LAWS OF NATURE AND THEIR SUPPORTING CASTS

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Abstract. It is an underappreciated fact within the philosophical literature on laws of nature that many scientific laws require the aid of a supporting cast of additional modelling ingredients (such as boundary conditions, material parameters, interfacial stipulations, rigidity constraints, and so on) in order to perform their traditional role in scientific inquiry. In this paper, I suggest that this underappreciated fact spells trouble for some recent reformulations of David Lewis's Best Systems Account (BSA) of laws of nature. Under the auspices of 'pragmatic Humeanism,' several philosophers have recently argued that the criteria of strength and simplicity that lay at the heart of Lewis's original formulation should be replaced with alternatives that are more sensitive to the role that laws play in scientific practice. Although the criteria that these philosophers put forward differ in a variety of ways, they are primarily concerned with the ability of laws to furnish us with predictions and encode information. This, I suggest, is a problem. If it is true that many scientific laws do not *on their own* perform some of the roles with which they are traditionally associated, then they are unlikely in isolation to make meaningful contributions to the predictive strength of a system or encode information about particular systems. Such laws are thus unlikely to end up in the best system, and so these accounts will have trouble conferring lawhood upon them.

When I was a kid, I enjoyed watching some old Disney cartoons that featured Donald Duck and his nephews Huey, Dewey, and Louie. In a few episodes, the somewhat incompetent Donald Duck finds himself serving in the post of Sheriff of Bullet Valley.[1] Although he warns his nephews against joining him on this dangerous adventure, the incorrigible Huey, Dewey, and Louie sneak along nonetheless, intent on helping their bumbling uncle with his various challenges. Donald Duck remains blissfully unaware of the secret machinations of his nephews, despite the fact that they are by and large responsible for the success he encounters in performing his various sheriff-related duties. Although Donald believes he has single-handedly succeeded in restoring law and order to Bullet Valley and its surrounds, the viewer realises that his tenure as Sheriff of Bullet Valley would have proved disastrous without his cabal of helpful nephews.

It strikes me as an underappreciated fact that in the course of scientific inquiry many laws behave much like Donald Duck. On first glance, it might seem that such scientific laws are capable of performing a variety of tasks all on their own (for example, providing us with accurate descriptions of the evolution of various

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[1]I should thank Mark Wilson for reminding me of these old cartoons.

systems through time). Little do we realise, the law's ability to perform such a task owes much to its own supporting cast of helpers. In the case of laws, it is things like boundary conditions, material parameters, interfacial stipulations, rigidity constraints, and so on that serve as the analogues of the helpful Huey, Dewey, and Louie.

In this paper, I will argue that this fact about the way that laws feature in scientific practice spells trouble for some recent formulations of the Best System Account (BSA) of laws of nature. Where David Lewis originally insisted that the laws of nature are the true generalisations that feature in the deductive system that best balances strength and simplicity with respect to an underlying Humean mosaic,[2] several philosophers have recently suggested that these criteria (of simplicity and strength) should be replaced with (or augmented by) others that are more sensitive to the role that laws play in scientific practice.[3] Although the criteria that these philosophers put forward differ in a variety of ways, they are primarily concerned with the ability of laws to furnish us with predictions and encode information. This, I suggest, is a problem. If it is true that many scientific laws do not *on their own* perform some of the roles with which they are traditionally associated, then they are unlikely in isolation to make meaningful contributions to the predictive strength of a system or encode information about particular systems. Such laws are thus unlikely to end up in the best system, and so these accounts will have trouble conferring lawhood upon them.

To be clear from the outset, I do not think that Humean views are *alone* in having difficulty accommodating the way in which laws rely on a variety of supporting cast members. Indeed, it strikes me that Humeans and non-Humeans alike in discussions of laws of nature *generally* overlook the fact that laws are typically only able to perform their familiar roles when embedded in the right kind of modelling environment. To this end, it seems to me that the problems I raise for various Humean accounts of laws in this paper are illustrative of a more *general* oversight in the literature on laws of nature. That being said, there are two good reasons to focus here on Humean accounts. The first is the fact that non-Humean accounts differ more widely from one another in character and structure than do Humean accounts, and so determining exactly *how* and *to what extent* this oversight affects such accounts is a delicate task. The second is that pragmatic Humeans tend to be more explicit than most about exactly how their account relates to the *role* played by laws in scientific practice, and so the problems that arise from considering the role played by constructions like boundary conditions can be seen most clearly in the context of these accounts. For these reasons, this paper will mainly focus on articulating the problems that this supporting cast dynamic raises for pragmatic Humean accounts. Once the shape of this problem becomes clearer, however, we will be in a position to consider how questions about the role and status of boundary conditions might impact our accounts of laws more broadly.

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[2]See Lewis [1973], [1983], [1986], [1994].

[3]In particular, Hicks [2017], Dorst [2019], Jaag and Loew [2020], Wilhelm [2022].

This paper, then, will proceed as follows. In §1, I briefly outline the traditional formulation of the BSA as well as the more pragmatically-inflected alternatives that have been proposed recently. In §2, I focus on a particular kind of construction on which scientific laws regularly rely: boundary conditions. Although in the philosophical literature it is common to see the term 'boundary conditions' employed as though it were more or less synonymous with 'initial conditions,' applied mathematicians and physicists often mean something far more substantial when they talk about boundary conditions.[4] §3 illustrates the essential role played by these more involved boundary conditions in allowing some scientific laws to perform the tasks traditionally associated with them by way of an case-study: the Navier-Stokes Equations. In §4, I outline the general problem that laws such as the Navier-Stokes equations present to the BSA with reference to Lewis's account. In particular, I argue that it is difficult to see how the BSA can render the verdict that the Navier-Stokes equations are, indeed, a law. In §5, I examine how this problem arises for the different attempts to reform the BSA along pragmatic lines by examining the details of the various proposals. In §6, I consider some of the differences between the kinds of boundary conditions required by the Navier-Stokes equations and explain why it is that the Humean cannot avoid the problem raised by simply accepting the verdict that the various boundary conditions turn out to be laws. Finally, in §7, I conclude by suggesting that addressing the problem outlined in this paper may require more radical reform to the BSA than simply providing new criteria for picking out the best system.

§1. The BSA and its Pragmatic Variants. David Lewis originally formulated his Best System Account in terms of the 'Humean mosaic,' which is simply supposed to be the totality of all the particular matters of fact about the universe. The idea is that we might consider various axiomatised deductive systems as attemptes to systematise as many of these particular matters of fact that make up the Humean mosaic as we can. Different systematisations may exhibit different virtues to different degrees. Some may be quite simple, perhaps in the sense that they contain relatively few axioms. Others may be quite strong, in that we can deduce many consequences from the axioms, or in that the axioms rule out many different possible worlds. In reality, Lewis suggests, we should want any systematisation of the mosaic to *balance* these competing virtues. As such, his account holds that a true generalisation is a law of nature if and only if it features as an axiom or theorem of the system that *best balances* the virtues of simplicity and strength. If there turn out to be several such systems, then the laws will be the true generalisations that feature in *all* of the best systems.

[4]It is worth mentioning here that boundary conditions are not the *only* example of the kind of dynamic between laws and supporting contructions that I am highlighting in this paper. For instance, *material parameters* (such as conductivity and viscosity) are constructions that allow us to capture the complex scale-dependent behaviours of some system (often a particular material) such that we may actually apply the relevant continuum-scale laws to that system. They do not simply report the initial values of variables that feature in certain continuum-scale equations. For more details on such material parameters, see Batterman [2013], Batterman and Green [2020], Batterman [2021].

Dorst [2019] helpfully points out that we might distinguish two components here. The first is the thought that the laws are those statements that feature in our best systematisation of something like a Humean mosaic. The second is an actual specification of what makes for the best system (and thus the laws). The idea here is that in evaluating the merits of various candidate systematisations of the Humean mosaic, we attempt to balance certain principles. Dorst calls this second component the 'nomic formula.' Thus Lewis's original nomic formula involves finding the best balance between strength and simplicity.

In recent years, a series of philosophers have suggested that these two components of Lewis's BSA can and should be separated from one another. According to such proposals, we ought to retain a broad commitment to the idea that the laws are those generalisations that feature in the best sytem while replacing Lewis's nomic formula with one that better reflects the role played by laws in scientific practice.[5] Thus Dorst [2019] suggests that the best system is the one with the highest predictive utility, Jaag and Loew [2020] argue that the best system encodes information in a way that is most cognitively useful for creatures like us, Hicks [2017] focusses on the fact that laws must facilitate predictions and explanations and be inferred from repeated experiment, and Wilhelm [2022] adds computational tractability to the list of principles that should appear in our nomic formula.

The general thrust of these recent attempts to reform the BSA is the thought that in developing an account of laws of nature we should pay more attention to the pragmatic role that laws play in scientific practice. As such, many of these recent proposals for alternatives to Lewis's BSA begin by asking a question like: what role do laws *actually play* in scientific practice? Once we have determined the salient role or roles, the thought is that we can adjust our nomic formula to ensure that whatever it is that our account declares the laws to be is capable of playing the role that laws play in scientific practice.

Althought these more pragmatically-inflected versions of the BSA strike me as clear improvements on Lewis's original formulation, the tale of Scrooge McDuck with which we began might indicate that there is a problem lurking here. If it is true that the tasks typically assigned to be performed by laws in scientific practice are actually performed by laws along with a substantial supporting cast, how much of an improvement can we make on the BSA by focussing on 'the role that laws play in scientific practice'? Answering this question will be the focus of the rest of this paper. In the meantime, however, it will be important to meet at least one member of the supporting cast and to see exactly how certain laws rely on them to furnish us with predictions, descriptions, and so on.

[5]That is not necessarily to say that the nomic formula proposed by Lewis has *nothing* to do with the epistemic practice of science. Indeed, he suggests that the system that best balances strength and simplicity "has the virtues we aspire to in our own theory building" (Lewis [1983, 41]). Nonetheless, Lewis restricts himself to a more schematic and abstract characterisation of the epistemic practice of science than would appeal to recent pragmatically-inclined Humeans.

§2. Boundary Conditions. Amongst the various kinds of constructions and modelling ingredients that feature in the supporting casts upon which many laws rely, *boundary conditions* stand out as particularly important and ubiquitous in scientific practice. As a result, it will be helpful to examine in at least some detail the way in which boundary conditions support scientific laws in their traditional tasks. Although boundary conditions are often associated with initial conditions, they play a distinct role in scientific practice and it will be especially important to understand exactly how the two differ from one another.

Indeed, philosophers in the literature on laws of nature (and, indeed, beyond) tend to assume that 'boundary conditions' are more or less *the same* as 'initial conditions.' The following passage from Bhogal and Perry [forthcoming, 17] illustrates this tendency:

> However, the best system is not a *purely* nomic entity. It contains non-nomic boundary conditions as well as laws. The best system is, roughly, the deductive closure of statements which best systematize the facts about the mosaic, balancing simplicity and informativeness. Nothing about that systematization requires that it only include *laws*; it may include contingent things, like the precise boundary conditions. In fact, such intuitively contingent boundary conditions seem like they will be required for the system to be informative. A system where the axioms are only the laws of Newtonian mechanics, for example, would not be particularly informative on its own – it needs the addition of boundary conditions specifying what objects there are, their mass, their velocity, and so on.

As another example, Hicks [2017, 1002] writes that the orthodox BSA "cannot differentiate laws from boundary conditions" before explaining how by contrast his own account delivers a "distinction between initial conditions and laws." That is, the task of differentiating laws from boundary conditions is seen as the same as that of differentiating laws from initial conditions. Jaag and Loew [2020, 2542] consider the question of why scientists distinguish laws proper from "mere boundary conditions," by which they mean information about the coordinates, masses and charges of various particles. By and large, one sees the term 'boundary condition' used either as though it were synonymous with 'initial condition' or as though boundary conditions were a particular *kind* of initial condition.

In reality, initial conditions and boundary conditions are two very different kinds of things. Granted, there may be some specific fields, such as point particle mechanics, within which boundary conditions tend to look very much like initial conditions. However by and large what physicists and applied mathematicians mean by 'boundary condition' is something above and beyond merely fixing the value of some parameter or parameters at some specified time. Boundary conditions in this more substantial sense are constraints on the values that a differential equation must take on the boundary region of the solution space of the relevant problem. They typically arise in the contexts of *boundary value problems* in which a core differential equation must be augmented by additional constraints before it admits of a unique or appropriate solution. These constraints, moreover, must typically apply *at all times t* and not merely at some specified initial or specific time. Indeed, they are often differential equations themselves.

Bursten [2021] helpfully distinguishes between the 'variable fixing' and 'structure specifying' role that such modelling ingredients can play. When philosophers, such as in the passages above, write about 'boundary conditions,' they are typically referring to something like what Hempel [1942, 36] calls 'determining conditions.' These determining conditions are statements that provide the information about the specifics of an event required for some universal hypothesis to properly apply to it. We might know, for instance, that some differential equation captures the way in which certain classes of populations grow, but before we can use that equation to model some *particular* population we need to know the initial population size, relevant growth rate, and so on. These kinds of conditions, then, are contingent facts that specify the value of certain parameters or variable that appear in the general equations for some kind of system.

Distinct from this variable fixing task, however, is the task of specifying the mathematical structure of the boundary of the space on which some differential equation is defined. Performing this *structure-specifying* task requires more than simply plugging in the right kind of values for the parameters of the system at hand (such as "the initial population consisted of n individuals," or specifying masses and velocities of particles, and so on). In many cases, solving the differential equation requires that we impose constraints on how the function that will emerge as a solution to the differential equation can evolve over time in the region of the boundary. For example, we might need to specify how the normal vector or derivative of some velocity field changes over time in certain directions. We will see how this works in more detail shortly, but for now it is simply important to note that without such boundary conditions our original equations often may not possess a 'solution' in any cogent sense.

The general point here is that laws of nature in their differential equation form rely on boundary conditions in a far more substantial fashion than is typically recognised.[6] The ability of some scientific laws to perform their central descriptive and predictive tasks depends on modelling ingredients, such as boundary conditions, which involve more than simply (as Bhogal and Perry write), "specifying what objects there are, their masses, their velocity, and so on." Before we consider whether the BSA is able to handle this fact about the way that laws operate in scientific practice, it will help to see exactly how it is that such boundary conditions play this more expansive structure-specifying role. To this end, we shall in the next section meet the Navier-Stokes equations and the boundary conditions with which they are typically augmented.

[6]There are, of course, exceptions. In addition to Bursten [2021] and Sykora [2019], Mark Wilson [2006], [2017] has repeatedly emphasised the way that conceptual and mathematical differences between boundary conditions and initial conditions are often overlooked. In a similar vein, Wolf and Read [2023] note that boundary conditions play an important structural role in our attempts to evaluate claims of empirical equivalence between dynamical theories.

§3. The Navier-Stokes Equations and Slip Conditions. The *Navier-Stokes equations*[7] are employed in a wide variety of scientific contexts involving fluid flow, such as ocean currents, weather patterns, the motion of water in pipes, blood flow, air flow over the wing of a plane, and so on. They have for quite some time been considered the correct formulation of the laws governing fluid motion. As Hermann von Helmholtz [1873] wrote:

> "As far as I can see, there is today no reason not to regard the hydrodynamic equations [of Navier and Stokes] as the exact expressions of the laws that rule the motion of real fluids."

In particular, the Navier-Stokes equations improved on the previously known Euler equations by correctly formulating the influence of fluid viscosity on fluid motion.[8]

Yet the Navier-Stokes equations on their own do not tell us how individual systems featuring fluid motion will behave. For that, they must be augmented with a variety of boundary conditions, some quite general and others more specific. Most prominently, we require a *slip condition*, which specifies the tangential component of the velocity of a fluid at the surface of flow along the stationary boundary. For instance, how does the contact between the walls of a pipe and flowing water impact the velocity of the fluid along the walls? Without such additional constraints, we are typically unable to solve the equations or find ourselves provided with incorrect values, depending on the system.

Typically, though not always, we must augment the Navier-Stokes equations with a *no-slip* condition, which sets this tangential component of the velocity to zero. In physical terms, this captures the fact that at the fluid-solid interface, the force of attraction between the fluid and solid particles is greater than that between the fluid particles themselves, owing to the fact that the effect of viscosity predominates at the boundary (see Rapp [2017, 244-245] and Schobeiri [2010, 234]). This specification of boundary structure allows the equations to apply in some concrete fashion to real systems. (Something like the no-slip condition is required to explain why dust accumulates on a stationary ceiling fan, for instance.)

It is important to note again that a boundary condition such as the no-slip condition is not simply a mere contingent fact that we plug into the equation expressing the relevant law. Indeed, Sykora [2019] has shown that the no-slip condition in particular is invariant under certain classes of interventions and enjoys quite broad empirical and theoretical support. The no-slip condition does not tell us what the velocity of any particular fluid particle is at any particular time, but rather provides a constraint on the way the velocity of the fluid particles in the boundary region must evolve for all times t. It is this added structure that

[7] One will occasionally see historical references to the singular *Navier-Stokes equation*, but modern terminology has settled on referring to the equations in the plural. Since they are vector equations they can, if necessary, be written as a series of equations in each of the component spatial directions.

[8] For more detail on the equations, what they look like, and what the various terms in them mean, see Moffatt [2015] and Batchelor [2000].

ensures that the task of solving the Navier-Stokes equations amounts to what Jacques Hadamard [1923] famously termed a "well-posed problem." This simply means that the model admits of a unique solution that changes continuously with the initial conditions. Without the inclusion of some kind of slip condition, we would be unable to find a unique solution (or sometimes any solution at all) to the Navier-Stokes equations for fluid systems.

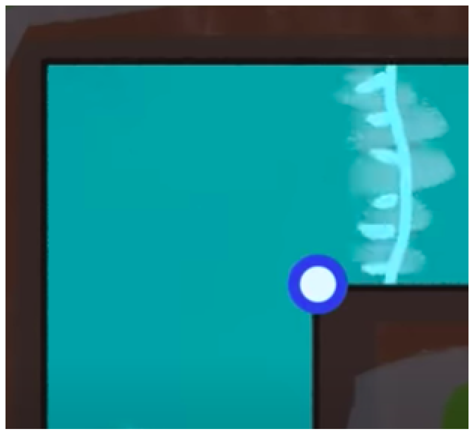

Figure 1. Without the appropriate boundary condition (in addition to the obligatory slip condition), the Navier-Stokes equations will predict that the velocity field diverges to infinity in the indicated inside corner.

In specific cases, other boundary conditions may be required. For instance, if we are interested in the way that fluid behaves after being poured out of a pipe, we require so-called *inlet/outlet conditions* before the Navier-Stokes equations can be properly applied to our system. The form of these conditions depends on the kind of inlet or outlet we have, though often something along the lines of $\frac{\partial u}{\partial t} + \bar{u}\frac{\partial u}{\partial x} = 0$ is required, where $\bar{u}$ denotes an averaged value of the velocity in a particular area. If our fluid flows along a surface that forms a right angle, on the other hand, the Navier-Stokes equations (plus the appropriate slip condition, of course) will predict that the fluid velocity along the inside corner is infinite (see Figure 1). This is a result not of some defect in the equations but rather of a lack of certain pieces of information required by the geometry of the problem. To accommodate such systems, we must also augment the equations with a *Neumann boundary condition* which specifies the derivative of the velocity at that point of the boundary.[9]

[9]More precisely, such additional boundary conditions serve as a kind of prerequisite for the numerical techniques we use to tame the singularities that the Navier-Stokes equations contain in cases involving sharp corners. Applied mathematicians and physicists commonly deal with singularities in the core differential equations of their model by way of a variety of numerical and 'semi-analytic' methods. In cases involving sharp corners, we must imposed further boundary conditions on our flow before we can employ such methods to extract information from the

In many circumstances we may be able to *apply* some set of equations very comfortably to some system without needing to *solve* them in their entirety; that is, without possessing an explicit *solution*. We may do this because it is extremely difficult to get our hands on explicit solutions, and so we might approximate a solution or treat the equations numerically or something along those lines. The sense in which the Navier-Stokes equations without the boundary conditions do not admit of a solution is different than this. Without the boundary conditions the application of the Navier-Stokes equations to a particular system will likely not amount to a *well-posed problem*. In such cases there is no sensible solution to be approximated or treated numerically in the first place. So when I say that certain boundary conditions are integral to our ability to solve the Navier-Stokes equations, I mean this in the sense that without the boundary conditions we do not even have a well-posed problem to solve, rather than the sense that the boundary conditions help us to apply the Navier-Stokes equations by allowing us to get our hands on *actual solutions*.

One final point is important here. In this context, whether the Navier-Stokes equations can be 'solved' in some case or another is not merely a matter of computational tractability. Even in more ideal cases the equations are often extremely intractable and must be handled using a complex toolkit of numerical methods and approximations developed by applied mathematicians. In the above cases, the Navier-Stokes equations themselves do not, without the appropriate boundary conditions, possess the right kind of structure to ensure that sensible solutions exist. This cannot be rectified simply by finding the correct information to add to the equations themselves. The Navier-Stokes equations *are* the correct laws for describing the motion of viscous fluid, and they can be verified as such by both empirical and theoretical considerations. The moral here is this: simply because some statement is a physical law of nature does not necessarily guarantee that it can be applied to any system at all without the addition of the boundary structure appropriate to that system.

§4. Boundary Conditions and the BSA. Why, then, might the way in which laws like the Navier-Stokes equations rely on boundary conditions present a problem for the BSA? The rough idea is that unless the required boundary conditions are included in our candidate system, the Navier-Stokes equations are unlikely to make any meaningful contribution to the strength of our system. In such a case the Navier-Stokes equations would be unlikely to end up in the best system and thus unlikely to come out as laws. On the other hand if we *do* include the boundary conditions in our system, then we run the risk of conferring lawhood on the entire supporting cast. It will perhaps help to see how this problem plays out for Lewis's 'strength and simplicity' formulation of the BSA and then think about the more pragmatic formulations that have appeared in recent years.

Navier-Stokes equations regarding our system. For examples of this approach, see Gupta, Manohar, and Noble [1981] and Deliceoğlu, Çelik, and Gürcan [2019].

Recall that according to Lewis, a generalisation is a law of nature if it appears as an axiom or theorem in the deductive system that best balances simplicity and strength with respect to the Humean mosaic. Yet as we have seen, the Navier-Stokes equations on their own are unlikely to make any contribution to the strength of some candidate system. Unless they are coupled with the appropriate boundary conditions, which may vary depending on the system at hand, there is very little that we will be able to *deduce* about any particular system (or very few possible worlds we can rule out) as a result of the Navier-Stokes equations. Indeed, as we saw, they may in fact provide *incorrect* results in such a case. Given that the inclusion of the Navier-Stokes equations in our system would result in at least a marginal decrease in simplicity with no real gain in strength, it would seem unlikely that the best system would include the Navier-Stokes equations on their own. In other words, if the boundary conditions are not included in our candidate systematisation, it seems unliklely that Lewis's BSA will declare the Navier-Stokes equations to be a law.

It might seem as though there is a simple solution here: we can simply add the boundary conditions to our system in order to ensure that the Navier-Stokes equations is in a position to contribute to its overall strength. However there are two problems with this move. The first is that there is an extraordinarily large (possibly infinite) number of boundary conditions that we would need to add to our system in order to accomplish this, appropriate to the various physical systems that we might encounter. This would seem to represent a pretty dramatic loss with respect to the simplicity of our system. Of course, much has been written about exactly how the trade-off between simplicity and strength is supposed to work in Lewis's BSA,[10] but it would seem that whichever way you slice it a system which includes both the Navier-Stokes equations and the full litany of boundary conditions they employ would likely be one of the least simple candidates on offer.[11]

Suppose, however, that we overcome this problem. That is, suppose that some system which includes the Navier-Stokes equations and the relevant boundary conditions turns out to be the one that best balances strength and simplicity. In that case, we have succeeded in conferring lawhood on the Navier-Stokes equations. Unfortunately, we may have gone too far. If any true generalisation that features in the best system comes out as a law of nature, then it would seem that our entire supporting cast of boundary conditions will turn out to be laws of

[10]For instance, objections have been raised by Hall [2015], Roberts [2008], and Woodward [2014].

[11]There is a related problem here worth mentioning. Given that different classes of systems will require different, incompatible boundary conditions, there is a risk that including all the requisite boundary conditions will render the system *inconsistent.* Perhaps the Humean might avoid this by suggesting that each boundary condition should be included in the system with a specification of the kinds of systems it is to be applied to and the kinds of laws it should combine with, but this seems once again to place a pretty heavy toll on the system's simplicity. Not only must our system include an enormous quantity of boundary conditions, but these boundary conditions are quite complicated specifications in and of themselves. It seems then even more implausible that such a system would count as simple enough to win the title of 'best system.' Thanks to an anonymous reviewer for suggesting this point.

nature. Granted, some of these, such as the no-slip condition, display a limited range of lawlike characteristics. Many others, however, such as the variety of inlet/outlet conditions, do not (these differences will be discussed further in §6). It would seeem to be a real problem for the BSA if the only way it could render the verdict that the Navier-Stokes equations were laws of nature was at the cost of declaring that *all* the boundary conditions also turn out to be laws of nature.

Of course Humeans, such as Bhogal and Perry as we saw earlier, tend to recognise that the best system will need to contain a variety of initial condition statements in order to ensure that the laws contained therein are informative. The mere fact that $f = ma$ features in our system, for instance, does not allow us to derive or deduce correct statements about the Humean mosaic unless we also include some information about "what objects there are, their mass, their velocity, and so on." Yet simply because such initial conditions feature in our best system does not on its own seem to mean that we run the risk of conferring lawhood on them. The laws, after all, are the *generalisations* that feature in the best system, and initial conditions statements about the masses and velocities of particular objects clearly do not seem to be generalisations.[12]

We might then ask: why are boundary conditions any different? The answer is that where initial conditions are particular, discrete pieces of information about the state of a system at a certain time, boundary conditions are differential equations themselves which impose ongoing restrictions on the evolution of our system. Unlike initial conditions, then, it does not seem that there is any clear reason not to regard these boundary conditions as *generalisations* in our system. The no slip condition, for instance, is a generalisation that relates the velocity of the fluid at the boundary to the shear rate at the boundary. As such, it seems that as long as such boundary conditions feature in the best system the defender of the BSA is committed to declaring that they are laws.

By talking about boundary conditions as *generalisations* here I do not mean to point to a mere *syntactic* difference. After all, if this distinction amounts simply to the difference between, for instance, statements that are universally quantified and those that are not, then it seems that we could simply stretch initial conditions into the right shape to count as a generalisation.[13] Such a distinction would not be able to robustly capture the difference between initial conditions and boundary conditions. When I say that boundary conditions (and laws) are generalisations where initial conditions are not, I mean that boundary conditions are general statements *about* the Humean mosaic and not the kind of thing we can think of as being *in* the mosaic.

There are, of course, a variety of ways that one might understand the facts that make up this mosaic, the predicates featured therein, and so on. Nonetheless, it seems right to say that whether or not we stretch them into the logical shape of a generalisation, statements roughly of the form "system $\mathcal{S}$ exhibits properly $\mathcal{P}$ at time t" are the kind of thing we should imagine as making up the mosaic.

[12] Some pragmatic Humeans, such as Dorst [2019], talk in terms of *principles* rather than generalisations.

[13] Thanks to an anonymous reviewer for pushing me on this point.

On the other hand, statements like "system $\mathcal{S}$ exhibits properly $\mathcal{P}$ *at all times* t" or "systems of class $\mathcal{C}$ exhibit properly $\mathcal{P}$ *at all times* t" seem clearly to be general statements *about* the mosaic rather than the kind of discrete fact we should expect to find *in* the mosaic. The suggestion then is that where initial conditions by and large are discrete facts that might feature *in* the mosaic, boundary conditions pose a special kind of problem because they are of the latter kind of more general statement *about* the mosaic.

That, at least, is the shape of the problem. There are two reasons, however, that it might not help to dwell on the implications of the role played by boundary conditions for Lewis's BSA in particular. The first is that there seems to be relatively broad consensus amongst Humeans that, for a variety of reasons, Lewis's particular nomic formula stands in need of revision.[14] In particular, many philosophers have argued on grounds totally unrelated to those that concern us here that Lewis's BSA does not sufficiently reflect facts about scientific practice.

The second reason is that Lewis insists on a far sharper distinction between fundamental and non-fundamental laws than do some of the BSA's recent reformers. On his view, laws must only make reference to an elite class of 'perfectly natural' properties, and so it is less clear that his account is intended to capture all of the laws of fluid dynamics at all. That is to say that since the Navier-Stokes equations make reference to macroscale material properties such as viscosity which are not on his view 'perfectly natural,' Lewis may have rejected them as not sufficiently fundamental and thus beyond the scope of his account.[15]

More recently, Humeans of various stripes have attempted to dispense with this aspect of Lewis's view.[16] In the absence of some strict naturalness constraint, however, it might seem difficult to insist on a sharp distinction between the 'fundamental laws' one's account is intended to cover and the 'non-fundamental' laws it is not.[17] The main point here is that where Lewis's machinery of perfectly natural properties might allow him to dismiss the Navier-Stokes equations as

[14]Of course, pragmatically-inclined Humeans such as Hicks [2017], Dorst [2019], Wilhelm [2022], and Jaag and Loew [2020] represent a big part of this consensus. But in addition, more orthodox Humeans such as Loewer [2007], [2020] and Cohen and Callender [2009] have suggested a variety of modifications to Lewis's nomic formula.

[15]It also seems to me that some distinction between fundamental and non-fundamental laws is likely to play a role in how some non-Humeans would respond to the problems raised by the relationship boundary conditions and laws. For instance, if one is a primitivist (such as Maudlin [2007]) and thus thinks that laws are primitive entities who perform the role of carrying the universe from prior states to subsequent states, then it would seem to be a real problem if the laws must rely on boundary conditions to accomplish this task. Presumably, then, such a primitivist would want to deny that laws like the Navier-Stokes equations are fundamental in some relevant sense. A similar line of thought would seem to apply to those who, like Emery [forthcoming], think that laws play some kind of metaphysical *governing role*. As I mentioned in §1, however, assessing the way in which the relationship between laws and boundary conditions impacts the viability of non-Humean views is beyond the scope of this paper.

[16]Most notably Cohen and Callender [2009] and Loewer [2007], [2020].

[17]Jaag and Loew [2020, 2526fn3], for instance, simply distinguish the "fundamental laws of physics" from the "so-called laws of the special sciences."

beyond the intended domain of adequacy of his account, this move does not quite seem available to the more pragmatically-inclined Humeans that are wary of positing such properties.[18]

§5. Alternative Nomic Formulas. Now that we have seen at least the broad shape of the problem posed by boundary conditions for the BSA, we can look in detail at how this might apply to some recent reformulations of Lewis's nomic formula.

5.1. Computational Tractability. Wilhelm [2022] argues that along with strength and simplicity we should consider *computational tractability* to be one of the theoretical virtues our best system should balance. A system X is more computationally tractable than another Y if X is "overall, more computationally useful than Y when it comes to performing numerical integrations, estimating infinite series expansions, constructing idealized models of phenomena, approximating exact solutions to equations of motion, and so on" (Wilhelm [2022, 3]). The idea is that laws of nature ought in practice to do more than simply rule out a large array of possible worlds. A system that is computationally tractable as well as strong not only rules out plenty of possibilities but also "gives us the tools to determine which worlds are eliminated" (Wilhelm [2022, 5]).

Does adding computational tractability to the list of theoretical virtues help with the problem of conferring lawhood on the Navier-Stokes equations? It does not seem to me that it does. Recall that the Navier-Stokes equations are computationally intractable not simply in the sense that they are difficult or computationally expensive to solve (though they are) but rather in that without the boundary conditions they do not present us with a well-posed problem to solve in the first place. Put differently, a system that contains the Navier-Stokes equations (but not the relevant boundary conditions) is no more useful when it comes to constructing idealised models of phenomena or approximating exact solutions to equations of motion than the same system with the Navier-Stokes equations removed. Such a system would then presumably be unlikely to be the one that strikes the best balance between the relevant theoretical virtues and thus the Navier-Stokes equations would be unlikely to come out as laws.

If we attempt to remedy this by adding the boundary conditions as axioms to our system, then we run into the same problems as we saw in the previous

[18] This point is borne out, I would suggest, by some of examples that these more pragmatic Humeans appeal to. Jaag and Loew [2020, 2530] mention the Wiedemann–Franz law which deals with macroscale thermodynamic properties such as thermal conductivity. Dorst [2019, 887] considers the ideal gas law, which again relates a variety of macroscopic properties of ideal gasses. Hicks [2017] discusses at various points different theories of planetary motion. It does not seem to me that any of these laws have a strong claim to be 'more fundamental' than the Navier-Stokes equations in such a way that would relieve these pragmatic Humeans from the burden of accounting for the details presented earlier. Indeed, given that we do not currently possess a truly fundamental physical theory, it would be quite philosophically awkward to appeal to a substantive characterisation of the role played by laws in contemporary scientific practice to motivate an account of laws that was only intended to capture some restricted subset of 'fundamental' laws.

section. First, any increase in computational tractability and/or strength will come at a significant cost to simplicity given the sheer number of boundary conditions we will require. Suppose that this can be overcome, we nonetheless risk conferring lawhood on the entire supporting cast of boundary conditions, since Wilhelm's account has it that any theorem of the best system comes out as a law of nature.[19] The underlying problem is that computational tractability is not a feature that laws in general exhibit in isolation. Many laws on an abstract level capture how certain systems behave but require the right kind of support before they can accomplish the tasks that Wilhelm collects together under the banner of computational tractability.

5.2. The Epistemic Role Account. Hicks [2017] argues that we should depart even more substantially from the criteria laid down by Lewis. The BSA, he suggests, focusses too much on the outputs of scientific inquiry and not enough on the inputs, such as experimentation. He rightly points out that the methodology of science is concerned with more than simply the organisation and unification of as many truths as possible. In particular, science "aims both at discovering truths that can be employed in a wide range of situations much smaller than the universe as a whole, and at marshalling empirical evidence to provide epistemic support for believing those truth" (Hicks [2017, 993]).

With this in mind, Hicks presents the *epistemic role account* (ERA), according to which

> "The laws of nature are those true statements that, as a group, are best suited to produce predictions and explanations and to be inferred from repeated observation and controlled experiments." (Hicks [2017, 995])

The 'output role' that the ERA identifies for laws is similar to the one that features in the BSA in that "science should output a set of generalizations that will enable us to easily deduce predictions and provide explanations" (Hicks [2017, 995]). Where the ERA differes from the orthodox BSA is in the importance it places on the 'input role' of laws, in that they must be the kind of thing we can infer from observation and experimentation. Hicks thus adds two extra requirements: the laws must be able to be observed in isolated subsystems of the universe, and the laws must be observable in isolation.[20]

In the next section, we will consider how the focus on the predictive role played by laws fares in light of the boundary conditions-related difficulties we have

[19] Given that Wilhelm's BSA holds that the laws are all of the *theorems* of the best system and not necessarily only the *generalisations*, one might wonder whether this means he faces some problem of unintentionally conferring lawhood on all the initial conditions as well. However Wilhelm does not include initial conditions *themselves* in any of his various candidate systems, instead considering what one could derive from his various candidate systems when they are 'supplemented' with sentences about initial conditions. Such an approach would seem strange in the case of boundary conditions, since they are not particular, discrete sentences about the state of some part of the mosaic at some time but generalisations of a kind with those that feature in the deductive systems under consideration.

[20] I take it that by 'observing a law' in this context Hicks means observing *particular instances* of the generalisation captured by the law.

discussed so far. In the meantime, it is worth thinking about Hicks's requirement that the laws of nature must be observable in isolation. In 1828, Antoine Cournot wrote of Claude-Louis Navier's (correct) formulation of what would come to be known as the Navier-Stokes equations that

> "M. Navier himself only gives his starting principle as a hypothesis that can be verified solely by experiment. If, however, the ordinary formulas of hydrodynamics resist analysis so strongly, what should we expect from new, far more complicated formulas?"

In essence, Cournot was complaining about the fact that it was at the time extremely difficult to subject the Navier-Stokes equations to empirical testing. The viability of some of the premises employed in Navier's derivation was difficult to ascertain, since it was unclear how the resulting equations could be applied to even simple systems (Darrigol [2005, 116-8]). Indeed Navier himself, although quite confident in the theoretical underpinnings of his equation, nonetheless conceeded that the formula "cannot suit the ordinary cases of application" (Darrigol [2005, 115]).

So what changed between this point and 1873 such that Helmholtz could as we saw earlier triumphantly declare that the Navier-Stokes equations were "the exact expressions of the laws that rule the motion of real fluids"? The answer is that physicists succeeded in determining the correct boundary conditions for several systems of central importance. As Darrigol [2005, 144] writes, the key reason that "as late as the 1860s the Navier-Stokes equation did not yet belong to the physicist's standard toolbox" was that a consensus had yet to emerge with regard to "the boundary condition, which is crucial in judging consequences for fluid resistance and flow retardation."

Indeed it was (unsurprisingly) George Stokes who realised that considerations of boundary conditions were key to the applicability of the Navier-Stokes equations to real fluid systems. In 1850 he employed the no-slip condition, which we met in §3, in order to extract from Navier's equation an array of correct predictions regarding the motion of fluid through a cylindrical pipe (Darrigol [2005, 142-3]). Despite the fact that a variety of molecular and non-molecular derivations of the Navier-Stokes equations had already been given, it was not until the work of Stokes that physicists were able to subject them to thorough empirical testing. Once the correct boundary conditions were found for certain central cases, physicists were able to understand more generally how to determine the boundary conditions appropriate to a wider class of systems.[21] It was exactly *this* development that inspired Helmholtz's optimistic declaration of 1873.

The moral of this historical interlude is that the reliance of some laws on the appropriate supporting cast can run all the way to questions of confirmation and testing. Some laws cannot be properly subject to experimental testing until the

[21] Indeed, developing techniques for producing boundary conditions for the Navier-Stokes equations and understanding their behaviour remains a very active area of modern mathematical research. See, for instance, Nordström and Svärd [2006], Kučera and Skalák [1998], Raymond [2007].

right boundary conditons (or material parameters, or rigidity constraints, and so on) are produced. Although Hicks [2017, 1000] is right to point out that scientific investigation is characterised by a "divide and conquer methodology of evidence gathering," it is too much to demand that "each part of the lawbook must be independently tested." Our ability to subject certain laws to empirical testing is contingent upon our ability to formulate the correct boundary conditions.

Hicks articulates the thought that we must be able to subject laws to isolated experimental testing in terms of the virtue of *modularity.* Roughly, a set of laws L is more *modular* than a set of laws L^* if various subsets of the laws in L apply to more subsystems of the universe than do the subsets of the laws in L^*. We can put the problem, then, as follows. Let L be some set of laws and L^* be the set of laws that we get by adding the Navier-Stokes equations to L. Then since L^* does not contain the boundary conditions, the addition of the Navier-Stokes equations will not allow L^* to apply to more subsystems than our original L. That is, the Navier-Stokes equations in isolation do not contribute to the modularity of our set of laws – L^* is just as modular as L. Of course, we could always *include* the boundary conditions, but in such a case we would run into the by now familiar problem of conferring lawhood on all the relevant supporting ingredients.

5.3. Prediction. One common thread that runs through several of the proposed alternative BSAs is the idea that *prediction* is one of the most important roles that laws play in scientific practice. As such, several of the alternative nomic formulas that feature in these BSAs hold that something like predictive utility is the key to determining the best systematisation of the Humean mosaic. Jaag and Loew [2020, 2534] propose that the best system is the one that is maximally cognitively useful to creatures like us, but insist that "the main cognitive function of the laws is facilitating predictions." Cognitive usefulness, then, is simply something like predictive utility. Dorst [2019, 886] suggests that "the primary pragmatic use of laws is predictive" and so his BSA centres around several desiderata such that "the system with the 'best balance' is the one with the highest predictive utility." Hicks [2017] also intends for his nomic formula to ensure that the best system is one suited to the predictive needs of agents operating in the world.

Of course, it is undoubtedly right to say that prediction lies at the heart of the overall role that laws of nature play in scientific investigation. Moreover, in articulating alternative nomic formulas framed around this predictive role, the pragmatic reformers of the BSA have shed light on the kinds of features that might allow laws to play this role. The question, however, is whether we can expect laws to play this predictive role in isolation, without the help of their supporting cast. If, as I have urged, we cannot in general maintain such an expectation, then we must ask whether these proposals too are faced with a problem similar to those we have seen in previous sections. Does a nomic formula centred on prediction allow us to confer lawhood on the Navier-Stokes equations without also conferring lawhood on the collection of sundry boundary conditions on which they rely?

Unfortunately, I do not think so. Suppose that you have some fluid system, the various parameters of which you are able to measure to an arbitrary degree of accuracy. Now take the Navier-Stokes equations and plug in the values produced by your measurements. Do the resulting equations tell you what the velocity field will look like a minute or two from now? No. The resulting equations will not have a solution unless you provide the right kind of boundary constraints (and perhaps also inlet/outlet conditions, depending on your system). In the terminology we introduced earlier, the problem will not be well-posed and so we will not even be able to *approximate* our way to a reliable solution. This is exactly the problem we saw in the historical interlude of the previous section: it was so difficult to subject the Navier-Stokes equations on their own to empirical testing because without a procedure for generating the appropriate boundary conditions one is not in a position to say what it is they predict of any particular system.

In articulating his Best Predictive System Account, Dorst [2019] outlines several desiderata, the best balance of which should ensure the highest predictive utility. One way of putting the point above is to say that in isolation the Navier-Stokes equations fail almost entirely to meet the first two (and arguably most important) desiderata: informative dynamics and wide applicability. Dorst requires that "the actual putative laws of nature joinly imply a dynamics for various systems" Dorst [2019, 887]. But any set of principles featuring the Navier-Stokes equations and not the boundary conditions will imply no such dynamics for fluid systems. Similarly, Dorst also includes as a desideratum that our dynamical principles apply to a wide variety of systems so that we need not gather additional information about particular subclasses of systems we might meet in different circumstances. Of course, the Navier-Stokes equations tolerate a wide variety of initial conditions relevant to all the possible fluid systems we might encounter. Yet as we have seen, this does not guarantee that the Navier-Stokes equations may be *applied* to that same wide variety of systems in the absence of the more specific boundary conditions required by the problem at hand.

At any rate, it seems that our familiar problem rears its head again. Without the boundary conditions the system (or set of principles) containing the Navier-Stokes equations will be no more predictively useful than the system without them. Yet if we include the boundary conditions in our set of principles then we must thereby admit them into the pantheon of laws.

5.4. A Difference in Roles. One might at this stage worry that the pragmatic Humean can avail themselves of a rather simple reply. Even if boundary conditions play an important and indispensible supporting role for some scientific laws, they nonetheless are *not* the kind of thing playing the central pragmatic or epistemic role associated with laws. We might think that there is a stark difference between playing the appropriate 'law role' in science and assisting some *other* generalisation as it goes about playing that role. Although they rely on boundary conditions in all kinds of complex ways, it is the Navier-Stokes equations *themselves*, and not those boundary conditions, which are responsible for the predictions and explanations that scientists are able to produce. On this line of thinking, then, the pragmatic Humean can admit that constructions like the no-slip condition play an integral role in *supporting* the Navier-Stokes equations and even admit that they will need to be included in any eventual best system without being forced thereby to *confer lawhood* upon the boundary conditions.[22]

There is something very intuitive about this suggestion. Indeed, the thought that there is an important distinction to be drawn between the supporting cast and the laws that play the starring role is *precisely* the source of our intuition that accounts of laws should be expected to confer lawhood on the Navier-Stokes equations and *not* on things like the no-slip condition. The problem, however, is that this intuitive distinction is exactly the kind of thing that we would want an account of laws to explain in the first place, and so is not the kind of distinction to which an account of laws should appeal. Given a set of regularities, the job of an account of laws is in part to tell us which are the laws and which are not. If the criteria offered by the account label some things laws that we intuitively recognise play a slightly different role, then it is no response at all to say: those things are not laws on my account because they only play a supporting role to the *real laws*. After all, it is the job of the account in the first place to tell us *what the real laws are*. From the perspective of an account of laws, the many regularities that obtain in the world do not come, as it were, pre-labelled.

It is also worth registering that scientifically speaking the question of the different roles played by central laws and boundary conditions can be quite subtle. If one were to merely write down the Navier-Stokes equations and the relevant boundary conditions as a set of equations, it would not be right to say that one could somehow immediately discern that the Navier-Stokes equations are the real laws and the boundary conditions merely supporting actors. The intuitive distinction that we draw between the Navier-Stokes equations and their boundary conditions is rooted in relatively complex and subtle facts about the way that these respective components come to be *used*. But recall that for pragmatic Humeans (and proponents of the BSA in general) it is the *system as a whole*, and not individual regularities, that we evaluate according to some list of pragmatic criteria. We do not ask whether the Navier-Stokes equations play some lawlike role in scientific practice but whether the *system containing them* best fulfils some criteria inspired by the role that laws play in scientific practice. If the only way to get the Navier-Stokes equations into the best system is to include some regularities that seem otherwise to play a different *individual* role in scientific practice, then perhaps this an indication that the general framework of the BSA is too coarse-grained to capture important distinctions between the roles played by the different components of the set of equations we must use to make predictions about fluid systems.

In short, this kind of reply puts the philosophical cart before the horse. There is almost certainly an important distinction to be drawn between the individual roles played by the Navier-Stokes equations and their attendant boundary conditions, but this is the kind of thing that ought to *emerge from* an account of laws rather than be *appealed to* by an account of laws. Moreover, the fact that

[22]Thanks to an anonymous reviewer for suggesting this reply.

pragmatic Humean accounts confer lawhood on *all* of the regularities that make it into the best system makes it hard to see how we would be able to recover such a distinction by imposing further conditions at the level of the *system*. We will return to this point again in §7.

5.5. Predictive Contexts. In §4, we asked why boundary conditions pose a problem for the traditional BSA that is distinct from the one posed by initial conditions. The answer was that where initial conditions may be construed as pieces of information *in* the Humean mosaic, boundary conditions (like laws) are instead general statements *about* the mosaic. As such, where the proponent of the BSA could plausibly admit initial conditions into their best system without conferring lawhood on them, this would not work in the case of boundary conditions.

In the context of the pragmatic Humean reformulations of the BSA, we might consider a different form of the suggestion that we can handle boundary conditions in the same way we handle initial conditions. Rather than thinking about the informativeness of a system in terms of how much it tells us about the mosaic, as Lewis did, some pragmatic Humeans may conceive of informativeness as the extent to which a system allows us to input relatively small amounts of information about the mosaic and get back larger amounts of information about the mosaic.[23] We may then imagine that we already possess the information about the mosaic that is relevant to our given predictive context, and that the best system will be the one that allows us to get the most out of this information. On this conception, there is no need to include initial conditions in our system at all, and thus no risk that they may end up counting as laws against our wishes. Rather, initial conditions are pieces of information about the mosaic that we *input into* a system of generalisations (or *to which we apply* a system of generalisations), and the laws will be the members of that system which allows us to maximise some list of pragmatic criteria. The question then is: why can't we simply think of boundary conditions in the same way?

There are two answers worth outlining, here. The first is similar to the reply offered in the case of the traditional BSA: boundary conditions are not the kind of thing we can easily construe as discrete pieces of information that we *input into* our set of laws. Distinguishing beteween the generalisations in our candidate system and the information we have on hand in some predictive context may make sense when we imagine that information to take the form "system $\mathcal{S}$ exhibits properly $\mathcal{P}$ at time t," but this distinction gets a bit murkier if the information we are 'inputting into' our system is of the more general form "system $\mathcal{S}$ exhibits properly $\mathcal{P}$ *at all times* t" or "systems of class $\mathcal{C}$ exhibit properly $\mathcal{P}$ *at all times* t." If we want to treat some statement S as the kind of thing that we merely 'plug into' some system of laws rather than needing to consider as part of the system itself, it would seem to me that S should be at least in

[23]This conception of the best system as in some sense 'amplifying' our knowledge of the mosaic is one that comes out most explicitly in Dorst [forthcoming] and Callender [2017]. Thanks to an anonymous reviewer for pointing this out.

the neighbourhood of a discrete piece of information about the Humean mosaic. Boundary conditions, I suggest, are not quite in this neighbourhood.

The second (and perhaps more interesting) reason that we cannot treat boundary conditions as we would initial conditions relates to the difference between the 'variable fixing' and 'structure-specifying' roles mentioned in §2. There is a difference between a statement specifying the *condition at the boundary* of some system in terms of the value of some variables and a *boundary condition* in a more involved mathematical sense. Whereas conditions at the boundary, like initial conditions, help us to specify the system to which we would like to apply our laws, boundary conditions help to provide the mathematical structure required to apply the laws at all. More specifically, they help to ensure that our attempts to apply certain laws to some system (or class of systems) amount to a *well-posed problem*. If you change the initial conditions (or conditions at the boundary), you change the system you are working with. If you change the *boundary conditions*, on the other hand, you change the nature of the predictive problem you are trying to solve. In this sense, boundary conditions, alongside laws, form part of the theoretical machinery that we use to turn particular bits of information into predictions, rather than simply being 'inputs' into that theoretical machinery. Unlike initial conditions, then, they should not be treated as information that we 'input into' some predictive system but rather as part of the predictive system itself.

§6. Boundary Conditions as Laws? The pragmatic Humean might at this stage wonder what is so bad about the possibility that their account delivers the result that the boundary conditions required by the Navier-Stokes equations are laws.[24] After all, I did mention that some of these boundary conditions, such as the no-slip condition, exhibit a limited range of lawlike characteristics. On the other hand, many of the boundary conditions on which the Navier-Stokes equations rely do not exhibit these characteristics. In particular, it will help to look at the differences between the no-slip condition and some of the inlet-outlet conditions required for certain systems.

It is worth noting to begin with that the no-slip condition applies to a relatively wide variety of systems, from fluid in pipes to air flowing around a ceiling fan. In these contexts, it serves as a generalisation that relates the velocity of the fluid at the boundary to the shear rate at the boundary. Moreover, such slip conditions remain invariant under quite a wide variety of interventions we might perform on our system.[25] For instance, if a slip condition is the appropriate one for a fluid-solid pair, then changing the size of the shape of the boundary in most ways will not affect the boundary conditions at all. Indeed, as long as the amount of slip is independent of the amount of shear, as it is in most cases, then physicists treat the amount of slip as a robust properly of a given fluid-solid pair (Lauga, Brenner, and Stone [2007, 1232]). That is to say that the slip condition for water flowing along a lead surface will apply whether the surface is a closed

[24] Thanks to Erica Shumener, as well as an anonymous reviewer, for raising this point.
[25] For a more detailed overview of the experimental data, see Sykora [2019, 15-21].

pipe, a container wall, an obstacle in a stream, and so on. The slip conditions for various fluid-solid pairs are thus invariant under considerable changes in boundary shape. Finally, as the discussion in §5.2 of Stoke's derivation shows, these slip conditions are not merely empirically measured but indeed enjoy a sort of theoretical support.

The above is not to suggest that the various slip conditions for the Navier-Stokes equations *certainly are* laws. Rather, it is simply to point out that *some* boundary conditions for the Navier-Stokes equations display some of the characteristics that we intuitively associate with laws: they have wide scope, they are invariant over a wide array of changes, and they enjoy some form of theoretical support (i.e. they are not brute empirical generalisations).

Inlet-outlet conditions, on the other hand, lack these features entirely. If there is additional liquid flowing *into* (or out of) our system, then working with the Navier-Stokes equations requires that we characterise via boundary conditions how this inflow (or out flow) behaves. By contrast with slip conditions, such inlet conditions are often very specific. The right inlet condition depends in sensitive ways on the specific geometry of the physical system and so our inlet conditions often have a very limited scope.[26] For this reason, they do not display particularly notable amounts of invariance under manipulations: small changes to the shape of the boundary can radically impact the suitability of a given inlet condition. Finally, we do not often possess good 'theoretical' methods for determining these inlet-outlet conditions and in such cases must employ heavily computational empirical methods to produce them.

If the slip conditions were the only boundary conditions required by the Navier-Stokes equations, then the pragmatic Humean may simply want to bite the bullet and accept that on their account slip conditions will turn out to be laws. The fact that these conditions exhibit some of the characteristics that we intuitively associate with lawhood may make this an acceptable price to pay. However conferring lawhood on the entire set of boundary conditions involves conferring lawhood on the inlet conditions as well, even though they display almost no intuitively lawlike behaviour. Indeed, in spite of their formal structure these conditions seem far more like particular, contingent facts than the kind of thing that any scientist would recognise as a law.

Part of the difficulty here, as we saw in §5.4, is that the pragmatic criteria with which the BSA operates are applied at the level of the *system as a whole*. This means that pragmatic Humeans who are happy to confer lawhood on slip conditions but want to avoid conferring lawhood on inlet-outlet conditions must outline some criteria for picking out the best system on which the slip conditions appear in the best system but the inlet-outlet conditions do not. The problem is that despite the fact that they may exhibit very different degrees of intuitively lawlike behaviour, they are *equally integral* to the ability of our system as a whole to play the pragmatic role we want it to play. For instance, both kinds

[26] Relatedly, we must often resort to highly computational methods appropriate to very specific circmunstances in order to determine these inlet-outlet conditions in the first place. For examples, see Galusinski, Meradji, Molcard, and Ourmières [2017] and Bruneau and Fabrie [1994].

of conditions are vital to the question of whether the task of solving the Navier-Stokes equations amounts to a well-posed problem or not. Removing the inlet-outlet conditions from our system would have just as negative an impact on the pragmatic capacity of our system as would removing the more 'intuitively lawlike' slip conditions.

In short, it does not seem promising for the pragmatically-inclined Humean to respond to the problem we have posed by simply embracing one horn of the dilemma and accepting the verdict that the boundary conditions for the Navier-Stokes equations are laws. Although this might seem an acceptable price to pay in some cases, it is clearly too high a price to pay in others. Moreover, both the more intuitively lawlike and less intuitively lawlike boundary conditions are equally vital to the ability of any system containing the Navier-Stokes equations to perform certain pragmatic tasks. As such, it is difficult to see how the pragmatic Humean could outline criteria for picking out the best system that would ensure that things like the slip conditions found their way into the best system (and thus were counted as laws), while things like inlet-outlet conditions did not.

§7. Conclusion: A Pragmatic Tension. So where does all of this leave these recent attempts to reform the BSA? By and large these alternative BSAs proceed by identifying features that laws of nature must possess in order to play the role that they do in scientific practice and then use these features to generate a new nomic formula while leaving in place the broader framework of the BSA. I have argued that there is a problem with this strategy, since many scientific laws require the assistance of (often quite complex) additional modelling ingredients before they are in a position to perform their central role in scientific inquiry. If this is right, then it is difficult to see how such alternative BSAs will be able to render the verdict that such laws are indeed laws. The Navier-Stokes equations, along with many others, will be left out in the cold.

Perhaps there are other strategies for generating nomic formulas that avoid this problem, but it does seem to me that the fact that laws do not always operate as lone wolves poses a broader challenge for the framework of the BSA. Recall that the BSA is primarily phrased in terms of the Humean mosaic, made up of particular matters of fact, and generalisations over that mosaic. Not every such generalisation is a law, however, and so the task becomes that of cleaving the laws proper from the accidental generalisations. In practice, however, scientists make constant use of modelling ingredients that occupy a somewhat messy continuum between full-blown laws and simple initial conditions-style matters of fact. The BSA faces the challenge of reconciling the fact that these ingredients do not seem to be (in most cases) appropriate candidates for lawhood with the fact that they play an integral role in scientific practice (and indeed in allowing laws to do the job that they do). It is not easy to see how a different set of criteria for picking out the best system, however motivated by an examination of the role laws play in scientific practice, will help us to handle the delicate interplay between scientific laws and their supporting casts.

Perhaps what the BSA needs here is some independent handle on the distinction between laws proper and their supporting casts. If the Humean were able to differentiate in some robust way between the generalisations eligible to be laws and those merely eligible to play supporting roles, then they could avoid the problems we encountered above by ensuring that their nomic formula applies only to the former kind of generalisation. Armed with a distinction between law-eligible and supporting-eligible generalisations, the BSA may then proceed along some of the pragmatic lines we have seeen in order to distinguish the accidental generalisations from the laws proper. On this line of thinking, even if we are forced to include some of the supporting cast members in our best system, they will not thereby turn out to be *laws* because we have in hand some independent distinction between the members of the best system eligible to be *laws* and those merely eligible to play supporting roles.

Maybe it is possible to draw such a distinction, but this would be no simple task. The slip conditions required by the Navier-Stokes equations are differential equations in their own right that apply at all times t to the velocity field describing the motion of fluid particles in the system. At the very least this seems to suggest that a mere syntactic criterion will not be enough to maintain such a distinction. Perhaps we can draw the distinction required by attending more closely to the roles played by laws proper *within* the broader modelling environments consisting of laws and their supporting casts, though I am not sure exactly how this might look. At the very least, drawing the distinction in such a way would require closer examination of the details of the role played by boundary conditions (and material parameters, and so on) in scientific practice than has been characteristic of the literature on laws of nature thus far.

There would also be something quite strange about this way of defending the BSA. In some sense the central claim of the BSA is that it is precisely the notion of membership in the best system that captures what it is to be a law and thus what it is to play a lawlike role in scientific practice. Perhaps it is true that the pragmatic Humean could respond to the difficulties surrounding boundary conditions by saying something like: although members of the supporting cast might find their way into the best system, there are finer distinctions between the role that various components of the system play in scientific practice to which we will need to attend in order to separate the laws from the non-laws. In some sense, this is probably right. But in another sense, we might ask: how much work is the notion of membership in the best system now doing in separating the laws from the non-laws? If we are denying lawhood to general statements about the Humean mosaic that find their way into the best system on the basis of more fine-grained considerations of the role played by different kinds of general statements in scientific practice, then why should we continue to work within the framework of the BSA? In such an event it would seem like it was these more fine-grained considerations that were doing the real work of separating the laws from the non-laws.

Of course, these issues must be worked through carefully, and doing so is beyond the scope of what I hope to achieve here. The point of this paper is to argue that the integral role played by supporting cast members such as boundary conditions in the scientific employment of laws presents a considerable obstacle to recent attempts to reform the BSA. The point of these concluding remarks is to tentatively suggest that getting around this obstacle might require more radical reform than simply switching out Lewis's old nomic formula for a more pragmatically-inspired one.

There is a way in which this, if true, is unsurprising. I think that Woodward [2014, 92] was right to say that

> "The appeal of the BSA does not, I believe, mainly derive from its demonstrated descriptive adequacy as a treatment of detailed aspects of scientific practice involving laws. It rather has to do with its overall fit with other ideas to which many philosophers are committed: two of these are a picture of scientific reasoning as involving a trade-off between simplicity and strength, and a 'Humean' programme of reduction of the nomic to the non-nomic. [...] This makes many philosophers think that something along the lines of the overall package must be right and perhaps that they ought to pay less attention than they should to the details of exactly how the account is supposed to work."

More recent defenders of the BSA have done an admirable job in attempting to refine the BSA so that the kinds of generalisations that feature in the best systems picked out by their nomic formula more closely reflect the laws that feature in scientific practice. But then again, one might wonder, as Woodward does, whether the ability to capture the methodology of modern science in all its complex, gory detail was ever part of the BSA's core appeal. Driven by commendable naturalistic scruples to demand more of the BSA in terms of descriptive adequacy to the methodology of modern science, we may find that the framework begins to collapse (or at least creak unpleasantly) under a kind of pressure it was never intended to withstand. Adding a kind of pragmatic inflection to the nomic formula is one thing, but reckoning with the fact that the scientific use of laws involves a far wider array of constructions than simply laws and pieces of the Humean mosaic may turn out to be another thing entirely.

Perhaps my pessimism will turn out to be misplaced. Either way, if we want to amend the BSA so that it provides us with a more descriptively adequate picture of scientific methodology, we must do more than simply ask ourselves what role laws (on their own) play in scientific practice. We must ask ourselves *how* they go about playing that role and whether, in fact, they require any help in doing so. If, as I have argued, they do, then the question is: does the BSA have the resources to recognise the supporting cast without inadvertently giving them all a star billing? It seems to me that the viability of the program of pragmatic reform of the BSA depends on the answer to this question.

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