The Composition of Forces

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Abstract

This paper defends a realist account of the composition of Newtonian forces, dubbed 'residualism'. According to residualism, the resultant force acting on a body is identical to the component forces acting on it that do not prevent each other from bringing about its acceleration. Several reasons to favor residualism over alternative accounts of the composition of forces are advanced. (i) Residualism reconciles realism about component forces with realism about resultant forces while avoiding any threat of causal overdetermination. (ii) Residualism provides a systematic semantics for the term 'force' within Newtonian mechanics. (iii) Residualism allows us to precisely apportion the causal responsibility of each component force in the ensuing acceleration. (iv) Residualism handles special cases such as null forces, single forces, and antagonistic forces in a natural way. (v) Residualism provides a neat picture of the causal powers of forces: each force essentially has two causal powers—the power to bring about accelerations (sometimes together with other co-directionnal forces) and the power to prevent other forces from doing so—exactly one of which is manifested at a time. (vi) Residualism avoids commitment to unobservable effects of forces: forces cause either stresses (tensile or compressive) or accelerations.

The way Newtonian mechanics handles the composition of several forces acting on a body—i.e. by vector addition as illustrated by the parallelogram of forces—is so straightforward and powerful that many attempts have been made to apply it to other areas. The composition of Newtonian forces has been used as a paradigm to shed light on the composition of colors,[1] on the composition of desires, motives, and other 'psychical forces',[2] on the composition of instrumental values or 'utilities',[3] on the composition of 'social forces',[4] on the composition of *prima facie* duties,[5] on the composition of the different partial grounds of a truth into a total ground,[6] on the composition evolutionary forces,[7] or on the composition of causal powers.[8]

Yet, suggestive as the parallelogram of forces has been, our understanding of its metaphysical underpinnings remains quite poor. In fact, most philosophers in the field believe that the composition of forces is a mere fiction—for its truth would entail that both component forces *and* resultant forces are real, a view that even realists about forces widely reject. As a result, Cartwright's ([1980]) oft-quoted claim that '*We* add forces (or the numbers that represent forces) when we do calculations. Nature does not 'add' forces.' has remained virtually unchallenged (contrary to the other claims she makes nearby).

I shall argue on the contrary that nature does add forces and, correspondingly, that both component forces and resultant forces are real. The picture I propose, residualism, equates resultant forces with those component forces that do not counteract each other. The paper is organized as follows.

Section 1 introduces the issue of the composition of forces and presents two main objections to the view that both component and resultant forces are real: *causal* overdetermination and overcounting. In answer to these objections, most realists about forces endorse frugal realism: the view that either component forces or resultant ones are real, but not both.

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[1] Grassmann ([1854]), furthered by Krantz ([1973] [1975]).
[2] Wundt ([1897], p. 186); Sidgwick ([1981], p. 112); Lewin [1938] ; see McLaughlin [1987] on Freud's approach to mental forces and resistance.
[3] Jevons ([1967], Ch. IV, p. 133), Fisher ([2006], Ch. 3).
[4] Pareto ([1935], vol. 4, § 2087ff.); Lewin [1951], see Gibson [1958] for criticism.
[5] Ross ([2002], pp. 28-29).
[6] Bolzano ([1929], §206, §211).
[7] Sober [1993].
[8] Mumford and Anjum ([2011b], Ch. 2).

Sections 2 and 3 argue against frugal realism. Section 2 argues that frugal realists are committed to an unsystematic semantics of the term ‘force’. Section 3 argues that frugal realists fail to apportion causal responsibility for accelerations among component forces.

Sections 4 and 5 assess two realist accounts of the composition of forces. Instead of choosing between component and resultant forces, both deal with overdetermination and overcounting worries by arguing that component and resultant forces are partly identical.

The first is primitivism about the composition of forces (section 4), which basically reads off the metaphysics of the composition of forces from its parallelogram representation. Primitivism is straightforward and ensures semantics systematicity. But it fails to apportion causal responsibility; and it is committed to there being null resultant forces, which even the friends of zero-value physical quantities have reasons to reject.

The second realist account of the composition of forces is residualism (section 5). It is argued that residualism accommodates all of the previous difficulties: residualism is immune to the overdetermination and overcounting threats; it provides a systematic semantics for the term ‘force’; it precisely apportions the causal responsibility of forces; and it is not committed to the reality of null forces. Moreover, residualism provides a clean picture of the causal powers of forces and of their manifestations, avoiding any commitment to unobservable effects.

1 Introduction: Component Forces or Resultant ones?

1.1 Component forces vs. resultant forces

An apple is falling from the tree. According to Newton’s second law of motion, its acceleration is determined by the forces exerted upon it. Assume that in that case only two forces act upon the apple: the gravitation exerted by the earth on the apple (represented by vector $\vec{G}$) and the resistance or friction exerted on it by the air (represented by vector $\vec{F}$). Following Newton’s second law, the vectorial sum of these two forces allows us to determine the acceleration of the body, given its mass:

$$\vec{F} + \vec{G} = \vec{R} = \mathrm{m}\vec{a}$$

$\vec{R}$ is the vectorial sum of $\vec{F}$ and $\vec{G}$. '$\vec{F}$' and '$\vec{G}$' stand for the component forces and '$\vec{R}$' stands for the resultant force. One might worry that Newton's second law is not explicitly about resultant forces:

> A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed. (Newton [1999])

Its straightforward mathematical translation appears to be:

$$\vec{F} = m\vec{a}$$

However, on the basis of Newton's corollaries 1 and 2 on the composition of forces and of the parallelogram law, it is clear that the 'motive force', $\vec{F}$ is meant to be a resultant force, the result of the vectorial sum of all the component forces. I shall therefore assume, quite standardly, that the second law, taken to the letter, bears on resultant forces,[9] and that it can be expressed mathematically as follows:

$$\sum \vec{F_i} = m\vec{a}$$

The vectorial summation of forces is typically represented thanks to the parallelogram of forces: the two component forces under consideration are taken to be adjacent sides of a parallelogram whose diagonal is the resultant force (Figure 1).

[9] The view that the second law also holds for each component force, separately, shall be discussed in 2.2.

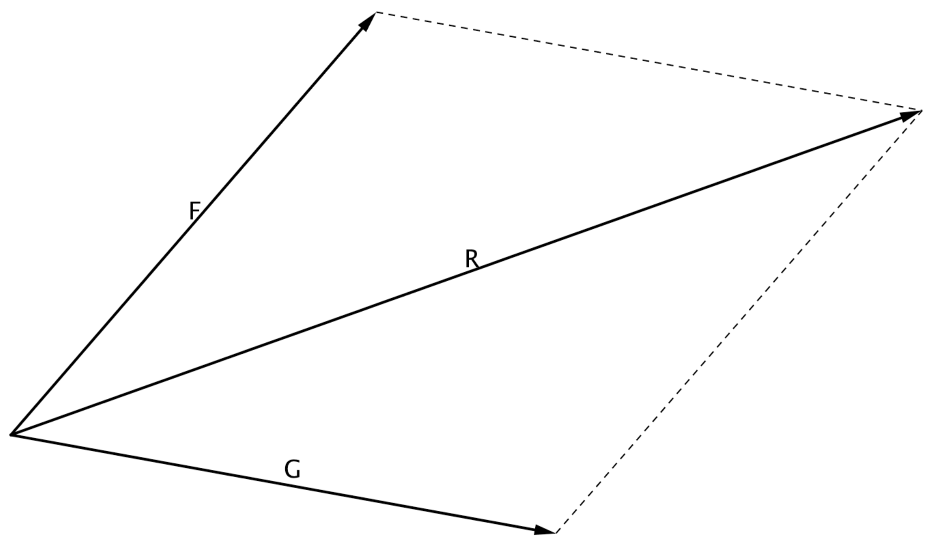

Figure 1: Parallelogram of forces

Discussions surrounding the parallelogram of forces have a gripping history (see Benvenuto [1985], [1991]; Lange, [2009], [2011]). They were mostly targeted at proving the parallelogram of forces. The question to be tackled here is slightly different, though connected. It bears on the metaphysics underlying the parallelogram: rather than trying to prove the parallelogram of forces—which shall be taken for granted—we shall ask: what, in nature, corresponds to this vectorial story? Are there real forces in nature corresponding to $\vec{F}$, $\vec{G}$, and $\vec{R}$? Which force-vectors stand for real forces in the parallelogram above? Here are four possible answers:

anti-realism: Neither '$\vec{F}$', '$\vec{G}$', nor '$\vec{R}$' refer to real forces. More generally, there are no real forces.

frugal resultant realism: '$\vec{R}$' represents some real force, but '$\vec{F}$' and '$\vec{G}$' don't. There are real resultant forces, but no real component forces (Cartwright [1983, pp. 54–73]; Wilson [2009]). Only the diagonal of the parallelogram stands for some force.

frugal component realism: '$\vec{F}$' and '$\vec{G}$' stand for real forces, but not '$\vec{R}$'. There are real component forces, but no real resultant forces. (Creary [1981]; Bigelow et al. [1988, n. 10]; Johansson [2004, pp. 167–8]; Molnar [2003, pp. 194–198]; Schrenk [2011]). Only the adjacent sides of the parallelogram represent some forces.

generous realism: '$\vec{F}$', '$\vec{G}$' and '$\vec{R}$' stand for real forces. Both component and resultant forces are real. One version of this view—residualism—is to be defended here; another version—primitivism about vectorial composition—is to be rejected.[10]

I shall not try to rebut anti-realism about all forces here (see Wilson, [2007]; Massin [2009] for recent attempts). This paper aims to reject both kinds of frugal realism so as to defend a version of generous realism. If there are forces, then there are both component and resultant forces.

Some caveats need to be addressed before starting. First, let me make clear what is meant here by 'real forces'. By 'forces', I mean properties or relations that have at least the following essential features:

> (a) Forces are properties of (or better, relations between) bodies that are distinct from the purely spatio-temporal properties and relations of these bodies. Forces do not reduce to the positions or accelerations of bodies, for instance.
>
> (b) Forces are vectorial properties, that is, they have both a magnitude and a direction.
>
> (c) Forces have the power to bring about the acceleration of the bodies upon which they act (the condition under which such a power manifests shall be spelled out later on).[11]

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[10] These four options are not exhaustive. 'Only $\vec{F}$ is real', for instance, is another logical possibility, but it would sounds *ad hoc* to grant the reality of $\vec{F}$ but not that of $\vec{G}$. Yet another possibility is to endorse one or the other options above depending on the case under consideration. Thus Bigelow and Pargetter ([1990]) endorse a kind of frugal realism according to which sometimes the resultant force which is real, sometimes the component forces, but never are both real together.

[11] Why do I say that forces have accelerating powers, instead of saying that they *are* accelerating powers? There are two reasons for this. First, as I shall argue below, there is a second kind of power essential to forces: that of preventing other forces from bringing about accelerations. But still, why not say that forces are pairs of causal powers, instead of saying that they have two causal powers?

Then, by ‘real’ force, I shall mean a force that mind-independently and irreducibly exists. A force is mind-independent if and only if it exists independently of any representation of it. A force is irreducible if and only if it cannot be reduced to non-forces.

Second, I shall only consider forces as they appear in Newtonian Mechanics (henceforth, NM). To the extent that better physical theories have superseded NM, what follows might be of only historical relevance. There are, however, two reasons to think that it might be of broader interest. First, as argued by Wilson [2007], NM might be considered a special science whose ontological commitments matter for contemporary metaphysics. Second, even if NM is of only historical interest, understanding the composition of forces in its context might still help us understand how non-Newtonian forces compose.

This leads to the second reason not to identify forces with (pairs of) causal powers. I assume that the two following claims are plausible:

i. Forces cause accelerations (if they exist, and under some conditions). Gravity causes bodies to fall.
ii. ‘x causes y’ conceptually entails ‘x has the power to cause y’. *Having causal powers* is a property that accrues to (possible) causes.

It follows from these that forces have accelerating powers. A first way to maintain that forces are causal powers would be to reject (i), by saying that the causes of accelerations are not the forces, but their bearers (typically: bodies, charges, masses). Such a position, which would put agent causation at its core (bodies being causes of accelerations) might well be worth exploring further, but I shall here stick with the more standard approach: since they causes accelerations, forces have powers, they are not powers.
A second way to maintain that forces are causal powers would be claim that we do not have to make a choice between having and being causal powers. Force could somehow have the causal powers they are. This move would be akin to the move made by bundle theorists of substances: on such approaches, substances have the properties they consist in.
Note, finally, that even if forces are not causal powers, they might still properly be said to be powerful properties (or better still, powerful relations), in contrast to being powers. The distinction between powers and powerful properties is easily overlooked but crucial if one is to mark the distinction between properties that are powers and properties that have powers. While powers are properties of possible causes; powerful properties are themselves possible causes. Powerful properties have causal powers, but on the other hand, there are strong reasons to think that causal powers lack causal powers. For instance, if a force has the causal power to cause some acceleration, what could be the causal power of that causal power? The power to redundantly cause the acceleration? Causal powers, as the relation of causation, are typically powerless (Massin [2009, §5]). Oddie ([1982]) and Armstrong ([1997 pp. 41–42]) discuss related worries with respect to the Eleatic principle. Summing up: forces are not causal powers but powerful properties, because they have causal powers.

Finally, one might object that the question addressed here—namely, which forces are real, component or resultant ones?—is a non-starter whose answer is, trivially, 'both'. For the concepts of component and resultant forces are interdependent: one cannot be grasped without the other. Hence, one might argue, frugal realism makes no sense: if only component forces were real, they would not be component in any sense; and if only resultant forces were real, they would not be *resultant* in any sense.

This straightforward argument for generous realism is, however, inconclusive. Frugal realists will sensibly grant that the concepts of component and resultant forces are interdependent, but they will insist that this interdependency occurs only at the representational level (the level of vector calculus) and has no counterpart in reality. It actually makes sense to ask which of the representational force-vectors do refer to real forces: $\vec{F}$ and $\vec{G}$ or $\vec{R}$? The claim that only component forces are real can then be read as a claim that only forces represented by component-vectors are real (and likewise for the claim that only resultant forces are real).

1.2 Two problems for generous realism

Most realists about forces are frugal realists. Though frugal resultant realists and frugal component realists disagree on which forces are real, they agree that resultant and component forces cannot be both real. The main worry for generous realism, frugal realists of both camps agree, stems from causal overdetermination. Developing an initial suggestion from Creary ([1981]), Wilson ([2009]) proposes a *reductio* of generous realism along these lines:

- P1 Resultant and component forces are real.
- P2 Resultant and component forces are wholly distinct.
- P3 The resultant force acting on a body is sufficient to cause its acceleration.
- P4 The component forces acting on it are jointly sufficient to cause its acceleration.
- C Resultant and component forces causally overdetermine their effects.

Generous realism about forces leads to regular causal overdetermination of the accelerations of bodies. If regular overdetermination is to be avoided, generous realism has to be dropped.

A less common objection to generous realism goes as follows (Hüttemann [2004, p. 105], to the best of my knowledge, is the only one to have pressed it). Newton's second law asks us to sum all of the forces that act on a body in order to determine its kinematic behavior. If the resultant force is one of the forces acting on the body on top of the component ones, it should be included in the summation. That is, the cause of the acceleration would be:

$$\vec{F} + \vec{G} + \vec{R}$$

which is clearly absurd: not only would the predicted acceleration have twice the intensity of the real acceleration, we would be also caught in a bad regress, since $\vec{S}$, the resultant of the sum of $\vec{F}$, $\vec{G}$ and $\vec{R}$ is also a real and distinct force exerted on the apple. *S* should therefore be added in turn, etc. We then get a second *reductio* of generous realism about forces:

P1	Resultant and component forces are real.
P2	Resultant and component forces are wholly distinct.
P3	The acceleration of a body is determined by the sum of all the wholly distinct forces acting upon it (Newton's second law).
C	The acceleration of a body is determined by the sum of the component forces and of their resultant.

These two problems for generous realism will have to be dealt with (namely by rejecting P2 instead of P1). I propose, however, that we bracket them for a while, so as to stress first the costs that frugal realism itself incurs. Frugal realism about forces faces two worries. The first stems from semantic unsystematicity, and the second from attribution of causal responsibility.

2 Against Frugality (1): Semantic Unsystematicity

A first objection to frugal realism runs as follows. Both component forces and resultant forces are mentioned in the laws of NM. Frugal realists have to say that some but not all occurrences of the term 'forces' stand for real forces. The three main kinds of laws at stake here are (i) the partial laws, such as the law of gravitation, which describe the different forces that act on a given body; (ii) the second law, which tells us that the resultant force acting on a body brings about an acceleration of that body in inverse proportion to its mass; and (iii) *the* parallelogram or composition law, telling us how component forces compose to produce resultant forces. Frugal component realists are typically happy with a straightforward reading of the partial laws, but urge that the second law should not be taken to the letter, for there are no resultant forces. Frugal resultant realists, for their part, accept that the term 'force' stands for forces in the second law, but deny that this is typically the case with partial laws. And all frugal realists are partly unhappy with the parallelogram law, since it mentions component and resultant forces.

The objection pressed here is that frugal realism leads to an unsystematic semantics of the term 'force' in NM. The objection is originally due to Wilson ([2009]) who presses it against frugal component realists. I fully concur, but will argue that her own frugal resultant realism is in the end also vulnerable to the same objection. NM, taken at face value, quantifies both over component and resultant forces. The objection from semantic unsystematicity asks: why should the term 'force' sometimes refer to real forces, and other times not? The anti-realist and the generous realist give consistent answers: the first says that no occurrence of the term 'force' ever refers to forces; the second says that all do. The frugal realist, on the other hand, proceeds willy-nilly: depending on the law at stake, he defends a literal or revisionary reading. Such a semantic unsystematicity culminates in the interpretation given to the parallelogram law: frugal realists have here to cherry-pick, within that same law, the occurrences of the term 'force' that stand for real forces. Let us see how the objection applies to each kind of frugal realism in turn.

2.1 The semantic problem for frugal resultant realism

The two forces acting on the falling apple—gravitation and air friction—have distinct *relata*: the air friction relates the air to the apple, while the gravitation relates the earth to the apple. Each receives a different nomic explanation: gravitation is described by Newton's law of gravitation, while air resistance or drag is a complex force, which fluid dynamics describes as being grounded in skin friction and form drag. These two forces not only have distinct *relata* and fall under distinct laws, they also have different properties. For instance, gravitation is a volumic force that acts at each inner point of the apple, while air resistance only acts on its skin. Taken to the letter, NM appeals to different component forces and types thereof, described by different partial laws. The partial laws, as far as forces are concerned, are laws such as Newton's gravitation's law, the drag equation of fluids dynamics, or Coulomb's law, which characterize forces of a specific kind. Unless the body is acted on by one force only (a special case to which we shall return later), partial laws bear on component forces.

Hence, partial laws constitute the main problem for frugal resultant realists. Those who reject the reality of component forces have to give some alternative story about how to understand these laws. Two main strategies are available, which mirror the difference between fictionalism and ersatzism about possible worlds.

The first—fictionalism—is to deny that partial laws state the facts, which amounts to rejecting the facticity of such laws, a position endorsed by the early Cartwright ([1983], pp. 54–73) about partial laws. Partial laws are then given an anti-realist reading: they are explanatory, but deprived of any counterpart in reality. One problem is that such an option sounds *ad hoc* if other laws are read realistically: why should some laws be given a realist, facticity-reading, and some others not? Why should the term 'force' be taken to stand for real forces when it occurs in the second law, but as a mere notational device when it appears in partial law? Such a semantic treatment of the term 'force' is erratic.

The second strategy—ersatzism—is to maintain the facticity of partial laws, but to argue that properly understood such laws do not bear on component forces but on some component forces' *surrogates*. Wilson ([2009]) proposes two ways of maintaining the facticity of partial laws while avoiding commitment to component forces. The first claims that partial laws bear on resultant forces; the second that partial laws bear on some specific kind of dispositions, distinct from forces (the varieties of frugal resultant realism are recapped in Figure 2).

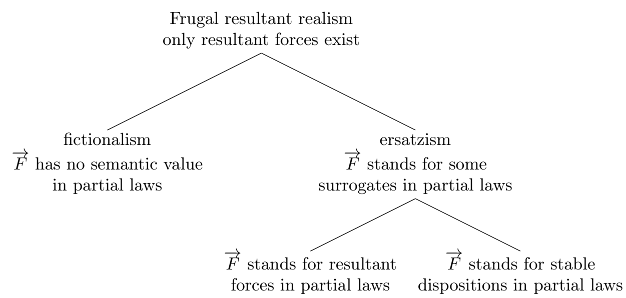

Figure 2: Varieties of frugal resultant realism

2.1.1 Do partial laws bear on resultant forces?

According to Wilson's first proposal, the law of gravitation does state the facts, but it is in the end about resultant forces. How can this be so, given that in the case of the falling apple, the law apparently concerns only one of the two forces acting on the apple? Wilson's ([2009], §4.1) idea is to deny that the law holds in such cases: the law holds only in isolated circumstances, in contrast to conjoined ones. This avoids any commitment to component forces, for in isolated circumstances, only one force acts on the body under consideration.

The problem with this proposal is that it amounts to restricting the scope of a physical law on philosophical grounds. Coulomb's law, or the law of gravitation, do not initially demand such restrictions. The latter, for instance, does not state that there is a gravitational force between two masses m_1 and m_2 at a distance r, namely: $F_g = \frac{Gm_1m_2}{r^2}$ *provided no other forces act on* m_1 *and* m_2. Such a *proviso*, absent from Newton's original law, should only be introduced as a last resort.

2.1.2 Do partial laws bear on dispositions distinct from forces?

But Wilson has an alternative proposal to save the facticity of partial laws, which she calls 'the disposition-based account of partial laws' (Wilson [2009], §4.2). The idea is that laws such as Coulomb's law bear on a kind of stable dispositions that manifest themselves under the form of electrostatic forces in isolated circumstances, while they have some other of kind of manifestation (not forces) in conjoined circumstances.

A first objection is that this proposal is a rather intricate interpretation of an apparently simple law. It introduces a kind of disposition that does not explicitly appear anywhere the original law and whose nature and manifestation remain obscure. If the choice is between (i) component forces or (ii) a type of disposition that manifests itself as a force in isolated circumstances and in another way (yet to be specified) in conjoined circumstances, the former is to be preferred, *ceteris paribus*. First, because their nature is better known: they satisfy the standard characterization of force (typically, vectorial properties having the power to bring about accelerations of the body they act upon). Second, because Coulomb's law, taken to the letter, is about a kind of force, not about a kind of disposition whose manifestation is a kind of force in isolated circumstances and something else in conjoined circumstances. Some strong reason is needed to introduce such unknown dispositions and to depart so radically from the literal reading of the law.

Second, it remains in the end unclear how Wilson's second proposal helps retrieve semantic systematicity. True, in all force-laws of NM, and in all circumstances, the term 'force' now stands for some real entities. But it no longer *means* the same across laws: in the second law, and in partial laws in isolated circumstances, 'force' means (resultant) force. But in conjoined circumstances, 'force' means something else: the non-force manifestation of the underlying disposition. True, the semantics of the term 'force' is now systematic to the extent that the term, within NM, always refers to something. But its semantics has become intensionally unsystematic: the sense of the term now shifts across contexts. All occurrences of 'force' in NM have a semantic value, but these semantic values are of strongly heterogeneous kinds.

Finally, one potential worry for these two proposals offered by Wilson is that both appeal to the distinction between 'isolated' and 'conjoined' circumstances. In order for such an appeal not to be self-defeating, one needs a definition of 'conjoined circumstances' that does not mention anything like 'circumstances in which several forces act'. This might prove tricky.

I believe Wilson could agree with the problems just raised for each of her two proposals: she might grant that all things being equal, it would be preferable to read partial laws, straightforwardly, as holding in all circumstances and as bearing on forces in all circumstances. But she will urge that all things are not equal: causal overdetermination is our killjoy. Because the cost of overdetermination is so high, we should accept non-straightforward readings of partial laws. However, as I shall argue, we can have both component and resultant forces without overdetermination. But let us see first why semantic unsystematicity is no less of a problem for frugal component realists.

2.2 The semantic problem for frugal component realists

Back to the falling apple. In order to determine its acceleration we need to vectorially add the two forces that act upon it (gravitation and air friction) so as to put the result of this summation on the left-hand side of Newton's second law.

The objection to frugal component realism on the basis of the second law is analogous to the objection to frugal resultant realism raised above on the basis of partial laws. In the same way that anti-realists about component forces, faced with partial laws, are led to semantic convolutions, anti-realists about resultant forces have to give non-straightforward readings of the second law, since it is *prima facie* about resultant forces. As with frugal resultants realism, there are two main ways of providing such an alternative story: denying the facticity of the Second law (fictionalism); or maintaining its facticity, but interpreting it as bearing on something other than resultant forces (erzatsism).

Of course, the fictionalist option readily leads to semantic unsystematicity: 'force' is claimed to have a semantic value when it occurs in partial laws, but to lack any semantic value when it occurs within the second law. Besides, since the second law literally bears on resultant forces, the fictionalist approach leaves the component forces without any connection to the accelerations of the body upon which they act. The resulting picture is hardly appealing: we get, on the one hand, component forces deprived of any known causal powers; and on other hand, accelerations deprived of any known causes, since there are no resultant forces, nor surrogates for them (acceleration can at best be predicted thanks to the second law).

So the best option for the frugal component realist is to argue that the second law does not bear on resultant forces but on something else. Two solutions have been proposed. According to the first, the second law bears on pluralities or sets of component forces. According to the second, the second law bears on individual component forces. But both fail, I shall now argue (the varieties of frugal component realism are recapped in Figure 3).

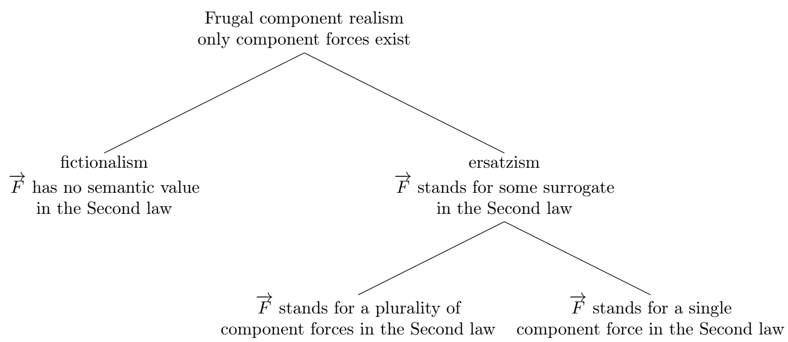

Figure 3: Varieties of frugal component realism

2.2.1 Does the Second law bear on sets of component forces?

Creary ([1981], p. 152) argues that *R* does not stand for a resultant force (which is 'merely a mathematical fiction'), but rather for the 'set' or plurality of all component forces acting on the body. Although component forces 'act together' so as to yield the acceleration of the body, the crucial point is that, for him (as for Cartwright) component forces do not combine vectorially with each other in nature (which would commit him to the reality of resultant forces).[12]

[12] Some of Creary's claims contradict this: he describes the vectorial mode of composition of component forces as 'simple and satisfying' (p. 151) and later on replaces 'sets' of of causes by causes standing in relations of '*reinforcement, interference*, and *predomination*' (p. 153). There are, I believe, two opposite strands in Creary's paper: the first one denies the reality of resultant forces, considers the vectorial composition story as a fiction, and maintains that all we have are sets of component forces : forces combined in a boolean way which 'act together'. The second strand of Creary's paper takes vectorial composition, or at least some non-boolean composition of forces (predomination, interference), with ontological seriousness and places the the composition of causes at the center of the stage. I think that these two lines of argument are incompatible (one cannot accept that component forces combine vectorially and yet deny the reality of resultant forces). Although I here equate Creary's position with the first one—frugal component realism—, I wish to

As pointed out by Wilson ([2009]), Creary's proposal is semantically unsystematic: on the face of it, the second law bears on one force, the resultant force (which is, by definition, unique). It does not bear on a plurality of forces. Why read the term 'force' as bearing on pluralities of forces when it occurs in the second law (which Creary calls 'law of causal action'), but as bearing on individual forces when it occurs in partial laws (which Creary calls 'laws of causal influence')?

This ersatzist reading of the second law also raises another worry. Suppose that lots of different forces act on a body. How is it that from such a chaos of component forces such a unitary acceleration ensues? True, we have the parallelogram law. But Creary's approach, denying real vectorial composition, entails that the parallelogram law is only an epistemological trick used to predict the direction and magnitude of the acceleration. Such an inference ticket is, ontologically speaking, utterly ungrounded. We are left with no metaphysical answer as to why a given unitary acceleration follows from a chaotic swarm of components forces. Since forces do not compose with each other vectorially prior to their bringing about the acceleration, the fact that they bring about the acceleration they do has to be metaphysically brute.

Of course, every explanation must come to an end: thus it could be that the composition of causes in NM is akin to the composition of causes in chemistry (*pace* Mill [1856], Bk III, Chap. IV, §2), or to the laws of statistical mechanics (that heat nomically depends on the mechanical kinetic energy of the constituent particles is, arguably, not to be explained further). But what makes us suspicious about this is precisely the availability of an alternative story in the case of forces: component forces first vectorially combine with each other so as to yield a single, real resultant force. Then, that single resultant force brings about a single acceleration, of the same direction and of proportionate magnitude. This picture—bracketing overdetermination and overcounting issues—is more natural and enlightening than the view that multiple component forces together bring about, without further ado, a single acceleration, usually of an entirely new direction.

2.2.2 Does the second law bear on individual component forces?

Another option for the frugal realist about component forces willing to save the facticity of the second law is to subscribe to the so-called 'dynamical account' of

stress that I largely agree with his second strand of thought (residualism, as we shall see, also gives a central role to the relation of interference between component forces).

the composition of forces. According to this account, each component forces brings about a component acceleration, which then composes with the other component accelerations to yield the resultant acceleration of the body. The dynamical account of the parallelogram of forces is to be to be contrasted with the so-called 'statical' account, to be defended here, according to which component forces compose with each other prior to, and independently of, the ensuing acceleration.[13] The dynamical account has many advantages. It allows the frugal realist about component forces to bypass the resultant forces while still giving an explanation of the direction and intensity of the final acceleration (an explanation that Creary's solution fails to provide, as we have seen). It gets rid of the problem of the composition of forces by passing the buck of vectorial composition to the accelerations they cause (component forces do not vectorially compose with each other, only the component accelerations that they each bring about do). And, last but not least, the dynamical approach gives a systematic semantic for the term 'force' across NM: all occurrences of the term, even in the second law, refer to component forces: the 'motive force' of the original formulation of the second law is read as referring to component forces, rather than to resultant ones.

Such a semantic systematicity, however, is bought at a high price: the introduction of component accelerations.[14] Given that a single body cannot simultaneously accelerate in different directions, nor at different rates, component accelerations have to be equated with potential accelerations, non-realized accelerations, or tendencies to accelerate (Armstrong [1988], p. 311; Johansson [2004], pp. 163 ff., Molnar [2003], pp. 194 ff.; Mumford [2009]). Such potential accelerations are problematic for two reasons at least.

First, component accelerations are unobservable entities, postulated for purely theoretical purposes. It should be stressed that they are more elusive than their contenders. Actual accelerations can be directly perceived and measured (by visual perception and accelerometers) and so, arguably, do component forces (by pressure

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[13] The dynamical account has a long history (see Benvenuto [1991], Ch. 4; and Lange [2009]). According to this account, the composition of forces obeys Mill ([1856], Bk III, Ch. IV)'s 'Principle of the Composition of Causes'—which, as Smith [2010] rightly points out, is in effect a principle of the composition of effects.

[14] McKitrick [2010] raises largely converging worries against the view that the manifestations of dispositions are contributions to effects (the view that component forces tend to bring about component accelerations being one prototypical instance of that view). Massin ([2009] §3.1) advances further objections to component accelerations and the dynamical reading of the second law.

perception and dynamometers, see below). But the component accelerations lying between them remain out of reach: no perceptual or technological apparatus seems able to directly measure them.

Second, the problem has only been shifted elsewhere, for given that both component and resultant accelerations are claimed to be real in the dynamical account, we now have to explain how nature adds accelerations. Such a realist commitment to the composition of accelerations is problematic for at least three reasons. (i) As argued in detail by Smith [2010], if accelerations are conceived of as temporally extended effects, it is objectionable that they can ever compose with one another. One might answer that component accelerations are strictly instantaneous. This however increases the air of mystery surrounding them, and leaves the two next worries unaddressed. (ii) If component and resultant accelerations have causal powers, we get a new threat of causal overdetermination. One answer could be that component accelerations, being potential, lack any causal powers. But again, this answer makes them even weirder: component accelerations are now potential, strictly instantaneous, powerless accelerations. (iii) Even if the overdetermination worry can be avoided, the overcounting issue recurs: if the resultant acceleration is the sum of all the accelerations of a body, why is it not itself one summand?

I suspect that one reason why potential accelerations have been found unproblematic is that we tend to think about them in terms of component forces. Component accelerations indeed look very much like component forces: (i) potential accelerations have a direction, (ii) they are not reducible to any actual changes of position (or derivative thereof), and (iii) they determine actual accelerations. Like forces, component accelerations are causally empowered vectorial entities that do not boil down to actual kinematical changes. But since, *ex hypothesis,* they are not forces, (but their unobservable effects), this way of thinking about them is misleading.

All in all, and unsurprisingly, frugal realists about forces have to either embrace semantic unsystematicity (if they endorse the fictionalist approach), or to make convolutions to try regain it, often in vain (if they endorse one erzasist approach).

3 Against Frugality (2): Causal Responsibility

3.1 Apportioning causal responsibility

The second objection to frugal realism stems from attributions of causal responsibility. Causal responsibility in NM is typically read as apportioned [Sober, 1988]. When several forces act together to bring about an acceleration, one might wonder what their respective contribution to the acceleration of the body is: is the acceleration due more to this or that component force, and in which proportion? Thus, in the case of the falling apple, the force responsible for the fall of the apple, intuitively, is gravitation, while air friction, on the contrary, is preventing the apple from falling even more quickly.[15]

One reason why attribution of causal responsibility matters stems for the common assumption that causal responsibility for action outcomes is a necessary condition for moral responsibility for such outcomes.[16] Suppose Tybalt is intentionally pushing Romeo towards a cliff while Juliet is trying to prevent him from falling by pulling him in the opposite direction. Pushing and pulling, let us here assume not too implausibly, are actions that consist in exerting forces on a body in order to change or maintain its kinematic behavior[17]. Tybalt intentionally exerts a force on Romeo that goes in the direction of the cliff. Juliet intentionally exerts an opposite force on him (we assume for simplicity that no other force acts on Romeo). As it happens, Tybalt is stronger than Juliet, and Romeo falls. Intuitively, Tybalt is morally responsible for the fatal step of Romeo partly because he is causally responsible for it: the force that caused Romeo's fall is the one he exerted. Sentencing Juliet would be a gross judicial mistake, for the force she exerted is not causally responsible for Romeo's fatal step: it did not contribute to Romeo's acceleration.

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[15] Like Sober [1988], I use 'causal responsibility' to express the contribution of a cause to its effect, i.e. the proportion in which it contributes to it effect. Kistler ([2006], Ch. 5; [2014] §4.6, 4.7) uses the expression differently. Having contrasted 'eventive' and 'factive' causal statements, he then distinguishes 'causal relations', which relate events, from 'causal responsibility', which relates facts. While fully agreeing with the importance of the distinction that Kistler is after, it seems to me that 'causal responsiblity' is a misnomer: 'causal explanation' might be a better term for such a relation between facts (as proposed by Schaffer [2015]).

[16] This is not uncontroversial though, see e.g. Reinach [2009], Sartorio [2007].

[17] Incidentally, pushes and pulls are straightforward commonsense counterexamples to the philosophical view that all basic actions are bodily movements. One can push and pull a door more or less strongly (= exert forces of different magnitudes), without it ever moving.

One objection to there being any asymmetry in causal responsibility here points out that it is the determinate acceleration of Romeo that has to be causally explained. But then, the objection goes, both Juliet and Tybalt incur causal responsibility for it. For Romeo clearly would not have accelerated at this rate towards the cliff had Juliet exerted no force upon him. Hence Juliet and Tybalt would in fact share causal responsibility for the determinate acceleration undergone by Romeo.

My answer to this worry is to deny that the counterfactual test above establishes causal responsibility. That the acceleration of a body would have been different in the absence of a given force does not entail that this force is causally responsible, even partly, for the acceleration of the body. Here is an argument to that effect. The forces causing an acceleration each have a precise *share* of causal responsibility: they are partly or fully responsible for the ensuing acceleration. In at least one case, their precise share of causal responsibility is straightforwardly calculated. Suppose Tybalt and Samson are both pushing Romeo towards the cliff, without encountering any opposition. Let us say that Samson's force is 10N and that Tybalt's force is 20N. Their forces being co-directional, the resultant force, in accordance with vector addition, will be 30N (20N + 10N). Sampson will then be responsible for one third of the resultant force that causes Romeo's acceleration (10/30), while Tybalt will be responsible for two thirds of it (20/30). It is hard to think of any better way to apportion their respective causal responsibility.[18]

Now consider again our first example, where Juliet, instead of joining forces with Tybalt, opposes him. Assume Juliet's force is also 10N. Since the forces Juliet and Tybalt exert are now opposite, the resultant force exerted on Romeo, according to vector calculus, will be of 10N (20N – 10N). What, then, could be the share of responsibility of Juliet and Tybalt in that force? If we follow the same method for calculating causal responsibility, we will have to say that Juliet's share of responsibility is –10/10, while Tybalt's one is 20/10. The problem is that this hardly makes sense: shares, proportions, can neither be negative nor greater than one. Juliet cannot be less than partly responsible, and Tybalt cannot be more than fully responsible for Romeo's acceleration.

So, if really we are to maintain that both Juliet and Romeo are causally responsible for Romeo's fatal step, then on natural assumptions about the apportionment of causal responsibility we are committed to saying that Juliet has a negative share of

[18] To prevent possible confusion, note that the fact that Romeo and Tybalt have here different degrees of *causal* responsibility does not entail that they have different degrees of *moral* responsibility (they might be equally blameworthy, or not).

responsibility and Tybalt a greater-than-one share of responsibility for Romeo's fall. This odd conclusion is readily avoided, on the other hand, if we accept that accelerations can be counterfactually dependent on component forces without these forces being causally responsible for them. Drawing on Sober [1988]'s distinction between 'making a difference in an effect' and 'contributing to an effect', we should say that although all component forces make a difference in the ensuing acceleration, not all of them contribute to that acceleration: Juliet's force made a difference in Romeo's acceleration, but it did not contribute to that acceleration (contrary to Tybalt's force, that both made a difference in, and contributed to the acceleration).[19]

Sober [1988] would object that in the case of NM (by contrast to biology), *making a difference in* and *contributing to* accelerations amount to one and the same thing. According to him, the counterfactual accelerations that each component force would have caused, had it acted alone, indicates its 'contribution' to the effect. NM, he claims, apportions causal responsibility by estimating the effect that each component force would have had in isolation. I disagree. To determine the share of causal responsibility of component forces by means of the accelerations each would bring about in isolation, one would need to determine the share of each component acceleration in the resultant actual acceleration. But the vectorial composition of accelerations is mute with respect to such ratios. Apportionment is a division, and we cannot divide vectors by vectors (except co-linear ones). (Non-flat) parallelograms—of forces or of accelerations—do not bring out the proportion of the components in the resultants. Consequently, knowing which acceleration a force would have caused in isolation is hardly of help in apportioning its causal responsibility in the actual acceleration. Counterfactuals about the effects of forces

[19]A referee of this journal pressed the following objection: consider an already-accelerating body being suddenly slowed down by a counteracting force: did the second force not contribute to the lessened acceleration? Two things might be said in reply. First, one needs to distinguish the lessened acceleration from the lessening of the acceleration, i.e. the deceleration. It is intuitively the case that the coming to act of the counteracting force contributed to the lessening of the acceleration. But what explains the lessened acceleration, i.e. what explains that the body continues to accelerate in the same direction in spite of the introduction of second counter-acting force, is still, arguably, (part of) the first force.

A second, compatible, answer is to insist that the contribution of a force to an acceleration has to be read as a direct contribution in the following sense: if a force directly contributes to an acceleration, it is not the case that it causes the acceleration by first causing something else. By contrast, the newly-introduced counter-acting force (here, Juliet's force) make a difference in the ensuing acceleration only by preventing the first force (here, Tybalt's force) from having its full kinematic effect.

in isolation, *pace* Sober, bring out that component forces make a difference in the actual acceleration, but they do not yet apportion their causal responsibility in that acceleration.

So NM, *per se*, does not apportion causal responsibility. To understand such an apportionment, we need to dig into the metaphysical underpinnings of the parallelograms.

3.2 Causal responsibility *versus* frugality

My claim is that frugal realist readings of NM fail to apportion causal responsibility. Frugal realists are not in a position to say that Tybalt's force, by contrast to Juliet's, is causally responsible for Romeo's acceleration.

This objection straightforwardly applies to frugal resultant realism. Since it denies that several forces can act at once on the same body, the very question of weighting the causal contributions of each component forces does not even arise. Taken apart from each other, neither Tybalt nor Juliet exerted any forces, the frugal resultant realist has it. All he can say is that the single resultant force that Juliet and Tybalt exerted together on Romeo was 100% causally responsible for his acceleration. Whether Tybalt's push or Juliet's pull was responsible for Romeo's fatal step, and to what extent, is a question that cannot be answered, because it cannot be asked.

Wilson might reply, in line with her second proposal, that a close question nevertheless makes sense. Her second proposal, to remind ourselves, is that partial laws hold in conjoined circumstances, but that in such cases they bear on some manifestations that are not forces. Tybalt and Juliet are not exerting forces, but they are nevertheless doing something. Why not weight the causal impact of these doings? Wondering about the causal responsibility of these non-force manifestations might indeed make sense. However, as long as we do not know what these manifestations are, or how they compose with each other, the whole story about their respective causal contributions remains to be seen.

Does frugal component realism fare better with respect to the causal responsibility objection? Consider first the frugal component realist in its fictionalist version: only component forces are real, since 'resultant forces' correspond to nothing in reality. Can such a frugal realist account for the different contributions of the forces at play? Hardly. Recall that for such a fictionalist, component forces have no known causal powers (for they do not fit in the second law), and accelerations have no known causes (the second law only predicts them by mentioning fictitious resultant forces). So for all we know, the force exerted by Tybalt had no acceleration-effect, and Romeo's step had no force-cause. The case against Tybalt collapses.

What about Creary's ersatzist approach, which replaces resultant forces by sets or pluralities of components forces? On this approach, what brought about Romeo's fatal step was the set of all the forces that acted upon Romeo. Romeo's step was caused by a set of two forces, the one exerted by Tybalt and the one exerted by Juliet. The problem is, Creary has no way of singling out which forces caused Romeo's fall and in what proportion. He might try to argue that Tybalt's force is more *similar* to the resultant force-vector we use when predicting the ensuing accelerations (they have the same direction), so that it is a more salient cause. But this is at best saving appearances, for if pressed by the judge to explain whether such predictions tell us anything about what really happened, he will have to answer negatively. In reality, he has to say, Tybalt was no more and no less responsible for the fall of Romeo than Juliet. Both, together, brought about Romeo's fall.

Consider, finally, the dynamical approach, which claims that the second law is not about resultant forces bringing about actual accelerations, but about each component force respectively bringing about a component acceleration. Tybalt, by exerting a component force, brought about a component acceleration of Romeo towards the cliff. Likewise, Juliet brought about a component acceleration of Romeo away from the cliff. This might at first sounds like an overwhelming case against Tybalt. But it isn't. For the component acceleration towards the cliff, which Tybalt admittedly brought about, is not Romeo's fatal step. That component acceleration is merely potential: it might have occurred without any fatal step ever occurring. We still don't know who is causally responsible for Romeo's actual acceleration, nor in which proportion.

Hence, no version of frugal realism proves able to apportion causal responsibility among component forces.

4 Primivitism about Vectorial Composition

We then face the following dilemma: if we endorse generosity about forces we get causal overdetermination and overcounting; if we endorse frugality about forces we get semantic unsystematicity and leniency toward criminals. Can we have our cake and eat it? The two arguments against generous realism (section 1.2) rely on a common premiss, P2, to the effect that component and resultant forces are wholly distinct. Could it be rejected? Could it be that component and resultant forces are partly identical? Could R, on the one hand, and F and G, on the other, be partly identical? The threats of overdetermination and overcounting would then vanish, paving the way for an acceptable version of generous realism.

There is reason for hope. In at least one simple case, such an identity between component and resultant forces is very likely to hold. Suppose an apple is in free fall (e.g. towards the moon). In that case only one force is exerted on it: the gravitation of the moon. Is this solitary force component or resultant? The answer given by frugal realists of both sides is doomed to be *ad hoc*: the intuitive answer is clearly 'both'. In the case where a single force is acting on a body, there is an identity between the component and the resultant force. Is there some way of arguing that identity remains at stake in more complex cases?

Mereological composition comes to mind, but can be rejected readily: the mereological sum of all component forces acting on a same body is not a resultant force. Resultant forces have a single direction and magnitude; mereological sums of component forces have no single direction, and perhaps no single magnitude (see Wilson ([2009], §3.3]. Russell ([1903], p. 477] makes the same point about component and resultant accelerations). The mode of composition of component forces into resultant ones is vectorial, and this cannot be captured by purely mereological means.

There are two better ways of getting such an identity: primitivism about vectorial composition and residualism. In this section I introduce and reject primitivism (I defend residualism in the last section).

4.1 Primitive vectorial composition

According to primitivism about vectorial composition (henceforth, primitivism *tout court*), vector addition is a primitive metaphysical mode of composition, on a par with mereological composition. The general idea is that, because vectorial summation is what it is, resultant forces are nothing over and above the resultant forces. The proposal comes from Alastair Wilson:

> [A] resultant force is nothing over and above the component forces, because of the nature of vector addition; just as the mereological sum of some objects is nothing over and above the objects because of the nature of mereological summation.[20]

[20] This quote comes from a commentary Wilson gave in reply to an earlier version of the present paper. Although Wilson appears to be the only one to have stated this proposal, the view that forces compose with each other in a non-mereological way had already been endorsed by Spurrett [2001]

This picture introduces a new primitive mode of composition, but the price might be worth paying. First, primitivism straightforwardly avoids causal overdetermination and overcounting issues. Second, primitivism nicely avoids issues pertaining to semantic unsystematicity: it is a version of generous realism, so that all occurrences of the word ‘force’ in NM do stand for real forces in nature. Finally, the metaphysics of the composition of forces can, under primitivism, be read off directly from the parallelogram of forces. In other words, the parallelogram of forces, which is a vectorial representation, can be taken at face value: it depicts the vectorial composition of forces as it truly happens in nature. Such simplicity is appealing.

There are however, two reasons not to accept primitivism as the true story about the composition of forces.

4.2 First objection to primitivism: causal responsibility

First, primitivism fails to properly attribute causal responsibility: in this respect it fares no better than frugal realism. When asked who is causally responsible for the fall of Romeo, Tybalt who pushed him, or Juliet who pulled him, and to which proportion, the upholder of primitivism can only say that it is a primitive fact that their forces composed in a way that brought about the fall. It is no more Tybalt’s force than Juliet’s that brought about the fall, but both of them, vectorially combined. We remain unable to point to the force exerted by Tybalt as that which is causally responsible for the fall of Romeo, and more generally to apportion the causal contribution of each force in the kinematic outcome.

4.3 Second objection to primitivism: null forces

Primitivism, second, entails that there are null resultant forces, distinct from a mere absence of forces. In case of static equilibria between different component forces (such as when exactly two antagonistic forces act on a body), the resultant force is null. Primitivism has it that such null resultant forces are vectorial compounds having non-null vectorial components. But then null resultant forces have to be *something*: a non-existent cannot be composed of existents, be it vectorially or

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and Mumford and Anjum ([2011b], pp. 42–3). In a similar way, Moore [2012] tries to shed light on such a *sui generis* mode of composition by equating resultant forces with maximally homogeneous mixtures of component forces (one issue is whether we should understand the composition of forces in terms of mixtures rather than the reverse. As mentioned above, Grassmann, [1854] defends the second option).

otherwise. A second reason why primitivism is committed to null resultant forces, *qua* distinct from absences of force, is that it would be *ad hoc* to say that among all possible primitive vector additions, some (the non-null ones) correspond to something real, but not others (the null ones). How bad is such a commitment to null resultant forces?

On the face of it, null resultant forces, *qua* distinct from absence of forces, are counterintuitive. The naïve way of describing such cases is to say that null force-vectors correspond to no forces in reality.[21] Compare this to temperatures. Things of zero temperature intuitively have a temperature. But the intuition about things of zero weight (weight being a force) is far less clear: we are tempted to say that such things have no weight rather a weight of 0, in the same way that we tend to think of things of zero height as having no height.

Besides, if we accept null resultant forces, we thereby have to say, in virtue of the second law, that some null acceleration of the body is caused by the null resultant force. Rather than diagnosing a causal relation between a null force and a null acceleration, wouldn't it be more natural to say that the body was not caused to accelerate, because no resultant force was acting upon it? Yet other zero-value physical magnitudes ensue, such as further effects of null accelerations. Moreover, once we have null resultant forces, why not have zero-component forces as well? For instance, if some particles have no mass (perhaps photons?) shouldn't we say that they exert zero gravitional forces on each other? Physical reality ends up being filled with zero accelerations, zero masses, zero interactions... That certainly *sounds* odd.

One might reply, however, that such intuitions are misleading. Balashov [1999] thinks so (and Wilson [2009] agrees). He maintains that null forces (and other zero-values physical quantities) are distinct from mere absences of forces. One key argument Balashov presents in favor of the existence of zero-valued physical quantities is his 'argument from composition': 'two or more *P*-hoods, he writes, cannot result in complete *P*-lessness'. Transposed to forces, the idea is that if a null resultant force results from the composition of non-null component forces, the null force has to be something rather than nothing.

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[21] This is of course not to deny that there is a difference between static cases of equilibrium between component forces, and static cases where no component forces are acting. The difference consists precisely in the presence or absence of (non-null) component forces, not in the presence or absence of null resultant forces.

I do not think that this argument goes through. Balashov is clearly right that if a compound has some components, the compound must also be real, even when it is of null value (as we have just been arguing, this is precisely why primitivism is committed to null resultant forces). However it cannot be assumed that what results from any mode of composition is a compound comprising the original components (a point emphasized by Spurrett [2001]). Not all modes of composition yield compounds. Consider subtractive modes of composition, e.g. wealth equals assets minus liabilities. The liabilities of an economic agent are clearly not parts or components of his wealth. His wealth is not a compound, but a residue: it is what *remains* once the debts have been subtracted. Hence there is nothing wrong in sticking with our intuition that having a wealth of zero corresponds to having no wealth. Reifying null residues would be a bad mistake. Nothing is left, not even a null something. For Balashov's argument from composition to go through, it has to be assumed that the mode of composition of forces is non-subtractive.[22] As will soon become clear, this is question-begging.

Finally, the strongest argument against zero forces, on top of their counterintuitivity, stems from vectorial direction. Zero-value scalar quantities are one thing, zero-value vectorial quantities quite another. For what could be the direction of a vector whose magnitude is null? No answer sounds satisfying. Consider a two-dimensional Cartesian coordinate system. The direction of a vector is there equated with the value of its angle from the abscissa, anticlockwise. What could be the direction of a zero-magnitude vector in such a system? One might think first that the direction of such a vector should be null, given that its magnitude is null as well. But in fact a null direction it just one direction among others: namely the one of the abscissa (whose direction is also that of some non-null vectors). There appears to be no reason to ascribe to null vectors the direction of the abscissa rather than any other. It is in fact hard to see why any trigonometric direction should be ascribed to such null-vectors: 0°, 30°, 90° etc. are all equally arbitrary answers. One might try: if a null-vector has a direction, it has to be a non-trigonometric direction, on pain of arbitrariness. But here we seem to be reaching the limits of the idea of a direction. In order to keep things intelligible, one should rather say that if something is at no angle from the abscissa—including 0°—then it has no direction. But if it has no direction, it is not a vector-quantity. So there

[22] One could reply that 'composition' is a misnomer in subtractive cases. Perhaps this is so. But then Balashov's mistake lies no more in the composition argument itself than in the application of that argument to non-compositional—subtractive—cases.

cannot be null resultant forces, for they wouldn't have any direction. Hence primitivism has to be rejected.

5 Residualism

5.1 Residualism introduced

How then should we understand the composition of forces? Let us recap the main *desiderata* we have met so far:

1. We want both component and resultant forces to be real (so as to avoid, e.g., semantic unsystematicity).
2. We do not want them to be wholly distinct (so as to avoid causal overdetermination and overcounting).
3. We want to avoid any appeal to unobservable manifestations of forces (such as component accelerations).
4. We want to be able to attribute, and to precisely ponder, the causal responsibility of each force at play in the ensuing acceleration (e.g. to blame Tybalt and not Juliet for the death of Romeo).
5. We want to say that when the vectors sum is null, there is no resultant force rather a null resultant force.
6. We want single forces to be at once component and resultant forces.

Residualism, I will now argue, meets all of these *desiderata*. Residualism is the following view:

residualism*:* the resultant force acting on a body is identical to the (sub-)component force(s) that do not prevent each other from bringing about the acceleration of the body.

More succinctly: resultant forces are the (sub-)component forces that do not counteract each other. Resultant forces are residual component forces. This picture therefore reverses the standard take on the issue: resultant forces are not compounds but residues. The right way to understand composition of forces is not additive, but subtractive: resultant forces are remaining component forces, namely those component forces that are not counteracted. They are like soldiers who survived the battle.

Key to residualism is the idea that forces have *two* essential kinds of causal powers. They have first the power to bring about the acceleration of the body they act upon. They have, second, the causal power of preventing other forces from bringing about accelerations of the body they act upon:

kinematic power: each force essentially has the causal power to bring about accelerations of the body it acts upon.

static power: each force essentially has the causal power to prevent antagonistic forces (same magnitudes, opposite directions) from causing the acceleration of the body it acts upon.

The conditions under which each of the two causal powers of forces manifest are simple: when the one is *on*, the other is *off*. When a force $\vec{F}$ does not meet any antagonistic force, it causes the corresponding acceleration of the body: it exerts its kinematic power. When, on the other hand, $\vec{F}$ meets an antagonistic force $\vec{G}$, then $\vec{F}$ prevents $\vec{G}$ from causing the acceleration it would have caused, had it acted alone (and reciprocally, $\vec{G}$ prevents $\vec{F}$ from bringing about the acceleration $\vec{F}$ would have caused, had it acted alone): it exerts its static power. The kinematic power of a force manifests unless some inhibitor prevents it from doing so. Its stimulus is an absence. The static power of force, by contrast, is triggered by the presence of an antagonistic force.

Forces, therefore, necessarily exert one and only one of their two powers. Forces are never sleeping: when their kinematic power remains latent, their static power manifests itself, and conversely. If a force does not prevent another force from causing an acceleration, it causes an acceleration. If it does not cause an acceleration, then it prevents another force from causing one.[23]

The residualist's overall strategy is to try to uncover fully counteracted and fully uncounteracted forces in all cases of composition of forces, so as to equate the resultant forces with the uncounteracted ones.

[23] See Fine ([1998], pp. 283-4) for a close picture of the relation between the disposition to warm and the disposition to cool.

5.2 Residualism at work

To see how residualism handles different cases of force composition and satisfies our *desiderata*, let us proceed from simple to more complex cases.

5.2.1 Antagonistic forces acting on a body

Let us start with the paradigmatic case of antagonistic forces, which lies at the heart of residualism. Antagonistic forces are pairs of forces acting on the same body, which have the same intensity but opposite directions (see Figure 4).[24]

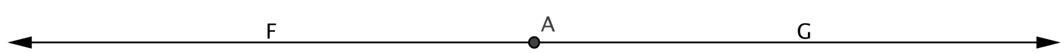

Figure 4: Antagonistic forces

Consider an apple hanging on the end of a flexible branch. The earth exerts a gravitational force on it, while the branch of the tree exerts an antagonistic force on it of the same intensity but in the opposite direction. As a result, the apple does not move. How should we understand the relation between such antagonistic forces?

[24] To prevent a possible confusion: because they act on a same body, antagonistic forces are emphatically *not* the action-reaction pairs described by Newton's third law.

According to residualism, the force exerted by the branch on the apple prevents the force exerted by the earth on the apple from causing the downwards acceleration it would have caused, had it been alone. And conversely, the gravitational force acting on the apple prevents the force exerted by the branch from causing the upwards acceleration it would have caused, had it been the only force acting on the apple. That is, the relation between antagonistic forces is a causal relation of mutual prevention. Antagonistic forces prevent each other from causing the acceleration each would have caused in isolation.

What then, is the resultant force in such a case? Residualism asks us to look for component forces that are not prevented from accelerating the apple. But there are *no* such forces in the present case, for the two forces completely counteract each other. This meets our 'no null forces' *desideratum* (5): when the vector summation gives a null result, that null resultant vector does not represent a null force, but nothing. There is no residual, uncounteracted, component force.

5.2.2 Single force acting on a body

When only one force acts on a body (see Figure 5), no force is prevented from bringing about an acceleration.

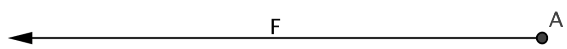

Figure 5: Single force

Hence the resultant force is just that component force itself, in accordance with *desideratum* (6). It is now easy to see how our two first desiderata are met as well. Because residualism equates resultant forces with some of the component forces, component forces and (non-null) resultant forces are real: this guarantees semantic systematicity. For the same reason, component are resultant are (partly) identical: this avoids all threats of overdetermination and overcounting.

5.2.3 Co-directional forces acting on a body

Consider now the case in which two forces of a same direction are acting on a body (see Figure 6).

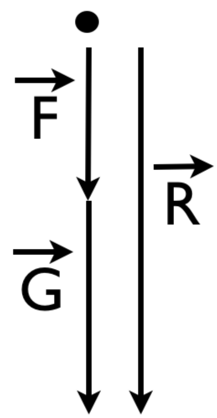

Figure 6: Resultant force of co-directional forces

In such a case, there is no opposition between component forces: they do not counteract each other. According to residualism, the resultant force here is nothing but the component forces together. But what does 'together' mean, here? Two options are open to us:

1. The first is to appeal to primitive vectorial composition, albeit not everywhere. Primitivism appeals to primitive vectorial composition in all cases; on the present proposal by contrast, primitive vectorial composition is not always needed. One case where it might be needed is the present one, of co-directional forces. But it in other cases, such as cases of antagonistic forces, it plays no role.
2. Skeptics about primitive vectorial composition might however put their hope in the fact that the two component forces here have the same direction. Hence, they might argue, their summation only concerns their magnitudes, so that standard mereological composition might be all we need here. Co-directional forces could add-up mereologically.

5.2.4 Opposite forces acting on a body.

Consider next our starting case: an apple is falling towards the earth, its fall being slowed down by the air friction ($\vec{F}$). The gravity ($\vec{G}$) and the air resistance go in opposite directions but have different magnitudes: because the air friction has a lower intensity than the gravitation in this case, the apple accelerates towards the earth. I shall call those kind of forces opposite but not antagonistic. Opposite forces are pairs of forces acting on a same body that have opposite directions and that do not necessarily have the same magnitudes (by contrast to antagonistic forces).

The air resistance prevents the gravitational force from exerting its full kinematic effect. In a vacuum, the apple would have accelerated at a higher rate towards the earth. According to the present hypothesis, we should interpret this as meaning that the air resistance prevents part of the gravitational force, $-\vec{\mathrm{F}}$, to exert its effect. The rest of the gravitational force, which is not counteracted by the air resistance, causes the acceleration. This unimpeded part of $\vec{G}$ is nothing but $\vec{R}$ the resultant force (see Figure 7).

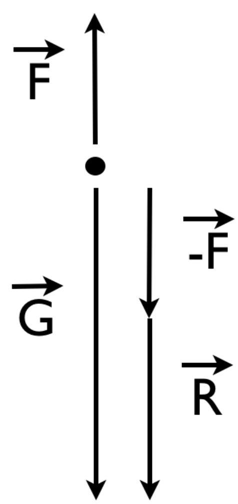

Figure 7: The resultant force of opposite non-antagonistic component forces

In such a case, we need to decompose one of the component forces into two co-directional sub-component forces, so as to uncover the (sub-)component forces that fully cancel each other out. In the present case, $\vec{G}$ must be decomposed into $-\vec{F}$ and $\vec{R}$, to see that $-\vec{F}$ and $\vec{F}$ prevent each other from bringing about any acceleration of the body. Only one force remains unimpeded: $\vec{R}$, which is to be equated with the resultant force.

As with the previous case, whether that decomposition of $\vec{G}$ into $-\vec{F}$ and $\vec{R}$ is mereological, vectorial or of some other sort is to be left open here. One version of residualism uses primitive vectorial composition, although in a piecemeal way; the other tries to get rid of it and insists, here as above, that the decomposition of co-directional forces is nothing but scalar decomposition.

In both cases, this general way of dealing with opposite forces satisfies our *desideratum* (4) about causal responsibility in the simple case of Romeo. The reason why Tybalt but not Juliet is 100% causally responsible of the fall of Romeo is that the resultant force that caused Romeo's fall ($\vec{R}$) is (a vectorial or mereological) part of the force Tybalt exerted upon Romeo ($\vec{G}$), but not a part of the force Juliet exerted upon him ($\vec{F}$).

5.2.5 Two non-colinear forces acting on a body

The tricky issue is to apply residualism to cases of non-colinear component forces. How is residualism to handle the (non-flat) parallelograms of forces? Taking up a traditional example, suppose a barge is pulled by two horses, on both sides of a canal. Each horse exerts a force ($\vec{F}$ and $\vec{G}$) on the barge, and mathematically the resultant force $\vec{R}$ corresponds to the diagonal of the parallelogram, having these two vectors as sides.

At first sight, it would seem hopeless to argue that $\vec{R}$ is a component of $\vec{F}$, $\vec{G}$, or both of them. The problem is not that there is no way of decomposing $\vec{F}$, $\vec{G}$, or both of them, in order make clear that $\vec{R}$ is a sub-component of them. The problem, on the contrary, is that there are infinitely many ways of doing this. Choosing one of them would be entirely *ad hoc*. I wish to maintain, though, that there is one, and only one, natural decomposition in such a case, which is represented in Figure 8.

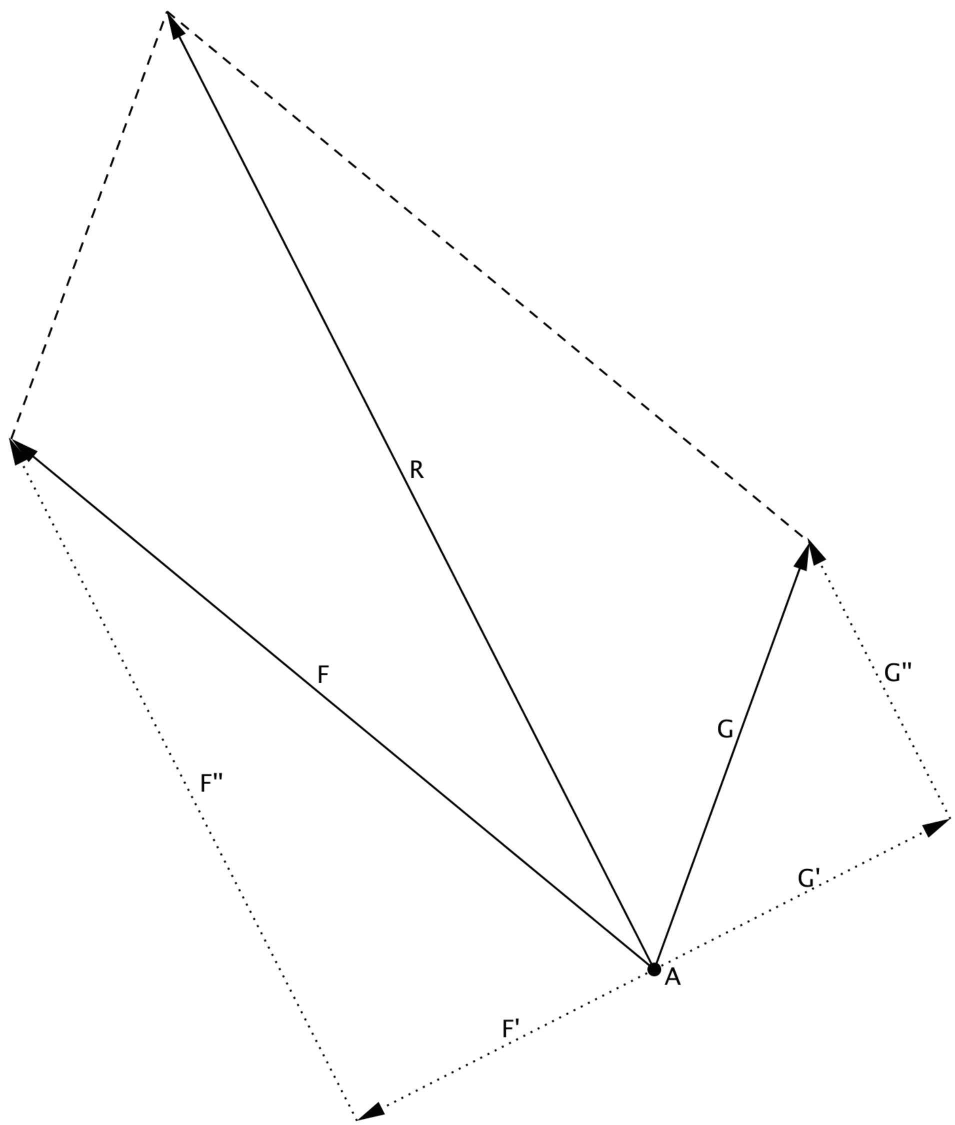

Figure 8: Composition of non-colinear forces: the sum of $\overrightarrow{F''}$ and $\overrightarrow{G''}$

The proposal is that we analyze each component force $\vec{F}$ and $\vec{G}$ into two sub-component forces: $\overrightarrow{F'}$ and $\overrightarrow{F''}$, on the one hand, and $\overrightarrow{G'}$ and $\overrightarrow{G''}$ on the other, so that:

1. $\overrightarrow{F'}$ and $\overrightarrow{G'}$ are antagonistic forces, orthogonal to the resultant force.

2. $\overrightarrow{F''}$ and $\overrightarrow{G''}$ are co-directional forces, whose (vectorial or mereological) sum gives the resultant force, $\vec{R}$.

5.2.6 Three and more non-colinear forces acting on a body

Finally, how does residualism handles cases of three non-colinear forces, $\vec{F}$, $\vec{G}$, and $\vec{H}$, acting on a body? One proposal would be to first sum $\vec{F}$ and $\vec{G}$, following the picture just given, and then to iterate that treatment so as to sum the resultant force of $\vec{F}$ and $\vec{G}$ to $\vec{H}$. That would be plainly ad hoc: we could as well have started by adding $\vec{G}$ and $\vec{H}$, or $\vec{F}$ and $\vec{H}$, which would have led us to diagnose distinct pairs of antagonistic forces in the set-up. Which antagonistic forces exist in nature should not depend on our additory whims.

One should rather sum all component forces at once. The way to do so is, here again, to resolve each component force into at most two sub-component forces, one parallel to the resultant force, the other perpendicular to it. We then end with two sets of sub-component forces: the one co-linear with the resultant forces, the other orthogonal to it. Necessarily, those orthogonal to the resultant forces fully cancel each other out (meaning that they are still there). Some of the forces co-linear with the resultant forces might also be antagonistic: the also cancel each other out. The resultant force is the one that remains. Figure 9 provides an example (for ease of presentation, I use Cartesian coordinates here, aligning the ordinate axis with the resultant force).

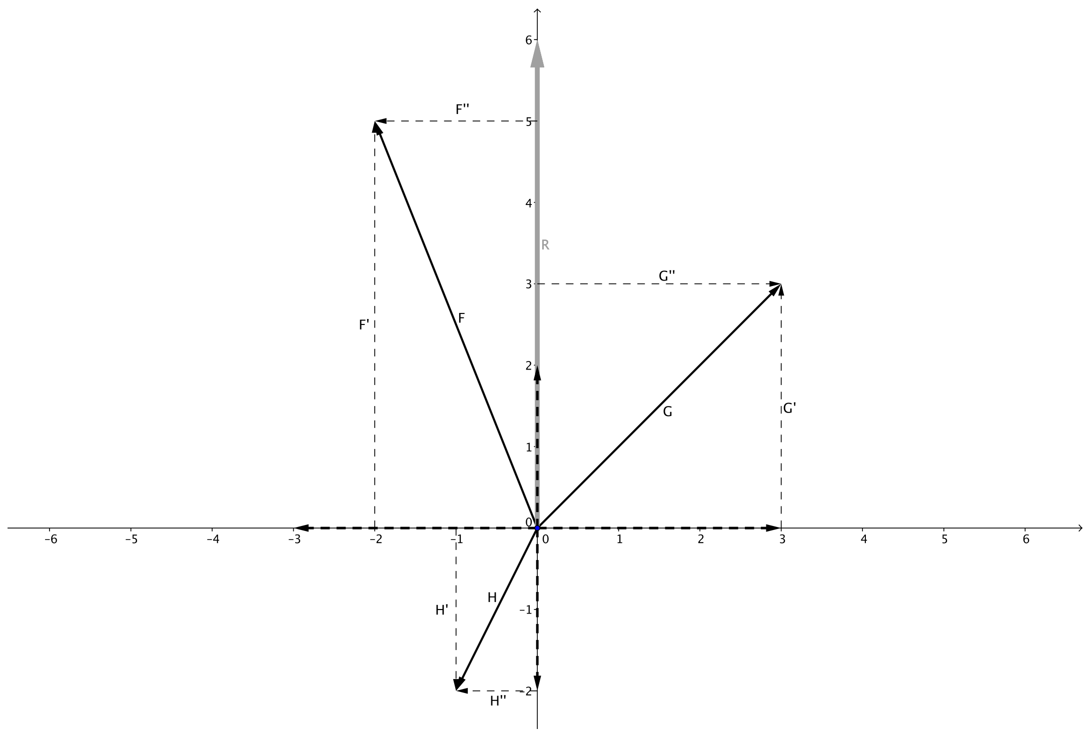

Figure 9: Composition of three non-colinear forces

We first decompose each component forces ($\vec{F}$, $\vec{G}$,$\vec{H}$) into two forces: one co-linear to the component force ($\overrightarrow{F'}$,$\overrightarrow{G'}$, $\overrightarrow{H'}$), the other orthogonal do it ($\overrightarrow{F''}$, $\overrightarrow{G''}$, $\overrightarrow{H''}$).

$$\vec{F} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \overrightarrow{F'} + \overrightarrow{F''} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

$$\vec{G} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \overrightarrow{G'} + \overrightarrow{G''} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

$$\vec{H} = \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \overrightarrow{H'} + \overrightarrow{H''} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

We then get two collections of co-linear forces, to which we apply the residualist's recipe: look first for antagonistic forces, then identify the resultant force with the forces that remains.

Let us start horizontally: $\overrightarrow{F''}$ and $\overrightarrow{H''}$, on the one hand, and $\overrightarrow{G''}$on the other hand, cancel each other out: $\overrightarrow{F''}$ and $\overrightarrow{H''}$ prevent $\overrightarrow{G''}$ from accelerating the body towards the right; and, conversely, $\overrightarrow{G''}$ prevents $\overrightarrow{F''}$ and $\overrightarrow{H''}$ from accelerating the body toward the left. These antagonistic forces are represented by the two dotted arrows on the *x*-axis.

Vertically, now, $\overrightarrow{H'}$ prevents a part of the sum of $\overrightarrow{F'}$ and $\overrightarrow{G'}$ to accelerate the body upwards (and conversely): these antagonistic forces are represented by the two dotted arrows on the *y*-axis. The resultant force, $\vec{R}$, is what remains of $\overrightarrow{F'}$ and $\overrightarrow{G'}$ once partly prevented from causing an upward acceleration by $\overrightarrow{H'}$.

5.3 Residualism under stress

One might bring up of four main objections to residualism: (i) The decompositions of forces it introduces, so as to uncover antagonistic forces, are *ad hoc*. (ii) Even if these decomposions are not *ad hoc* they still remain vectorial, so that residualism is circular. (iii) Such decompositions lead to inconsistencies, to the extent that one and the same force might be composed of different forces at different times. (iv) Finally, by ascribing two causal powers to forces, instead of one, residualism incurs an important metaphysical cost.

5.3.1 *Ad hoc*?

Not all parallelograms are borne equal, residualism entails: flat parallelograms are the central cases, in virtue of which non-flat ones should be understood. Why give such priority to antagonistic forces and, more generally, to co-linear ones? Why should tugs of war be more paradigmatic than horses-drawn boats? Why not treat all parallelograms alike, as do primitivists (who take all of them at face value) and frugal realists (who dismiss all of them)?

The answer is that antagonistic forces are indeed special, in a least three respects.

1. Antagonistic forces are physically remarkable. This readily appears as soon as one gives up the simplificatory assumption equating bodies to points. Antagonistic forces acting on extended bodies correspond, within continuum mechanics, to normal stress: pressures and tensions. To put a rod under pressure or compression, two antagonistic inward forces have to be exerted on each end; to put it under tension two antagonistic outward forces have to be exerted on each end. To be under normal stress one has to be acted on by antagonistic forces. The resultant force acting on (the whole of) an extended

body generates no stress in it. A body in free fall undergoes neither pressure nor tension.

2. Antagonistic forces are epistemologically special: contrary to co-directional forces, they can be directly measured and perceived. Consider again a body in free fall. One can infer the direction and magnitude of the gravitational force acting upon it, from its mass and its acceleration. But if one is to directly measure the gravitational force that acts upon it—through a scale or dynamometer—one needs to counteract that force by stopping the body. Free-falling scales are useless, hopelessly indicating '0kg'. Uncounteracted forces—which are (part of) resultant forces—cannot be directly measured, contrary to antagonistic forces.[25]

For the same reason, the perception of force is dependent on there being at least two forces acting against each other: only antagonistic forces can be directly perceived. Gravitation cannot be felt in free fall (although its kinematic effect might be seen). This, again, is more apparent when considering extended bodies: antagonistic forces are there equivalent to pressures or tensions, whose perception is both physiologically and psychologically well documented (Massin [2010]; de Vignemont and Massin [2015]). If the mechanoreceptors dedicated to pressure and tension perception within our skin and muscles were acted on by only one force, they would accelerate without transmitting any action potential to the brain.

3. Antagonistic forces are conceptually special: first because the causal concept at play here is the negative concept of prevention, while the causal concept needed to understand the kinematic effects of resultant forces is that of 'positive' causation. Second, because the concept at play in antagonistic forces is that of *mutual* prevention: this is not to say that prevention is a symmetrical relation, but only that each antagonistic force prevents the other from having the effect it would have had in isolation. Hence we have two instances of the prevention relation in cases of antagonistic forces, while we have only one instance of 'positive' causation in the case of resultant forces.

It is plausibly because of this specificity of antagonistic forces that most attempts, mentioned in the introduction, to generalize the composition of forces to other areas have given a central role to the antagonistic cases (a point underlined by Gibson, [1958], and exemplified, notably, by Mumford and Anjum [2011b]'s vector model

[25] Earman, Roberts, and Smith [2002, p. 287] make a similar point.

of causation). They are right: we should not be egalitarians with respect to compositions of forces. Antagonistic forces do constitute a special case. Contrary to frugal realism and primitivism, which treat them as cases among others, residualism gives them the prominent place they deserve.

With this in hand, one can now see why the residualist's account of the barge's case is not *ad hoc*. In Figure 8 above, $\overrightarrow{F'}$ and $\overrightarrow{G'}$ are real (component) forces present in the set-up. They can be measured by a dynamometer; and, assuming the barge is a continuous rigid body acted upon only by $\overrightarrow{F'}$ and $\overrightarrow{G'}$, it is along this line that it is under (tensile) normal stress. That is, the barge is in a state of tension induced by $\overrightarrow{F'}$ and $\overrightarrow{G'}$ cancelling each other out. If you want to reinforce the barge so that it does not split because of the forces exerted by the horses, you have to put glue, metal plates, or whatever along the line of action of these forces.

Correspondingly, the forces that do not cancel each other out—$\overrightarrow{F''}$ and $\overrightarrow{G''}$, which constitute the resultant force—are the ones causally responsible for the acceleration. By uncovering and subtracting the antagonistic forces, residualism allows one to determine precisely the degree of causal responsibility of each component force in the ensuing acceleration (in accordance with our fourth *desideratum*). $\overrightarrow{F''}$ and $\overrightarrow{G''}$ correspond respectively to the causal contribution of each horse to the acceleration of the barge. Thus, the ratio of the magnitude of $\overrightarrow{F''}$ to the magnitude of $\vec{R}$ gives the share of causal responsibility of $\vec{F}$ in the acceleration of the barge. Thanks to residualism, we can therefore not only attribute full causal responsibility in simple cases (such as the Romeo case where the force exerted by Tybalt is 100% causally responsible for Romeo's fatal step), but apportion causal responsibility exactly when non-colinear forces together bring about the acceleration of a body.

5.3.2 Circular?

The second objection to residualism is that it explains the composition of forces by appealing to yet other (de)compositions of forces, making it circular. In the case of co-directional forces, as we saw, one might reply that the composition at stake is mereological: the result of two co-directional forces acting on a same body might amount to their mereological sum, for all that has be added up here is just the (scalar) magnitude of the forces.

But in the barge case, where residualism diagnoses orthogonal sub-component forces, this move is not possible anymore. Residualism claims that $\vec{F}$ (in figure 8) is composed of $\overrightarrow{F'}$ and $\overrightarrow{F''}$ (and likewise for $\vec{G}$, composed of $\overrightarrow{G'}$ and $\overrightarrow{G''}$), which are not co-linear: such a vectorial composition cannot be reduced to any scalar composition. So isn't residualism committed to accepting primitive vectorial decomposition anyway?

The first line of answer is to accept that residualism is committed at some stage to primitive vectorial composition: the resolution of component forces into sub-component ones is a primitive case of (de)composition of forces. This move is not self-defeating in so far as residualism does not pretend to analyze the nature of all composition of forces, but only to describe what happens, in nature, when forces meet each other. Still, one might worry that if primitive vectorial composition has to enter the stage anyway, if would be far simpler to endorse primitivism right away. But as we saw, there are strong reasons to favor residualism over straightforward primitivism: contrary to primitivism, residualism avoids ontologizing null forces, it captures the specificity of antagonistic forces, and it apportions the causal responsibility of each component force in the ensuing acceleration. Hence, appealing to primitive vectorial composition only to decompose component forces so as to diagnose the antagonistic and co-directional forces is not pointless.

Since the second answer to this objection also constitutes an answer to the next objection, let me introduce that objection first.

5.3.3 Inconsistent?

Suppose a body is acted on by one force $\vec{F}$ only until $t1$, at which point a second (non-colinear) force $\vec{G}$ begins to act upon it (e.g. suppose one horse starts pulling the barge before the other). Before $t1$, $\vec{F}$ is a solitary force—at once component and resultant—and is not composed of any actual sub-forces. But after $t1$, $\vec{F}$, on the residualist's picture, has orthogonal sub-component forces, $\overrightarrow{F'}$ and $\overrightarrow{F''}$. Which components a force has, and whether it has components, therefore depends on the presence or absence of concurrent forces. Is it problematic?

If, as just suggested in reply to the circularity objection, $\vec{F}$ is, after $t1$, nothing over and above $\overrightarrow{F'}$ and $\overrightarrow{F''}$ in virtue of the nature of vectorial composition, such a context-sensitivity of the composition of $\vec{F}$ might be found inconsistent. At $t1$, $\vec{F}$

indeed lacks any decomposition, while at $t1$ $\vec{F}$ is nothing over and above its vectorial decomposition into $\overrightarrow{F'}$ and $\vec{F}''$. How can a force remain the same force if its essential components change over time? Perhaps relative identity might come to the rescue, but it would be better for residualism not to be committed to such a controversial move.

There is, however, a second way to answer both the circularity and inconsistency objections, which, albeit at a price, avoids any commitment to primitive vectorial composition and to the view that one force might have different components depending on the context. Rather than seeing the sub-component forces appealed to in residualism as some primitive vectorial parts or constituents of the component forces, one might consider such sub-component forces as resulting from the breakage of these component forces. On this approach, the body on which concurrent forces acts cause them to break into sub-forces. Bodies acts as force-breakers.[26] What is meant by breaking here is that when two non-colinear forces $\vec{F}$ and $\vec{F}$ act on a same body, as in Figure 8, each splits into two orthogonal forces (respectively $\overrightarrow{F'}$ and $\overrightarrow{F''}$; and $\overrightarrow{G'}$ and $\overrightarrow{G''}$), so that $\vec{F}$ and $\vec{G}$ are not there anymore. Thus, $\overrightarrow{F'}$ and $\overrightarrow{F''}$ are not in any strict sense (non-mereological) parts or components of $\vec{F}$, for $\vec{F}$ has been destroyed. If such a story holds, the circularity and inconsistency objections vanish. There is no threat of circularity, for instead of explaining the vectorial composition of several component forces in terms of the vectorial composition of their sub-component forces, we explain it in terms of the breakage of those component forces. Second, since $\vec{F}$ is replaced during breakage by $\overrightarrow{F'}$ and $\overrightarrow{F''}$, there is no time at which $\vec{F}$ actually consists of $\overrightarrow{F'}$ and $\overrightarrow{F''}$, vectorially combined, so the constitution of $\vec{F}$ does not vary over time.

Despite its advantages, such a solution is not without cost. Either breakage occurs instantaneously, at the exact moment in which the second force $\vec{G}$ begins to act on the body, or it occurs later. If it occurs later, there is a time at which $\vec{F}$ and $\vec{G}$ are concurrent forces acting on the body, without yet composing with each other: assuming the existence of such a state, just on theoretical grounds, is quite speculative. If, on the hand, breakage is instantaneous, then one has to say that $\vec{F}$ and $\vec{G}$ never act together on the body: the only forces that act together on it are $\overrightarrow{F'}$

[26] I owe this expression to Benj Hellie.

and $\overrightarrow{F''}$, and $\overrightarrow{G'}$ and $\overrightarrow{G''}$. Such a version of residualism is then revisionist in some important respect: it has it that the forces that truly compose with each other are not the forces that we first thought were composing with each other. It is still a version of generous realism to the extent that both the component forces ($\overrightarrow{F'}$ and $\overrightarrow{F''}$, $\overrightarrow{G'}$ and $\overrightarrow{G''}$) and the resultant forces exist together, but the component forces that are real are not the ones we diagnosed in the first place ($\vec{F}$ and $\vec{G}$): these are not real.

In sum, the (de)composition of forces to which residualism appeals so as to uncover antagonistic forces, might either (i) be conceived of as a primitive vectorial (de)compositions, or (ii) be analyzed in mereological terms (for co-directional forces) and causal terms or 'breakings' (for the orthogonal sub-component forces).

5.3.4 Costly?

A last objection to residualism is that the static causal power it attributes to forces on top of their kinematic one constitutes an important cost (which neither frugal realism nor primitivism incur). On top of being an additional kind of power attributed to forces, one might fear that its manifestation is no less mysterious than the component accelerations introduced by the dynamical account.

The static power of forces is indeed a cost of residualism. But note first that that this cost is well controlled, for as we saw, we know precisely when each of the two causal powers of a force manifests itself, namely, when the other one does not.

Second, the manifestation of the static causal power of forces is no less observable than the manifestation of their kinematic causal power. As we saw, manifestations of the kinematic causal power of forces are accelerations, which that can be seen; and manifestations of the static causal power of forces amounts to stresses (pressures and tensions), which can be felt. Pressures and tensions, we must remember, are perfectly occurrent—albeit non-kinematical—states of bodies, which can be directly measured and perceived, through pressure sense, muscular sense, or dynamometers. Hence, in accordance with our third *desideratum*, there is no hidden effect or manifestation of forces. Better, thanks to the on/off structure of the two causal powers of forces, a force always does something observable: either it manifest itself as a state of stress, or as a state of acceleration of the body it acts upon.

There is a final reason to accept the cost of these two causal powers of forces. Thanks to their on/off structure these powers can be given a conditional analysis that seems immune to fink-like objections (Martin[1994]). Their conditional analysis runs as follows:

kinematic power: A force $\vec{F}$ is disposed to cause the acceleration of the body x it acts upon $=_{\text{df}}$ if $\vec{F}$ does not meet any antagonistic force, then $\vec{F}$ causes the acceleration of x.

static power: A force $\vec{F}$ is disposed to prevent another force $\vec{G}$ from bringing about an acceleration of the body $x =_{\text{df}}$ if $\vec{G}$ is an antagonistic force acting on x, then $\vec{F}$ prevents $\vec{G}$ from bringing about the acceleration of x.

Both powers might prove unfinkable. Imagine a sorcerer, who, each time a force is solitary, casts a spell on that force so as to prevent it from bringing about the corresponding acceleration. Hasn't he thereby finked the kinematic power of that force? Not if our sorcerer has only Newtonian skills. For under NM, the only way to prevent a force from exerting its kinematic power is to introduce an antagonistic force, which amounts to falsifying the manifestation condition of that kinematic power. The kinematic power will not manifest, simply because it will not have been triggered. Likewise, the only way to prevent a force from exerting its static power is to remove all antagonistic forces around it, which amounts to falsifying the condition of manifestation of that power. The trick is that any candidate fink will have to falsify the condition of manifestation of the power in question. If true, there is no possible Newtonian fink to the causal powers of forces.

What about non-Newtonian finks? Couldn't a good sorcerer free himself from the second law and prevent the solitary force from bringing about an acceleration without ever introducing a counteracting force? Not if the second law is part of what makes forces what they are, as is implicitly assumed when we say that forces essentially have powers to bring about accelerations.

Relatedly, assuming the truth of the second law, if the conditions of manifestation of the kinematic power are met—if the force is acting alone on the body—it is metaphysically necessary that the body will accelerate. If it does not accelerate, then it is metaphysically necessary that an antagonistic force is acting on it. It is then, on the present picture, a metaphysical necessity—if NM is true—that each force will either bring about an acceleration or prevent another force from doing so. Mumford and Anjum [2011a] argue that a weaker, *sui generis* dispositional modality is required to capture the relation between dispositions and their manifestations.[27] In the case of Newtonian forces, metaphysical necessity might be all we need. This is not to say that there is no contingency. Whether or not there are other forces around might be contingent. But this is the contingency of the existence of forces, not the contingency of the dispositional modality.

6 Conclusion

We have been assessing three ways of handling the composition of forces: frugal realism (which denies the reality of either component or resultant forces), primitivism about vectorial composition (which reads off the metaphysics of the composition of forces directly from the parallelogram of forces), and residualism (which equates resultant forces with non-counteracted component forces). Each of these three stories avoids the threats of causal overdetermination and overcounting. Frugal realism does so by endorsing anti-realism about either resultant or component forces. Primitivism and residualism do so by endorsing partial identity between component forces and resultant forces. Each comes with its problems.

Frugal realism is semantically unsystematic; it fails to account for the share of causal responsibility of each component force in the ensuing acceleration, and it equates solitary forces with either component or resultant ones, while they are intuitively both at once.

Primitivism about vectorial composition is committed to the reality of null resultant forces and also fails to apportion causal responsibility.

Residualism incurs three main costs. First, it entails that forces essentially have two causal powers: that of bringing about accelerations, and that of preventing other forces from doing so. Second, it is committed to the naturalness of some decompositions of component forces into sub-component ones. Relatedly, residualism is either committed to primitive vectorial decomposition at some point; or to explaining how component forces causally break up into sub-component ones.

The benefits of residualism, I have argued, outweigh its costs. First because these costs are reasonable (we know precisely when and how each of the two powers of forces manifest; and the naturalness of some decomposition of forces is well-

[27] This idea of a weaker dispositional modality was anticipated by some dispositional accounts of *ceteris paribus* laws. See Schrenk [2007] for a critical overview.

grounded from a mechanical point of view), and second, because a unified semantic treatment of the term 'force' in NM, a precise apportionment of causal responsibility, a diagnosis of solitary forces as being at once component and resultant, a denial of the reality of null forces, a clear explanation of the specificity of antagonistic forces, and the guarantee that forces always have actual and observable manifestations, are worth the price.

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