the philosophy of logic to sociology. As we show in this section, these concerns are ultimately misplaced.

2. The Practice-Based Approach

Practice-based approaches are distinguished, first, by their dissatisfaction with more traditional philosophical approaches towards a research field and then, second, by their positive proposal for how to go about rectifying these alleged shortcomings.

The approach first emerged in the philosophy of science in the 1960s, in response to perceived inadequacies with traditional philosophical approaches towards the scientific disciplines (Soler et al. 2014). In particular, these traditional approaches were criticised on the basis of producing accounts of the sciences which were:

(i) *Too idealised*, in virtue of being based upon *a priori* reflections of what we *want* the sciences to look like, or what they *should* look like given our preconceptions of the fields, rather than reflecting the realities of research in these fields;

(ii) *Over simplistic*, in failing to reflect the plurality of the fields' aims and methodologies;

(iii) *Too present-centred,* by falling foul of a tendency to produce Whig histories, presuming that the fields' histories are a story of smooth and unstoppable progress up to the present state of affairs; and

(iv) *Too end-product focused,* by concentrating on the properties of final theories, and thereby neglecting the important processes which led to the discovery of these results, including communal processes.

An early example of these concerns, and advocacy of a more practice-orientated approach are found in Kuhn's (1962) *The Structure of Scientific Revolutions*, in which Popper's (1959) falsificationism is criticised on several of the scores above. Popper's account of the scientific method is denounced for both idealising scientific methodology by presenting a naïve picture of scientific progress as a continual chain of evermore informative theories that perpetually

become falsified and for being too present-centred, by presuming that the aims and norms for the evaluation of past scientific theories were the same as those of contemporary science.

Further, Kuhn highlights how past accounts of scientific methodology had been deficient by neglecting important features of scientists' research activities, such as the designing and testing of experimental equipment, and their use in measuring constants. These failures to take seriously the various roles of experimentation within the sciences beyond the direct testing of hypotheses were further emphasised by Hacking (1983) and other 'New Experimentalists', who generally criticised past accounts of scientific methodology for paying too little attention to the rich variety of activities constituting the actual scientific method.

Taking inspiration from practice-based research in the philosophy of science, similar work in the philosophy of mathematics began around the start of the twenty-first century (Carter 2019; Hamami & Morris 2020),[2] with traditional approaches to the philosophy of mathematics being criticised for possessing too idealised a picture of mathematics, with mathematical knowledge conceived wholly in terms of theorems evidenced by formal proofs (Corfield 2003).

Contrary to this traditional view, it has since been argued on the basis of actual mathematical proofs that mathematical understanding progresses in many ways, including through the abundant use of informal proofs, whose positive epistemic features cannot be reduced to those of formal proofs (Larvor 2012; Tanswell 2015), and in the case of some areas of mathematics such as geometry, through visualisation (Giaquinto 2007).

Further, in virtue of being too concerned with philosophically foundational issues, such as the metaphysics of mathematical objects and resulting epistemological puzzles, traditional philosophy of mathematics has produced an unjustifiably simplistic picture of the mathematical enterprise, neglecting important features of contemporary mathematics, including the appraisal of definitions (Tappenden 2018) and the use of diagrams (Giardino 2017).

Conjoined with these criticisms of traditional approaches to the philosophies of science and mathematics, the practice-based approach possesses a positive story of how the philosophy of these fields should then proceed to rectify these faults. This solution requires both a re-evaluation of our *aims* when providing a philosophical account of a field, and a modification in the *methods* we should use to realize them.

Rather than attempting to construct grand unified theories of the essential nature of the sciences and mathematics conforming to established preconceptions about the goals of rational enquiry, viable metaphysical accounts of reality, or justificatory norms, we should aim instead to produce accounts which: (i) reflect the reality of research in these fields; (ii) recognise the plurality of aims and methodologies found across them; (iii) situate results in the field within their proper historical context; (iv) recognise the development of, and changes in, the methodological norms within the fields; and, (v) give equal attention to the processes of discovery as the properties of the final products (Corfield 2003; Soler et al. 2014). Only by reorientating our aims in the philosophy of these fields can we ensure we don't fall foul of the methodological mistakes currently existent in the literature.

Successfully meeting these aims, however, requires embracing new methods. In particular, to ensure that the philosopher's proposals reflect real-life research in the field, rather than simply being based upon what we would *expect* or *prefer* the research fields to look like given certain epistemological or metaphysical presumptions, more time must be spent looking in detail at how scientists and mathematicians go about achieving their research goals.

Much of this work will take the form of case studies, whether this be an in-depth study of the activities of an individual researcher or research group, or a wider study of the norms within a particular sub-field. However, historiographic studies are also commonly used, in order to trace the development of a particular prominent concept, or track evolution of the methodological norms within a field (Krantz 2011), and studies from cognitive science are sometimes used to inform an account of how theories are evidenced or selected for (Giaquinto 2007).

These proposed alterations to the aims and methods of the philosophy of logic have a notable impact upon the viable generality of the philosopher's conclusions, as one can only be informed with up-to-date research in so many areas of the sciences. Given the practical constraints placed upon her reliable sources of information, and the recognition that there is no principled restriction on the plurality of aims and methodologies that a field can possess, the practice-orientated philosopher of science and mathematics will need to normally resist the temptation to make *generalized claims* about scientific or mathematical methodology. Instead, their results will be restricted to proposals about the methodological norms found within certain sub-fields of a discipline (often within a certain time period).

[2] Though in some cases earlier—see Lakatos (1976).

These results can then form the basis of a more general hypothesis about the methodology of a wider scope of sciences (and time periods), to be tested with further detailed case studies. Consequently, constructing any kind of detailed picture of even a sub-field of the sciences or mathematics across time becomes a collaborative project, not one for an individual philosopher.

An excellent example of the contrast between traditional and practice-based approaches to the philosophy of a field is found in the debate over what constitutes a scientific explanation. Traditionally, accounts of (scientific) explanation assumed that a *general theory* of explanation could be provided, and proposals were often motivated and evaluated on the basis of: (i) a few simple paradigm examples, often taken outside of a scientific context, and (ii) intuitions regarding what constitutes a suitable explanation.

For instance, consider Hempel's (1965) famous defence of the *Deductive-Nomological* (DN) model. Taking as his starting point three brief examples of explanations—the change in a mercury thermometer when immersed in hot water, the appearance of an oar being bent when submerged under water, and the law of refraction—Hempel attempts to "abstract [from these] some general characteristics of scientific explanation" (1965: 247). These general characteristics themselves being justified on the basis of certain presumptions about the essential role of causation and nomic expectability within explanations, the importance of *laws* in accounting for this causation, and further the need for logical consequence to enable this nomic expectability. Hempel's defence of the DN model exemplifies what is known as a "top-down" approach to scientific explanations (Braillard & Malaterre 2015), characterised by its attempt to infer what constitutes a scientific explanation from certain presumptions and intuitions about what explanations *should look like* without consideration of detailed realistic examples from the sciences, and made upon the presumption that there is a structure that *all* scientific explanations should conform to.

Advocates of the practice-based approach have criticised this "top-down" approach to scientific explanations on the basis that it unjustifiably assumes an "unwarranted essentialism about the nature of explanations across the sciences" (Woody 2015: 80), which fails to pay adequate attention to paradigm cases of scientific explanation. Indeed, what we find with essentialist accounts of explanation is that they typically struggle, blighted by contrived examples disconnected from scientific contexts, while deeming a significant number of scientific explanations illegitimate in virtue of their failure to conform to the proposed essentialist features of explanation (Woody 2015).

In comparison, since philosophers of science dropped the essentialist assumption and began constructing accounts of scientific explanation from the "bottom-up", starting with instances of explanation from various sub-fields, research in the area has flourished with theories of the wide variety of forms of explanations found across the life (Brigandt 2013), medicinal (Qiu 1989), and physical sciences (Fisher 2003) being produced. Particularly striking is the fact that the practice-based approach's critique of traditional essentialist accounts of scientific explanation is seemingly so embedded now within the contemporary literature on scientific explanation that, without being explicitly cited, the prevailing presumption is that one should build a case for what constitutes an explanation within a sub-field of science based upon detailed case studies from that field (Bokulich 2011; Machamer et al. 2000).[3]

What goes for the question of what constitutes a scientific explanation holds equally for other topics within the philosophies of science and mathematics. Rather than attempting to answer these questions from the "top-down", beginning with certain philosophical presumptions about the topic or simple intuitive examples, we instead work from the "bottom up", beginning with examples of relevant practice from the pertinent field.

By embracing these new aims and methods, the practice-based approach has shown itself to have two benefits over traditional approaches to the fields, at least when it comes to the philosophies of science and mathematics (Carter 2019; Hamami & Morris 2020; Soler et al. 2014).

Firstly, the approach is able to provide more insightful answers to established and prominent philosophical questions about scientific and mathematical methodology than traditional "top-down" approaches. Clear examples of this benefit come not only from the aforementioned debate over the nature of scientific explanations, but the

[3] Of course, this does not preclude comparisons being made and connections drawn between the forms of explanation found across the sciences (see, for instance, recent work on model explanations—Bokulich 2017). The important point is that these potential elements of similarity are no longer assumed to be the *essential features* of the explanatory enterprise within science, such that anything which fails to possess these properties is in essence not an explanation.

increased understanding over what constitutes a proof within mathematics (De Toffoli 2020) and the detailed accounts of the role model building plays across a range of sciences (Gelfert 2016; Weisberg 2013).[4]

Secondly, the approach has opened up new important research questions about the fields that were previously neglected using traditional "top-down" methods. This has been achieved through a combination of the approach's method facilitating a more detailed consideration of the activities of practitioners, and further using these activities as a means to motivate new philosophical questions, rather than simply imposing established philosophical questions upon the fields of enquiry.

For instance, the approach has led to the investigation of: (i) the various sources of evidence which mathematicians rely upon, such as visualisation (Giaquinto 2007) and computer-aided proofs (Avigad 2008); (ii) what constitutes scientific understanding (de Regt 2017); (iii) the characteristics that mathematicians look for when choosing formal notations (De Toffoli 2017); (iv) the role of (interdisciplinary) collaboration within the sciences (Andersen 2016); and (v) the theoretical virtues mathematicians prize within a piece of mathematics (Rota 1997).

As we'll go onto see in the rest of this entry, much of what holds for the use of practice-based approaches within the philosophies of science and mathematics holds equally for the practice-based approach to the philosophy of logic. The approach has clear advantages over traditional "top-down" approach when it comes to understanding the aims, methodology, and epistemology of logic. As a result, philosophers of logic should be less reticent to embrace it.

3. Shortcomings within Traditional Philosophy of Logic

The first point to note in motivating a practice-based approach towards the philosophy of logic is that many of the same perceived shortcomings identified within traditional discussions in the philosophies of science and mathematics are also found in the philosophy of logic. The concern is that debates about logic are often built upon historical presumptions about its subject matter, rather than being informed by research in the area of enquiry itself. What results are accounts of logic that not only miss important activities and aims within the research area, but go so far as to problematize actual logical research by deeming it to be disciplinarily unsuitable (Martin 2021b). We highlight here four common shortcomings in the literature, resulting from the use of "top-down" approaches:

1) It tends to produce accounts of logic which *are far too idealised*, distorting our understanding of the field.

Accounts of the nature of logic are often detached from the realities of logical research, based upon expectations about logic's subject matter, methodology, and epistemology given long-standing philosophical presumptions about the properties of logic, what constitute proper standards of rationality, and viable sources of evidence. What results is a picture of logic at odds with that practiced by logicians.

A particularly prominent example of this shortcoming is found in the epistemology of logic, over how we come to establish the correct laws of validity. Philosophical tradition has it that evidence about validity must be both non-inferential and *a priori*. *Non-inferential*, for otherwise one would need to presume the validity of at least *some* rules of inference to establish the reliability of the inferences which partially constitute one's justification for the logical rules (Haack 1976). *A priori* because, firstly, no observable states of affairs directly demonstrate that a rule of inference is valid, and secondly, the possibility of *inferring evidence* for particular logical laws from empirical evidence is precluded by the non-inferentiality of logical evidence. Thus, if these two starting assumptions are correct, we must have *unmediated a priori* access to the truth of logical laws about validity.

Motivated by these initial epistemological presumptions, modern epistemology of logic has been dominated by two foundationalist accounts of logical knowledge—*logical rationalism* and *semanticism*—both of which agree that we have unmediated *a priori* access to the logical laws of validity, but simply disagree on the source of this *a priori* justification (Martin & Hjortland 2022).

While according to rationalists, evidence of validity is constituted of intuitions facilitated by a quasi-perceptual intellectual faculty, with which one simply *sees* that a particular logical law is true or inference valid

[4] The latter is one of clearest successes of the practice-based approach, and discussions of model-building are some of the clearest uses of the approach, with proposals explicitly appealing to *compatibility with scientific practice* as a criterion for success (Weisberg 2013: Ch. 3).

(BonJour 1998), semanticists deny the need to posit a novel cognitive faculty to accommodate logical knowledge. Instead, we gain evidence for logical laws directly through linguistic proficiency. In virtue of understanding the meaning of the constituent terms of a logical law or inference, we automatically become justified in assenting to its truth or validity (Ayer 1936). In other words, logical laws are epistemically analytic (Boghossian 1996).

Both rationalism and semanticism are examples of positions within the philosophy of logic motivated through a "top-down" approach, in which an attempt is made to infer how knowledge of validity *could be possible* on the basis of certain prominent standing presumptions about the nature of logic, rationality, and viable sources of evidence.

Unfortunately, both proposals are significantly divorced from how logicians actually go about evidencing their theories of validity. Rather than any pretense to have unmediated access to the laws of logic, logicians engage in a painstaking process of theory construction and testing informed by (amongst other things) those inferences made by mathematicians within informal proofs, the logico-semantic paradoxes, and certain results in mathematics (Martin & Hjortland 2021; Priest 2016; Russell 2015; Williamson 2017).

Why and how these sources of evidence are treated as a reliable guide to the laws of validity are interesting and difficult questions, but they are at threat of being completely passed over by these idealized accounts of logic's epistemology. In virtue of failing to come to grips with the complexities of logic's epistemology, rationalism and semanticism serve as prime examples of the dangers of embracing a "top-down" approach to the philosophy of logic, at least when it comes to understanding logic's methodology.

2) Its conclusions are often *too synchronically homogeneous*, leading to unjustified generalisations and neglect of prominent research topics.

In virtue of being preoccupied with traditional philosophical concerns, or too focused on certain uses of logic deemed "philosophically important", a significant portion of those projects that logicians are engaged in, and the uses these logics are put to, are neglected in the current literature. For instance, the use of Kripke frames to model mental attitudes within multi-agent systems in AI (Wooldridge 2009), Church's type theory to model ethical and legal reasoning (Benzmüller et al. 2020), and dynamic logics to model the semantics of complex linguistic phenomena (Keshet 2018). What results from the omission of these prominent uses of logic (often, outside of philosophy departments) is not only a diminished and poorer view of the diversity and richness of the field as a whole, which is bound to impact our appreciation of its wider aims and methods, but also significant opportunities to inform longstanding philosophical debates.

To give just one example, based upon the proclamations of Kant and Frege that logic is concerned with how we *ought to* reason, philosophers have been interested in establishing whether (and how) logic's laws are normative for reasoning (see entry on NORMATIVITY OF LOGIC). In the main, this work has focused exclusively on drawing a connection between monotonic logics and norms for reasoning. However, there is a whole research area of non-monotonic logics, *logics of belief revision,* which explicitly attempts to model norms of belief revision (Hansson 2022). While this does not in and of itself show that alethic monotonic logics fail to express norms for reasoning in some sense, it does lead to significant opportunities to compare the relative goals, motivations and types of evidence used within monotonic logic and logics of belief revision research programmes, which would greatly inform the present debate about the normativity of logic. This is a clear example where a myopic view of what constitutes the field of logic can lead to missed opportunities when it comes research in the philosophy of logic.

3) Its proposals are often *ahistorical* (leading to implicit *diachronic homogeneity*).

In virtue of the propensity within contemporary philosophy of logic to build idealised accounts of logic based upon prominent presumptions about its aims and methods, what tends to result is an essentialist account of logic which gives the impression that logic as a field is diachronically homogeneous. Yet, just like other research areas, logic is a social enterprise with continually changing priorities and techniques. Thus, contemporary philosophy of logic is not only prone to unjustified simplifications when it comes to drawing conclusions about the methods and research goals of contemporary logical research (unjustified *synchronic homogeneity*), but also about the historical status of logic (unjustified *diachronic homogeneity*).

A prominent example of this ahistoricality is the presumption that that there is some canonical purpose for logic which has persisted through time (Cook 2010: 495; Priest 2006: 196). This presumption is contrary to the varying

priorities of distinct schools of thought within the research field throughout the development of logic, from Ancient Greece to the humanist tradition during the Renaissance period, from the empiricist movement with Bacon and Locke which emphasised the importance of logic as an organon of science and criticised syllogistic (formal/schematic) logic for its failure to serve this role, to the re-emergence of the importance of formal logic as a calculus for reasoning with Leibniz and Wolff.[5] For instance, far from the logic studied within a contemporary discrete mathematics course, it was common for Stoic and Epicurean studies of logic to include not only a canonical (or dialectical) element, but also rhetoric and the evaluation of definitions (Diogenes Laërtius 1925: VII.40-8 & X.29-31).

Even if one wished to extract from these far broader historical arenas of logic what we might call logic *proper* (in the modern sense), as is sometimes done within Kant's work on the *general logic*, thereby isolating Aristotle's study of the syllogistic in the *Posterior Analytics* from his other works traditionally gathered together in the *Organon*, drawing too close a connection between the field then and now is likely to be hasty. For instance, the means through which syllogistic forms were justified is vastly different to how contemporary monotonic alethic logics are evidenced. Neither the logico-semantic paradoxes nor the inferential moves made by mathematicians played any role within the wider motivations or justification of the syllogistic forms, although both are prominent sources of evidence in contemporary philosophical logic.[6] Further, contemporary logics are not only interested in simply categorising (in)valid argument forms and bringing valid arguments under schematised generalisations. They also wish to *explain why* a given argument is valid with an informative semantics (see entry on EXPLANATIONS IN LOGIC).

Such diachronic homogeneity may seem surprising given the excellent work that is done in the history of logic. However, unfortunately, contemporary philosophy of logic is prone to not take advantage of these resources, as was earlier the case in the philosophies of science and mathematics.

4) It is *too end-product focused*.

Contemporary philosophy of logic concentrates primarily on the properties of *logics*, rather than on the processes which led to the formulation and justification of these logics. This has quite significant consequences, not only because it can lead to us neglecting the many important techniques and methods which logicians use to develop, evidence, and apply their systems, but it can result in hasty conclusions about the field.

For instance, following Haack (1978), there has been a tradition to understand (dis)agreement within logic in terms of the ways in which logics themselves can (dis)agree, whether in terms of their theorems, rules of inference, or structural features. In light of this, and the fact there are currently multiple live disagreements over the correct or best logic, whether in philosophical logic over the best alethic logic or in belief revision theory, this has led some (Resnik 1999) to conclude that the persistent disagreement between candidate theories in the field is evidence that logic is more akin to ethics than the sciences. Yet, this emphasis on (the lack of) consensus over the end-product of logic neglects the significant areas of agreement between logicians, including over what constitute suitable methods within the field, relevant sources of evidence, and the relative strengths and weaknesses of the available theories (Martin 2021c).[7]

Being too-end focused also leads us to neglect interesting research questions related to the development of techniques and theories within logic, which allow us to properly assess the connection between the methodological norms of logic and other research areas, notably those of science. Such systematic exploration is important if we are properly to assess the claims of prominent proposals within the philosophy of logic, such as anti-exceptionalism about logic.

---

[5] For a brief, but informative, account of the development in the perceived aims of logic, see Lu-Adler (2018: Ch. 3).

[6] For instance, it was well known from the 16th century onwards that syllogistic logic could not accommodate important steps in Euclidean proofs (Mugnai 2010). Yet, this was not enough during that period to motivate a revision in the logical theory.

[7] This concern over a lack of consensus within logic also notably exhibits the faulty presumption that progress in a field is constituted or evidenced by consensus, although no contemporary account of progress in the sciences embraces such a view. This highlights another weakness within contemporary philosophy of logic: that it is prone to be somewhat naïve about the sciences, although they serve as a useful source of analogy in order to understand logic. For more on this, see the entry on ANTI-EXCEPTIONALISM ABOUT LOGIC.

So much then for the brief negative case against the traditional "top-down" approach to the philosophy of logic. These examples serve to show that philosophy of logic has a tendency to suffer from some of the shortcomings that were historically found in the philosophies of science and mathematics. It is one matter, though, to criticise the status quo, another to propose a solution to the underlying problems. The question then is what the practice-based approach can do to address these concerns. This is the topic we move onto consider now.

4. Towards a New (Practice-Based) Approach

The fundamental rationale for re-orientating the philosophy of logic towards a practice-based approach is to avoid the perceived shortcomings present within traditional philosophy of logic. In essence, the proposal is that one achieves this by giving primary importance to the actual activities of logicians; building one's account of logic's objectives and methodology up gradually from paradigm instances of practice in the field. In this sense, the practice-based approach towards the philosophy of logic takes it lead from the successes of similar approaches in the philosophies of science and mathematics.

How exactly, though, does this practice-based approach differ from traditional approaches to the philosophy of logic, and why should we expect it to avoid the aforementioned shortcomings that have putatively plagued the field? The approach's focus on the actual activities of logicians is exemplified by four methodological features, each of which allow it to address the concerns identified in the previous section.

Distinct Methodological Starting Point

Firstly, the practice-based approach has a distinct *methodological starting point* to traditional approaches. Philosophy of logic traditionally has used a "top-down" approach, beginning with certain philosophical presumptions about the subject. These could take the form of assumptions about logic itself, such as its privileged status in virtue of its laws being formal, wholly general and necessarily true, or wider philosophical assumptions, such as established accounts of what constitutes knowledge or rational standards of enquiry. From these postulates, one then attempts to *infer* viable accounts of logic's aims, epistemology or methods, with the adequacy of any proposal ultimately tested against the background of these presumptions and on the basis of the theory's ability to respect them.

In contrast, the practice-based approach operates a "bottom-up" approach, beginning with case studies of instances of practice from the field that are pertinent to one's research question. From these initial case studies, tentative conclusions are drawn and hypotheses proposed, to then be tested against further case studies. The aim is to steadily build up a detailed theory of particular elements of the field through a process of testing proposals against an ever-increasing sample of case studies.

The approach, then, is not a mere blind *description* of the individualised activities of practitioners. Like most good enquiries, it is *purpose driven*. There will be some particular facet of the field that we wish to understand better, whether this be the virtues belief revision theorists look for in their logics, or the comparative role of judgements over cases within epistemic and deontic logics. Further, we may even have some concrete hypothesis about the field that we wish to test, due either to pilot studies or some standing philosophical presumptions. The important point, though, is that if we wish to find support for these proposals, we ultimately need to source them from instances of logical practice, most directly in the form of case studies.

Differing Evidential Priority

This brings us onto the second difference between the practice-based approach and more traditional "top-down" approaches to the philosophy of logic. In those instances there is a clash between prominent practices within the field of enquiry and the background philosophical assumptions, as far as the practice-based approach is concerned *evidential priority* (in most cases) ought to be given to the current practice within the field when it comes to establishing philosophical conclusions about its aims, methodology or subject matter. In other words, the practices

of experts within the field are treated as the most reliable evidence we have to understand the field's aims, epistemology and methods; certainly, more reliable than philosophical presumptions.[8]

In contrast, it is not uncommon within traditional philosophy of logic to dismiss cases of logical practice as irrational or unviable if they clash with certain philosophical conclusions. This could be dismissing the possibility of empirical evidence rationally informing a logical theory on the basis that it contravenes the requirement that logical evidence is wholly *a priori* or non-inferential (BonJour 1998), or rejecting the *bona fides* of monadic second-order logic because the plural quantification it relies upon is not "ontologically innocent" (Linnebo 2003). In both of these cases, we have an argument that a possible logical practice or product is unviable because it contravenes a certain philosophical expectation about logic, regardless of whether these practices are widely found within, or these products are widely studied by, the community.

A particularly nice illustration of the evidential priority afforded background philosophical commitments over logical practice when they happen to clash comes from the debate over the extent to which logical laws are *constitutive of rational thought* (Leech 2015). One consequence of the view that they *are* constitutive of rational thought is that if one fails to adhere to the (correct) logical laws, then one is not reasoning rationally. This, of course, has repercussions for the possibility of rational disagreement in logic. After all, if by definition a logician who fails to adhere to the correct logical laws (whatever these are) cannot be reasoning rationally, their possibility of engaging in a *rational* disagreement with peers over the correct logic is precluded.

However, this consequence runs contrary to what we find in the literature. Not only do we find advocates of competing logics debating with one another over the comparative strengths and weaknesses of their candidates, but debates over the validity of important logical laws, such as *modus ponens* and the disjunctive syllogism (Anderson & Belnap 1975; McGee 1985). While for advocates of a practice-based approach, this clash with logical practice spells a blow to *constitutivism*, the constitutivist themselves seems content to admit that those who challenge the logical laws putatively constitutive of rational thought cannot be providing rational considerations, *even if* these challenges are taken seriously by their peers (Martin 2021c).

Consequently, the practice-based approach differs from the traditional "top-down" approach not only in its methodological starting point, but also in the *evidential priority* it accords logicians' practices and philosophical background assumptions when the two clash.

Generality as Virtue or Requirement

Thirdly, the approaches differ in their attitudes towards the *generality* of their claims about logic. While the practice-based approach sees great virtue in being able to provide informative generalisations about the aims and methods of logic as a whole, such generality is considered as merely one theoretical virtue and not a requirement of true claims about the field. Such generality should not be blindly prioritised over the *accuracy* of our accounts of the field. In particular, given that the practice-based approach recognises that the aims and methods of the field supervene upon the collective actions and decisions of its practitioners, it should not be a surprise if there is variation in many of these facets across both research programmes in the field at any one time and time-periods.

The same attitude is not true of those claims normally found within the philosophy of logic. Given that, traditionally, philosophers have been focused on attempting to extract the *nature of* logic from ruminations on the concept LOGIC, or what we would *like* the field or its objects of enquiry to look like, its conclusions are rarely bounded in this fashion. For instance, *formality* is a necessary requirement for logics, not just of this period or a particular research period, but *tout court* (Sher 2016). Similarly, for *topic-neutrality*, *generality* and the *necessary truth* of logical laws. Indeed, it is often thought that it is logic's essential possession of these properties which differentiates its subject matter from those of other research areas, such as the sciences (Martin & Hjortland 2022).

The same propensity for unbounded generalisations in the philosophy of logic can be found in discussions over the *aims* or *methodology* of logic, caused by the attempt to identify these features of the field based upon our

[8] Why do we say in *most* cases? We must be open to the possibility that individual members of the community can make methodological mistakes, and thus not reflect the general methodological norms of the field. Thus, just as with any empirical finding, we should not be too hasty as to draw dramatic conclusions from individual cases. This potential concern can be addressed through considering a range of case studies and subsequently identifying outliers.

philosophical expectations or wishes. For instance, that logic is inherently concerned with truth-preservation (Beall & Restall 2006), although there prominent logics which are not; for instance, the preservationist logics of Jennings & Schotch (1984).

Generality within our conclusions about any topic is something to strive for—it allows our theories to be more informative and increases their predictive power. However, according to the practice-based approach, such generality should be a *goal* of our enquiry and not an axiom of our methodology.

In reply, one might rightly highlight that an important lesson we have learnt from the philosophy of science is that, just as we shouldn't forsake accuracy for generality, so we shouldn't always forsake generality for accuracy. A theory which attempts *total accuracy* without being able to draw general conclusions across a range of cases will be as useless as the 1:1 map in Borges' (1975) *On Exactitude in Science*.

This is true, and the practice-based approach happily acknowledges the point. After all, a theory without any generality at all, which restricts itself to just accommodating the given data at any one time, will never contain any of the predictive power we desire from our theory. Indeed, as far as the practice-based approach is concerned, it is only by one's proposals possessing such predictive power that they can even be suitably assessed, by testing them against further case studies. Without such predictive power beyond the initial cases, we would never be able to determine whether they were representative of the communal norms or not.

The important point, though, is that while it is totally responsible to offset generality against accuracy, and so introduce idealisations into one's account of the field, this idealisation should be a *conscious process* conducted for the sake of another theoretical virtue, such as predictive power. It should not be mistaken for providing us with some insight into the *essence* of logic. Just as with scientific models, idealisation can become theoretically dangerous once we forget it is purpose driven and just that—an idealisation.

Scope of Enquiry

Finally, the *practice-based approach* differs from traditional philosophy of logic in terms of the *scope* of its objects of enquiry. Traditional philosophy of logic typically concentrates on established philosophical questions and how they relate to logic, be it the rationality *of logic*, the metaphysics *of logic*, or the epistemology *of logic*. This is another manifestation of the epistemic priority given to philosophical considerations over those of logic, though this time in terms of the types of questions philosophy of logic even addresses, rather than the answers it provides.

As far as the *practice-based approach* is concerned, the purpose of the philosophy of logic is not to use logic as an instructive case study to justify one's choice philosophical theory, or to show how logical knowledge would be possible in light of one's favoured epistemology. This would, unjustifiably, suggest that logic as a field of research is somehow subservient to our wider philosophical theories and aims. Yet, just as with the empirical sciences and mathematics, logic is an important and diverse field of research that ought to be examined and analysed in its own right, and on its own field-specific terms.

Thus, according to the practice-based approach, we ought to explore the philosophical repercussions of the full diversity of logicians' activities, beyond what has traditionally been considered to be of philosophical importance. This could include placing greater importance on the activities of logicians within pure logic (Priest 2006; Martin 2022), often neglected by philosophers of logic although it constitutes a large portion of the field, the purposes that logical systems are put to *outside of* philosophy departments, and the processes used to develop logics, rather than focusing almost exclusively on the final products of the field.

As with the other methodological differences between the practice-based approach and traditional philosophy of logic, this difference in attitude is explained by the fact that according to the practice-based approach actual logicians' practices have primacy, and it is these practices which should lead our philosophical analysis of logic, not traditional philosophical assumptions. Ultimately, it is the job of the philosophy of logic to understand logic as we find it, just as it is the goal of the philosophy of science to understand science as it is.[9]

[9] Note, this does not mean that philosophy can never suggest improvements to the aims, methods, or otherwise of logic. As we shall see in the following section, this is a perfectly respectable philosophical project and not in tension with the practice-based approach. Rather, the important pitfall to avoid is mistaking one's normative account for what logic *should* aim to do, or how we *should* go about discovering the correct logical laws, as an account of what these aims currently are and how we go about discovering the laws at present. The distinction is, of course, important. For once one recognises that one's proposal

Combined, these elements of the practice-based approach allow us to address those concerns raised against traditional philosophy of logic. In virtue of using a "bottom-up" approach and basing its conclusions upon detailed case studies of actual practice in the field, there is less risk of producing accounts of logic which over-idealise and distort the realities of the field. Even if are our initial enquiries are led by some underlying philosophical presumption we may have about the properties of logic, its objects of study, or its relationship to other research areas, we are equally assured that these initial presumptions should not determine our conclusions about the field, given the *epistemic priority* which the approach gives to the actual activities of practitioners.

The approach's failure to require its conclusions to be wholly general also help in this regard, ensuring that its claims are not too hasty and thereby deeming whole swathes of the field disciplinarily inappropriate or mistaken. This particular feature of the practice-based approach's methodology also allows us to make more nuanced claims about the plurality of aims and methods found across either varying research programmes or time-periods. This is a benefit that could not possibly be afforded by traditional top-down approaches. No rumination on the concept of LOGIC or its traditional properties will provide us with an appreciation of the various aims of different research programmes in contemporary or historical logic, nor how these diverging aims are associated with different appropriate methodological and epistemological norms.

Finally, by widening the scope of enquiry and grounding the philosophy of logic's research questions in the practices of logicians, the approach allows us to focus on important but currently neglected research questions about logic, such as the methodological processes used by logicians in the development and assessment of logics.[10]

5. Prominent Concerns

So far, we've focused on the putative problems associated with more traditional "top-down" approaches to the philosophy of logic, and how the practice-based approach helps to resolve these with a new methodology for the *philosophy of* logic. However, even if we admit the initial problems and recognise that the practice-based approach would help solve them to an extent, we might still be hesitant to embrace the approach. After all, we might think that this new approach brings with it such unwanted consequences that we are better sticking with the admittedly flawed, but still less troublesome, traditional approach. Or, we might think that regardless of its virtues, the practice-based approach is actually wholly unsuitable to address many of the questions we want to ask about the philosophy of logic.

Given that it is concerns such as these which, in my experience, have led to researchers being hesitant to embrace the practice-based approach, it is important that we attempt to address them. We'll finish this entry by looking at three such concerns that I regularly hear levelled against the practice-based approach. They are by no means the only concerns that can be brought against the approach, but they are among the most common. As we'll see, each is ultimately misplaced.

1. Part of the underlying rationale for the practice-based approach is the fact that the field of logic is socially constructed. However, if we admit that the aims, methodology and epistemology of a research field are socially constructed, aren't we also committed to the subject matters of the field themselves being social constructions?

The fact that the goals and methodological norms of a research field are the product of social decisions on the part of the field's researchers does not mean that the *phenomena* these research areas study are themselves social constructs. The metaphysical status of these phenomena will depend on the subject matter of the research area itself.

While the current research goals, methodology, and epistemology of astrophysics are a result of the collective activities and decisions of its practitioners, many of the phenomena it studies—stars, cosmic rays, plasmas—actually do exist. In contrast, not only are the current research goals, methodology, and epistemological norms of descriptive

---

is normative rather than descriptive, one takes on the additional burden of showing *why* the current practice (whether that be in terms of aims or methods) is inadequate, even if it appears to have brought significant successes.

[10] For an argument that the practice-based approach has already been able to deliver on some of these promises, see Martin (2022).

linguistics a result of the collective activities and decisions of its practitioners, but the phenomena it studies (namely, the properties of natural languages) are *also* the result of such collective activities and decisions.

Further, sometimes, to the surprise of its practitioners, the putative phenomena of a research area fail to exist at all, as in the case of phrenology and natal astrology. However, the status of the (non-)existence of these putative phenomena in no way alters the fact that the research goals, methodology, and epistemology of these (subsequently discredited) research areas were the results of collective activities and decisions on the part of its participants.

Thus, regardless of whether the phenomena a field studies exists (or not), and in what capacity, the field's research goals, methodological procedures, and epistemology themselves supervene upon the activities and decisions of its practitioners. Importantly, then, the status of a research field as a social construction (albeit often a totally rational social enterprise) is independent of whether the field's subjects of enquiry are social constructions.[11]

One consequence of this realization is that the motivation for a practice-based approach towards logic (or, any research field) is independent of one's views on the metaphysics of the field's subject matter. One can be a metaphysical realist (Sider 2011), conventionalist (Warren 2020), or expressivist (Resnik 1999) about logic while embracing a practice-based approach to the philosophy of logic. Indeed, given that we would expect the activities of logicians to be somewhat different given these various interpretations of logic's subject matter, it would make sense for advocates of these positions to embrace a practice-based approach in order to inform their proposals.

*2. Is the practice-based approach still philosophy? We want to know what we **should** do with regards to logic, not just what we **actually do**. Yet, the practice-based approach only provides us with conclusions regarding the latter. The approach collapses the distinction between philosophy and sociology.*

At root, this concern is based upon two misconceptions: firstly, that the practice-based approach can only ever serve to *describe current practices*, not to help in the process of *improving* those practices, and secondly, that it makes sense to talk about how logic *should be done* in isolation from its current aims and methods. Let us deal with these points in turn.

The practice-based approach does not preclude us from highlighting areas in which the aims or methods of the field can improve. Recognising the socially constructed features of a research field does not stop us from passing judgement upon their activities or achievements. However, it does require us before passing such judgement to understand the field's research goals and methods as they stand, and further assess their methods and achievements in light of these goals. After all, it is no good to criticise a hammer for failing to be a chisel.

Once we understand a field's current research goals, it is very much a live option to argue that the field's current methodological norms are unsuitable to realise these goals, or that they *have* at present failed to realise them. Further, we may even criticise the field's current research goals themselves, whether on the basis that they no longer serve the needs of our community, or because they are hopefully imprecise. These are, of course, just the kind of criticisms we find of previous schools of logical thought in Bacon, Frege, and more modern research programmes, such as the preservationists (Jennings & Schotch 2009). Yet, both of these normative conclusions presuppose a suitable understanding of the current state of the art, which is best achieved through a practice-based approach.

These points also lead us to addressing the second misconception, that we can have some direct access to how logic should be done, or what it aims should be, regardless of the current state of play. This attitude is indicative of falling foul of an *essentialist fallacy* not uncommon in philosophy, that there is some *true*, *correct*, or *proper* subject matter, methodology, or epistemology of a field of research regardless of the particular interests or goals of the practitioners in the field at a given time.

Falling foul of this fallacy does not require one to deny that changes *have* taken place in terms of the field's subject matter, methodology, or epistemology, but simply that these norms should be judged against a fixed standard of what the *correct* subject matter, methodology, or epistemology of the field is. Thus, the closer the field gets to this correct subject matter, methodology and epistemology, the better condition it is in. If it deviates from any of these standards, then it is erring in some sense.

Yet, there are no God-given set of research goals, subject matters, methodological or epistemological norms which define the field as *properly understood*. Rather, one's case must be based on particular concerns with the field's current practices. Perhaps the field's goals are no longer useful or precise enough given new technical

[11] A similar reply to this concern is found in Burgess (1990).

innovations, their techniques too time consuming for their resulting utility, or their results in contradiction with findings from other extremely successful fields of science. As is often the case with proposals about the *purpose* of a research field or its *proper aims*, the proof of the pudding will be in its eating: what benefits are accrued by embracing the research plan, and how much support it amasses among members of the field. It cannot be imposed *ex nihilo* based upon essentialist considerations through ruminations on the concept of LOGIC.

Thus, ironically, far from stopping us from providing interesting normative conclusions about the field of logic, the practice-based approach is actually the most efficient way of facilitating these normative conclusions.

3. *The practice-based approach focuses on the **field** of logic, but in the philosophy of logic we want to know about **logic itself**.*

The root of this concern, and its mistake, is the same as that above: that we can have some other, more direct, access to the nature of logic than through understanding the aims, methods and results of the field. Talk of *logic itself* suggests some essential subject matter of logic independently of the aims of the field, such that nature or some Platonic realm has designated these objects or subject matters as "logical". Yet, there is no such privileged ahistorical subject matter of logic, just as there is no ahistorical subject matter of biology, psychology, or physics. It would be hopeless to try to talk about the *subject matter* and *laws of physics themselves*, without reference to the subject matter of the field of physics and the laws it proposes, through the relevant activities of physicists.

The example of these other research areas also serves to highlight the dangers of attempting to fix the reference of *logic itself* on the basis of proclamations from founding figures. This proposal has been (somewhat implicitly) used in the debate over the *primary* or *canonical* purpose of logic and is ill-advised. Roy Cook (2010: 495), for instance, has suggested that we should consider the "codification of logical consequence in natural language" to be the primary purpose of logic on the basis of the views of founding figures of the field, such as Aristotle and Tarski. But Aristotle also suggested that science ought to aim at providing teleological explanations, and few scientists now take this activity seriously. Techniques change, purposes change, even if not everything is in flux.

It is an interesting question across the abstract and empirical sciences what historical connections exactly are sufficient for talking about the continuity of a particular research field, whether it be physics, mathematics, or logic. However, as with personal identity, it is clear that no one element is both fixed and substantive enough to constitute some essence of a research area. Appealing to the views of founding figures in an attempt to establish this essence is simply to fall foul of the *embryonic* fallacy—the presumption that an activity has the same aims and purposes as when it was initially developed—and would equally require us to admit that the primary purpose of astronomy is to provide planetary data for the higher art of astrological predictions.

However convenient it would be to have some direct access to the nature of "logic itself" by ruminating on the concept LOGIC or appealing to the words of founding figures, understanding the subject matter of logic through the activities of its practitioners is the best route we have. Again, at root, this is the rationale for the practice-based approach.[12]

[12] **Acknowledgments:** I am grateful to audiences at the European Network for the Philosophy of Logic, the University of Bergen, and the University of Copehagen for their feedback on earlier versions of this paper, particularly Leon Commandeur, Silvia De Toffoli, Filippo Ferrari, Ole Hjortland, Ulf Hlobil, Franci Mangraviti, and Gil Sagi. Research for this paper was supported by a PNRR grant, under the European Union's NextGenerationEU research and innovation programme, as well as the DFG (Deutsche Forschungsgemeinschaft) project no. 288923097, FOR 2495

References

Andersen, H. (2016), 'Collaboration, Interdisciplinarity, and the Epistemology of Contemporary Science', in *Studies in History and Philosophy of Science A* 56: 1–10.

Anderson, A. R. and Belnap, N. D. (1975), *Entailment: The Logic of Relevance and Necessity, Vol. I* (Princeton University Press).

Avigad, J. (2008), 'Computers in Mathematical Inquiry', in P, Mancosu (ed.), *The Philosophy of Mathematical Practice* (Oxford University Press), 302–16.

Ayer, A. J. (1936), *Language, Truth and Logic* (Dover).

Beall, Jc and Restall, G. (2006), *Logical Pluralism* (Clarendon Press).

Benzmüller, C., Parent, X. and van der Torre, L. (2020), 'Designing Normative Theories for Ethical and Legal Reasoning: LOGIKEY framework, methodology, and tool support', in *Artificial Intelligence* 287: 103348.

Boghossian, P. A. (1996), 'Analyticity Reconsidered', in *Noûs* 30: 360–91.

Bokulich, A. (2011), 'How Scientific Models Can Explain,' in *Synthese* 180: 33–45.

Bokulich, A. (2017), 'Models and Explanation,' in L. Magnani and T. Bertolotti (eds.), *Springer Handbook of Model-Based Science* (Springer), 103–18.

BonJour, L. (1998), *In Defense of Pure Reason* (Cambridge University Press).

Borges, J. L. (1999), 'On Exactitude in Science', in *Collected Fictions,* trans. by A Huxley (Penguin), 325.

Braillard, P. A. and Malaterre, C. (2015), 'Explanation in Biology: An introduction,' in P. A. Braillard and C. Malaterre (eds.), *Explanation in Biology* (Springer), 1–28.

Brigandt, I. (2013), 'Explanation in Biology: Reduction, pluralism, and explanatory aims', in *Science & Education* 22: 69–91.

Burgess, J. (1990), 'Epistemology & Nominalism', in A. D. Irvine (ed.), *Physicalism in Mathematics* (Kluwer Academic Publishers), 1–16.

Carter, J. (2019), 'Philosophy of Mathematical Practice: Motivations, themes and prospects', in *Philosophia Mathematica* 27: 1–32.

Cook, R. T. (2010), 'Let a Thousand Flowers Bloom: A tour of logical pluralism', in *Philosophy Compass* 5: 492–504.

Commandeur, L. (2022), 'Against Telic Monism in Logic', in *Synthese* 200. Online first: https://doi.org/10.1007/s11229-022-03497-1

Corfield, D. (2003), *Towards a Philosophy of Real Mathematics* (Cambridge University Press).

De Regt, H. W. (2017), *Understanding Scientific Understanding* (Oxford University Press).

De Toffoli, S. (2017), 'Chasing' the Diagram—The Use of Visualizations in Algebraic Reasoning, in *The Review of Symbolic Logic* 10: 158–86.

De Toffoli, S. (2020), 'Reconciling Rigor and Intuition', in *Erkenntnis* 86: 1783–1802.

Diogenes Laërtius (1925), *Lives of Eminent Philosophers (2 Volumes)*, trans. by. R. Hicks. Loeb Classical Library (Harvard University Press).

Dutilh Novaes, C. (2012), 'Towards A Practice-Based Philosophy of Logic: Formal languages as a case study', in *Philosophia Scientiæ* 16: 71–102.

Fisher, G. (2003), 'Explaining Explanation in Chemistry', in *Annals of the New York Academy of Sciences* 988: 16–21.

Gelfert, A. (2016), *How to Do Science with Models: A philosophical primer* (Springer).

Giaquinto, M. (2007), *Visual Thinking in Mathematics: An epistemological study* (Oxford University Press).

Giardino, V. (2017), 'Diagrammatic Reasoning in Mathematics', in L. Magnani and T. Bertolotti (eds.), *Springer Handbook of Model-Based Science*. *Springer Handbook of Model-Based Science* (Springer), 499–522.

Haack, S. (1976), 'The Justification of Deduction', in *Mind* 85: 112–9.

Haack, S. (1978), *Philosophy of Logics* (Cambridge University Press).

Hacking, I. (1983), *Representing and Intervening* (Cambridge University Press).

Hamami, Y. and Morris, R. L. (2020), 'Philosophy of Mathematical Practice: A primer for mathematics educators', in *ZDM Mathematics Education* 52: 1113–26.

Hansson, S. O. (2022), 'Logic of Belief Revision', in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*. Online: https://plato.stanford.edu/archives/spr2022/entries/logic-belief-revision/

Hempel, C. G. (1965), 'Aspects of Scientific Explanation,' in *Aspects of Scientific Explanation and Other Essays in the Philosophy of Science* (Free Press), 331–496.

Jennings, R. E. and Schotch, P. K. (1984), 'The Preservation of Coherence,' in *Studia Logica* 43: 89–106.

Jennings, R. E. and Schotch, P. K. (2009), 'Paraconsistency: Who needs it?' in P. Schotch, B. Brown, and R. Jennings (eds.), *On Preserving: Essays on preservationism and paraconsistent logic* (University of Toronto Press), 17-31.

Keshet, E. (2018), 'Dynamic Update Anaphora Logic: A simple analysis of complex anaphora', in *Journal of Semantics* 35: 263–303.

Krantz, S. G. (2011), *The Proof is in the Pudding: The changing nature of mathematical proof* (Springer).

Kuhn, T. (1962), *The Structure of Scientific Revolutions* (University of Chicago Press).

Lakatos, I. (1976), *Proofs and Refutations: The Logic and Mathematical Discovery*, J. Worrall and E. Zahar (eds.), (Cambridge University Press).

Larvor, B. (2012), ‘How to Think about Informal Proofs’, in *Synthese* 187: 715–30.

Leech, J. (2015), ‘Logic and the Laws of Thought’, in *Philosophers' Imprint* 15/12: 1–27.

Linnebo, Ø. (2003), ‘Plural Quantification Exposed’, in *Noûs* 37: 71–92.

Lu-Adler, H. (2018), *Kant and the Science of Logic: A historical and philosophical reconstruction* (Oxford University Press).

Machamer, P., Darden, L., and Craver, C. F. (2000), ‘Thinking about Mechanisms’ in *Philosophy of Science* 67: 1–25.

Martin, B. (2021a), ‘Anti-Exceptionalism about Logic and the Burden of Explanation’, in *Canadian Journal of Philosophy*, 51: 602-18.

Martin, B. (2021b), ‘Identifying Logic Evidence’, in *Synthese* 40: 9069–95.

Martin, B. (2021c), ‘Searching for Deep Disagreement in Logic: The Case of Dialetheism’, in *Topoi* 40: 1127-38.

Martin, B. (2022), ‘The Philosophy of Logical Practice’, in *Metaphilosophy* 53: 267-83.

Martin, B. (2024), ‘Reflective Equilibrium in Logic’, in *Synthese* 58. Online first: https://doi.org/10.1007/s11229-023-04480-0

Martin, B. and Hjortland, O. T. (2021), ‘Logical Predictivism’, in *Journal of Philosophical Logic* 50: 285–318.

Martin, B. and Hjortland, O. T. (2022), ‘Anti-Exceptionalism as Tradition Rejection’, in *Synthese*. Online first: 10.1007/s11229-022-03653-7

McGee, V. (1985), ‘A Counterexample to Modus Ponens’, in *Journal of Philosophy* 82: 462–71.

Mugnai, M. (2010), ‘Logic and Mathematics in the Seventeenth Century,’ in *History and Philosophy of Logic* 31: 297–314.

Payette, G. and Wyatt, N. (2018), ‘How Do Logics Explain?’ in *Australasian Journal of Philosophy* 96: 157–67.

Peregrin, J. and Svoboda, V. (2017), *Reflective Equilibrium and the Principles of Logical Analysis: Understanding the Laws of Logic* (Routledge).

Popper, K. (1959), *The Logic of Scientific Discovery* (Hutchinson).

Priest, G. (2006), *Doubt Truth to be a Liar* (Clarendon Press).

Priest, G. (2016), ‘Logical Disputes and the a Priori,’ in *Logique et Analyse* 59: 347–66.

Qiu, R.-Z. (1989), ‘Models of Explanation and Explanation in Medicine’, in *International Studies in the Philosophy of Science* 3: 199–212.

Resnik, M. D. (1999), ‘Against Logical Realism’, in *History and Philosophy of Logic* 20: 181–94.

Rota, G. C. (1997), ‘The Phenomenology of Mathematical Beauty’, in *Synthese* 111: 171–82.

Russell, G. (2015), 'The Justification of the Basic Laws of Logic', *Journal of Philosophical Logic* 44: 793–803.

Sher, G. (2016), *Epistemic Friction: An Essay on Knowledge, Truth and Logic* (Oxford University Press).

Sider, T. (2011), *Writing the Book of the World* (Clarendon Press).

Soler L, *et al*. (2014), 'Introduction', in L. Soler, S. Zwart, M. Lynch, and V. Israel-Jost (eds.), *Science After the Practice Turn in the Philosophy, History, and Social Studies of Science* (Routledge), 1–43.

Tajer, D. (2022), 'Anti-exceptionalism and Methodological Pluralism in Logic', in *Synthese* 200. Online first: https://doi.org/10.1007/s11229-022-03698-8

Tanswell, F. (2015), 'A Problem with the Dependence of Informal Proofs on Formal Proofs,' in *Philosophica Mathematica* 23: 295–310.

Tappenden, J. (2008), 'Mathematical Concepts: Fruitfulness and Naturalness,' in P. Mancosu (ed.), *The Philosophy of Mathematical Practice* (Oxford University Press), 276–301.

Warren, J. (2020), *Shadows of Syntax* (Oxford University Press).

Weisberg, M. (2013), *Simulation and Similarity: Using models to understand the world* (Oxford University Press).

Williamson, T. (2017), 'Semantic Paradoxes and Abductive Methodology', in B. Armour-Garb (ed.), *The Relevance of the Liar* (Oxford University Press), 325–46.

Woody, A. I. (2015), 'Re-orienting Discussions of Scientific Explanation: A functional perspective', in *Studies in History and Philosophy of Science* 52: 79–87.

Wooldridge, M. (2009), *An Introduction to MultiAgent Systems*, 2nd ed. (John Wiley & Sons).