

The Duality of the Universe

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Abstract

It is proposed that the physical universe is an instance of a mathematical structure which possesses a dual structure, and that this dual structure is the collection of all possible knowledge of the physical universe. In turn, the physical universe is then the dual space of the latter.

1 Introduction

The purpose of this paper is to propose that the physical universe, and all knowledge of the physical universe, are related to each other by a mathematical duality relationship. Whilst this proposal was inspired by the hypothesis in Majid (2000, 2007) and Heller (2004a,b) that the physical universe is *self*-dual, the proposal made here suggests, on the contrary, that the dual space of the physical universe possesses a distinct structure to that of the physical universe. The idea proposed in this paper does, however, share with Heller's work the hypothesis that the physical universe is an instance of a mathematical structure. Equivalently, it can be asserted that the physical universe is isomorphic to a mathematical structure (Tegmark, 1998). This doctrine can be dubbed 'universal structural realism'.

In particular, the proposals in this paper draw upon concepts from mathematical category theory, and the notion of a mathematical dual. A category consists of a collection of objects such that any pair of objects has a collection of 'morphisms' between them. The morphisms satisfy a binary operation called composition, which means that you can tack one morphism onto the end of another. In addition, each object has a morphism onto itself called the identity morphism. For example, the category Set contains all sets as objects and the functions between sets as morphisms; the category of topological spaces contains all topological spaces as objects, and has continuous functions as morphisms; and the category of smooth manifolds contains all smooth manifolds as objects and has 'smooth' (infinitely-differentiable) maps as morphisms. However, the definition of a category does not require the morphisms to be special types of functions, and the objects need not be special types of set.

Categories can be related by maps called 'functors', which map the objects in one category to the objects in another, and which map the morphisms in one

category to the morphisms in the other category, in a way which preserves the composition of morphisms. Furthermore, one can relate one functor to another by something called a ‘natural transformation’. Suppose that o is an object in a category A , and suppose that X is a functor from A to another category B , and Y is a functor from A to another category C . X maps o to $X(o)$, an object in category B , and Y maps o to $Y(o)$, an object in category C . A natural transformation N from X to Y defines the image of X in Y . Hence, a natural transformation is defined by a family of maps, $N_o : X(o) \rightarrow Y(o)$, for all the objects o in the category A .

There is a special type of category called a monoidal category \mathcal{C} which is equipped with a unit object $\underline{1}$ and an associative product functor $\otimes : \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$, such that $V \otimes \underline{1} \cong V \cong \underline{1} \otimes V$, for any $V \in \text{Obj}(\mathcal{C})$.

Given a vector space V over a number field \mathbb{F} , the dual space is defined to be the space V^* of linear functionals $\phi : V \rightarrow \mathbb{F}$. V^* is also a vector space over \mathbb{F} , hence one can take the dual of the dual V^{**} . This is actually isomorphic to the original vector space, $V^{**} \cong V$, because each element $v \in V$ defines a linear functional $v : V^* \rightarrow \mathbb{F}$ by the stipulation that $v(\phi) = \phi(v)$.

In the case of a commutative group G , the dual \hat{G} is defined to be the set of homomorphisms $\phi : G \rightarrow S^1$ into the set of complex numbers of unit modulus. This coincides with the set of one-dimensional unitary representations of G , and it transpires that in a general context, the dual space is a space of representations.

In particular, the form of duality proposed in this paper is a generalisation of that established by the Tannaka theorem for non-commutative groups. Given such a group G , the dual object is the category $\Pi(G)$ of finite-dimensional unitary representations, a monoidal category whose product is the tensor product of representations. A representation ϕ of the category $\Pi(G)$ is defined to be such that it associates with each object $T : G \rightarrow \text{End}(V)$ in $\Pi(G)$ an endomorphism of the target space of T ,

$$\phi(T) \in \text{End}(V) .$$

With each $g \in G$, one can define such a representation ϕ_g as follows:

$$\phi_g(T) = T(g) \in \text{End}(V) .$$

The Tannaka theorem demonstrates that the map $g \mapsto \phi_g$ is an isomorphism between G and $\Gamma(\Pi(G))$, the space of all representations of $\Pi(G)$. Hence, taking the space of all representations of $\Pi(G)$ to be the dual of $\Pi(G)$, the dual of the dual is isomorphic to the original group, $\Gamma(\Pi(G)) \cong G$.

2 Epistemology is the dual of metaphysics

Equipped with these concepts, we now proceed to outline the main idea of the paper as a series of propositions. Our first proposition is that:

Conjecture 1 *The structure of the physical universe is such that any two models of this structure are isomorphic. Such a structure corresponds to a category \mathcal{C} containing only one object \mathfrak{U} .*

Our second proposition is that:

Conjecture 2 *The representations of \mathfrak{U} are provided by the collection of functors $\text{Funct}(\mathcal{C})$ from \mathcal{C} into other categories. $\text{Funct}(\mathcal{C})$ is the dual space of \mathfrak{U} .*

This is simply a generalisation of the familiar concept of a linear group representation. One can think of a group G as a category with only one object, in which all the morphisms (the group elements) are isomorphisms. Under this perspective, a linear group representation $T : G \rightarrow \text{End}(V)$ is a functor from the single-object category into the category of vector spaces. Such a functor maps the single object to a vector space, and maps the group elements, (the morphisms of the single object), into the morphisms of the vector space V , (the linear operators $\text{End}(V)$). Generalising this, a linear representation of an arbitrary category is a functor from that category into the category of vector spaces, and a general representation of an arbitrary category is a functor from that category into some other category.

Our third proposition is:

Conjecture 3 *Knowledge of the physical universe corresponds to a representation of the universe, hence all possible knowledge of the universe corresponds to the collection of all possible representations of the universe. The collection of all possible representations of the universe corresponds to the collection of functors $\text{Funct}(\mathcal{C})$, hence the collection of all possible knowledge of the universe corresponds to $\text{Funct}(\mathcal{C})$. The collection of all possible knowledge of the physical universe is the dual space of the universe.*

We shall elaborate on this proposition below. For now, suffice to note that knowledge of the universe corresponds not necessarily to homomorphisms of the structure \mathfrak{U} , or its substructures, but, more generally, to functors into objects in other categories.

Our fourth proposition is that

Conjecture 4 *The dual $\Gamma(\text{Funct}(\mathcal{C}))$ of the dual is isomorphic to \mathfrak{U} .*

Given a collection of functors $\text{Funct}(\mathcal{V})$ of a category \mathcal{V} , a representation of the collection of functors assigns to each functor $T : \mathcal{V} \rightarrow \mathcal{W}$ an object in the target category \mathcal{W} . Specifically, for each object $v \in \text{Obj}(\mathcal{V})$, there is a representation ϕ_v on the collection of functors defined as follows:

$$\phi_v(T) = T(v) \in \text{Obj}(\mathcal{W}) .$$

Thus, for a category \mathcal{V} , the space of representations of the space of representations, is isomorphic to \mathcal{V} .

Our fifth proposition is that

Conjecture 5 *Knowledge of all the knowledge of the physical universe corresponds to a representation of $\text{Funct}(\mathcal{C})$. Given that the collection of all such representations is isomorphic to \mathfrak{A} , it follows that all knowledge of all knowledge of the physical universe is isomorphic to the physical universe itself \mathfrak{A} . Thus, one can re-construct the structure of the physical universe from the collection of all knowledge of the physical universe. The physical universe is the dual space of the collection of all knowledge of the physical universe.*¹

If we take metaphysics to be the most general study of the physical world, and epistemology to be the most general study of all knowledge of the physical world, then epistemology is the most general study of the dual space of the physical world. Epistemology is the most general study of the space dual to that which metaphysics provides the most general study of. In this sense, epistemology can be said to be the dual of metaphysics.

The third proposition, that defines knowledge to consist of mathematical representations of the physical universe by means of functors, is sufficiently general to encompass the various different types of cognitive representation present within our culture. C.S Peirce proposed a tripartite division of representational ‘signs’ into ‘icons’, ‘indices’, and ‘symbols’. Peirce held that icons resemble what they represent, indices are causally connected to what they represent, and symbols are arbitrary labels for what they represent, (see Schwartz 1995, p536-537). Each of these types of representation can be considered to be functors. A physical object, or the state of a physical object, can be represented by a mapping f if either:

1. The object/state is a structured entity \mathcal{M} , which is the domain of a mapping $f : \mathcal{M} \rightarrow f(\mathcal{M})$ defining the representation. The range of the mapping, $f(\mathcal{M})$, will also be a structured entity, and the mapping f will be a homomorphism with respect to some level of structure possessed by \mathcal{M} and $f(\mathcal{M})$.
2. The object/state is an object x in a category \mathcal{C} , and the mapping $f : \mathcal{C} \rightarrow f(\mathcal{C})$ defining the representation is a functor. As a special case, if \mathcal{C} is a set \mathcal{M} , and the object/state is an element $x \in \mathcal{M}$, then the mapping $f : \mathcal{M} \rightarrow f(\mathcal{M})$ defining the representation will simply be a map between sets.

In the first case, \mathcal{M} and $f(\mathcal{M})$ can be treated as objects belonging to different categories, and the mapping f can be treated as the restriction to \mathcal{M} of a functor between those categories.

The first type of representational mapping corresponds mathematically to a homomorphism. A scale-model of a Formula 1 car, or the topological map

¹Note that although it is postulated here that the dual space of the physical universe is distinct from, and non-isomorphic to the physical universe itself, as Majid (2007, p15) points out, given a structure X and its dual \hat{X} , one can form a self-dual structure by simply taking the cartesian product $X \times \hat{X}$.

of the London underground, exemplify this type of representation. The homomorphism between a particular Formula 1 car and its wind-tunnel model is the restriction of a functor between the category of all Formula 1 cars and the category of their wind-tunnel models.

The second type of representational mapping, in contrast, doesn't correspond to a homomorphism between a thing and its representation. In this case, the representational functor or mapping $f : \mathcal{C} \rightarrow f(\mathcal{C})$ can be defined by either (i) an objective, causal physical process, or by (ii) the decisions of thinking-beings.

The primary example of type-i non-homomorphic representation is the perceptual representation of the external world by brain states. Taking the example of visual perception, there is no homomorphism between the spatial geometry of an individual's visual field, and the state of the neuronal network in that part of the brain responsible for vision. Nevertheless, the correspondence between brain states and the external world is not an arbitrary mapping, but a correspondence defined by a causal process involving photons of light, the human eye, the retina, and the human brain. The correspondence exists independently of human decision-making.

The primary example of type-ii non-homomorphic representation is the representation of a physical system provided by a digital computer simulation. A contemporary computer represents a physical system by electronically encoding the numerical representation provided by mathematical physics. Numbers are represented by segments of computer memory called 'words', typically consisting of several bytes. There is no homomorphism between a number and the electronic state of a word of computer memory; each number is merely an element in the domain of a mapping which maps numbers to the electronic states of computer memory. There are many ways to represent a number by the state of a word of computer memory. Moreover, the same electronic states of computer memory can represent things other than numbers, such as character symbols, or images and sounds. The correspondence between numbers and states of computer memory is dependent upon the interpretational decisions taken by the humans who program and operate the simulation.

3 Mental representation

The proposal that knowledge consists of various types of mathematical representation, incorporates the representational theory of the mind (RTM), an approach to the mind-brain relationship which falls under the aegis of functionalism. Let us briefly digress, then, to explain this approach to the mind-brain relationship.

Functionalism is one possible reaction to the identity theory of the mind-brain relationship. The identity theory claims that minds can be reduced to brains in the sense that mental properties, states and processes can be *defined* in terms of brain properties, states and processes. Functionalism rejects this, but endorses the weaker notion of supervenience, which holds that any change in the higher level properties, states and processes of a composite system, must

correspond to a change in the lower level properties, states and processes. The idea is that there can be no difference in the higher-level state of a composite system without a difference in the lower-level state, otherwise there would be a one-many correspondence between the lower-level states and higher-level states. In these terms, the mind clearly supervenes upon the brain: each brain state determines a unique mental state, and each change in mental state requires a change of brain state. However, supervenience does not require the higher-level description to be definable in terms of the lower-level description. In particular, supervenience does not require the properties of the higher-level description to be definable in terms of the properties of the lower-level description. When supervenience is combined with this claim of irreducibility, the resulting credo is often termed ‘emergentism’. Emergent states and properties are higher level states and properties which supervene upon lower level states and properties, but which are not definable in terms of those lower-level properties.

Functionalism accepts that the mind and the brain exist; it accepts that the mind cannot be identified with the brain; and it accepts that the mind supervenes on the brain. Functionalism claims that the mind is a set of functionalities and capabilities, at a higher level of description than the brain. Thus, although functionalism accepts that the mind cannot be identified with the brain, it still contends that the mind can be objectively characterised.

The notion that the identity of the mind is defined by a set of functionalities and capabilities, leads to the notion of substrate-independence, the claim that the mind could supervene upon multiple substrates, of which the brain just happens to be one example. Neurophysiology demonstrates how brain structure supports the functionalities and capabilities of the mind, hence neurophysiology demonstrates how the structure of the brain supports these mental structures. However, functionalism argues that there are multiple substrates which could support such mental structures, hence one cannot identify the mind with the brain.

From the perspective of structural realism, functionalism holds that the mind possesses a structure which cannot be identified with the structure of the brain. In other words, functionalism holds that the structure of the mind is non-isomorphic to the structure of the brain. This is consistent with the proposal that the structure of the physical universe has a dual structure, which is non-isomorphic to the physical universe.

Functionalism holds that a mental state has functional relationships to perceptual stimuli, behavioural responses, and other mental states. i.e., a mental state maps current perceptual stimuli to behavioural responses and the next mental state. Thus, in mathematical terms, one can treat a mental state as a function

$$I \rightarrow S \times O ,$$

where I is the set of input states (the perceptual states), S is the set of mental states, and O is the set of output states, (the behavioural responses). By implication, the set of mental states is then a set of such functions, and this set

presumably possesses some structure.

Functionalism, however, should not be conflated with ‘behaviourism’, which claims that mental states have no ‘internal’ content, and are *nothing but* maps between perceptual stimuli and behavioural responses. Functionalism claims that mental states are a lot more than such maps. We shall now consider two specific functionalist approaches to the mind: the ‘top-down’ approach provided by the conjunction of the representational theory of the mind (RTM) and the computational theory of the mind (CTM), and the ‘bottom-up’ approach of *connectionism*. In particular, we shall attempt to highlight the structural aspects of these approaches.

The RTM attempts to provide a functionalist account of ‘intentional’ mental states. These are states, such as beliefs and desires, in which the attention of the mind is directed towards something, called the ‘content’ of the intentional state. The RTM claims that an intentional mental state is a type of functional state that involves a relationship between the thinker and the symbolic representation of something. If a thinker holds the belief that ‘the cat is on the mat’, then the thinker is held to be in one type of functional relationship to a symbolic representation of ‘the cat is on the mat’. If a thinker holds the desire that ‘the cat is on the mat’, then the thinker is held to be in a different type of functional relationship to the same symbolic representation. Beliefs and desires differ by virtue of the fact that they cause different succeeding mental states and behavioural responses. The RTM considers mental processes such as thinking, reasoning and imagining to be sequences of intentional mental states.

Many advocates of the RTM claim that the mental representations which provide the content of beliefs, desires, and other intentional states, possess an internal structure. They hold that this internal system of representation has a set of symbols, a syntax, and a semantics, collectively termed the language of thought. There are rules for composing the symbols into expressions, propositions, and mental images, hence the content of an intentional state can be said to possess a symbol structure, and one might call this the infrastructure of an intentional state. The computational theory of mind (CTM) is then the conjunction of the RTM with the claim that mental reasoning is the formal, syntactical manipulation of such symbols (Horst 2005). Applying the main proposal in this paper, an intentional mental state belongs to a category of intentional states, which are related by a functor to the things they represent.

In contrast with the CTM, connectionism does not attempt to model the mind by ascribing a structure to intentional states at the outset. Instead, connectionism adopts a ‘bottom-up’ approach, modelling the mind with ‘neural networks’, abstractions from the network of nerve cells and synapses in the human brain. In the connectionist approach, the foremost structures are those possessed by the neural networks, not those possessed by sets of intentional states, or those possessed by the symbols which purportedly compose the contents of intentional states.

A neural network consists of a set of nodes, and a set of connections between the nodes. The nodes in a neural network possess activation levels, the connections between nodes possess weights, and the nodes have numerical rules

for calculating their next activation level from (i) the previous activation level, and (ii) the weighted inputs from other nodes. The pattern of connections and activation levels can be thought to provide a lower-level of description than that provided by sets of intentional states and symbol structures. However, under certain circumstances, the pattern of connections and activation levels is deemed to provide a representation of things, hence the connectionist models of the mind can still be subsumed under the aegis of the RTM. It should also be noted that connectionism (arguably) contrasts with the CTM in the sense that thinking can take place at a sub-symbolic level.

These functionalist accounts of the mind entail that the mind *can* be incorporated into structural realism. They claim that the mind possesses a structure, albeit a higher-level structure than that of the brain. All functionalist accounts accept that the mind supervenes on the brain, but they reject the idea that the higher-level structure can be defined in terms of the lower-level structure. One can accept that the mind is essentially subjective, but one can still hold that it possesses a structure, a distinct structure from the structure of the brain. The structure may well be a non-spatial structure, which cannot be reduced to any of the structures which characterise the brain, but it is, nevertheless, the structure of subjective experience. Intentional mental states are not observable by means of the sense organs, are directly accessible only to their owners, and do not occupy space, yet they nevertheless possess a structure which functionalism and the RTM sets out to capture. The proposal made in this paper entails that this structure is part of the dual structure of the physical universe.

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