

**A PENETRATING QUESTION IN THE HISTORY OF
IDEAS: SPACE, DIMENSIONALITY AND
INTERPENETRATION IN THE THOUGHT OF
AVICENNA***

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What is space? Is it an empty, three-dimensional extension that is distinct from, even ontologically prior to, any bodies that supposedly occupy it, and so is in a sense absolute? Or, conversely, is space in some sense relative to and ontologically dependent upon bodies: either as indicating relations between bodies or as some theoretical fiction, useful for locating bodies *vis-à-vis* one another and describing their spatial relationships? Another question: why is it that two bodies apparently cannot interpenetrate one another, that is, why can they not simultaneously occupy the same spatial coordinates? Is impenetrability some irreducible fact about the physical stuff of our world? Or can it be explained in terms of some more basic feature of our world?

Any one familiar with the history of early modern philosophy and science will not find these questions or similar ones new. Indeed, these questions have a long pedigree extending back at least as far as Plato and Aristotle and then proceeding through the classical period and Middle Ages into the modern era. With respect to the ancient Greek and medieval Latin milieus the variety and philosophical depth of discussion about these questions have been amply testified to in Richard Sorabji's

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magisterial work *Matter, Space and Motion*,¹ for the ancient period, and Edward Grant's equally magisterial work *Much Ado about Nothing*,² for the medieval Latin period. What has not been written is an account of medieval, Arabic philosophers' discussions of these issues. The present paper attempts to fill part of this lacuna by considering Avicenna's thoughts on space, dimensionality and interpenetrability.

The broader historical and philosophical context of Avicenna's discussion concerns providing a proper philosophical and scientific account of place (Gr. τόπος, Ar. *makān*) that began with Aristotle, continued through Aristotle's later Greek commentators and ultimately reached Avicenna. In what follows I do not pretend to present the whole debate surrounding place; rather, I concentrate on one small sub-set of arguments within the broader complex of issues associated with place. This sub-set concerns the notion of interpenetration: first as it relates to physical bodies alone, and second as it relates to a physical body's interpenetrating what one might call 'absolute space', *i.e.*, some self-subsisting three-dimensional extension that, as it were, contains physical bodies and yet is distinct from the bodies it contains.

Towards addressing these issues, in the first section I begin with a brief summary of Aristotle's account of place as well as his and certain later Aristotelians' arguments against the idea that absolute space exists. In the second section I turn to the criticism of Aristotle's view by the Neoplatonic philosopher, John Philoponus, considering primarily his critique of Aristotle's argument against identifying place with space and Philoponus' reason for thinking that there is absolute space. This historical background provides the context for Avicenna's own account of dimensionality, interpenetration and his return to Aristotle's account of place as opposed to the Neoplatonic notion of absolute space. Thus, in the third section I consider Avicenna's refutation and analysis of space as something possessing dimensions that is distinct from any occupying body. Here I reconstruct three arguments that Avicenna uses

¹ Richard Sorabji, *Matter, Space and Motion, Theories in Antiquity and Their Sequel* (Ithaca, 1988).

² Edward Grant, *Much Ado about Nothing* (Cambridge, 1981); and for a summary of the most salient arguments also see his "The principle of the impenetrability of bodies in the history of concepts of separate space from the Middle Ages to the seventeenth century", *Isis*, 69 (1978): 551–71.

to deny that there is anything like absolute space. Of these three, the first two rely heavily upon an Aristotelian outlook that a contemporary audience may not accept; however, Avicenna's final argument invokes an account of conceptual analysis and mathematical objects that has certain affinities with the thoughts of some contemporary modal metaphysicians and mathematical constructivists or intuitionists. In this argument, Avicenna questions the very legitimacy of moving from imagination to real possibility as well as the Platonism that apparently underlies a theory of absolute space, and so, I suggest, he makes a real contribution to contemporary philosophical debates about the nature of space.

I. ARISTOTLE

In book IV of his *Physics*, Aristotle identifies six criteria that a philosophically and scientifically adequate account of place must meet. These are

1. the place of a physical body must be the first thing that surrounds that body;
2. the place must be distinct from the body occupying it;
3. the place must be neither greater nor less than the body occupying it;
4. place must be something separable from the body occupying it, *i.e.*, when a body undergoes local motion, it must move from one place to a different place;
5. place must possess the attributes of 'up' and 'down'; and
6. a body should naturally move to and remain in its proper or natural place.³

Having provided these six criteria for place, Aristotle then considers four candidates for place: either it is a body's (1) matter, (2) its form or shape (*μορφή*), (3) the extension between its extremes (*διάστημα*), *i.e.*, the space it occupies, or (4) the extremes, or, more exactly, the innermost (unmovable) limit of that which surrounds.⁴ Aristotle's argument for his preferred account of place attempts to show that the first three candidates fail to meet one or more of the initial criteria, and then to conclude that place is the fourth candidate, namely, the first unmovable limit of that which surrounds. Aristotle

³ *Physics*, IV 4, 210b34–211a6.

⁴ *Ibid.*, 211b5–9.

quickly eliminates matter and form, since they are neither distinct from the body, and so fail to meet criterion (2), nor separable from the body, and so fail to meet criterion (4).

Aristotle's reasoning for eliminating the third candidate, *i.e.*, the extension or the space occupied by a body, is not so clear. His criticism does not obviously show that extension fails to meet one of the proffered criteria, but to show that one or more philosophical absurdities arise when one identifies place with extension. Unfortunately, there is no consensus, either among ancient or contemporary commentators, exactly how to reconstruct Aristotle's reasoning here.⁵ I quote the passage in full:

If some extension were natural and something remaining, then the places in it would be infinite; for since the water and air exchange positions all the parts in the whole will do the same as all of the water in the vessel. Further, place will be together with the changing thing so that there will be a different place of the place and many places will be together. Further, the place of the part in which it is moved, when the whole vessel exchanges positions, is not different, but is the same; for the air and the water (or the parts of the water) exchange one position for another in that which they are, but not in that place in which they came to be, which is a part of the place that is the place of the whole heavens.⁶

My own suspicion is that the notion of extension Aristotle is considering and attempting to refute here is something like 'absolute extension' or 'space' *in a plenum*. Although the notion of a plenum does not appear to do any work in his refutation of this conception of place, it would distinguish the present notion from the idea of 'absolute void space', *i.e.*, some independent, self-subsisting three-dimensional extension that contains body, but is nonetheless distinct from and in a sense ontologically prior to the bodies that it contains. Still, I take it

⁵ Cf. Themistius, *In Aristotelis Physica paraphrasis*, ed. Heinrich Schenkl (Berlin, 1900), pp. 114, 7–118, 9; Simplicius, *In Aristotelis Physicorum libros quattuor priores commentaria*, ed. Hermann Diels (Berlin, 1882), pp. 574, 19–578, 13; Philoponus, *In Aristotelis Physicorum libros quinque posteriores commentaria*, ed. Hieronymus Vitelli (Berlin, 1888), 553, 12–555, 27, among ancient commentators. Cf. also William D. Ross, *Aristotle's Physics*, A Revised Text with Introduction and Commentary, special edition (Oxford, 1998), pp. 55–6 and 572–4; Hippocrates G. Apostle, *Aristotle's Physics* (Grinnell, IA, 1969), pp. 245–6; Edward Hussey, *Aristotle, Physics* (Oxford, 1983), pp. 115–16; Sorabji, *Matter, Space and Motion*, pp. 76–8; Helen S. Lang, *The Order of Nature in Aristotle's Physics, Place and the Elements* (Cambridge, 1998), pp. 87–90; and Benjamin Morison, *On Location: Aristotle's Concept of Place* (Oxford, 2002), pp. 122–32, among modern commentators.

⁶ *Physics*, IV 4, 211b19–29.

that one advocating extension or absolute space as a thing's place, even in a plenum, probably is thinking of something like an interval or distance between a body's limits, which at the very least is conceptually distinct from any body that happens to occupy that interval or distance. In that case, though, absolute space, at least in principle, would be capable of existing devoid of body, even if it never does so, and so absolute space might be viewed as akin to a void. If extension or absolute space is akin to a void, then this might explain why Aristotle's present argument is so underdetermined; for he will spend the next four chapters (*Physics*, IV 6–9) developing some of his most technical arguments against the idea of a void (κενόν), and some of these arguments, as we shall see, apply to absolute space even in a plenum.

Aristotle's specific argument that I have in mind attempts to show that a space distinct from the bodies that purportedly occupy it, whether that space involves a plenum or void, is superfluous.⁷ Briefly, Aristotle has one consider a wooden cube from which one has abstracted all of its various tangible features except its magnitude. One is then asked to imagine this cube placed in some space. Since the space is not a body, it will not be displaced when the cube enters it, and so the space must penetrate the cube; however, in this case there is no principled way to distinguish the extension of the space from the extension of the cube in it. Thus, concludes Aristotle, "if [the extension] is no different from the place [of the body], then why should one make the place of the body something over and above (παρά) the volume of each [body], if the volume is passive?"⁸ This argument, in one variation or another, would be decisive for many, though not all, of Aristotle's later commentators in their denial that a body's place is the space that it occupies.

Themistius, one of Aristotle's earlier Greek commentators, expanded this argument in his paraphrase of the *Physics*, with his version of the argument playing at least as great a role in later Greek, Arabic and Latin discussions of absolute space as Aristotle's own argument. Themistius' version runs roughly as follows.⁹ He assumes that body is defined as whatever has three

⁷ *Ibid.*, IV 8, 216a26–b21.

⁸ *Physics*, IV 8, 216b13–15.

⁹ Themistius, *In Phys.*, 134, 1–135, 8. Alexander's commentary unfortunately is no longer extant; however, based on quotations in Simplicius' commentary on the

dimensions and necessarily that no two bodies can interpenetrate one another. Now, if there were absolute space, understood as a three dimensional extension, it would necessarily be a body, given the assumed definition of body; however, then, when a body occupies some portion of absolute space, two bodies would interpenetrate one another, a consequence that was hypothesized to be necessarily false. I shall refer to this argument and its variants as the ‘interpenetration argument’.

The argument assumes, presumably as first principles, two propositions: (1) body is whatever is three dimensional, or in other words, tri-dimensionality is a necessary and sufficient condition for being a body; and (2) no two bodies can interpenetrate. Both premises were called into question during the classical period. For instance, certain Stoics maintained that bodies do interpenetrate one another; however, since the Stoic position was to play no discernible role in later debates concerning interpenetration in the Arabic world, and indeed may not even have been known by Arab philosophers, I shall say no more about it. Themistius’ premise, which received by far the most attention, especially among later Neoplatonic commentators, was the claim that three dimensionality is a necessary and sufficient condition for being a body. This premise received some authoritative support from Aristotle, who in both the *Physics* and the *De Caelo*, claimed that body (σῶμα) was whatever is divisible in three dimensions, as well as from Euclid, who defined a solid (στερεόν) as what has three dimensions.¹⁰ Still, this account of body drew heavy fire from later Neoplatonic thinkers, who for various reasons, not always coinciding, wanted to replace Aristotle’s definition of place as the innermost limit of the containing body with a definition that made an object’s location in three-dimensional absolute space its place. Thus, the Neoplatonic philosophers’ preferred course for disarming the interpenetration argument was to deny the premise that tri-dimensionality was a necessary and sufficient condition for being a body.

Physics and Alexander’s own treatise, *De Mixtione*, the ascription of this argument to Alexander seems reasonable; see Simplicius, *In Phys.* 657, 35–658, 12 and Alexander, *De Mixtione*, in *Alexander of Aphrodisias on Stoic Physics*, ed. and trans. Robert B. Todd (Leiden, 1976), V, p. 218, 15–24 and VI, p. 219, 13–22.

¹⁰ Aristotle, *Physics*, III 5, 204b20 and *De Caelo*, I 1, 268a7–10; Euclid, *Elements*, II, defn. 1.

II. PHILOPONUS

It was partially in the spirit of disarming the interpenetration argument that the Neoplatonic philosopher, John Philoponus, drew a distinction between immaterial and material extension.¹¹ One can explain ‘immaterial extension’ in contemporary terms as akin to a Cartesian coordinate system with three axes in which the set of coordinates occupied by an object indicates that object’s place. Inasmuch as this coordinate system might be considered independent of, indeed even ontologically prior to, any object mapped onto it, it represents absolute space. In contrast, ‘material extension’, as opposed to immaterial extension, might be thought of as the bulk or internal dimensions inherent in any given body, and so is never independent of the body of which it is that body’s dimensions.¹²

Philoponus arrived at this distinction by performing a simple thought experiment, which he claims to be no different procedurally than Aristotle’s own method of abstraction by which Aristotle arrived at his distinction between form and matter.¹³ Thus, Philoponus has one think about a certain plenum, for instance, a spherical body. He then has one imagine the elimination of the body and asks, ‘What is left?’ He thinks that one will obviously be imagining an empty, immaterial extension. “For it was evident that [one could] everywhere send out a straight line from the center to the circumference. Thus what is that through which we send out the straight line but empty extension extended in three [dimensions]?”¹⁴ Based upon this thought experiment Philoponus claims that one necessarily

¹¹ For an extremely plausible account of Philoponus’ motivation lying behind his theory of space see David Sedley, “Philoponus’ conception of space”, in Richard Sorabji (ed.), *Philoponus and the Rejection of Aristotelian Science* (London, 1987), pp. 140–53.

¹² That Philoponus’ theory of absolute space was to find fertile ground in Arab lands can be seen in the tenth- and eleventh-century Arab mathematician and scientist Ibn al-Haytham’s treatise on place, *Fī al-makān wa-al-zamān*. In this treatise Ibn al-Haytham reaffirms Philoponus’ position that place is ‘the imagined interval between opposing points’ and repeats Philoponus’ critiques of Aristotle’s position. For the text and French translation of Ibn al-Haytham’s defense of his theory of place and ‘Abd al-Laṭīf al-Baghdādī’s criticism see “Al-Baghdādī critique d’Ibn al-Haytham”, in Roshdi Rashed, *Les mathématiques infinitésimales du IX^e au XI^e siècle*, 4 vols. (London, 2002), vol. IV: *Méthodes géométriques, transformations ponctuelles et philosophie des mathématiques*, pp. 901–53. I am extremely grateful to the anonymous referee for this very rich reference.

¹³ Philoponus, *In Phys.*, 574, 13–575, 20.

¹⁴ *Ibid.*, 574, 17–19.

draws a real distinction between the bulk or volume inherent in a physical body and a finite, three-dimensional extension that, though perhaps never empty of body, considered in itself is self-subsistent and so could in theory exist independent of any body.¹⁵

Let me be clear that the purpose of Philoponus' thought experiment here is *not* to show that such a void space actually exists – it can hardly accomplish this task. He merely wants to show that the notion of absolute space is possible *contra* the conclusion of the interpenetration argument; for if there is a necessary distinction, even a conceptual one, between immaterial and material extensions, then the interpenetration argument no longer can be demonstrative. Philoponus subsequently argues that Aristotle's preferred candidate for place, namely, the innermost limit of the containing body, is itself philosophically and scientifically inadequate and so Philoponus concludes by disjunction from Aristotle's original set of four candidates that place must be identified with extension. Philoponus' further arguments against Aristotle's account of place would take us beyond the scope of the present paper, but suffice it to say that Philoponus thought Aristotle's account of place in terms of an innermost limit of a surrounding body was philosophically untenable.¹⁶

¹⁵ *Ibid.*, 578, 5–579, 18.

¹⁶ Philoponus offers five arguments against Aristotle's account of place, which can in turn be classified under two rubrics: (1) problems connected with place as a surface and (2) problems connected with the place of the heavens and their motion (*In Phys.*, 563, 26–567, 29). Four of the arguments involve ridiculing the suggestion that place is a surface (ἐπιφάνεια), a term that Philoponus surreptitiously introduces into his presentation of Aristotle's account, but that Aristotle himself only uses once in his entire discussion of place and even then only in his presentation of the aporia (*Physics*, IV 1, 209a8). Nowhere, that I can discern, does Aristotle describe his preferred account of place in terms of 'surface'; rather, he describes it as an 'extremity' (ἔσχατον) or 'limit' (πέρας). Given his careful avoidance of the term 'surface', it seems reasonable to conclude that whatever Aristotle might have meant by 'limit', he did not mean surface and certainly not a two-dimensional, mathematical plane surface as Philoponus frequently describes it; for a careful analysis of what Aristotle does mean by a limit see Lang, *The Order of Nature*, pp. 91–113. By far Philoponus' strongest argument is the one concerning the place of the heavens and their motion. The argument takes the form of a dilemma: either the heavens have a place or they do not. If they have a place, then according to Aristotle's definition of place as the innermost limit of a containing body, there must be something outside the heavens that surrounds them, and this both Philoponus and Aristotle deny. In contrast, if the heavens do not have a place, then their diurnal motion cannot be with respect to place; however, the only recognized types of motion are motion

Armed with the distinction between material and immaterial extension, Philoponus attacks the basis of the interpenetration argument. Again the argument is that necessarily whatever has three dimensions must be a body, and so if a body were in a self-subsistent, three-dimensional space, two bodies would interpenetrate one another. Philoponus' response simply is to say that defining body solely in terms of dimensionality, and so making the possession of three dimensions a necessary and sufficient condition for being a body, is not a necessary proposition, in which case the argument employing this premise will not be demonstrative. For if having three dimensions essentially and by definition belonged to being a body, then it would be impossible to conceive that something possesses three dimensions with the elimination of body, and yet this is exactly what the thought experiment had shown one could do. His point might be clearer with an example: human is essentially and by definition a rational animal, and as such one simply cannot conceive that something is a human, if one has eliminated, for instance, animality. Since the same relation does not hold between body and dimensionality, as the thought experiment is intended to show, the two cannot be related essentially and by definition.

Moreover, since the two are not so related, nothing precludes a material extension, *i.e.*, a body, from being in an immaterial extension or absolute space. To maintain otherwise is simply special pleading. Thus, for example, just as an immaterial line can be applied either to another immaterial line or even to a physical edge without issue, and an immaterial plane can be applied either to another immaterial plane or to a physical surface without issue, it would be arbitrary to claim that an immaterial three-dimensional space could not be applied to another three-dimensional extension, whether immaterial or material, *i.e.* a physical body.¹⁷ What makes two bodies incapable of interpenetration, claims Philoponus, is not their dimensionality, but their mutual materiality.¹⁸ The door was now open for identifying place with absolute space, and Philoponus happily entered.

with respect to quantity, quality and place. Clearly the heavens' diurnal motion is not a change in either quantity or quality, and so it must be with respect to place, but it was assumed that the heavens do not have a place.

¹⁷ Philoponus, *In Phys.*, 558, 19–559, 18 and 560, 17–561, 5.

¹⁸ *Ibid.*, 559, 9–18.

III. AVICENNA

This introduction to Aristotle and some of his Greek commentators provides the historical and philosophical background to Avicenna's account of place, dimensionality and interpenetration. Avicenna's account of place comes in his own *Physics*, which roughly follows the arrangement of Aristotle's *Physics*. Moreover, Avicenna had access to both Themistius' paraphrase of the *Physics* and a paraphrastic edition of Philoponus' *Physics* commentary.¹⁹ Thus, he was aware of the classical debate within the Greek Aristotelian tradition surrounding the subject of place. Although Avicenna's own account is in fact a defense of the original Aristotelian position that place is the innermost limit of a containing body, Avicenna's attitude is not due to a deference or subservience to Aristotle. Avicenna arrived at Aristotle's position only after a thoughtful reconsideration of the issues at hand and a belief that the notion of absolute space is ultimately philosophically and scientifically untenable, primarily, though not exclusively, on the grounds of his own revised versions of the earlier classical arguments that we have considered. His version of the interpenetration argument would involve a careful, philosophical defense of the initial premises of the argument in light of the concerns raised by Philoponus. In addition Avicenna would need to undertake an in-depth analysis of matter, which Philoponus had claimed was the true cause of two, extended bodies' not being able to interpenetrate.

Philoponus' strategy again for undermining the interpenetration argument was to mark a real distinction between a material and an immaterial extension. Given this distinction he could maintain that possessing extension was only a necessary condition for being a body, not a necessary *and* sufficient condition. Thus, when a physical body, *i.e.*, something with material extension, occupies a region of absolute space, *i.e.*, something with immaterial extension, two bodies are not

¹⁹ The Arabic edition of Themistius' commentary is no longer extant; however, Philoponus' Arabic commentary is preserved; see *Aristūṭālīs, al-Ṭabī'a*, ed. 'Abd al-Raḥmān Badawī, 2 vols. (Cairo, 1964/65). Paul Lettinck has provided an analysis of this edition of the *Physics* as well as providing correspondences between the Arabic passages ascribed to Yaḥyā and Philoponus' original Greek; see Paul Lettinck, *Aristotle's Physics and Its Reception in the Arabic World* (Leiden, 1994).

interpenetrating one another. What makes a body impenetrable is its matter, which conceptually an immaterial extension must lack. Thus nothing in principle, argued Philoponus, precludes something with immaterial extension interpenetrating something with material extension.

Avicenna ultimately identifies three problems with Philoponus' position. The first problem involves the very distinction that lays at the heart of Philoponus' defense, namely the distinction between material and immaterial extension.²⁰ Avicenna criticizes the distinction in a fashion much like Aristotle's claim that an extension over and above the body's own volume would be superfluous. For Avicenna, the distinction between material and immaterial extension indicates neither a real distinction nor even a formal distinction between the two. At best there is only a rational distinction, or what later Latin scholastics would term a *rationalis ratiocinatis*, i.e., a distinction that has no foundation in reality, but exists solely in the intellect. This strong claim comes from considering the formal accounts of the two supposed extensions or intervals, one material the other not. In both cases, Avicenna maintains, all one means by the interval or extension is a continuity that is susceptible to division between two containing limits.

What we mean when we speak of the individual interval that is between these two things is that it is this indicated continuity between the two that receives the one division. So whatever is between this limit and that limit is this interval that is between the two limits. Thus, whatever is this interval between the two delimiting limits is necessarily a single individual and nothing else, not one interval and another interval. Consequently, between this limit and that limit there is not one interval belonging to the body and another interval.²¹

One should note that the limits Avicenna invokes here are not the limits inherent in the contained body, nor is there any mention that the limits must somehow belong to a body. In short, the limits, Avicenna mentions, are completely general and could be used to delimit either a material or immaterial extension. A concrete example will make clear Avicenna's point: consider a solid sphere. According to Philoponus there is a distinct material extension, which corresponds with the

²⁰ *Šifā'*, *al-Ṭabī'īyyāt*, vol. 1: *al-Samā' al-Ṭabī'ī*, ed. S. Zāyed (Cairo, 1983), I.7, 119.9–120.14. I have also consulted the edition edited by Ġa'far Āl Yāsīn (Beirut, 1996).

²¹ *Ibid.*, 119.15–120.3.

sphere's inherent volume. In addition to which, there is also an immaterial extension, which corresponds with the sphere's place and is distinct from the bodily sphere that occupies it. In both cases, Avicenna claims, the extension is simply the measure between the containing limits. Hence, the two purported extensions are formally one and the same thing; for insofar as one simply considers the sphere's dimensionality, one ignores the specific and accidental forms belonging to the sphere and considers only the extension bounded by its limits.

Avicenna implicitly invokes a principle of individuation for formally identical things, which he states clearly later in the chapter. The principle was hinted at by Aristotle and then stated clearly by Alexander of Aphrodisias, another of Aristotle's early Greek commentators, a principle, I might add, that became a common place in the later Aristotelian tradition.²² The principle states that for numerically distinct objects of the same species, their matter is what distinguishes them.

It is known that the things that agree with respect to [their] natures and that are not divided into kinds by differentiae in their substance are not multiplied because of their essences, but are multiplied only through the multiplication of the materials predicated of them, and when they have identical material, then they cannot be multiplied at all.²³

For instance, Plato and Aristotle are formally identically in that they both have the same species form, humanness. What individuates Plato from Aristotle, then, is that the form of humanness belongs determinately to *this* flesh and bone, *i.e.*, the matter, in the case of Plato, but belongs determinately to *that* flesh and bone in the case of Aristotle. In short, whenever two things are formally identical, but numerically distinct, then it is their matter that individuates them. Philoponus' supposed immaterial extension or absolute space, however, is *ex hypothesi* without matter. Consequently, Philoponus' absolute space cannot in principle be numerically distinct or have numerically distinct properties from the material extension of a body and its properties purportedly occupying the space, since, as Avicenna has shown, the two intervals or extensions are formally identical. In effect, then, there is only one extension, not two.²⁴

²² Alexander of Aphrodisias, *Fī Mabādi' al-kull* (*Alexander of Aphrodisias on the Cosmos*), ed. and trans. Charles Genequand (Leiden, 2001), p. 86.

²³ *Ṭabī'īyyāt*, I.7, 122.13–15.

²⁴ *Ibid.*, 122.8–18.

Avicenna's second critique of Philoponus' absolute space or immaterial extension begins along much the same lines as the above argument.²⁵ Whereas Philoponus maintained that one line can be applied to another and so there can be two lines existing together in the same place, indeed even a myriad of lines interpenetrating one another, Avicenna simply denies that there are multiple lines in the same place.²⁶ There is but a single line for the reason already given, namely, there is no principle by which the purportedly multiple lines could be individuated. The case, however, is different with respect to immaterial and material extensions; for one is supposedly no longer considering mathematical objects, *i.e.*, constructs of the intellect, but two purportedly self-subsisting things that exist independent of the mind and one another. Avicenna denies the possibility of this case by presenting a revised version of the interpenetration argument.

Avicenna begins by assuming as an empirical fact that two physical bodies, or material extensions, cannot interpenetrate, and Philoponus would concur. Thus, in the case of composite bodies, their not interpenetrating must be explained either by reference to their form or matter. Philoponus, we have seen, identified the matter as the cause of the two things' not interpenetrating. Although Avicenna denies that matter is what precludes interpenetration, he agrees with Philoponus that the cause of this impossibility is not a body's specific form; for two bodies would still be prevented from interpenetrating despite the absence of any species-specific properties.²⁷ Now a body thought of as devoid of any species-specific properties might be termed 'absolute body'. Although for Avicenna no absolute bodies exist in the external world, *i.e.*, bodies that are not some definite species, the notion of absolute body is conceivable, *i.e.*, one can think of something as just being a body without simultaneously thinking of it as being of some definite kind. Consequently, one should be able to give an account of why two absolute bodies could not interpenetrate without making reference to any specific or accidental forms that the body must in fact actually have.

Avicenna next considers whether matter alone can account for the impossibility of two absolute bodies' interpenetrating

²⁵ *Ibid.*, 120.15–122.8.

²⁶ *Ibid.*, 121.1–2.

²⁷ *Ibid.*, 121.2–4.

and denies that it can, at least in the proper sense of opposing interpenetration.²⁸ Avicenna's argument against matter's alone precluding interpenetration presupposes a number of points about Avicenna's own notion of matter. Although I cannot hope to do justice to the many intricacies of his view, hopefully the following broad and rather coarse-grained outline will suffice for our purposes.²⁹

Prime matter or matter considered in itself (*hayūlā*; the Greek ὕλη), according to Avicenna (and recent studies suggest that Philoponus would agree),³⁰ is nothing more than a potentiality to receive a form or forms.³¹ Potentialities for Avicenna have no independent existence, which is to say that for Avicenna there are no 'pure potentialities' in the world, but only the potentialities belonging to a certain subject or bearer (*ḥāmīl*) of those potencies.³² It is simply a contradiction to say that there is something actually existing that has no actuality, but is purely potentiality. Consequently, prime matter construed as a potentiality to receive a form or forms likewise cannot be self subsistent;³³ rather, one's understanding of prime matter comes from closely attending to certain real dispositions in the world, where those disposition, when put in the proper causal relations, become actual. This close attention to certain features to the exclusion of others Avicenna terms

²⁸ Avicenna acknowledges that there is a sense in which matter precludes interpenetration, namely, by way of negation, but this is in the sense that sound precludes being seen. It is not that sound actively opposes being seen; rather, there is a category mistake involved (*ibid.*, 121.5–12). Matter, Avicenna shall argue, is simply not the sort of thing of which interpenetration or non-interpenetration can be properly predicated.

²⁹ The most important studies of Avicenna's theory of matter include Arthur Hyman, "Aristotle's 'first matter' and Avicenna's and Averroes' 'corporeal form'", in A. Hyman (ed.), *Essays in Medieval Jewish and Islamic Philosophy* (New York, 1977), pp. 335–56; Elisabeth Buschmann, *Untersuchungen zum Problem der Materie bei Avicenna* (Frankfurt am Main, 1979); and most recently Abraham D. Stone, "Simplicius and Avicenna on the essential corporeity of material substance", in Robert Wisnovsky (ed.), *Aspects of Avicenna* (Princeton, 2001), pp. 73–130.

³⁰ See Frans de Haas, *John Philoponus' New Definition of Prime Matter* (Leiden, 1997).

³¹ The following account is drawn primarily from *Ṭabī'īyyāt*, I.2, 13.4–15.5.

³² See *Šifā'*, *Ilāhiyyāt*, ed. Georges C. Anawati *et al.*, 2 vols. (Cairo, 1960), IV.2, 182.7–15 and *Ṭabī'īyyāt*, III.11, 232.15–234.3 for discussions of potentialities' not being self subsistent.

³³ *Ṭabī'īyyāt*, I.2, 14.12–14.

‘abstraction’.³⁴ Thus in some sense prime matter is a hypothetical limit of a process of abstraction that a full analysis of physical bodies requires. Given prime matter’s close association with potentiality and receptivity, when the matter has any positive or actual determinations, and so is capable of being acted upon or acting in some qualified way, these actions or affections are the result of some actualized form by which the matter becomes possessed of certain positive attributes or powers. Therefore, since matter, considered solely as lacking any form, is only a potentiality to receive forms, and it is the form or forms that give it any positive determinations, matter itself could never actively do anything, such as in the present case oppose interpenetration. Considered in itself matter is indifferent to receiving any possible interval or extension, and so in principle it could even simultaneously receive multiple intervals or extensions.³⁵

According to Avicenna, the first form by which the matter is a body (or perhaps we should say absolute body) is that form by which it has three dimensions.³⁶ This form Avicenna terms the ‘form of corporeality’, which in effect is simply that by which the matter has three dimensions and so is localized and thus is susceptible to receiving other forms. Avicenna tells us:

Thus the form of corporeality is either something prior to the rest of the forms that belong to natural bodies and their genera and species or it is something joined to them and not separate from them. So this [principle] that belongs to the body the way wood belongs to the bed [*i.e.*, the matter] also belongs to the rest of the things possessing those forms in this way,³⁷ since each of them has determinate existence together with the [form of] corporeality in it.³⁸

What is important for us in Avicenna’s discussion of matter is that the matter considered in itself is simply a potentiality to

³⁴ For discussions of abstraction in Avicenna’s psychological works see Farīd Jabre, “Le sens de l’abstraction chez Avicenne”, *Mélanges de l’Université St. Joseph de Beyrouth*, 50 (1984): 281–310; and Dag N. Hasse, “Avicenna on abstraction”, in Wisnovsky (ed.), *Aspects of Avicenna*, pp. 39–72.

³⁵ *Ṭabṛ’iyyāt*, II.7, 121.16–122.2.

³⁶ It would seem that the dimensionality Avicenna has in mind here is not of determinate dimensions, which would follow on the accidental form of quality and so be ontologically posterior to a given species; rather, it is of some bare minimum required for both conceiving a body and a body’s receiving further forms; see Stone, “Simplicius and Avicenna”, pp. 100–1.

³⁷ Reading *bi-hādīhi al-manzila* with Āl Yāsīn for the Cairo’s *li-hādīhi al-manzila*.

³⁸ *Ṭabṛ’iyyāt*, I.2, 14.1–4.

receive forms and as such has no positive determinations or powers. Also, important is that the first form matter receives is the form of corporeality by which it has dimensions.

Bearing in mind this all too brief sketch of the relation between matter and dimensionality, one can better understand Avicenna's revised version of the interpenetration argument. He first considers the cause of why two bodies or material extensions do not interpenetrate, where the bodies are simply two absolute bodies considered in abstraction from any particular species or accidental form. Again, there is nothing untoward in limiting the discussion solely to absolute bodies, since Philoponus also accepts that any two material extensions could not interpenetrate. Based on Avicenna's earlier analysis of matter, however, there will not be two bodies unless the matter is informed by the form of corporeality through which it will have an interval or dimensionality. In this case the bodies are two composites that have matter and the form of corporeality (or tri-dimensionality) as their sole constituents. Since the two bodies considered as composites preclude interpenetration, the cause of their impenetrability must be either one of, or perhaps both of, the composite's constituents, namely, either the matter or the form by which the matter has dimensions. The matter, though, cannot be the cause, since considered in itself it has no positive determinations or powers at all, but is simply susceptible to receiving a form or forms, and so can contribute nothing to explaining the two bodies' being actually impenetrable. Moreover, since the matter plays no part in explaining impenetrability, the explanation of this fact need not require the composite of *both* matter and form. Consequently, the cause of a body's or material extension's precluding interpenetration must be its form of corporeality alone, *i.e.*, that by which it has dimensions or extension.

Avicenna now applies the point drawn from the case of two absolute bodies' not being able to interpenetrate to the case of a material extension's interpenetrating an immaterial extension. Avicenna has already argued that formally the accounts of immaterial extension and material extension are identical: both indicate a continuity that is susceptible to division between two containing limits. Since it is the form of dimensionality that precludes interpenetration, not the matter, and yet both material and immaterial extension have this same form, then an immaterial dimension must also preclude the

interpenetration of a material extension owing to its very dimensionality. Therefore, since the two extensions cannot interpenetrate, an immaterial extension simply never could be the place of a material body as Philoponus wants; for an immaterial extension would always be separate from the body that it was posited to contain as that body's place.

In the original form of the interpenetration argument suggested by Themistius, it was simply assumed as a first principle that a body is defined in terms of possessing three dimensions, and so tri-dimensionality was both a necessary and sufficient condition for being a body. Philoponus responded by drawing a distinction between material and immaterial extension, and so argued that tri-dimensionality is merely a necessary condition, but not a sufficient condition, for being a body. In contrast to the original interpenetration argument, Avicenna's version nowhere invokes the questionable premise that a body is by definition whatever has three dimensions. Instead, Avicenna's version simply makes having the form of tri-dimensionality, and thus corporeality, the first form a body has. He subsequently argues that when one considers the impenetrability of two absolute bodies, *i.e.*, the composites of matter and the corporeal form only, the matter cannot explain why the two cannot interpenetrate, since matter considered in itself is without any positive determinations. Thus, what precludes two bodies' interpenetrating is that very form by which the bodies have dimensions; however, this very form belongs to Philoponus' absolute space or immaterial extension. Hence, if dimensionality is what precludes interpenetration, as Avicenna's argument implies, an immaterial extension, *contra* Philoponus, must preclude a material extension's interpenetrating it, and so space cannot be the place of a body.

This concludes Avicenna's discussion explicitly dedicated to Philoponus' account of absolute space or immaterial extension. For the most part it is a reworking and refinement of Aristotle's and Themistius' earlier arguments. I dare say that Philoponus' view of absolute space has a greater resonance with one's pre-philosophical intuitions about space than Avicenna's critiques with their explicit appeal to an Aristotelian ontology, which not all will accept. Avicenna, however, returns to the subject of absolute space in a different context and provides yet another argument against the very procedure by which Philoponus arrived at his notion of immaterial extension. In

this later discussion, Avicenna does not appeal to points in Aristotelian metaphysics; instead he invokes an account of conceptual analysis and mathematical objects that has certain affinities with the thoughts of some contemporary modal metaphysicians and mathematical constructivists or intuitionists (or perhaps better, ‘anti-Platonist mathematicians’).

Thus we take up Avicenna’s third criticism of absolute space. Consider the procedure by which Philoponus arrived at the possibility of absolute space or immaterial extension. He has one think about some physical body and then imagine the elimination of that body; he then asks what is left. He believes that one will necessarily be thinking of a three-dimensional empty interval, distinct from the body that had occupied it, which he in turn identifies with absolute space or immaterial extension. The validity of this procedure, he tells us, is supported by the fact that it is through such a method of conceptual analysis and abstraction that Aristotle arrived at his distinction between form and matter considered in itself; for matter is something conceptually distinct from form, even if it never in fact exists without form.³⁹

At II.9 of his *Physics*, Avicenna focuses on the method of analysis (*taḥlīl*) used by Philoponus. Avicenna has two complaints about Philoponus’ method of arriving at the possibility of absolute space:⁴⁰ one, his application of the method of analysis involves a step not validly licensed by the very method he purports to adopt; and two, even granting Philoponus the illicit step there is no guarantee that his absolute space has the modal status that he claims it does, namely, as some possible extra-mental, subsisting extension at least capable of existing independent of bodies.

For Avicenna analysis involves a systematic isolation of one feature of an object after another, where one then attends only to that isolated feature of the object *qua* that feature.⁴¹ The *qua*-operator picks out that feature of the object that is the subject of special attention and allows one to concentrate on the properties and powers that the object has *qua* possessing

³⁹ Philoponus, *In Phys.*, 574, 27–575, 15.

⁴⁰ *Ṭabīʿiyyāt*, II.9, 141.8–142.7.

⁴¹ In many salient ways Avicenna’s discussion of analysis parallels comments by Alexander of Aphrodisias in *Quaestio* II.14 concerning ‘abstraction’ (ἀφαίρεσις); for an English translation see *Alexander of Aphrodisias; Quaestiones 1.1–2.15*, trans. Robert W. Sharples (Ithaca, NY, 1992). I am grateful to an anonymous referee for this observation.

that feature and that feature alone. He sums up the function of analysis thus:

Analysis marks a distinction owing to features whose existence truly is in the composite, but are confused in the view of the intellect. Thus some of them are separated from others through their potency and definition, or some of them indicate the existence of something. So when [the intellect] closely attends to the state of some of them, it moves from it to another, where in this case ‘abstraction’ means to ignore some [feature] and turn from it to another.⁴²

Avicenna is quite clear that ‘abstraction’ (*raf‘*) here does not involve the actual elimination (*i‘dām*) of the features not being attended to;⁴³ for the actual elimination of form in the analysis of matter would in fact entail the elimination of the very matter that is being considered, since nothing can actually exist as pure potentiality.⁴⁴ In short, the proper application of analysis only involves isolating certain features for special attention; it does not involve imagining that those aspects to which one is not attending are actually eliminated. Yet, this is exactly what Philoponus does in the final step of his analysis; he eliminates the body and imagines what is left.

Indeed, continues Avicenna, one may be simply imagining the limits of a containing body as what is left, or, if one only considers a single body in isolation from any containing body, then one may be simply imagining the non-existing body, not some void space.⁴⁵ Avicenna here is not making the trivial point that as a matter of happenstance one might have in mind a different object of imagination than what Philoponus imagines; rather, he is making the more substantive point that one might describe that very same object of imagination from a philosophically different vantage point. Philoponus’ analysis, though, required that his thought experiment necessarily lead to a distinction between immaterial and material extension in order to show that absolute space is at least possible. If Avicenna is correct, however, then Philoponus’ distinction

⁴² *Tabī‘iyyāt*, II.9, 142.4–6.

⁴³ Avicenna’s preferred term for abstraction is either *tağrīd* or *tağarrud* particularly in his psychological works; however, he uses other terms as well and in the present context he seems to be viewing *taḥlīl* and *raf‘* as at the very least akin to abstraction. This may be a concession to the Arabic translation of Philoponus’ χωρίζω and its derivatives, especially with reference to Avicenna’s use of *raf‘*; see *Aristūṭālīs, al-Tabī‘a*, p. 395.

⁴⁴ *Tabī‘iyyāt*, II.9, 141.11–14.

⁴⁵ *Ibid.*, 141.15–17.

is not necessary, since the object of imagination in the present case need not be an empty, three-dimensional space. Avicenna's move is isomorphic to the way contemporary metaphysicians argue against the necessity of a given state of affairs: a state of affairs is *not* necessary just in case there is a possible world where the referents of the state of affairs occur, but the state of affairs itself does not obtain. For example, it is not necessary that Aristotle is a philosopher; for there exists a possible world, where the referents exist, namely Aristotle and philosophers, and yet Aristotle is not a philosopher. Similarly, Avicenna's point is that the object of imagination in the thought experiment can equally be described as either the limit of a containing body or a non-existing body, neither of which requires that one is necessarily imagining Philoponus' absolute space. In other words, if Philoponus' thought experiment were to lead to a necessary distinction between material and immaterial extension, as it purports to do, then there should be no competing description of the result of the thought experiment, but there is. In short, nothing in Philoponus' thought experiment requires that the object of imagination necessarily is of a self-subsisting extension existing independent of a body, a point that was needed if Philoponus was to show that absolute space is at least possible.⁴⁶

⁴⁶ Avicenna makes an analogous move in his *Letter to the Vizir Abū Sa'd*, when discussing the reality of the infinite. The infinite, he tell us, is "a certain quantity or something possessing a quantity of which anything you take of it you still find something other than what you have taken and you never reach something beyond which there is not something". He then continues that one may judge that the whole of this quantity exists all at once, in which case there is an actual infinite, or one may judge that it does not exist all at once, but successively, in which case there is a potential infinite. Merely understanding the concept of the infinite, then, does not tell one whether there is an actual infinite or only a potential infinite; what is required in order to decide between these two options, Avicenna concludes, is a rigorous proof that precludes one of them. See *Lettre au Vizir Abū Sa'd*, ed. and trans. (in French) Yahya Michot, *Sagesses musulmanes*, 4 (Beirut, 2000), pp. 27–9.

In many ways Avicenna's section on the reality of the infinite in this letter reads like a detailed exposition of Aristotle's *Physics* III 8. There Aristotle criticized five arguments put forth for the actual existence of the infinite; the most relevant one for the present purposes is that since numbers, geometrical magnitudes and what is thought to be beyond the heavens is never exhausted, these must be an actual infinite (III 4, 203b23–25). Aristotle responds with the truism that the mere conceptualization of a thing does not entail that it is. Avicenna's argument, though, seems to be a stronger one and not the obvious truism of Aristotle, namely, for Avicenna the mere conceptualization of a thing does not entail its *necessary* possibility. Avicenna develops and argues for this thesis, as we shall see below, in both his *Physics* and *Letter to the Vizir Abū Sa'd*.

Avicenna's next complaint grants that even if one is imagining some interval or void space, such an imagining does not guarantee that the object of imagination is really possible.

Despite all of this, let us grant that this interval is something assumed in the estimative faculty (*al-wahm*) when a body or bodies are eliminated, but then on account of what is one aware that this imagining is not wrong to the point that what follows from it is not impossible, and whether this supposition is truly possible to the degree that what does follow from it is necessary? This speaker could demand that the basis of [this claim] is the estimative faculty and whatever the estimative faculty requires is necessary. The matter, though, is not like that; for many of the existing conditions are different from what is imagined.⁴⁷

For Avicenna the estimative faculty or imagination simply need not track real possibility, and this is a point that even Philoponus concedes.⁴⁸ For instance, according to both Philoponus and Avicenna the cosmos is necessarily finite. There is nothing, understood absolutely, other than our universe, and thus it is not philosophically or scientifically appropriate to talk of 'beyond' or 'outside' of the universe. Yet one certainly can imagine 'something' beyond our universe; one even can imagine there is infinite void space beyond our own cosmos, a view that Philoponus himself explicitly disavows.⁴⁹ Consequently, Philoponus' analysis that led to absolute space not only cannot guarantee the necessary distinction between immaterial and material extension, which his conception of absolute space needs, it also cannot insure that the notion of immaterial extension is in fact really possible at all.⁵⁰

I hazard that for Avicenna, absolute space is nothing more than a hypothetical construct closely akin to a mathematical object. Although Avicenna had a deeper respect for mathematics than many of his other fellow medieval Arabic *falāsifa*, he was not a mathematical Platonist, *i.e.*, one who posits a realm

⁴⁷ *Ṭabṛ'īyyāt*, II.9, 142.1–4.

⁴⁸ For a general discussion of the estimative faculty in Avicenna see Deborah L. Black, "Estimation (*wahm*) in Avicenna: The logical and psychological dimensions", *Dialogue*, 32 (1993): 219–58. For a discussion of the limits of the estimative faculty see Avicenna, *Šifā'*, *Kitāb al-Burhān*, ed. 'Abd al-Raḥmān Badawī, 2 edn. (Cairo, 1966), I.4, 18.3–22.

⁴⁹ Philoponus, *In Phys.*, 565, 8–9.

⁵⁰ A similar point again is made in Avicenna's *Letter to Vizir Abū Sa'd*, namely, the account (*ma'nā*) provided by the estimative faculty is not something we need to accept as a real possibility (33).

of mathematical objects ontologically distinct from the physical cosmos. Instead, Avicenna was more closely aligned with an Aristotelian approach to the foundations of mathematics.⁵¹ He saw mathematical objects as mental constructs abstracted from concrete physical objects, where the physical objects themselves set the constraints on the constructions that the mathematician can make.⁵² In one very important respect, then, the science of mathematics is subordinate to the science of physics; for mathematical objects are derived by abstraction from physical objects, *i.e.*, composites of form and matter. As such physics is ontologically prior to mathematics in that if there were no physical objects, there would be nothing from which to abstract mathematical objects. Thus, although the notion of absolute space might provide a useful fiction for locating objects and talking about their spatial relation *vis-à-vis* one another, absolute space cannot provide the ontologically prior conditions for either those things' being located or their spatial relations; for the notion of absolute space is a derived concept abstracted from naturally located bodies, and so presupposes the ontological priority of naturally located, physical objects.⁵³ Thus, for Avicenna:

All of the dimensions that are posited in [the body] between its limits as well as the limits [themselves] and its shapes and positions are features that do not belong to it as subsisting things; rather, they are derived from [the body's] substance.⁵⁴

Unlike Avicenna's earlier arguments, the present argument does not explicitly appeal to an Aristotelian ontology; rather, it questions the very legitimacy of moving from imagination to real possibility as well as the Platonism that apparently underlies a theory of absolute space. Thus, to the degree that one

⁵¹ See Ali A. al-Daffa and John Stroyles, "Ibn Sīnā as a mathematician", in Aydin Sayili (ed.), *İbn Sīnâ, Doğumunun Bininci Yılı Armağanı* (Ankara, 1984), pp. 67–140, especially section VI, "Foundations".

⁵² For discussions of Aristotle's view of mathematical objects see Ian Mueller, "Aristotle on geometrical objects", *Archiv für Geschichte der Philosophie*, 52 (1970): 156–71; Jonathan Lear, "Aristotle's philosophy of mathematics", *The Philosophical Review*, 91 (1982): 161–92; Mario Mignucci, "Aristotle's arithmetic", in Andreas Graeser (ed.), *Mathematics and Metaphysics in Aristotle* (Bern, 1987), pp. 175–211; Edward Hussey, "Aristotle on mathematical objects", *Apeiron*, 24 (1991): 105–31; and Michael White, "The metaphysical location of Aristotle's Μαθηματικά", *Phronesis*, 38 (1993): 166–82.

⁵³ For a discussion of the ontological priority of physical substances to their location see *Šifā', Ilāhiyyāt*, II.3, 72.4–73–11.

⁵⁴ *Ibid.*, II.2, 63.14–15.

finds Avicenna's concerns about Platonism and how to construe modal metaphysics compelling, or at the very least offering up a significant philosophical challenge, so also should one see Avicenna's argument as making a real contribution to contemporary philosophical debates about the nature of space.