Does an elementary particle have a unique intrinsic state?

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Abstract

J.M.G. Fell and other authors have asserted that an elementary particle has only one 'intrinsic' state. I will argue that this claim is not consistent with the mathematical structures and objects used to represent an elementary particle in relativistic quantum theory.

1 Introduction

To understand the issue at stake, let us begin with some background on the representation of elementary particles in modern mathematical physics.

In relativistic quantum theory, the two basic types of thing which are represented to exist are matter fields and gauge force fields. A gauge force field mediates the interactions between the matter fields. Relativistic quantum theory is obtained by applying quantization procedures to classical relativistic particle mechanics and classical relativistic field theory. The quantization procedure can be broken down into first-quantization and second-quantization. In first-quantization, it is possible to represent interacting fields in a tractable mathematical manner. The first-quantized approach is empirically adequate to the extent that it enables one to accurately represent many of the structural features of the physical world. Second-quantization, quantum field theory proper, is required to generate quantitatively accurate predictions, but quantum field theory proper is incapable of directly representing interacting fields.

In the first-quantized theory, a matter field can be represented by a cross-section of a vector bundle, and a gauge force field can be represented by a connection upon a principal fibre bundle. This is rather curious because the matter fields are obtained by quantizing the point-like objects of classical relativistic particle mechanics, whilst, at first sight, the gauge fields have undergone no quantization at all. If one treats the first-quantized matter fields as classical fields, and if one treats the matter fields as interacting with classical gauge fields, then there is no inconsistency. However, on both counts, such a treatment may be misleading. Given that the matter fields in the first-quantized theory are the upshot of quantizing classical particles, they are interpretable as wave-functions i.e vectors in a quantum state space. One of the outputs from the first quantized theory is a state space for each type of elementary particle, which becomes the so-called 'one-particle subspace' of the second-quantized theory. The vector bundle cross-sections which represent a matter field in the first-quantized theory, are vectors from the one-particle subspace of the second-quantized theory. The connections which represent a gauge field can be shown, under a type of symmetry breaking called a 'choice of gauge', to correspond to cross-sections of a direct sum of vector bundles, (Derdzinski 1992, p91). The cross-sections of the individual direct summands are vectors from the one-particle subspaces of particles called 'interaction carriers', or 'gauge bosons'. Hence, neither the matter fields nor the gauge fields of the first-quantized theory can be unambiguously treated as classical fields. Given these complexities, the terms 'particle' and 'field' will be used interchangeably throughout this paper, without the intention of conveying any interpretational connotations.

A particle is an elementary particle in a theory if it is not represented to be composed of other particles. All particles, including elementary particles, are divided into fermions and bosons according to the value they possess of a property called 'intrinsic spin'. If a particle possesses a non-integral value of intrinsic spin, it is referred to as a fermion, whilst if it possesses an integral value, it is referred to as a boson. The elementary matter fields are fermions and the interaction carriers of the gauge force fields are bosons. The elementary fermions number six leptons and six quarks. The six leptons consist of the electron and electron-neutrino (e, ν_e), the muon and muon-neutrino (μ, ν_μ), and the tauon and tauon-neutrino (τ, ν_τ). The six quarks consist of the up-quark and down-quark (u, d), the charm-quark and strange-quark (c, s), and the top-quark and bottom-quark (t, b). The six leptons have six anti-leptons, $(e^+, \overline{\nu}_e)$, $(\mu^+, \overline{\nu}_\mu)$, $(\tau^+, \overline{\nu}_\tau)$, and the six quarks have six anti-quarks $(\overline{u}, \overline{d})$, $(\overline{c}, \overline{s})$, $(\overline{t}, \overline{b})$. These fermions are partitioned into three generations. The first generation, (e, ν_e, u, d), and its anti-particles, is responsible for most of the macroscopic phenomena we observe. Triples of up and down quarks bind together with the strong force to form protons and neutrons. Residual strong forces between these hadrons bind them together to form atomic nuclei. The electromagnetic forces between nuclei and electrons leads to the formation of atoms and molecules. (Manin 1988, p3).

Free matter fields ('free particles') are matter fields which are idealized to be free from interaction with force fields. To specify the free elementary particles which can exist in a universe. i.e. the free elementary 'particle ontology' of a universe, one specifies the *projective*, infinite-dimensional, irreducible unitary representations of the 'local' symmetry group of space-time.

The large-scale structure of a universe is represented by a pseudo-Riemannian manifold $(\mathcal{M}, g)$. The dimension n of the manifold $\mathcal{M}$, and the signature (p, q) of the metric g, determine the largest possible local symmetry group of the space-time. The automorphism group of a tangent vector space $T_x\mathcal{M}$, equipped with the inner product $\langle \ , \ \rangle = g_x(\ , \ )$, defines the largest possible local symmetry group of such a space-time, the semi-direct product $O(p, q) \circledS \mathbb{R}^{p,q}$. If there is no reason to restrict to a subgroup of this, then one specifies the possible free elementary particles in such a universe by specifying the projective, infinite-dimensional, irreducible unitary representations of $O(p, q) \circledS \mathbb{R}^{p,q}$.

In the case of our universe, the dimension $n = 4$, and the signature $(p, q) = (3, 1)$, indicating three spatial dimensions and one time dimension. An n-dimensional pseudo-Riemannian manifold such as this, with a signature of $(n-1, 1)$, is said to be a Lorentzian manifold. Each tangent vector space of a 4-dimensional Lorentzian manifold is isomorphic to Minkowski space-time, hence the automorphism group of such a tangent vector space is the Poincare group, $O(3,1) \circledS \mathbb{R}^{3,1}$, the largest possible symmetry group of Minkowski space-time. In the case of our universe the actual local space-time symmetry group is a subgroup of the Poincare group, called the *restricted* Poincare group, $SO_0(3,1) \circledS \mathbb{R}^{3,1}$. The projective, infinite-dimensional, irreducible unitary representations of the restricted Poincare group correspond to the infinite-dimensional, irreducible unitary representations of its universal covering group, $SL(2, \mathbb{C}) \circledS \mathbb{R}^{3,1}$. Hence, one specifies the free elementary particle ontology of our universe by specifying the infinite-dimensional, irreducible unitary representations of $SL(2, \mathbb{C}) \circledS \mathbb{R}^{3,1}$.

It is assumed, or reasoned, that the free particle ontology of a universe equals the interacting particle ontology. In other words, although a realistic representation of particles involves representing their interaction with force fields, it is assumed, or reasoned, that the set of particle types which exists in a universe can be determined from the free particle ontology.

It is also assumed, or reasoned, that representations of the *local* symmetry group of space-time are an adequate means of determining the free elementary particle ontology. One could reason that elementary particles exist at small length scales, and the strong equivalence principle of general relativity holds that Minkowski space-time, and its symmetries, are valid on small length scales. i.e. the strong equivalence principle holds that the global symmetry group of Minkowski space-time is the local symmetry group of a general space-time. One can choose a neighbourhood U about any point in a general space-time, which is sufficiently small that the gravitational field within the neighbourhood is uniform to some agreed degree of approximation, (Torretti 1983, p136). Such neighbourhoods provide the domains of 'local Lorentz charts'. A chart in a 4-dimensional manifold provides a diffeomorphic map $\phi : U \to \mathbb{R}^4$. If $\mathbb{R}^4$ is equipped with the Minkowski metric, a local Lorentz chart provides a map which is almost isometric, to some agreed degree of approximation, (ibid., p147). Unless the gravitational field is very strong, one can treat each elementary particle as 'living in' the domain of a local Lorentz chart within a general space-time $(\mathcal{M}, g)$. Unless the gravitational field is very strong, the fibre bundles employed in relativistic quantum theory are assumed to be fibre bundles over Minkowski space-time. This is done with the understanding that the base space of such bundles represents the domain of an arbitrary local Lorentz chart, rather than the whole of space-time. Hence, with the exception of the regions where the gravitational field is very strong, the elementary particles which exist in a general Lorentzian space-time still transform under the global symmetry group of Minkowski space-time, namely the Poincare group, or a subgroup thereof.

With the exception of regions where the gravitational field is very strong, a fully realistic representation of each individual elementary particle would begin with a Lorentzian manifold $(\mathcal{M}, g)$ which represents the entire universe, and would then identify a small local Lorentz chart which the particle 'lives in'. The particle would then be represented by the cross-sections and connections of vector bundles over this small local Lorentz chart. In terms of practical physics, this would be an act of representational *largesse*, but in terms of ontological considerations, it is important to bear in mind.

Where the gravitational field is very strong (i.e. where the space-time curvature is very large), it is no longer valid to assume that the gravitational field is uniform on the length scales at which elementary particles exist. Note that because gravity is geometrized in general relativity, it is consistent to speak of *free* elementary particles in a gravitational field. Where the gravitational field is very strong, it is not valid to assume that free elementary particles transform under the global symmetry group of Minkowski space-time. Where the gravitational field is very strong, elementary particles are represented, in the first-quantized theory, by fibre bundles over general, curved space-times. Again, this is done with the understanding that the base space of such bundles represents a small region of space-time, rather than the whole universe. These considerations weaken the assumption that the representations of the Poincare group are an adequate means of determining the free elementary particle ontology in a universe. However, one might still be able to reason that the identity of elementary particles remains unchanged by a strong gravitational field, hence one can identify the free elementary particle ontology by studying the ontology under less extreme conditions. Suppose, for the sake of argument, that the particle ontology does change in a region of curved space-time: in a general curved space-time, there might well be no isometry group at all, hence the possible elementary particles in such a space-time region could not be classified by the irreducible unitary representations of that region's space-time symmetry group. If the free elementary particle ontology is not determined in all regions of space-time by the irreducible unitary representations of the local symmetry group in the regions of weak gravitational field, then one would have to abandon a classification scheme based upon representations of space-time symmetry groups.

Given the absence, in general, of a symmetry group for a curved space-time, practitioners of quantum field theory in curved space-time take the linear field equations associated with particles of mass m and spin s in the Minkowski space-time 'configuration representation', and generalise them to curved space-times. The solutions of these equations can be considered to represent first-quantized free particles of mass m and spin s in curved space-time. Whilst the solutions of these linear equations correspond to unitary irreducible representations of the space-time symmetry group in the case of Minkowksi space-time, no such correspondence exists for the generalised equations. Moreover, for $s > 1$, there are reasons for thinking the solutions to these equations do not satisfy physical criteria. For example, such equations do not have a well-posed initial-value formulation (Wald 1984, p375). If particles of $s > 1$ can exist in regions of weak gravitational field, one presumes they can wander into regions of curved space-time, hence one should not conclude that $s > 1$ particles cannot exist in curved space-time. It is possible that the generalised equations provide the physically correct description of $s > 1$ particles in curved space-time, but do not provide the same degree of tractability as their Minkowski space-time counterparts. Alternatively, it is possible that the correct representation of $s > 1$ particles in curved space-time has not yet been found.

The irreducible unitary representations of $SL(2,\mathbb{C}) \circledS \mathbb{R}^{3,1}$ are parameterized by mass m and spin s. One can present these representations in either the momentum representation (the Wigner representation), or the configuration representation. In the Wigner approach, free particles of mass m and spin s correspond to vector bundles $E^{\pm}_{m,s}$ over mass hyperboloids/light cones $\mathscr{V}^{\pm}_m$ in Minkowski (energy-)momentum space $T^*_x\mathcal{M}$. It is the Hilbert spaces $\mathscr{H}_{m,s}$ of square-integrable cross-sections of these vector bundles $E^{\pm}_{m,s}$ which provide the irreducible unitary representations of $SL(2,\mathbb{C}) \circledS \mathbb{R}^{3,1}$.

In the configuration representation, each irreducible unitary representation is constructed from a space of mass-m solutions, of either positive or negative energy, to a linear differential equation over Minkowski space-time $\mathcal{M}$. The Hilbert space of a unitary irreducible representation in the configuration representation is provided by the completion of a space of mass-m, positive or negative energy solutions, which can be Fourier-transformed into square-integrable objects in Minkowski energy-momentum space.

Whilst the Wigner approach deals directly with the irreducible unitary representations of $SL(2,\mathbb{C}) \circledS \mathbb{R}^{3,1}$, the configuration space approach requires two steps to arrive at such a representation. In the configuration space approach, for each possible spin s, one initially deals with a non-irreducible, mass-independent representation of $SL(2,\mathbb{C}) \circledS \mathbb{R}^{3,1}$. For each spin s, there is a finite-dimensional vector space V_s, such that the mass-independent representation can be taken as the space of cross-sections $\Gamma(\eta)$ of a vector bundle η over $\mathcal{M}$ with typical fibre V_s.

The representations of $SL(2,\mathbb{C}) \circledS \mathbb{R}^{3,1}$ upon such $\Gamma(\eta)$ can be defined by a combination of the finite-dimensional irreducible representations of $SL(2,\mathbb{C})$, and the action of $SL(2,\mathbb{C}) \circledS \mathbb{R}^{3,1}$ upon the base space $\mathcal{M}$. The complex, finite-dimensional, irreducible representations of $SL(2,\mathbb{C})$ are indexed by the set of all ordered pairs (s_1, s_2), (Bleecker 1981, p77), with

$$(s_1, s_2) \in \frac{1}{2}\mathbb{Z}_+ \times \frac{1}{2}\mathbb{Z}_+ \ .$$

In other words, the irreducible representations of $SL(2,\mathbb{C})$ form a family $\mathscr{D}^{s_1,s_2}$, where s_1 and s_2 run independently over the set $\{0, 1/2, 1, 3/2, 2, ...\}$. The number $s_1 + s_2$ is called the spin of the representation.

Now one can define, for each possible spin s, an infinite-dimensional, mass-independent representation of $SL(2,\mathbb{C}) \circledS \mathbb{R}^{3,1}$ upon $\Gamma(\eta)$. Letting $\psi(x)$ denote an element of $\Gamma(\eta)$, the representation is defined as

$$\psi(x) \to \psi'(x) = \mathscr{D}^{s_1,s_2}(A) \cdot \psi(\Lambda^{-1}(x-a)) \ ,$$

where it is understood that $A \in SL(2,\mathbb{C})$, $a \in \mathbb{R}^{3,1}$, Λ is shorthand for $\Lambda(A)$, and Λ is the covering homomorphism $\Lambda : SL(2,\mathbb{C}) \to SO_0(3,1)$.

These non-irreducible, mass-independent representations do not correspond to single particle species. Each space of vector bundle cross-sections represents many different particle species. To obtain the mass m, spin-s irreducible unitary representations of $SL(2,\mathbb{C})$ Ⓢ $\mathbb{R}^{3,1}$ in the configuration representation, one introduces linear differential equations, such as the Dirac equation or Klein-Gordon equation, which contain mass as a parameter. These differential equations are imposed upon the cross-sections in the non-irreducible, mass-independent, spin-s representation. Each individual particle species corresponds to cross-sections for a particular value of mass.

In terms of the Wigner representation, first quantization is the process of obtaining a Hilbert space of cross-sections of a vector bundle over $\mathscr{V}_m^{\pm}$. In terms of the configuration representation, first quantization is the two-step process of obtaining a vector bundle over $\mathcal{M}$, and then identifying a space of mass-m solutions.

There are two mathematical directions one can go after first quantization. Firstly, one can treat the Hilbert space obtained as the 'one-particle' state space, and one can use this Hilbert space to construct a Fock space. This is the process of second quantization. One defines creation and annihilation operators upon the Fock space, and thence one defines scattering operators. One can use the scattering operators to calculate the transition amplitudes between incoming and outgoing free states of a system involved in a collision process. Calculation of these transition amplitudes requires the so-called 'regularization' and 'renormalization' of perturbation series, but these calculations do enable one to obtain empirically adequate predictions. Nevertheless, a Fock space is a space of states for a free system. In the configuration representation, the space of 1-particle states is a linear vector space precisely because it is a space of solutions to the *linear* differential equation for a free system.

Although one could use either the Wigner representation or the configuration representation, second quantization conventionally uses a Wigner representation for the one-particle Hilbert spaces.

Given the single-particle Hilbert space $\mathscr{H}_{m,s}$ for a bosonic system, the Fock space is

$$\mathscr{F}_{m,s} = \bigoplus_{n=0}^{\infty} \mathscr{H}_{m,s}^{\odot n} \ ,$$

where $\mathscr{H}_{m,s}^{\odot n}$ is the n-fold symmetric tensor product of $\mathscr{H}_{m,s}$.

Given the single-particle Hilbert space $\mathscr{H}_{m,s}$ for a fermionic system, the Fock space is

$$\mathscr{F}_{m,s} = \bigoplus_{n=0}^{\infty} \mathscr{H}_{m,s}^{\wedge n} \,,$$

where $\mathscr{H}_{m,s}^{\wedge n}$ is the n-fold anti-symmetric tensor product of $\mathscr{H}_{m,s}$.

In both cases $\mathscr{H}^0 = \mathbb{C}^1$, the so-called vacuum sector, containing a distinguished non-zero vector $1 \in \mathbb{C}^1$, called the vacuum vector.

The irreducible, unitary representation of $SL(2, \mathbb{C}) ⓢ \mathbb{R}^{3,1}$ on the single-particle space extends to a unitary representation on the Fock space, albeit a non-irreducible representation.

The other mathematical direction one can go, which conventionally uses the configuration representation, is to treat first-quantization as an end in itself. In the fibre bundle approach, a mass m, spin s particle can be represented by the mass-m cross-sections of a spin-s bundle η. This mass-independent bundle η can, following Derdzinski (1992), be referred to as a *free-particle bundle*. One can associate a vector bundle δ with a gauge field, which can, again following Derdzinski, be referred to as an *interaction bundle*. One can take the free-particle bundle η, and with the interaction bundle δ, one can construct an *interacting particle bundle* α. The mass-m cross-sections of this bundle represent the particle in the presence of the gauge field. This is the route of the first-quantized interacting theory. The first-quantized interacting theory is not empirically adequate, and it is not possible to subject the first-quantized interacting theory to second-quantization because the state space of an interacting system is not a linear vector space; in the configuration representation, the space of states for an interacting 1-particle system consists of vector bundle cross-sections which satisfy a *non-linear* differential equation. Hence, there is no Fock space for an interacting system.

2 Intrinsic states and elementary particles

The state of a physical object is the set of all properties possessed by that object. Let us agree to define an intrinsic property of an object to be a property which the object possesses independently of its relationships to other objects, and let us also agree to define an extrinsic property of an object to be a property which the object possesses depending upon its relationships with other objects. If the value of a quantity possessed by an object can change under a change of reference frame, then the value of that quantity must be an extrinsic property of the object, not an intrinsic property. The value of such a quantity must be a relationship between the object and a reference frame, and under a change of reference frame, that relationship can change.

When the intrinsic state of an object doesn't change, it means that the intrinsic properties of the object don't change. The extrinsic properties of an object, its relationships with other objects, in particular its relationships with a reference frame, can change even if the intrinsic properties of the object don't change. Hence, the intrinsic state of an object can remain unchanged even though the overall state of the object, taking into account its extrinsic properties, does change.

Recall that each free particle corresponds to a unitary representation of the local, external (space-time) symmetry group $SL(2,\mathbb{C}) \circledS \mathbb{R}^{3,1}$. In the 'passive' approach to external symmetries, $SL(2,\mathbb{C}) \circledS \mathbb{R}^{3,1}$ acts upon the set of (local) inertial reference frames. Each $g \in SL(2,\mathbb{C}) \circledS \mathbb{R}^{3,1}$ maps a reference frame σ to a reference frame $g\sigma$. For each type of free particle, the group element g is represented by a unitary linear operator T_g on a Hilbert space. If v is the state of a system as observed from a reference frame σ, then $w = T_g v$ will be the state of the system as observed from the reference frame $g\sigma$. J.M.G. Fell argues that if v and w are a pair of unit vectors in a Hilbert space such that $w = T_g v$ for some $g \in SL(2,\mathbb{C}) \circledS \mathbb{R}^{3,1}$, then "in a sense," v and w, "(or rather the rays through them) describe the same 'intrinsic state'...for the transition from one state to the other can be exactly duplicated by a change in the standpoint of the observer," (Fell and Doran 1988, p30-31).

One can propose that the intrinsic properties of a physical system are those which are invariant under the action of the physical symmetry group, and the extrinsic properties are those which are not. Implicitly assuming that spatial translations belong to the physical symmetry group, Sternberg argues that "we may wish to regard a particle as unchanged if we pick it up and place it down at some other location...Thus, the location of a particle...is not an 'intrinsic' property of a particle," (Sternberg 1994, p148-149). In his well-known text *Symmetry*, Weyl appears to propose that intrinsic properties are those which are invariant under the action of the symmetry group, (1952, p127-133). For example, believing at the time that spatial reflections were physical symmetries, Weyl asserts that the physical symmetry group "contains the reflections because no law of nature indicates an *intrinsic* difference between left and right," (ibid., p129, with my italics).

Weyl, however, appeared to conflate the notion of intrinsic properties with the notion of objective properties, stating that "objectivity means invariance with respect to the group of [physical] automorphisms," (ibid., p132). When Weyl speaks of two congruent squares in the same plane which "may show many differences when one regards their relation to each other," (ibid., p127), he then states that "if each is taken by itself, any objective statement made about one will hold for the other," (ibid., p128). In other words, Weyl believes that identity of intrinsic properties entails identity of objective properties.

If one accepts that the extrinsic properties of objects, the relationships between objects, can also be objective, then one would reject the claim that the only objective properties of objects are those which are invariant under the action of the physical symmetry group. For example, the speed of a particle is an objective extrinsic property of a particle, an objective relationship between that particle and a reference frame. The fact that the speed of a particle is not invariant under a change of reference frame does not entail that the speed of a particle is not an objective property.

Whilst Weyl seems to propose that the intrinsic properties of an object are those which are invariant under the action of the physical symmetry group, Wigner proposed that an elementary particle corresponds to an *irreducible* representation of that symmetry group. In combination, these ideas entail that the intrinsic properties of an elementary particle are those which are invariant under an irreducible representation of the physical symmetry group. The idea which I wish to address in the rest of this paper is a distinct but closely related idea, most clearly expressed by J.M.G. Fell in the introduction to a mathematical text. Fell claims that an elementary particle has only one 'intrinsic' state, (Fell and Doran 1988, p29-32).[1] To claim that an object has only one intrinsic state, means that it can only possess one particular set of intrinsic properties.

This idea can be found elsewhere in the physics and philosophy of physics literature, albeit not necessarily in such an explicit form. For example, Steven French equates state-independent properties such as mass, charge and spin, with intrinsic properties, (French and Rickles 2003, p3 and p11), and equates state-dependent properties with non-intrinsic properties (French and Rickles 2003, p18, and French 2000, Section 4). If one equates intrinsic properties with state-independent properties, then this entails that there can only be one intrinsic state. There is, indeed, a distinction between the state-independent properties of a particle, which define the particle type, and the state-dependent properties of a particle, which are variable for a fixed type of particle; and there are, indeed, some intrinsic properties, such as mass, charge and spin, which are state-independent, but this does not entail that all intrinsic properties are state-independent.

Fell adopts Wigner's notion that the irreducibility of a representation is the defining characteristic of an elementary particle representation, and argues that the group action is "essentially" transitive upon the state space of such a representation. He argues, therefore, that an elementary particle has only one 'intrinsic' state. "It can never undergo any intrinsic change. Any change which it *appears* to undergo (change in position, velocity, etc.) can be 'cancelled out' by an appropriate change in the frame of reference of the observer. Such a material system is called an *elementary system* or an *elementary particle*. The word 'elementary' reflects our preconception that, if a physical system undergoes an intrinsic change, it must be that the system is 'composite', and that the change consists in some rearrangement of the 'elementary parts'," (Fell and Doran 1988, p31). Fell implies that when an elementary system is observed to undergo a change within some reference frame σ, it is the particle's relationship to the reference frame σ which changes, not any of the particle's intrinsic, non-relational properties.

The first objection to this argument is that the unitary, irreducible representations of $SL(2, \mathbb{C}) \circledS \mathbb{R}^{3,1}$ in the one-particle, first-quantized theory can be used to represent stable, composite systems as well as elementary systems. A composite system of spin s and mass m can be represented, in the configuration space approach, by those cross-sections of a spin-s free-particle bundle which provide mass m solutions to the relevant differential equation. The Hilbert space constructed from these cross-sections is, under Fourier transform, the Hilbert

[1] Private communication with R.S.Doran

space for a spin s, mass m particle in the Wigner approach. Thus, a composite system of spin s and mass m can be represented by Wigner's spin s, mass m, unitary, irreducible representation of $SL(2, \mathbb{C}) \circledS \mathbb{R}^{3,1}$. One might conclude from this that the unitary, irreducible representations of $SL(2, \mathbb{C}) \circledS \mathbb{R}^{3,1}$ specify not merely the possible free elementary particles which can exist in a universe, but all the possible free stable particles which can exist in a universe, whether they be elementary or composite. One might also conclude that the irreducibility of a representation of $SL(2, \mathbb{C}) \circledS \mathbb{R}^{3,1}$ does not entail the elementarity of the corresponding particle. Moreover, if irreducibility entails only one intrinsic state, then stable, composite systems would also have only one intrinsic state.

The second objection is that the Fock spaces used to represent elementary particles in the second-quantized theory do not possess irreducible representations of $SL(2, \mathbb{C}) \circledS \mathbb{R}^{3,1}$. This objection is mitigated by the fact that Fock spaces are often interpreted as multi-particle spaces. However, if one were to represent a scattering event between two elementary particles, such as an electron and a photon, one would use a Fock space $\mathscr{F}_{m_e,1/2}$ to represent the electron, a Fock space $\mathscr{F}_{0,1}$ to represent the photon, and each free asymptotic incoming state, and each free asymptotic outgoing state of the joint system, would be represented by a vector in the tensor product $\mathscr{F}_{m_e,1/2} \otimes \mathscr{F}_{0,1}$. The definition of a scattering operator on the tensor product Fock space then enables one to calculate transition probabilities between asymptotic incoming states and asymptotic outgoing states. It is a matter of interpretation whether Fock spaces represent aggregates of elementary particles, (something which could be appropriately dubbed a quantum *field*), or whether they simply provide a 'black-box' instrument for calculating transition probabilities between the incoming and outgoing free states of individual elementary particles.[2]

The argument that irreducibility itself entails only one intrinsic state is flawed anyway. The argument only has plausibility if one thinks in terms of classical particle mechanics. In quantum theory, there is no reason why the irreducibility of a particle representation should entail that there is only one intrinsic state. The field-like aspects of particles in quantum theory make for an infinite-dimensional state space. This is true in non-relativistic quantum mechanics, first-quantized relativistic quantum theory and second-quantized relativistic quantum theory. Because a free elementary particle is represented in the first-quantized theory by an *infinite-dimensional* irreducible representation of $SL(2, \mathbb{C}) \circledS \mathbb{R}^{3,1}$, the finite-dimensional space-time symmetry group cannot act transitively upon the state space. $SL(2, \mathbb{C}) \circledS \mathbb{R}^{3,1}$ is ten-dimensional, hence the orbits of its action on a state-space are, at most, ten-dimensional. Given that the state spaces are infinite-dimensional, this means that there is an uncountable infinity of orbits of the symmetry group $SL(2, \mathbb{C}) \circledS \mathbb{R}^{3,1}$. If one accepts that intrinsic properties are those invariant under the symmetry group, this means that each different orbit corresponds to a different intrinsic state. There are many changes in the state of an elementary particle which cannot

[2]See Chapter Six of Teller (1995) for a more detailed discussion of scattering in quantum field theory.

be cancelled out by a change in observational standpoint. In fact, there is an uncountable infinity of such changes! This is essentially because the state of an elementary particle, (in the first-quantized, one-particle theory), is represented by a field-like object, a cross-section of a vector bundle, and the value of the cross-section can change in an independent fashion at different points of space-time. A change of reference frame, in the special relativistic sense mandated by $SL(2,\mathbb{C}) \circledS \mathbb{R}^{3,1}$, is a more rigid, global transformation. $SL(2,\mathbb{C})$ acts transitively[3] upon the set of one-dimensional subspaces in the typical fibre of a free particle bundle η, but the transformation

$$\psi(x) \mapsto \psi'(x) = \mathscr{D}^{s_1,s_2}(A) \cdot \psi(\Lambda^{-1}(x-a))$$

permits only a global SL-symmetry in each fibre, and a global shift in reference frame, a global shift in the field values assigned to coordinate quadruples. The idea that an elementary particle has only one intrinsic state is destroyed by the infinite-dimensional nature of particle representations in quantum theory.

Mathematically, it is quite possible to introduce an infinite-dimensional group of external symmetries. Each fibre of a free particle bundle η is equipped with an $SL(2,\mathbb{C})$ structure, hence one has an automorphism bundle $SL(\eta)$, consisting of all the automorphisms in each fibre of η. The typical fibre of $SL(\eta)$ is isomorphic to $SL(2,\mathbb{C})$. The space of cross-sections $\mathcal{E} = \Gamma(SL(\eta))$ is the group of vertical bundle automorphisms of η. $\mathcal{E}$ provides an infinite-dimensional group which acts upon the cross-sections of the free-particle bundle η. Given a cross-section $\psi(x)$ of η, and an element $a(x)$ of $\mathcal{E}$, the cross-section is simply mapped to $a(x)\psi(x)$. It seems reasonable to call $\mathcal{E} = \Gamma(SL(\eta))$ a group of external (space-time) symmetries because it provides a double cover of $\Gamma(SO_0(T\mathcal{M}))$, the infinite-dimensional group of local oriented Lorentz transformations. This is the group of vertical automorphisms of the oriented Lorentz frame bundle. The latter consists of all the orthonormal bases $\{e_\mu : \ \mu = 0,1,2,3\}$ of the tangent spaces at all the points of the manifold $\mathcal{M}$, such that each e_0 is a future-pointing, timelike vector, and such that each $\{e_i : \ i = 1,2,3\}$ is a right-handed triple of spacelike vectors. This principal fibre bundle has the restricted Lorentz group $SO_0(1,3)$ as its structure group. A cross-section of the automorphism bundle $SO_0(T\mathcal{M})$ selects a linear isometry of the tangent space at each point, and thereby maps an oriented Lorentz frame at each point into another oriented Lorentz frame.

To reiterate, whilst $SL(2,\mathbb{C}) \circledS \mathbb{R}^{3,1}$ does act upon cross-sections of η as well as the base space, Minkowski space-time $\mathcal{M}$, it does not act transitively upon the space of cross-sections. Given that $SL(2,\mathbb{C})$ acts transitively upon the set of one-dimensional subspaces in the typical fibre of η, and given that the choice of SL-symmetry in the infinite-dimensional group $\mathcal{E} = \Gamma(SL(\eta))$ is locally variable, one needs to take a combination of $\mathcal{E}$ with a group of transformations of the base space $\mathcal{M}$, to obtain a group which does act transitively upon the space of cross-sections. Consider $SL(2,\mathbb{C}) \circledS \mathbb{R}^{3,1}$, treated purely as point transformations of the base space. In this sense, $SL(2,\mathbb{C}) \circledS \mathbb{R}^{3,1}$ consists

[3]Private communication with Shlomo Sternberg

of the 'active' counterparts of the group of transformations between inertial reference frames. The combination of $\mathcal{E}$ with this group acts transitively upon the set of cross-sections in η representing free particle states. Hence, if $\mathcal{E}$ and $SL(2,\mathbb{C})$ Ⓢ $\mathbb{R}^{3,1}$ were both physical symmetry groups of a free elementary particle, then a free elementary particle, (in the first-quantized theory), would only have one intrinsic state.

Recall, however, that a free particle bundle η houses many different particle species. The various $\mathscr{H}_{m,s}$ which are constructed out of cross-sections of η are not invariant under the action of the infinite-dimensional group $\mathcal{E}$. Whilst one particular particle may be represented by the space constructed from the mass m, positive-energy solutions of a differential equation in η, the group $\mathcal{E}$ is more than capable of mapping such cross-sections into objects which solve that differential equation for a different mass value, or which don't solve the equation at all. The automorphism group of each $\mathscr{H}_{m,s}$ is the unitary group $\mathscr{U}(\mathscr{H}_{m,s})$, into which $SL(2,\mathbb{C})$ Ⓢ $\mathbb{R}^{3,1}$ is mapped, as manifested under Fourier transform in the Wigner representation. $\mathcal{E}$ is not a group of automorphisms of any $\mathscr{H}_{m,s}$, even if it is the group of vertical automorphisms of η.

Note that Fell includes changes of velocity, i.e. accelerations, amongst the things which can be cancelled out by a change of reference frame. This implies that Fell is not merely thinking of the transformations between inertial reference frames provided by $SL(2,\mathbb{C})$ Ⓢ $\mathbb{R}^{3,1}$, but general coordinate transformations. It also implies that he considers an interacting elementary particle to only have one intrinsic state. By definition, a free particle cannot undergo acceleration, hence a representation of $SL(2,\mathbb{C})$ Ⓢ $\mathbb{R}^{3,1}$ is quite adequate to define a free particle.

In the case of an interacting, first-quantized, elementary fermion, one forms, in the simplest case, an interacting particle bundle $\eta \otimes \delta$, and in contrast with the free-particle case, one *does* use the infinite-dimensional group of vertical automorphisms of δ as a physical symmetry group. This is a significant difference between external symmetries and internal symmetries. The internal symmetry group is the *infinite-dimensional* group of cross-sections $\mathcal{G} = \Gamma(G(\delta))$ of an automorphism bundle $G(\delta)$. This means that any change in the internal degrees of freedom of an interacting particle, even if the change occurs in an independent fashion at different points in space-time, can be cancelled out by an internal symmetry (gauge transformation). This allows the group of internal symmetries to act transitively upon the infinite-dimensional space of internal states of an interacting particle. The gauge groups $SU(3)$, $U(2)$, $SU(2)$, and $U(1)$ act transitively[4] upon the set of one-dimensional subspaces in the typical fibres of the relevant interaction bundles, and because an internal symmetry is, in each case, a locally varying cross-section of the corresponding $G(\delta)$, the infinite-dimensional group $\mathcal{G} = \Gamma(G(\delta))$ of internal symmetries acts transitively upon the space of internal states of an interacting elementary particle. It is the external degrees of freedom which prevent an elementary particle, free or interacting, from having only one intrinsic state.

[4]Private communication with Shlomo Sternberg

To reiterate, an interacting elementary particle can undergo accelerations, so in addition to $\mathcal{E}$ and $\mathcal{G}$, one would require general coordinate transformations to cancel out all possible changes. Given that the base space $\mathcal{M}$ is Minkowski space-time, one can assume that all the physical reference frames correspond to global charts. The general coordinate transformations between physical reference frames in Minkowski space-time form an infinite-dimensional subgroup of the diffeomorphism group of $\mathbb{R}^4$, $Diff(\mathbb{R}^4)$. The active counterparts of these particular coordinate transformations form an infinite-dimensional subgroup of $Diff(\mathcal{M})$. The groups of vertical bundle automorphisms, $\mathcal{E}$ and $\mathcal{G}$, can be combined with this subgroup of $Diff(\mathcal{M})$. For an interacting elementary particle to have only one intrinsic state, $\mathcal{E}$, $\mathcal{G}$ and this subgroup of $Diff(\mathcal{M})$, would all have to be physical symmetry groups. The fact that this subgroup of $Diff(\mathcal{M})$ is not a physical symmetry group entails that an acceleration is an intrinsic change of state.

Fell claims that a composite object can possess different intrinsic properties at different times, but he also appears to hold what philosophers would call an 'endurantist' notion of the persistence of an object through time. The endurantist position holds that the same whole object is capable of possessing a property at one time, and not possessing that property at another time. One popular endurantist account of change holds that properties which are capable of being possessed by an object at one time, and not being possessed at another time, are properties which are possessed in relation to certain times, (Weatherson 2002, Section 1.1). As Hawley puts it, "objects change by standing in different relations to different times," (Hawley 2004, Section 3). If moments of time correspond to the state of other objects in the universe, then one might argue that properties which are possessed by an object in relation to certain times, must be extrinsic properties. Under this argument, then, the endurantist position entails that all properties capable of change must be extrinsic properties. Under the endurantist view, one might have to concede that the changing properties of all objects, composite or elementary, are extrinsic properties.

To render the notion of variable intrinsic properties consistent with endurantism may require one of the following lines of attack: one might argue that an object can possess an 'internal' clock, hence the claim that an object can only possess a changing property in relation to certain times does not entail that such properties are only possessed by an object depending upon its relationships with other objects. In addition, one might argue that a changing property can be an intrinsic property even if the times at which it is possessed by an object are relationships between that object and other objects in the universe. The intrinsic-ness of a property, one might argue, is not affected by the relationships which are necessary to define the times at which it is possessed.

There is an alternative to endurantism, dubbed the 'perdurantist' view, which holds that an object has temporal parts, and different temporal parts can possess different properties. In particular, under the perdurantist view the different temporal parts can possess different intrinsic properties. On the perdurantist view, the persistence of an object through time is analogous to the extension of an object in space, and the different temporal parts can possess different properties just as much as the different spatial parts of an object can possess different properties, (Hawley 2004, Section 1).

In perdurantism, the ascription of a property to an object at a particular time corresponds to the ascription of a property to a temporal part of a 4-dimensional object. The proposition 'x possesses F at time t' means that 'x' is a 4-dimensional object which has a temporal part 't' possessing the property 'F'. Quentin Smith describes the notion of temporal parts in these terms: "If an object x is a whole of temporal parts, then x is composed of distinct particulars, each of which exists at one instant only, such that whatever property x is said to have at a certain time is [possessed by] the particular (temporal part) that exists at that time," (Smith 1995, p84). With the notion of temporal parts, an object can be defined to undergo change if "one temporal part of x possesses a certain property F at one time and...another temporal part of x does not possess F at another time," (ibid., p84). Smith contrasts the 'temporal parts' notion of change with the endurantist notion of change that "the particular that possesses the property at one time is identical with the particular that does not possess the property at another time," (ibid., p84).

One might claim that any object, composite or elementary, can possess different intrinsic properties at different times. If endurantism cannot be rendered consistent with the notion of variable intrinsic properties, then this, and Fell's claim that only a composite object can possess different intrinsic properties at different times, is inconsistent with endurantism, but consistent with perdurantism.

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