Interpretive Analogies Between Quantum and Statistical Mechanics

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Abstract

Interpretive analogies between quantum mechanics and statistical mechanics are drawn out by attending to their common probabilistic structure and related to debates about primitive ontology and the measurement problem in quantum mechanics.

1 Introduction

That there are conspicuous interpretive analogies between quantum mechanics and statistical mechanics has surely not escaped notice. Yet surprisingly little has been written down concerning them. Indeed, most interpretive work on the two theories has proceeded entirely independently. The aim of this paper is to be rather more explicit in drawing out these analogies than has been so far done in the literature.

There is one relatively well-known exception to these observations however. Several workers in the field of quantum foundations have been motivated to adopt their favored interpretation of quantum theory, Bohmian mechanics, by an approach to classical statistical mechanics which they trace to Boltzmann. The so-called "Boltzmannian" interpretation of statistical mechanics is in many respects similar to the interpretive picture afforded by Bohmian mechanics. Both interpretations, in their most familiar versions, hold that the basic entities of the world are particles moving deterministically in space and time, just as in classical particle mechanics.

Both go beyond classical particle mechanics by introducing probabilities, via either macroscopic states or quantum states respectively. As it happens, since the particle dynamics is deterministic in both cases, the only way that properly probabilistic indeterminism can enter these interpretive pictures is in setting initial conditions or in the realization of a particular deterministic history of particle motions. Accordingly, in statistical mechanics one sees the invocation of an initial "past state" (Albert, 2000) and in quantum mechanics the positing of a "quantum equilibrium" initial state (Dürr et al., 1992).

Yet the possible interpretive analogies between quantum mechanics and statistical mechanics do not end there, and I believe that much insight may be gained by applying ideas developed in one context to the other. (Callender, 2007) offers an example of how this might be done. Exploiting the aforementioned analogy between Boltzmannian statistical mechanics and Bohmian mechanics, Callender applies ideas and strategies from the former to address similar interpretive challenges that arise with the latter.

Rather than approach the two theories' interpretive landscapes interpretation by interpretation, I set forth a novel framework within which these interpretations can be placed. Since it would not be possible to do full justice to all the interpretive possibilities available, this framework must be restricted somewhat. Hence I choose to characterize the probabilistic elements of the two theories ontically. This is reasonable, I think, for it ensures that the probabilities in the theories are objective and physical; other interpretations that purport to be physical and objective can presumably be obtained by appropriately modifying the account I provide here.

My principal claim is that different interpretations of these two statistical theories differ by what is deemed stochastic in the theory. I suggest that the stochasticity of a probabilistic theory can be manifested in a system's initial conditions, its dynamics, or in the realization of its observable properties. This furnishes a nice typology for interpretations of statistical mechanics and quantum mechanics, one where the analogies between corresponding interpretations are made manifest. A number of popular interpretations fit neatly into this framework as well.

This account also has the virtue of making clear how central the role of probability is in interpreting the two theories. Indeed, I argue that a "measurement problem" of sorts can even be contrived for statistical mechanics, in analogy with the famous measurement problem of quantum mechanics, which demonstrates that much of the quantum measurement problem can be understood as a matter of making sense of probabilities in statistical physics. Although I certainly do not want to claim that the quantum measurement problem is only a matter of interpreting probability, I do believe that setting out the analogous problem in statistical mechanics affords us an interesting perspective on how we think about theories. I also show how the considerations raised here intersect with debates over primitive ontology in quantum mechanics.

Thus the plan of the paper is as follows. I begin in §2 with the Boltzmann-Bohm connection, as it is familiar and will be useful in setting the stage for the ensuing discussion. In §3 I motivate and introduce my interpretive framework, situating various interpretations of statistical mechanics and quantum mechanics within it along the way. In §4 I connect the discussion of primitive ontology in quantum mechanics to the interpretive framework of this paper. Finally, in §5 I suggest the aforementioned statistical mechanical measurement problem in analogy with the quantum measurement problem, showing how the interpretations of probability discussed in §3 "solve" the measurement problems of both theories. §6 is a brief conclusion.

2 Boltzmannian-Bohmian Interpretations of Mechanics

The key idea behind Boltzmannian statistical mechanics and Bohmian quantum mechanics, according to their supporters, is that these theories are fundamentally about individual microscopic systems, usually assumed to be collections of particles. Whereas standard "Gibbsian" statistical mechanics is often understood to invoke a fictitious ensemble of microscopic systems, and standard quantum mechanics a "wave function" in place of classical particles or fields, Boltzmannian and Bohmian mechanics are said to have a *clear ontology*. This is thought to be a virtue, for "the first step in the construction of a physical theory is to establish what are the mathematical entities (particles, fields, strings,...) with which one intends to describe physical reality" (Allori and Zanghì, 2004, 1744).

While it may be a stretch to say that particles and fields are mathematical entities, one does certainly represent them with mathematical entities. So let us follow this guidance and begin with a mathematical description of the physical reality which these theories purport. This physical reality is essentially the same as what is described by classical particle mechanics. It is set within the classical conception of Euclidean space Σ and time τ under the appropriate mathematical description. At each instant $t \in \tau$ every particle $i \in \mathbf{N}$ (for $\mathbf{N}$ labeled particles) has some location $x_i \in \Sigma$ in space, which one may describe with a map $(\tau, \mathbf{N}) \to \Sigma$ (so $(t, i) \mapsto x_i$). This map specifies, in effect, all the physical facts of a particulate world.

Mechanics is not usually specified in these terms however, as one generally makes use of a possibility space to specify the kinematics and the dynamics of a system. A configuration $q = (x_1, x_2, \ldots, x_N)$, for example, specifies the location x_i of each particle i at a given time in the abstract configuration space $Q = \Sigma^N$. By combining the previous maps, i.e. $(\tau, \mathbf{N}) \to \Sigma$, together one obtains a new map $\tau \to Q$, so $t \mapsto q = (x_1, x_2, \ldots, x_N)$.

If one models classical spacetime as a four-dimensional manifold $\Sigma \times \tau$, then each particle traces out a continuous trajectory γ_i (the image set of a curve with pre-image τ) in spacetime parameterized by the time coordinate, i.e. a map $N \to \Sigma \times \tau$, so that $i \mapsto \gamma_i$. In the extended configuration space $Q \times \tau$, the set of individual trajectories for all the particles is a single continuous trajectory $\gamma \subset Q \times \tau$, which is built up out of all the particles and times. This (actual) trajectory in extended configuration space, like $(\tau, \mathbf{N}) \to \Sigma$, is also sufficient to capture all the empirical content of conventional classical particle mechanics for a specific system.

In classical mechanics the dynamics of a system is generally assumed to be deterministic. Since deterministic laws in classical mechanics require the specification not only of the position of each particle at a time but also its velocity (or momentum), one must make use of a (time-indexed) phase space Γ_t, which has twice the degrees of freedom of Q: half of them are the positional degrees of freedom and half the velocital or momental degrees of freedom. Deterministic dynamics can be represented as a collection of maps indexed by time t, i.e. for a "microstate" $(q, p) \in \Gamma_t$, $\phi_t : (q, p) \mapsto \gamma$, which is to say that given only a time and a microstate (point in phase space) one may recover the complete trajectories of all the particles. In the usual case the time subscript may be dropped, as the time-indexed phase spaces are isomorphic. In this case there is a bijection between microstates in the time-independent phase space Γ and trajectories in extended phase space $\Gamma \times \tau$.

I introduce the term "microstate" for a state in phase space because in Boltzmannian statistical mechanics one adds to the classical picture above the notion of a macrostate (Frigg, 2008). As microstates are characterized by certain microscopic properties of particles (position and momentum or velocity), macrostates are similarly characterized by macroscopic properties (like temperature). In the Boltzmannian picture they are realized by microstates in the following sense: if M is the collection of the macrostates of some system, then there is a projection map $\pi : \Gamma \to M$ which partitions the phase space Γ into macroregions associated bijectively with the macrostates. Thus, if one knows the macrostate of a system, one does not automatically know its underlying microstate; However, given a microstate the corresponding macrostate of the system is determined. Furthermore, the macrostate dynamics is entirely determined by the microstate dynamics in virtue of this reductive relation: a trajectory ν through $M \times \tau$ is fully determined by the projected microstate dynamics $\pi \circ \phi_t$ (as applied to the phase space of microstates). Much as in classical particle mechanics, a trajectory in the extended phase space is sufficient to capture all the empirical content of Boltzmannian statistical mechanics.

Bohmian mechanics also goes beyond the classical picture, but it adds quantum states in the form of quantum wave functions, which are particular (L^2) complex-valued functions on configuration space $Q \to \mathbf{C}$. The wave function dynamics, which is entirely independent of the particle dynamics, is given by the Schrödinger equation. I again treat the dynamics as a collection of maps indexed by time t: for a quantum state $\psi \in \mathcal{H}$, where $\mathcal{H}$ is the Hilbert space of wave functions, $U_t : \psi \mapsto \iota$, where ι is a trajectory through $\mathcal{H}$ (modulo overall phase factors). Importantly, the particle dynamics in Bohmian mechanics is not given by the classical dynamics from above. Instead it depends only on the particle configuration (so, not on the state in phase space) and on the wave function (the particles are said to be "guided" by the wave function). Thus the particle dynamics of Bohmian mechanics, usually represented by the so-called guidance equation, can be given as a collection of maps indexed by time t and quantum state ψ, i.e. for a configuration $q \in Q$, $\phi_t^\psi : q \mapsto \gamma$. Exactly as in classical particle mechanics, however, a trajectory in the extended particle configuration space is sufficient to capture all the empirical content of Bohmian mechanics. After all, according to Bell (1982, 996) "in physics the only observations we must consider are position observations, if only the positions of instrument pointers."

Putting these two interpretations side by side, one sees that both Boltzmannian statistical mechanics and Bohmian quantum mechanics have the following shared features. First, the basic ontology of the interpretations includes particles moving in space and time—plus some extra structure: macroscopic states or quantum wave functions (which may or may not be part of the basic ontology, depending on how these are interpreted). Second, the dynamics of the particles and extra ontology is fully deterministic. Thus the core of the two theories is evidently classical, with only slight modifications needed to account, respectively, for macroscopic phenomena (temperature, the approach to equilibrium, etc.) or quantum phenomena (spin, entanglement, etc.).

What has been overlooked so far in my presentation is that both theories are also probabilistic, in particular that their empirical content is captured principally by statistics of observables. In statistical mechanics the relevant empirical frequencies are of the macroscopic properties; in Bohmian mechanics they are of particle positions (and other quantum observables that are ultimately reducible to particle positions). Probabilities are implemented in the theory in somewhat different ways, but the statistics come out looking very similar, as we will now see.

In Boltzmannian statistical mechanics probability distributions are associated with macrostates, more accurately with finite-measure macroregions of the microscopic phase space. The initial probability distribution ρ_0 of a system in an initial macrostate M_0 is stipulated to be uniform with respect to the Liouville measure μ associated with the underlying phase space and to have this uniform support only on the macroregion $\pi^{-1}M_0$ identified with the macrostate M_0:

$$\rho_0 = \frac{\mu(\pi^{-1}M_0)}{\mu(\Gamma)}. \tag{1}$$

The dynamics of this probability distribution is fully determined by the deterministic particle dynamics. Using my notation, $\phi_t \circ \rho_0 = \rho_0 \phi_t$.

I have also neglected to mention so far the observables which one uses to calculate the empirical content of the theory. In statistical mechanics these are macroscopic observables, since it is presumed that microstates are epistemically (or at least practically) inaccessible. These should naturally be defined on macrostates, but it is just as well to define them on phase space, since one could easily pull macroscopic observables defined on the macrostates back onto phase space with the projection map π. In Boltzmannian statistical mechanics any observable O should be understood as a map $\Gamma \to \mathbf{R}$. The expectation value of O when the system is in the state with probability distribution $\rho = \phi_t \circ \rho_0$ is given by the following formula:

$$\langle O \rangle_\rho = \int_\Gamma \rho \, O \, \mathrm{d}\mu. \tag{2}$$

The expectation value sums together the values of the observable O in each microstate weighted by the probability associated to that microstate.

In quantum mechanics the wave function plays proxy for the probabilistic content of the theory. The initial probability distribution is assumed by conventional Bohmians to be in "quantum equilibrium". This means the probability distribution ρ_0 on configuration space is such that the probability of a particular configuration is given by the map $q \mapsto |\psi(q)|^2$ for initial wave function ψ, i.e. $\rho_0 = |\psi|^2$. If this is the case, then the guidance equation consistently gives the dynamics of ρ_0 as well, so that one can write the probability distribution at time t as $\rho = \phi_t \circ \rho_0$.[1]

[1] By "equivariance" (Dürr et al., 1992, 855) ρ, determined by pushing forward the particle dynamics flow onto the space of probability distributions, will be equal to $|U_t\psi|^2)$, as determined by the wave function dynamics.

Quantum mechanical observables (like position q) are usually treated as operators on the space of quantum states. The expectation value for an observable O in quantum mechanics is

$$\langle O \rangle_\rho = \int_Q \rho \, O \, \mathrm{d}q. \tag{3}$$

In Bohmian mechanics quantum observables are understood to be ultimately reducible to position measurements, so they are not to be interpreted "realistically" (Daumer et al., 1996). Nevertheless their formal use is completely unobjectionable to the Bohmian. That caveat noted, the similarity between the two theories' empirical content in terms of statistical moments is plainly evident, particularly when described in the formalism chosen here. This of course should not at all be surprising, since both quantum mechanics and statistical mechanics are essentially theories of probabilistic dynamical systems.

Now that the common formal and fundamental ontological structure of the two interpretations is manifest, as well as the empirical content thereof, I turn to a central philosophical issues one is naturally led into by interpreting the two theories in these ways. This issue concerns the introduction of probability into what are otherwise fully deterministic theories. If the particle configurations determine all observable properties, then what can probability be other than a measure of uncertainty over what the actual configurations are? Then again, since the empirical content of the theories is entirely contained in statistics of observables, how can uncertainty be physically relevant to which data are observed? In statistical mechanics such conflicting concerns have been referred to as the "paradox of deterministic probabilities" (Loewer, 2001; Winsberg, 2008; Lyon, 2011). Bohmian mechanics, naturally, suffers from the same worries (Callender, 2007).

Many philosophers interested in the question of how to interpret physical probabilities have been motivated by a "Humean" approach to laws and chances popularized by Lewis (1973, 1981, 1983) and defended by Loewer (1996, 2001, 2004) in the context of statistical mechanics. The Humean is skeptical of strong ontological interpretations of laws, chances, and other things which would inflate ontology beyond simple, "familiar" entities, e.g. localized entities moving around in space and time. Although the Humean accepts that laws, chances, etc. play an important explanatory role in science, he argues that these things objectively reduce to his preferred ontology of occurrent events.

To avoid the conflict between deterministic laws and indeterministic yet objective probabilities in statistical mechanics, Loewer argues that statistical mechanical probabilities are assigned to possible microscopic initial conditions. There is, one might loosely say, initial chances for a system (or the whole universe) to begin in an initial microstate, but that microstate's evolution is ever after deterministic. The Humean of course does not believe that there was an actual "chancy" event which brought about the initial state of the system (or universe). He claims merely that the best, objective systematization of the actual, occurrent facts allows one to describe the world in just this way. Obviously, setting such Humean scruples aside, one can give a realist interpretation of these chances as well.

A similar solution is offered by the Bohmians. Dürr et al. (1992) propose an initial condition for the universe, the already mentioned "quantum equilibrium". In effect, they suppose that the initial chance for the particles of the universe to begin in an initial configuration is given by $|\Psi|^2$, the norm squared of the wave function of the universe.[2] Although, again, one does not have to believe, in a Humean spirit, that there was such an actual chancy beginning to the universe, one may of course do so (Callender and Weingard, 1997).

In both cases, certain ontological assumptions, common to Boltzmannian statistical mechanics and Bohmian quantum mechanics, lead to a similar interpretive issue (and approaches to resolving it). The

[2] Dürr et al. (1992, 1995) have a slightly more sophisticated view than this, invoking the notion of typicality (which Goldstein (2001, 2012) invokes also in statistical mechanics), but I find the available criticisms of "typicality measures" convincing (Frigg, 2009) and so will not discuss the notion here.

common structure of the theories makes this almost inevitable. Presenting things as I have done in this section (and as Bohmian-Boltzmannians more or less do as well) puts the primary emphasis on the ontological assumptions being the basis of the analogies between the two theories. But it is not at all necessary to put the emphasis here. One could also begin with the particular interpretation of the laws and probabilities in the two theories favored by the Bohmian-Boltzmannians, i.e. with the idea that probabilities pertain to the chances of particular initial states or histories of observable properties of the systems described by the theories, such that the post-random trial evolution of the systems can be deterministic. One can then observe that this interpretation allows one to give a realistic interpretation of microstates in the two theories in terms of particles, should one wish to "insist that 'particles' means particles" (Dürr et al., 1995, 137).

Although proponents of Boltzmannian statistical mechanics and Bohmian quantum mechanics take their "clear" ontological starting point as a point in their favor (Allori and Zanghì, 2004), this seems to be so only if one thinks that point particles are somehow intuitive and concrete objects of experience. One may reasonably demur. In any case, whatever one's views on this matter, I urge that a different starting point than the starting point urged by Allori and Zanghì is of considerable philosophical interest as well. Indeed, I argue in the next section that the common probabilistic structure of statistical mechanics and quantum mechanics is at the heart of interpretive analogies between the two theories in a variety of interpretations, independently of considerations of primitive ontology. The particular way that Boltzmannians and Bohmians interpret their theories is, I claim, just a particular choice of how to characterize probabilities.

3 Stochasticity in Physics

Classical statistical mechanics and quantum mechanics are theories of statistical physics. In short, their predictive empirical content is captured by probabilistic statistics. This is in contrast to most classical theories, like particle mechanics, general relativity, and thermodynamics, whose empirical content is not essentially statistical and does not rely on probabilities. The empirical content of a theory is central epistemic importance. Yet this content does not wear its ontological interpretation on its sleeve (despite what Bell suggests about position observations in the previous quotation). Thus, *pace* Allori and Zanghì, from an epistemic point of view the first step in the interpretation of a physical theory is to determine what the empirical content of the theory is, and then determine how that content represents physical reality.

Since this task evidently depends on understanding the role of probability in the theories of interest, something must first be said about the interpretation of probability. In presuming an ontic interpretation of physical probabilities, I suppose that probabilities reflect some element of randomness or chanciness in the world. I emphasize that this is not meant as an endorsement of a particular interpretive position about the "nature" of probabilities, nor do I commit to any particular characterization of ontic probabilities in terms of dispositions, propensities, or what have you. I adopt this point of view only to avoid the morass of debate surrounding traditional interpretations of probability, engaging with which would be unnecessary and distracting. For philosophical reasons, e.g. Humean scruples, one may wish to back away from a fully ontic picture, of course, but that is a separate issue as well.

Given this basic assumption, I suggest that there is only a few distinct options available for incorporating realistic probabilities in quantum mechanics and statistical mechanics (cf. (Maudlin, 2007b)).

As before, I will confine attention to the simplest statistical moment, the mean or expectation value. If a system only had a finite number of states, then its expectation value for some real-valued observable would be the sum of its observable values for these states, each weighted by the probability of each state. Since in statistical mechanics and quantum mechanics one has a continuum of states, an expectation value is an integral.

In classical statistical mechanics the "statistical state" of a system is conveniently taken to be a probability distribution ρ on phase space Γ and an observable is a real-valued function O on phase space. One might equivalently take the full probability measure μ as the statistical state. The probability measure μ associated with phase space Γ is a map from Lebesgue-measurable subsets $\mathcal{L}$ of Γ to the real-number interval $[0, 1]$ which satisfies the usual axioms. It is defined for Lebesgue-measurable sets $U \in \mathcal{L}$ by $U \mapsto \int_U \rho \, \mathrm{d}\mu$, where $\mathrm{d}\mu$ is the natural Liouville measure on the phase space. It is due to this definition $\rho : \Gamma \to [0, 1]$ can stand in for the full probability measure $\mu : \mathcal{L} \to [0, 1]$ as the statistical state of the system.

Recalling formula (2) from the previous section, the expectation value of O when the system is in state ρ is

$$\langle O \rangle_\rho = \int_\Gamma \rho \, O \, \mathrm{d}\mu, \tag{4}$$

where the integral is over all phase space points $x \in \Gamma$. For the moment I wish to resist naively interpreting the points $x \in \Gamma$ as microscopic states, in particular as configurations of particles with specific momenta. To make this interpretation less inviting, let us confine attention to a particular observable $O : \Gamma \to \mathbf{R}$ and look at the probability space associated with its image set $O[\Gamma]$. The Lebesgue measurable sets $\mathcal{L}$ are pushed forward onto $O[\Gamma]$ using O; I denote collection of subsets as $\mathcal{L}_O$. The statistical state ρ is pushed forward onto $O[\Gamma]$ as well using O; the probability distribution ρ_O is just $O_* \rho$. Using the Lebesgue measure $\mathrm{d}\omega$ on $\mathbf{R}$ as a standard of integration, one can incorporate the probability distribution into another integration measure $\mathrm{d}O_\rho$ by setting it equal to $\rho_O \, \mathrm{d}\omega$. One may then write the expectation value of any observable O as

$$\langle O \rangle_{\rho_O} = \int_{O[\Gamma]} \omega \, \mathrm{d}O_\rho, \tag{5}$$

where ω is an observable outcome, i.e. $\omega \in O[\Gamma]$. Thus the expectation value is the integral of each outcome value ω weighted by its probability. All explicit reference to underlying microstates is thereby removed and the empirical content of the theory is manifest, solely in terms of an observable and a statistical state.

This last expression of statistical mechanical expectation values is formally similar to expectation values in quantum mechanics. If one assumes that O is a self-adjoint operator on the Hilbert space $\mathcal{H}$ and ψ is a particular pure quantum state, then the probability distribution ρ_O on the possible observable outcomes $O[\mathcal{H}]$ (the eigenvalues ω of O) is determined by applying the Born rule $\langle \psi, O_\omega \psi \rangle$, i.e. the inner product of ψ and the projection operator O_ω (that projects onto the eigenspace associated with ω) applied to ψ. Again, letting $\mathrm{d}O_\rho$ be the measure incorporating the probability distribution ρ_O, the quantum expectation value is expressible essentially as before:

$$\langle O \rangle_{\rho_O} = \int_{O[\mathcal{H}]} \omega \, \mathrm{d}O_\rho. \tag{6}$$

In conventional interpretations of quantum mechanics there are, of course, no microscopic states interpreted in terms of particles. Quantum mechanics transparently presents us with the empirical content of the theory in the same form derived above for statistical mechanics, namely solely in terms of observables and statistical states.

Putting the empirical content of the two theories in this abstract form may strike some readers as obscurantist. It does, however, make manifest what an expectation value is: a probabilistically weighted average of possible observable outcomes. From this interpretively neutral point one can then ask questions about its representational content, e.g. "what is it that makes observable outcomes a matter of probability?"

Before considering that question, time evolution should also be incorporated into the picture. Let us assume that the states, or, rather, our proxies for states, the probabilistic integration measure $\mathrm{d}O_\rho$, evolve in time. I will represent time evolution with a time evolution map T^t which takes states (probability distributions) to states (probability distributions) with a time step of t. I defer saying more about whether the dynamics is deterministic or indeterministic for the moment. Incorporating time evolution into the expectation value yields

$$\langle O \rangle_\rho^t = \int_{O[\cdot]} \omega \, \mathrm{d}O_\rho^t, \tag{7}$$

where $\mathrm{d}O_\rho^t = T^t \rho_O \, \mathrm{d}\omega$.

Now we may ask the question. In order to answer it, keeping in mind the assumption that probabilities be interpreted ontically, one must identify some aspect of the system as involving an element of chance. *Our* use of statistics for such systems is epistemic, of course, for we do not know precisely what the system will do (although we do know what it will do statistically). Our epistemic use of statistics depends for its success on the relevant probabilities being objective. Assuming that objective probabilities are ontic, i.e. chancy, the objectivity of probabilities is underwritten by it being (in some sense) random what the system will do—with respect to a set of possibilities and in proportion to the probabilities of those possibilities obtaining. These probabilities for outcomes give rise to (more or less) stable frequencies which expectation values and other statistical moments are intended to capture.

Thinking about the elements contained in the expectation value formula, there are, on the face of it, three physical aspects of a statistical system (classical or quantum) that could ground these probabilities: the initial conditions, the dynamics, or the observable properties themselves. Let us consider each of these in turn.

First, if one supposes that what is random about a physical system (as described by these two theories) is its initial conditions and nothing else, then it follows that every observable property O of the system has a determined value for all times ever afterwards: for every observable O there is a history of observable outcomes $\tau \to \mathbf{R}$. Nothing else is left to chance but an initial random trial or chance event. In this case expectation values and other statistical moments do not fully represent the system during its evolution. Everything observable about the system is given by the observable history, which may of course diverge from the statistical state's expectation values. For this reason the statistical state ρ does not represent the actual state of the system.

The statistical state hence represents uncertainty over the actual realized outcome of the trial. One might even suppose that there are deterministically-evolving "hidden variables" which represent the actual state of the system, i.e. determine the observable properties and are also fixed by the random trial. The statistical state in this case represents our (lack of) knowledge of this underlying, actual microstate. The initial statistical state is set by the initial chances, from which it inherits its objectivity, and its dynamics can be derived from the dynamics of the microstates.

This interpretation of stochasticity fits extant interpretations of statistical mechanics and quantum mechanics neatly. In Boltzmannian statistical mechanics the hidden variable are classical mechanical states, and in Bohmian quantum mechanics they are particle configurations. The outcome of the initial chance event is a particular initial particle state. This state evolves deterministically such that at every time all observable properties of the system are determined. As observers, we assume that we know the initial probabilities associated with the initial states, i.e. ρ_0, but not the initial microstate. With ρ_0, however, we can calculate statistics for the system since we know how it evolves in time. In this way, the "initial chance" story makes sense of all the formalism introduced in the previous section, albeit with the proviso that this story may be modified by fictionalizing or otherwise eschewing initial chances (as most interpreters have done).

Second, if one supposes that what is random about a physical system is the operation of its dynamics, then it follows that observable histories are not determined given any part of that history. In this case, since the system's evolution is not assumed to be deterministic, one can allow that the statistical state is the actual state of the system. If one does that, then the dynamical maps T_t incorporated into the expectation value formula are indeterministic—at least some of the time the statistical states of the system evolves indeterministically.

From this interpretive point of view expectation values and other statistical moments do purport to represent the system during its evolution. As this evolution is (partially) random one cannot expect exact agreement between the histories of observable outcomes and the expectations generated by the statistical predictions.

There are many ways this indeterministic evolution could be implemented. Indeed, one might think that the initial chances interpretation is just a special case of this interpretation, where all the indeterminism is located at an initial time. There are several much clearer examples of this interpretation of stochasticity in quantum mechanics and statistical mechanics however. In quantum mechanics approaches with indeterministic dynamics are called collapse theories (Ghirardi, 2016). In the Ghirardi-Rimini-Weber (GRW) approach, for example, quantum states evolve according to the Schrödinger equation most of the time, but the state of the system occasionally experiences indeterministic "collapses" to different quantum states. Other versions allow for continuously indeterministic evolution, e.g. continuous spontaneous localization. In classical statistical mechanics too there are various "stochastic dynamics" approaches that implement an indeterministic evolution of some kind (Uffink, 2007, §7). Although the details of how stochastic mechanisms are incorporated into dynamics vary between quantum mechanics and statistical mechanics (and even within the theories), it is not to my purpose to examine them here; rather it is to draw attention to the manifest analogy between interpretations in the two theories which follow this interpretation of stochasticity.

Third, if one supposes that what is random about a physical system is its observable properties, then it follows again that observable histories are not determined. It is possible to hold on to a deterministic state dynamics however. One lets the maps T_t be deterministic, but conceives of the realization of actual properties of a system as a stochastic process. As in the previous case, a realistic interpretation of statistical moments is possible: statistical states characterize the probabilistic information required to describe the chances of particular property values being realized, although these statistical states evolve deterministically.

It might seem that little differentiates the previous two interpretations. There are some important differences though. In one case the state dynamics is indeterministic, but the observable outcomes are determined by the state (observables are not random variables; states are). In the other case the state dynamics is deterministic, but the observable outcomes are random (observables are random variables, states are not). In deterministic systems there can be no bifurcation like this, as one individuate states by the actual properties of the system. Thus there is an important ontological question to answer about the nature of states and observables for indeterministic systems like those of quantum mechanics and statistical mechanics.

It is less obvious how to identify interpretations which follow this last approach to interpreting stochasticity, although I do believe they exist (or could be developed). In statistical mechanics some approaches appear to conform to the basic idea of deterministic state dynamics with stochastic observables, such as those programs that Uffink (2007) calls "statistical thermodynamics". Also some aspects of Prigogine's or Khinchin's approach (Sklar, 1993) appear to fit with the interpretive philosophy given here. One could also argue that Gibbs' approach to equilibrium statistical mechanics could be readily characterized in these terms, when one drops unrealistic talk of "ensembles" and takes seriously the idea that statistical mechanical states are probability measures. Ontologically speaking, what links all of the mentioned interpretations together is a desire to avoid talking about the nature of microstates, which can easily be done on this interpretation of stochasticity, for it only requires the realistic interpretation of the empirical content of statistical mechanics in terms of statistical states and observable properties.

It is perhaps less obvious which interpretations of quantum mechanics are compatible with this way of conceiving of probabilities. Recall, however, that the many worlds interpretation eschews a primitive ontology of deterministically evolving particles and maintains a deterministic state dynamics. Although most proponents of the interpretation appear to favor a "functional" interpretation of quantum probabilities, it would seem that a realistic interpretation of probability which locates the stochasticity of a quantum system in its observables is possible. What probabilities are in many worlds quantum mechanics has been a matter of much debate (Greaves, 2007; Saunders, 2010), and it would be too much of a diversion to enter into the debate here. I am content merely to suggest that there are interpretations of quantum mechanics which fall under this third category as well, analogous to the programs in statistical mechanics mentioned in the previous paragraph. Other ideas, e.g. an "ensemble" interpretation of quantum mechanics, may also be seen as falling under it too (allowing of course that probabilities may not be treated as chances as I assume in this paper).

These then are three distinct ways of conceiving of the stochasticity of statistical physics, assuming an ontic interpretation of probability. The starting point was looking at the statistics which determine the empirical content of such theories, in particular focusing on the simplest—expectation values. These can be cast in a suggestive form by making use of the common dynamical structures of statistical mechanics and quantum mechanics. On the face of it, stochasticity can be associated with one of three different components. These three components were the initial conditions, the dynamics, and the observables of a physical system. A choice of one of these suggests a particular interpretation (or class of interpretations) of statistical mechanics and of quantum mechanics.

The first leads to Boltzmannian statistical mechanics and Bohmian quantum mechanics, the analogy between which was described in the previous section. Whereas there I began with the common ontological posit of particles moving in space and time, I argued in this section that it is also possible to view the analogy between the two theories as coming from the choice of an interpretation of stochasticity. The second choice leads to another obvious analogy, namely between stochastic dynamics in statistical mechanics and collapse theories in quantum mechanics. The third choice leads to what I will call the stochastic observables interpretation. Although the link of this interpretation with well-established approaches in statistical mechanics and quantum mechanics is less apparent, I suggested that Gibbsian statistical mechanics and many worlds quantum mechanics, among others, could be understood in this way.

4 Primitive Ontology

The interpretive analogies between quantum mechanics and statistical mechanics which I introduced in the previous section should suggest that many interpretive problems in one theory carry over to the other. In this section I give the first of two examples of this by exporting the present debate concerning "primitive ontology" in quantum mechanics (Belot, 2011; Ney and Phillips, 2013; Allori et al., 2014; Esfeld, 2014) to statistical mechanics.

My aim in this section is not to survey the entirety of this debate or favor one view over another. It is to show how substantially the same issues arise in statistical mechanics, once it is recognized that there are similar interpretive options available there. To this end I focus on two problems raised by Belot (2011): the "macro object problem" and what I will call the "extra ontology problem".

An interpretation of quantum mechanics that specifies properties possessed by regions of spacetime has come to be called a primitive ontology (Belot, 2011). The version of Bohmian mechanics described above, according to which the material ontology of a quantum system consists in particles (Esfeld et al., 2014), has a primitive ontology, as do collapse interpretations (Maudlin, 2007a) that include an ontology of "flashes" (Esfeld and Gisin, 2014) or "matter densities" (Egg and Esfeld, 2015). If one supposes that quantum physics provides a fundamental description of reality, then according to proponents of a primitive ontology, it should describe a world of stuff and things moving around in space and time. Interpretations of quantum mechanics that suppose that the basic quantum description of the world is in terms of the quantum state, and that macroscopic reality emerges from particular quantum states, lack a primitive ontology. Among these views are wave function realism (Albert, 2013; North, 2013) and spacetime state realism (Wallace and Timpson, 2010).

That there has been no discussion in this vein in statistical mechanics is owed to the issue being regarded as settled. Insofar as one demands a realist interpretation of statistical mechanics, its ontology is thought to consist of one kind of entity: classical particles. I have argued, however, that the different interpretations of stochasticity imply that there are analogous interpretations of statistical mechanics to the interpretations of quantum mechanics concerning which this debate has arisen. One can therefore ask what the ontology of these interpretations is, as well as how the ontologies of the two theories relate.

So, let us turn to the two problems mentioned above. Firstly, Belot's "macro object problem" poses the challenge of explaining how quantum mechanics provides truth conditions for the assignment of properties to ordinary macroscopic objects. If macroscopic objects are simply composed of microscopic objects and their properties are entirely reducible to the properties of the latter, then interpretations which adopt a primitive ontology, like Bohmian mechanics, would seem to have an important advantage in solving the macro object problem. The specification of microscopic properties in spacetime, after all, is just one relation away from specifying macroscopic properties in spacetime. Interpretations of quantum mechanics without a primitive ontology, by contrast, evidently must furnish a more complicated story for how macroscopic objects possess the properties that they appear to possess.

Let us try to pose the macro object problem in classical statistical mechanics. How does this theory provide truth conditions for the assignment of properties to ordinary macroscopic objects? The usual answer is that macroscopic objects (boxes of gas) are simply composed of microscopic objects (molecules of gas) and their properties are entirely reducible to the latter. To give this answer, however, one must supplement the statistical state with an actual microstate. Just as collapse interpretations and Bohmian mechanics possess a primitive ontology in quantum mechanics which can address the macro object problem, so too can stochastic dynamics and Boltzmannian approaches be described as having a primitive ontology, one which can address the macro object problem in this context. Evidently, other interpretations of statistical mechanics without a primitive ontology will have to furnish a more complicated story in order to address the problem.

Secondly, Belot challenges primitive ontologists in quantum mechanics to make sense (ontologically speaking) of the quantum state. He claims that their interpretations are dualistic insofar as their ontologies apparently have two kinds of entity: the primitive ontology of particles (or what have you) and the quantum state (or wave function). Interpretations of quantum mechanics without a primitive ontology do not have this dualism.

This extra ontology problem can and has been raised in classical statistical mechanics. Recall that the empirical content of the theories I have been discussing, formally and epistemically-speaking, requires only a set of observables, a physical state in the form of a probability measure, and a dynamics for that state. In quantum mechanics this state is the quantum state and in statistical mechanics it is the statistical state. Whether one opts for a primitive ontology or not, interpreters of statistical mechanics should be able to answer the same challenge: how can one makes sense of the statistical state? Indeed, I already showed how one is led to this question in the Boltzmannian and Bohmian interpretations by assuming a primitive ontology; it is the paradox of deterministic probabilities.

Let us go a bit further and examine briefly how one might go about solving the extra ontology problem for dualistic interpretations. Belot provides a few suggestions for interpreting the wave function—it is a field (of sorts), a law (of sorts), or a property (of sorts). Let us suppose that a statistical state in statistical mechanics could be interpretable in the same ways and look at them in turn.

If one thinks of the probability distribution ρ as a field, then in all cases so far discussed it is a field defined on some abstract space, e.g. phase space, therefore making a connection to three-dimensional reality quite difficult. This is also a problem raised against the wave function in quantum mechanics. However it seems to be even more of a problem in statistical mechanics, since, assuming the Boltzmannian primitive ontology picture for example, it is clear that ρ plays no role in the dynamics of the physical particles (as in the paradox of deterministic probabilities).

If one thinks of the statistical state as instead encoding nomological facts, in analogy to interpretations that take the wave function to do so (Callender, 2015), then it is not so clear what nomological facts it is encoding. The main complaint in quantum mechanics is that wave functions are contingent, since the theory allows that there could be different initial wave functions and wave functions also evolve in time. This is unlike legitimate laws which hold with physical necessity, such as Hamilton's equation or the Schrödinger equation (Brown and Wallace, 2005; Belot, 2011). Initial statistical states are also plainly contingent in the same way and for the same reasons.

Finally, if one thinks of the statistical state as encoding dispositional properties of a primitive ontology, then it runs afoul of the microscopic physics (insofar as it is deterministic). In Bohmian quantum mechanics the relevant dispositional properties are the velocities of the particles, since the wave function is responsible for determining them via the guidance equation. In statistical mechanics the particles already possess determined velocities by the microscopic dynamics, so the exact same interpretation is not even possible.

Widening our scope, we can see what consequences the various interpretations of stochasticity have on primitive ontology. If one opts for the initial chances approach, then one can posit a primitive ontology with deterministic dynamics, but then one does not have an easy answer for the extra ontology problem, as suggested by the foregoing brief discussion (which of course is not to say that there is no answer). If one instead opts for the stochastic dynamics approach, then one can still posit a primitive ontology, but it is now possible to characterize the statistical state as encoding dispositional properties since the microdynamics is not necessarily deterministic. Finally, one may opt for the stochastic observables approach, in which case one can characterize the statistical state as encoding dispositional properties of the macroscopic ontology, but then one must face the challenge of the macro object problem (as does stochastic dynamics approach should one eschew supplying a primitive ontology to it).

It will not bear investigating these issues further here. I believe this discussion is sufficient to show, however, how primitive ontology, probability, and determinism of laws become interestingly entangled in statistical physics. The typology of interpretations I have provided in this paper hopefully indicates some of the important conceptual connections relevant to discussing these issues further.

5 The Measurement Problem

The second interpretive issue which I wish to consider briefly in relation to interpreting statistical mechanics is the quantum mechanical measurement problem. Perhaps surprisingly, a similar problem can be raised in statistical mechanics. I will suggest that the measurement problem in quantum mechanics is, at least to a certain extent, a metaphysical problem about how to interpret the stochasticity of the theory, in which case it should be no surprise that there is an analogous "measurement problem" in statistical mechanics. I am quite willing to concede that there is more to the quantum measurement problem than just a problem of interpreting probabilities in the theory. Yet I believe some insight into the problem may be gained by stripping the aspects that are not distinctly quantum mechanical away and drawing out the analogy between quantum mechanics and statistical mechanics more explicitly.

I will make use of a well-known presentation of the measurement problem found in (Maudlin, 1995), the "problem of outcomes". Maudlin argues that the following three plausible-sounding statements are inconsistent in quantum mechanics:

1. The quantum state of a system is complete, i.e. it is sufficient for specifying all of the physical properties of a system.

2. The quantum state always evolves in accord with a deterministic equation, viz. the Schrödinger equation.

3. Measurements of observable properties have determinate outcomes, i.e. measurements reveal that the system possesses definite properties.

The argument for the inconsistency of these claims is familiar. A generic quantum state is a superposition. Assuming 1 and 2 it follows that a system in a superposition will not in general have determinate outcomes. Assuming 1 and 3 it follows that the system cannot have evolved determinstically. Assuming 2 and 3, it follows that the system's state must be supplemented. According to Maudlin, rejecting 1 leads one to Bohmian mechanics; rejecting 2 leads one to collapse interpretations; rejecting 3 leads one to the many worlds interpretation.

Let us attempt a similar argument in the context of statistical mechanics. I claim that the following three statements are inconsistent:

1. The statistical state of a system is complete, i.e. it is sufficient for specifying all of the physical properties of a system.

2. The statistical state always evolves in accord with a deterministic equation, e.g. the Liouville equation.

3. Measurements of observable properties have determinate outcomes, i.e. measurements reveal that the system possesses definite properties.

A generic statistical state is essentially a classical superposition. Assuming 1 and 2 it follows that a system in such a state cannot have determinate outcomes. Assuming 1 and 3 it follows that the system cannot have evolved deterministically. Assuming 2 and 3 it follows that the system's state must be supplemented. According to the argument of this paper, rejecting 1 will lead to Boltzmannian statistical mechanics, rejecting 2 will lead to stochastic dynamics; rejecting 3 will lead to stochastic observables.

Presented this way, the analogy between the two measurement problems is strong. At least in this formulation of the measurement problem, the basic issue is how one interprets the stochasticity of statistical physics. Although there is no doubt that quantum mechanics brings with it new interpretational issues, Maudlin's "problem of outcomes" at least is not one of them, I claim, for the states of statistical mechanics suffer from the same indeterminateness as quantum states, and the possible solutions to the two problems follow essentially the same lines.

Some will be little impressed with this argument. To them the analogy is already obvious and so too the solution. The success of statistical mechanics, as the popular story goes, *demonstrates* that the systems it treats are composed of microscopic entities and bears out the atomic hypothesis. There is no question, according to this dogma, that one should reject the first statement, since systems possess microscopic states and properties, namely those of atoms and molecules.

Given the interpretive possibilities available, I believe these claims are far too sanguine. Metaphysical underdetermination is rife in physics as much as anywhere, and it is best to see what options there are before deciding on which ontology to favor. In any case, even if there was no reason to question the ontological presuppositions of classical statistical mechanics in the distant past, then surely the acceptance of quantum mechanics as our best microscopic physics should lead us to consider what the microscopic entities of classical statistical mechanics *really are*. It is by no means sure that they are classical particles.

Finally, I think it worth remarking that the parallel measurement problems suggest the question of how solutions of one relate to solutions of the other. Although it is possible that one might favor differing solutions for the two theories, it does seem natural to prefer common solutions to both problems, for philosophical reasons or otherwise. Indeed, the Boltzmann-Bohmians see it as an advantage of their position that they have a common primitive ontology in their favored interpretations; it might be seen as an additional virtue that they solve the measurement problems in a consistent way. If it is good wisdom to pursue parallel solutions, then the collapse theorist might look to stochastic dynamics approaches in statistical mechanics, and perhaps doing so might put give a useful perspective on the quantum to classical limit. The same may hold as well for the statistical thermodynamics approach and some version of the many worlds interpretation of quantum mechanics.

6 Conclusion

I have drawn attention to analogies between interpretations of classical statistical mechanics and quantum mechanics centered on an important issue at the heart of these analogies: the interpretation of stochasticity in the two theories. Rather than discuss probability in all its many traditional interpretations, I focused on realist, ontic interpretations of probability, where probabilities are understood to be related to some kind of fundamental randomness in the world. With this interpretational stance, answering the question of what probability does in statistical theories comes down to locating a source of stochasticity in the theory. I cast statistical mechanics and quantum mechanics in a common abstract form, starting from the epistemically significant empirical content of the theories, the statistics of observables. Stochasticity, I argued, could be associated with three aspects of the theories: its initial conditions, its dynamics, or its observable properties. As it turns out, the usual realist interpretations of the two theories can be seen to line up closely with these choices. The most well-known analogy, that between Boltzmannian statistical mechanics and Bohmian quantum mechanics, takes the first option; stochastic dynamics approaches in statistical mechanics and collapse interpretations in quantum mechanics take the second; the third option is unusual in that it has been for the most part overlooked, although I suggested that Gibbsian statistical mechanics and the many worlds interpretation of quantum mechanics might be understood to take this option were one to give their probabilities an ontic interpretation.

To show that this way of thinking about the two theories is fertile I discussed its consequences for two major interpretational issues in quantum mechanics: primitive ontology and the measurement problem. I argued that debates about these issues could be reprised in the context of statistical mechanics. Moving the debate about primitive ontology into statistical mechanics puts to question the nature of a statistical state: is it an object, a law, a property? I also argued that at least one common rendering of the quantum measurement problem translates directly into statistical mechanics, which I took to reveal that to some extent the quantum measurement problem is essentially a problem of making sense of its probabilities. I showed that analogous interpretations of the two theories solve the two measurement problem in essentially the same way. This raises the question of how quantum mechanics and statistical mechanics are related, since one might suppose that these two measurement problems should be solved in the same way for the sake of consistent interpretations.

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