PHILOSOPHY SOUTH

The truth-functional hypothesis does not imply the liar's paradox[1]

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ABSTRACT

The truth-functional hypothesis states that indicative conditional sentences and the material implication have the same truth conditions. Haze (2011) has rejected this hypothesis. He claims that a self-referential conditional, coupled with a plausible assumption about its truth-values and the assumption that the truth-functional hypothesis is true, lead to a liar's paradox. Given that neither the self-referential conditional nor the assumption about its truth-values are problematic, the culprit of the paradox must be the truth-functional hypothesis. Therefore, we should reject it. In this paper I argue that, contrary to what Haze thinks, the truth-functional hypothesis is not to blame. In fact, no liar's paradox emerges when the truth-functional hypothesis is true; it emerges only if it is false.

Keywords: material implication, truth-functional hypothesis, liar's paradox.

The truth-functional hypothesis states that indicative conditional sentences and the material implication have the same truth conditions. Haze (2011) has rejected this hypothesis on the basis that it gives rise to a liar's paradox. In this short paper, I argue that, contrary to what Haze claims, no liar's paradox emerges from the truth-functional hypothesis. I reconstruct Haze's reasoning in the first section and argue against it in the second.

The material implication and the liar's paradox

Haze (2011) parts from the following self-referential conditional:

If (1) is true, (1) is false.

Call the antecedent of (1) "A", and its consequent "C". Haze thinks it is plausible to assume that (1) is neither true nor false. He then argues that the following supposition is intuitively true:

(S) Under the assumption that (1) is neither true nor false, A and C are both false.

(S) is based on the following intuitive reasoning: A states that (1) is true, and C states that (1) is false. Both are inconsistent with the assumption that (1) is neither true nor false. Thus, if the assumption is correct, A and C must be false.

Now, the problem is that if the truth-functional hypothesis—hereafter "TFH"—is correct, the reasoning above cannot hold. If TFH is correct and A and C are both false, then (1) must be true, since any material implication is true when both antecedent and consequent are false.

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However, if (1) is true, A must be true, since it states that (1) is true, and C must be false, since it states that (1) is false. But under TFH, (1) must be false, for it has a true antecedent and a false consequent. Nevertheless, if (1) is false, A must be false, since it states that (1) is true. But in this case, (1) must have a false antecedent and a true consequent, and therefore it must be true according to the truth-functional hypothesis. However, if (1) is true….

Thus, according to Haze, if we accept the reasoning supporting (S) together with TFH, we must conclude that (1) generates a liar's paradox. Both (S) and the reason that supports it seem to be correct and do not, by themselves, commit us to the liar's paradox, since we could have discarded (1) as neither true nor false, and still accept that A and C had truth-values. Everything suggests that TFH is the culprit. Thus we must, Haze concludes, discard it.

Down (and out of) the Liar's hole

A well-known feature of the material implication is that when the antecedent and the consequent of a conditional have truth-values, the conditional has truth-values: if the antecedent is true and the consequent is false, the conditional is false. The material implication is true in all remaining combinations of truth-values. According to TFH, these features of the material implication extend to indicative conditionals in general.

Here is a problem with Haze's argument: his argument only works under the assumption that TFH is false. Given the acceptance of TFH, (1) has the same truth conditions as the material implication. However, as its truth table attests, material implications are functions that always have truth-values. Thus, if (1) does not have truth-values, it cannot be a material implication by definition.

A *reductio* helps to support the argument above. From the assumption that (1) is a material implication, it follows that (1) is not a material implication: If C is true, then (1) is false, but since (1) is a material implication, it can only be false when A is true and C is false. However, we have already assumed that C is true. Therefore, (1) cannot be false. On the other hand, if A is true, then (1) is true. But in this case, A is true and C is false, and (1) cannot be true when A is true and C is false. Thus, (1) cannot be true either. Therefore, (1) can neither be true nor false. Thus, (1) is not a material implication.

Yet another reason against considering (1) a material implication is this: (1) can only have truth-values if A and C have truth-values, and A and C can only have truth-values if (1) has truth-values. In the event that (1) is neither true nor false, A and C cannot be either true or false, since had they had truth-values, (1) would also have had truth-values. Which suggests that it is not possible that the acceptance of (1) together with TFH and (S) lead straight to a liar's paradox: a liar's paradox situation would emerge *only if* A and C had truth-values, but that is not possible if (1) is a material implication. Since Haze failed to support the claim that (1) is a material implication, his claim that a liar's paradox emerges is unfounded.

Someone could object that my argument assumes that the material implication is, by definition, a two-valued function.[3] However, it is plausible to think that the material implication occurs in truth-functional logics with more than two truth-values, and hence isn't, by definition, a two-valued function.[4] This objection, however, misses the target. The argument works even under the assumption that the material implication isn't a two-valued function. The only assumption my argument needs is that the truth-value of a compound truth-function results from the truth-values of its components. Haze's argument does not work because it oscillates between the thesis that the truth-value of a compound sentence is a function of the truth-value of its sub-sentences[5] and the thesis that (1) has no truth-value. In order to make a compelling case against the material implication, you cannot have both.

Haze would need to explain why the truth-function alone generates the liar's paradox. The only way to do this is by attributing truth-values to A and C. But since intuitively A and C are neither true nor false, (1) won't have truth-values either. The temptation is to go back and attribute truth-values to A and C based on this information, but that would be incoherent because the information depends on A and C not having truth-values in the first place. The paradox is stopped in its tracks.

References

HAZE, T. 2011. A Liar Paradox of Material Implication. Sprachlogik. A philosophy blog with a focus on logic and language. Available at: http://sprachlogik.blogspot.com.br/2011/02/liar-paradox-of-material-implication.html ___ Accessed on: July 10, 2011.

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[3] Thanks to an anonymous reviewer for calling my attention to this point.

[4] Consider, for example, Łukasiewicz's three-valued logic (L3) where 'indeterminate' is an additional truth-value. The only circumstances where the connective that represents the conditional is false is when the antecedent is true and the consequence is false. In the other circumstances in which the truth-values are only "true" and "false", the conditional is true. The conditional is indeterminate in all remaining circumstances involving the truth-value "indeterminate". In other words, even in this three-valued logic, the connective preserves the truth-conditions of the material implication.

[5] The argument need not assume that sentences are truth-bearers. One could, without any loss, substitute 'proposition' for 'sentence'.