DOING WELL ENOUGH IN AN ANDERSONIAN-KANGERIAN FRAMEWORK

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I recast the DWE ("Doing Well Enough") deontic framework as an Andersonian-Kangerian modal framework and explore its metatheory systematically.

To what extent can the framework, "Doing Well Enough", for supererogation and related notions of common-sense morality (Mares and McNamara 1997, and McNamara, 1990, 1996a-c) be recast in an Andersonian-Kangerian modal framework ("A-K Framework") and studied systematically? To answer this question, I will introduce three propositional constants and use these to define analogues to all the deontic operators employed in the DWE-ish logics (subject to a later qualification regarding the main *indifference* operator of DWE). Since the recast logics will all be normal modal logics, we can easily prove an analog of the Fundamental Theorem for Canonical Models for normal modal logics, and then employ this theorem in generating completeness proofs. A few primary metatheorems are proven and employed to generate 241 determination results (soundness and completeness results) for a salient subset of the class of DWE-AK logics. In Section I, I briefly sketch Standard Deontic Logic and its recast in an A-K framework. In Section II, I identify the DWE framework. In Section III, I recast the DWE framework in an A-K framework and note some contrasts between the two frameworks and some limits on the recast. Finally, in Section IV, I turn to the metatheory of the DWE A-K framework.

1 BACKGROUND: SDL AND AK-SDL

1.1 Standard Deontic Logic (SDL)

Assume that we have a language of classical propositional logic with one additional primitive unary operator: "**OB**" for "it is obligatory that". Then, SDL, may be axiomatized as follows:

<u>SDL</u>:	A1: All Tautologies	R1: If $\vdash$ p and $\vdash$ p → q then $\vdash$ q
	A2: **OB**(p → q) → (**OB**p → **OB**q)	R2: If $\vdash$ p then $\vdash$ **OB**q
	A3: **OB**p → ¬**OB**¬p	

Regarding SDL's expressive powers, we might add the following:

<u>Defs</u>: **PE**p =df ¬**OB**¬p ("it is permissible that")
IMp =df **OB**¬p ("it is impermissible that")
GRp =df ¬**OB**p ("it is gratuitous/non-obligatory that")
OPp =df ¬**OB**p & ¬**OB**¬p ("it is optional that")

So SDL is just the normal modal logic "D" or "KD" with a suggestive notation. I assume that the reader is acquainted with the usual Kripke-style possible world semantics for normal modal logics. Recall that the only constraint on the accessibility relation for KD is seriality: every world has some (morally acceptable) world accessible to it.

1.2 An Andersonian-Kangerian Recast of SDL

Assume that we have a language of classical modal propositional logic with an additional special deontic propositional constant: "d" for "Morality's demands are (all) met". Then Andersonian-Kangerian SDL, "AK-SDL", may be axiomatized as follows:

AK-SDL: A1: All Tautologies

A2: $\Box(p \to q) \to (\Box p \to \Box q)$

A3: $\Diamond d$

R1: If $\vdash p$ and $\vdash p \to q$ then $\vdash q$

R2: If $\vdash p$ then $\vdash \Box q$

So AK-SDL is just the normal modal logic K with A3 added. (Aqvist 1984 is an excellent general source on the recast of SDL-ish deontic logics as Andersonian-Kangerian modal logics.) A3 is interpreted as telling us that it is possible that all of morality's demands are met. In import, it is similar to (though stronger than) the “No Conflicts” axiom, A3, of SDL itself. All of SDL's deontic operators are defined operators in AK-SDL:

Defs: $\mathbf{OB}p =df \Box(d \to p)$ $\quad$ $\mathbf{GR}p =df \Diamond(d \,\&\, \neg p)$

$\mathbf{PE}p =df \Diamond(d \,\&\, p)$ $\quad$ $\mathbf{OP}p =df \Diamond(d \,\&\, p) \,\&\, \Diamond(d \,\&\, \neg p)$

$\mathbf{IM}p =df \Box(p \to \neg d)$

So in AK-SDL, to say that *p is obligatory* is to say that p is necessitated by morality's demands being met, to say that *p is permissible* is to say that p is compatible with morality's demands being met, etc. Proofs of SDL-ish wffs are then just K proofs of the corresponding modal formulae involving "d". Here is a proof of $\mathbf{OB}p \to \neg\mathbf{OB}\neg p$:

1. Assume $\neg(\mathbf{OB}p \to \neg\mathbf{OB}\neg p)$.	[For Reductio]
2. That is, assume $\neg(\Box(d \to p) \to \neg\Box(d \to \neg p))$	[Def of "**OB**"]
3. So $\Box(d \to p) \,\&\, \Box(d \to \neg p)$.	[Propositional Logic]
4. So $\Box(d \to (p \,\&\, \neg p))$.	[3--Derived K rule]
5. But $\Diamond d$	[A3]
6. So $\Diamond(p \,\&\, \neg p)$.	[4 and 5--K rule]
7. But $\neg\Diamond(p \,\&\, \neg p)$.	[K-Theorem]
8. So $(\Box(d \to p) \to \neg\Box(d \to \neg p))$	[2-7, Propositional Logic]
9. So $\mathbf{OB}p \to \neg\mathbf{OB}\neg p$	[8--Def of "**OB**"]

1.3 Semantics for AK-SDL

We define the frames as follows:

F is an AK-SDL Frame: $F = \langle W,R,DEM\rangle$ such that: 1) W is a non-empty set; 2) R is a subset of W x W; 3) DEM is a subset of W; (4) $\forall i \exists j(Rij \,\&\, j \in DEM)$.

DEM is to be thought of as that subset of worlds where all of morality's demands are in fact met. Corresponding to seriality for SDL, clause (4) above validates A3, by ensuring that there is always some *accessible* world where morality's demands are all met.

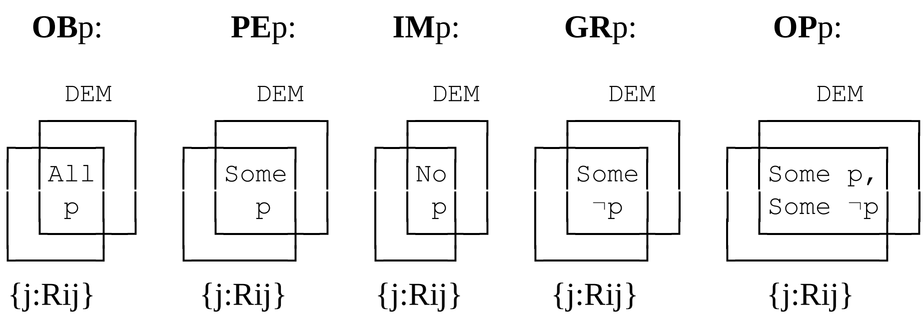
A model, and truth, can be defined in the usual way:

$\underline{M \text{ is an AK-SDL Model}}$: M = <F,V>, where F is an AK-SDL Frame, <W,R,DEM>, and V: Propositional Variables x W -> Power(W).

Basic Truth-Conditions at a world, i, in a Model, M:
[SL]: (Standard clauses for Propositional Logic.)
[$\Box$]: $M \models_i \Box p$ iff $\forall j$(if Rij then $M \models_j p$).
[d]: $M \models_i d$ iff $i \in$ DEM.

Derivative Truth-Conditions:
[$\Diamond$]: $M \models_i \Diamond p$: $\exists j$(Rij & $M \models_j p$)
[**OB**]: $M \models_i$ **OB**p: $\forall j$[if Rij & $j \in$ DEM then $M \models_j p$]
[**PE**]: $M \models_i$ **PE**p: $\exists j$(Rij & $j \in$ DEM & $M \models_j p$)
[**IM**]: $M \models_i$ **IM**p: $\forall j$[if Rij & $j \in$ DEM then $M \models_j \neg p$]
[**GR**]: $M \models_i$ **GR**p: $\exists j$(Rij & $j \in$ DEM & $M \models_j \neg p$)
[**OP**]: $M \models_i$ **OP**p: $\exists j$(Rij & $j \in$ DEM & $M \models_j p$) & $\exists j$(Rij & $j \in$ DEM & $M \models_j \neg p$)

The derivative deontic truth-conditions, relative to i, can be pictured as follows:



Plainly, the normative status of p at i depends on p's relationship to *the intersection* of the i-accessible worlds and the worlds where morality's demands are met. If the intersection is permeated by worlds where p holds, *p is obligatory*; if it contains some p-world, *p is permissible*; etc.

2 STANDARD DWE ("Doing Well Enough")

2.1 The Language of DWE

Assume that we have a language of classical propositional logic with these additional primitive unary operators:

OBp: It is *OBligatory* that p (or "S must see to it that p").
MAp: Doing the *MaXimum* involves seeing to it that p (or "S ought to see to it that p").
MIp: Doing the *MiNimum* involves seeing to it that p (or "The least S can do involves seeing to it that p").
INp: It is *INdifferent* to see to it that p (or "S's seeing to it that p is a matter of moral indifference").

Suppose that I have promised to contact you to conduct some business, and I am thereby obligated to do so. Imagine that I can conduct our business by emailing you, calling you, or stopping by. (Imagine these are the only ways to conduct the promised business.) Note that these are ordered in such a way that the response is increasingly personal. Now it is not difficult to imagine that the moral worth of these actions might match the extent to which the response is personal. Suppose it does. Assuming you would not let me conduct our business twice, the three alternatives are exclusive. Then it is *obligatory* that I contact you in one of the three ways, but no one in particular, since any one of the three will discharge my obligation to contact you. Now if I choose to discharge my obligation in the minimally acceptable way, I will do so by email rather than by telephone or in person. So *doing the minimum involves* emailing you. On the other hand, if I conduct the business in person, I will have discharged my obligation in the optimal way. *Doing the maximum (what morality recommends) involves* stopping by your place. Finally, we can easily imagine that nothing of moral worth hinges on whether I wear my Nikes or not when I contact you. So wearing them is *a matter of moral indifference*.

We note the following defined operators, and their intended readings:

Defs:	**PE** =df ¬**OB**¬.	(It is *PErmissible* that p.)
	IM =df **OB**¬.	(It is *IMpermissible* that p.)
	GR =df ¬**OB**.	(It is *GRatuitous* that p.)
	OP =df ¬**OB** & ¬**OB**¬.	(It is *OPtional* that p.)
	SI =df ¬**IN**.	(It is *SIgnificant* to see to p.)
	SU =df **PE** & **MI**¬.	(It is *SUpererogatory* to see to p.)
	PS =df **PE** & **MA**¬.	(It is *Permissibly Suboptimal* to see to p.)

Continuing with our example, note that although the three alternatives, conducting the business by email, phone, or in person, are not on a par morally speaking, each is still *morally optional*. For each, the agent is permitted to do it or to refrain from doing it. Now we saw that doing the minimum involves e-mailing you. But suppose that rather than e-mailing you, I either call or stop by. Both of the latter alternatives are *supererogatory*. In each case, I will have done *more than I had to do*--more than I would have if I had done the minimum permitted. On the other hand, if I do not stop by, I will have done something sup-optimal, but, since emailing you and calling you are each nonetheless permissible, each is *permissibly suboptimal*. Finally, although each of the three ways of contacting you is optional, none is without *moral significance*. For whatever option I take of the three, I will have done something supererogatory or I will have done only the minimum; in either case, I will have done something with *moral significance*.

2.2 DWE Semantics

The frames are defined as follows:

F = < W, A, $\leq$ > is a DWE-Frame:
1) W is non-empty;
2) A $\subseteq$ W^2 and A is serial;
3) $\leq$ $\subseteq$ W^3: a) (k $\leq_i$ j or j $\leq_i$ k) iff (Aij & Aik), for any i,j,k in W;
b) if j $\leq_i$ k and k $\leq_i$ l then j $\leq_i$ l, for any i,j,k,l in W.

(Similar structures are employed for different purposes in Aqvist, 1992 and in Jones and Porn, 1985.)

In DWE Frames we have a set of worlds, and a relation--interpreted here as relating worlds to their morally *acceptable* alternatives. (Note we did not say a morally *ideal* alternatives.) As 2) indicates, for each world, there is a morally acceptable alternative--seriality holds. Finally, we have a set of world-relative orderings, and 3) implies that for each world i, the associated i-relative ordering is confined to the i-acceptable worlds, and it is a weak-ordering: it is reflexive, connected and transitive. Notice that we allow for worlds being *tied* in the i-relative ordering of the i-acceptable worlds, so we can talk sensibly of "levels" of i's acceptable worlds. We can represent all this as follows:

```
                                /\
                                ||   \
  at least one i-acceptable -> *|      weakly ordered
                                ||     i-acceptables
   a level of i-acceptables -> -||-  /
                                \/
```

The vertical bar represents the weakly ordered i-acceptable worlds. The horizontal line through the bar indicates a "level" of i-acceptables (an equivalence class with respect to equi-rank). The asterisk indicates there is always an i-acceptable world.

The notions of an assignment and a model are then easily defined:

<u>*P* is an Assignment on *F*</u>: *F* = <W, A, ≤ > is a DWE-Frame and *P* is a function, P: PV -> Power(W), defined on PV (Propositional Variables).

<u>*M* = <*F P*> is a DWE-Model</u>: *F* = < W, A, ≤ > is a DWE-frame and *P* is an assignment on *F*.

Truth-conditions are then given as follows, where $j =_i k =df\ j \leq_i k\ \&\ k \leq_i j$.

<u>Basic Truth-Conditions at a world, i, in a Model, *M*</u>:
0) (Conditions for variables and truth-functional connectives)
1) $M|=_i \mathbf{OB}p$: $\forall j(\text{if } Aij \text{ then } M|=_j p)$.
2) $M|=_i \mathbf{MA}p$: $\exists j(Aij\ \&\ \forall k(\text{if } j \leq_i k \text{ then } M|=_k p))$.
3) $M|=_i \mathbf{MI}p$: $\exists j(Aij\ \&\ \forall k(\text{if } k \leq_i j \text{ then } M|=_k p))$.
4) $M|=_i \mathbf{IN}p$: $\forall j[\text{if } Aij \text{ then } \exists k(k =_i j\ \&\ M|=_k p)\ \&\ \exists k(k =_i j\ \&\ M|=_k \neg p)]$.

<u>Derivative Truth Conditions</u>:
5) $M|=_i \mathbf{PE}p$: $\exists j(Aij\ \&\ M|=_j p)$.
6) $M|=_i \mathbf{IM}p$: $\forall j(\text{if } Aij \text{ then } M|=_j \neg p)$.
7) $M|=_i \mathbf{GR}p$: $\exists j(Aij\ \&\ M|=_j \neg p)$.
8) $M|=_i \mathbf{OP}p$: $\exists j(Aij\ \&\ M|=_j p)$ and $\exists j(Aij\ \&\ M|=_j \neg p)$.
9) $M|=_i \mathbf{SI}p$: $\exists j[Aij\ \&$ either $\forall k(\text{if } k =_i j \text{ then } M|=_k p)$ or
$\forall k(\text{if } k =_i j \text{ then } M|=_k \neg p)]$.
10) $M|=_i \mathbf{SU}p$: $\exists j(Aij\ \&\ M|=_j p)\ \&\ \exists j[Aij\ \&\ \forall k(\text{if } k \leq_i j \text{ then } M|=_k \neg p)]$.
11) $M|=_i \mathbf{PS}p$: $\exists j(Aij\ \&\ M|=_j p)\ \&\ \exists j[Aij\ \&\ \forall k(\text{if } j \leq_i k \text{ then } M|=_k \neg p)]$.

We can represent these truth-conditions (relative to a world i) as follows:

```
OBp:      PEp:       IMp:     GRp:      OPp:
 ^
/\\       /\\        /\\      /\\       /\\
|| all    || some    || no    || not    || some p
*|        *|         *|       *| all    *|  and
||  p     ||  p      || p     ||  p     || some ¬p
\//       \//        \//      \//       \//

MIp:      MAp:      SUp:      PSp:      INp:      SIp:
 ^         ^                            ^
/\        /\        /\        /\        /\        /\
||        ||\ all   ||        ||\ all   ||\       ||\
||        *|/  p    |* - p    *|/ ¬p    || all    || not
||        ||        ||        ||        *|        *| all
*|\ all   ||        *|\ all   |* - p    || |p|    || |p|
||/  p    ||        ||/ ¬p    ||        ||/       ||/
\/        \/        \/        \/        \/        \/
```

A "^" under an operator indicates that it is primitive. An "all |p|" indicates that both p-worlds and ¬p-worlds occur among all the associated <u>levels</u>.

Where "*****" ranges over **OB**, **MA**, **MI**, the associated **DWE Base Logic** is:

A0. All tautologous DWE-wffs;
A1. *****(p → q) → (*****p → *****q)
A2. **OB**p → (**MI**p & **MA**p)
A3. (**MI**p v **MA**p) -> **PE**p
A4. **IN**p → **IN**¬p
A5. **IN**p → (¬**MI**p & ¬**MA**p)
A6. **OB**(p → q) & **OB**(q → r) & **IN**p & **IN**r .→. **IN**q
R1: If |- p and |- p → q then |- q
R2: If |- p then |- **OB**p.

<u>Metatheorem:</u> The DWE-Base logic is determined by the class of DWE-models (Mares and McNamara 1997).

3 DWE RECAST: AN AK-DWE FRAMEWORK

To what extent can we recast the DWE framework in the Andersonian-Kangerian way? I will now present a *general* Andersonian-Kangerian framework in which, I believe, we can provide the best recast of the DWE framework that is possible.

3.1 AK-DWE Language

Assume that we have a language of classical modal propositional logic with the usual primitive necessity operator: "☐" for "it is necessary (or inevitable) that". We now add that we have *three* distinguished deontic propositional constants:

d: morality's demands are met (or “nothing unacceptable is done”)
x: the moral maximum/optimum is done (or “morality's demands are met maximally”/ “what morality recommends is done”)
n: the moral minimum is done (or “morality's demands are met minimally”)

We then introduce the following definitions:

Defs: $\Diamond$p: $\neg\Box\neg$p	**MA**p: $\Box(x \rightarrow p)$
OBp: $\Box(d \rightarrow p)$	**MI**p: $\Box(n \rightarrow p)$
PEp: $\Diamond(d$ & p)	**SU**p: **PE**p & **MI**$\neg$p
IMp: $\Box(d \rightarrow \neg p)$	**PS**p: **PE**p & **MA**$\neg$p
GRp: $\Diamond(d$ & $\neg$p)	**SI**p: **MA**p v **MA**$\neg$p v **MI**p v **MI**$\neg$p
OPp: $\Diamond(d$ & p) & $\Diamond(d$ & $\neg$p)	**IN**p: $\neg$**MA**p & $\neg$**MA**$\neg$p & $\neg$**MI**p & $\neg$**MI**$\neg$p

The definitions on the left are as before. Regarding the rest, *doing the maximum involves* seeing to p if p is necessitated by meeting morality's demands optimally (by doing what morality recommends); *doing the minimum involves* seeing to p if it is necessitated by meeting morality's demands in a minimal way; it is *supererogatory* to see to p if doing so is compatible with meeting morality's demands but incompatible with doing so in a minimal way. Similarly, p is *permissibly suboptimal* if seeing to p is compatible with meeting morality's demands, but incompatible with doing so in the maximal way. Finally, p is a matter of *moral indifference* if p, as well as $\neg$p, is compatible with meeting morality's demands minimally, and with meeting them maximally; and p is a matter of *moral significance* if it is not a matter of moral indifference, as just construed.

3.2 Semantics for AK-DWE

For generality, we will define frames using a minimum of constraints on the semantic notions associated with the deontic constants. We can add constraints back in later.

F is a AK-DWE Frame: *F* = <W,R,DEM,MAX,MIN> such that: 1) W is a non-empty set; 2) R is a subset of W x W; 3) DEM U MAX U MIN is a subset of W.

Aside from a set of worlds and an accessibility relation, we have three distinguished subsets of worlds: DEM, the worlds where morality's demands are all met, MAX, the worlds where morality's demands are all met in an optimal way, and MIN, the worlds where morality's demands are all met in a minimal way. The remaining semantic notions are defined as follows:

M is an AK-DWE Model: *M* = <*F*,V>, where *F* is an AK-DWE Frame, <W, R, DEM, MAX, MIN>, and V: Propositional Variables x W -> Power(W).

Basic Truth-Conditions at a world, i, in a Model, *M*:
[SL]: (Standard clauses for Propositional Logic)
[$\Box$]: $M \models_i \Box p$ iff $\forall j$(if Rij then $M \models_j p$).
[d]: $M \models_i d$ iff $i \in$ DEM.
[n]: $M \models_i n$ iff $i \in$ MIN.
[x]: $M \models_i x$ iff $i \in$ MAX.

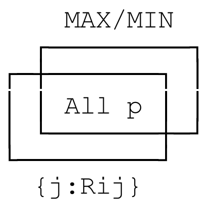
Derivative Truth Conditions:
[$\Diamond$]: $M \models_i \Diamond p$: $\exists j$(Rij & $M \models_j p$)
[**OB**]: $M \models_i$ **OB**p: $\forall j$[if Rij & $j \in$ DEM, $M \models_j p$]
[**PE**]: $M \models_i$ **PE**p: $\exists j$(Rij & $j \in$ DEM & $M \models_j p$)
[**IM**]: $M \models_i$ **IM**p: $\forall j$[if Rij & $j \in$ DEM, $M \models_j \neg p$]

[**GR**]: $M \models_i$ **GR**p: $\exists j(Rij \,\&\, j \in \text{DEM} \,\&\, M \models_j \neg p)$
[**OP**]: $M \models_i$ **OP**p: $\exists j(Rij \,\&\, j \in \text{DEM} \,\&\, M \models_j p) \,\&\, \exists j(Rij \,\&\, j \in \text{DEM} \,\&\, M \models_j \neg p)$
[**MA**]: $M \models_i$ **MA**p: $\forall j[\text{if } Rij \,\&\, j \in \text{MAX}, M \models_j p]$
[**MI**]: $M \models_i$ **MI**p: $\forall j[\text{if } Rij \,\&\, j \in \text{MIN}, M \models_j p]$
[**SU**]: $M \models_i$ **SU**p: $\exists j(Rij \,\&\, j \in \text{DEM} \,\&\, M \models_j p) \,\&\, \forall j[\text{if } Rij \,\&\, j \in \text{MIN}, M \models_j \neg p]$
[**PS**]: $M \models_i$ **PS**p: $\exists j[(Rij \,\&\, j \in \text{DEM} \,\&\, M \models_j p] \,\&\, \forall j[\text{if } Rij \,\&\, j \in \text{MAX}, M \models_j \neg p]$
[**IN**]: $M \models_i$ **IN**p: $\exists j(Rij \,\&\, j \in \text{MAX} \,\&\, M \models_j p) \,\&\, \exists j(Rij \,\&\, j \in \text{MAX} \,\&\, M \models_i \neg p)$
$\&\ \exists j(Rij \,\&\, j \in \text{MIN} \,\&\, M \models_i p) \,\&\, \exists j(Rij \,\&\, j \in \text{MIN} \,\&\, M \models_i \neg p)$
[**SI**]: $M \models_i$ **SI**p: $\neg\exists j(Rij \,\&\, j \in \text{MAX} \,\&\, M \models_j p)$ or $\neg\exists j(Rij \,\&\, j \in \text{MAX} \,\&\, M \models_i \neg p)$
or $\neg\exists j(Rij \,\&\, j \in \text{MIN} \,\&\, M \models_i p)$ or $\neg\exists j(Rij \,\&\, j \in \text{MIN} \,\&\, M \models_i \neg p)$

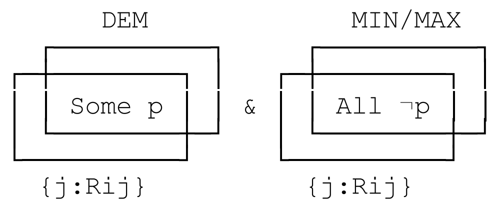
<u>Truth in a model</u>: $M \models p$ iff $M \models_i p$, for every $i \in W$ in M.
<u>Validity in a Model Set</u>: $C \models p$ iff $M \models p$, for every model M in C.

Truth-conditions for the distinctive DWE-ish operators can be pictured as follows:

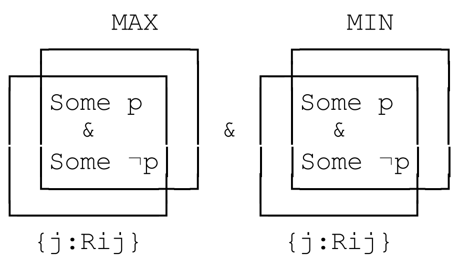
|=i MAp/MIp:



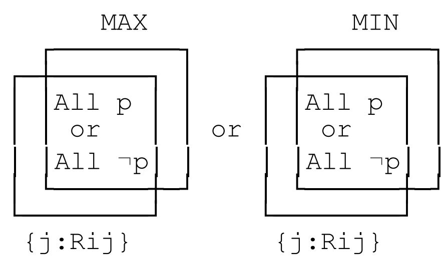
|=i SUp/PSp:



|=i INp:



|=i SIp:



3.3 A Partial Answer to Our Recast Question

To what extent is the recast above successful? First of all, for each deontic operator of DWE, we have a plausible intuitive analog operator in AK-DWE--with one qualification to be noted in a moment. So we have already come a long way toward being able to recast the expressive resources of DWE in an Andersonian-Kangerian framework. Nonetheless, there are some limits or qualifications.

3.3.1 Maximality and Minimality in DWE and AK-DWE

In DWE, we can have models in which the limit assumption is not satisfied: a world i's acceptable worlds can be ranked higher and higher without end. The truth-conditions for **MA** reflect this fact, since they do no tell us that **MA**p is true at i iff p is true at all the best ranked i-relative worlds, but rather iff there is some i-acceptable world, j, such that, from there on up, all the i-acceptable worlds are p-worlds. This, unlike the truth-at-all-the-bests clause, allows for models where {p: |=i **MA**p} is an inconsistent set. Now obviously, given our semantics for AK-SDL, {p: |=i **MA**p}, that is, {p: □(*x* → p)}, will always be a consistent set, provided that MAX is non-empty. (This, by the way, is why A3 of AK-SDL is stronger in import than A3 of SDL itself. With a connected ordering semantics for **OB** that allows for violations of the limit assumption, it need not be the case that morality's demands can be all (i.e. collectively) met, even though A3 of SDL is still universally valid on such a semantics.) In a sense, at the semantic level, we can think of MAX as representing the morally best worlds without the benefit of an ordering. (Similar remarks can be made regarding **MI**p.) Now it turns out that if we restrict ourselves to DWE models in which there is always a best ranked i-acceptable world, no new wffs are validated, so the choice between the two clauses makes no difference regarding validities. The next contrast reflects a more substantial limit on our recast.

3.3.2 DWE Indifference and AK-DWE Indifference

In DWE, **IN**, is a *primitive* notion in its own right with no full parallel here. For in standard DWE, the following is a theorem (and it is valid in all DWE models):

$$\mathbf{IN}p \rightarrow (\neg\mathbf{MA}p \,\&\, \neg\mathbf{MA}\neg p \,\&\, \neg\mathbf{MI}p \,\&\, \neg\mathbf{MI}\neg p).$$

However, the converse is not a theorem in the DWE base logic, (nor is it valid in all DWE models). In contrast, in AK-DWE, **IN** is a defined operator, and that definition yields this equivalence (E) immediately:

$$\text{E:}\ \ \mathbf{IN}p \leftrightarrow \neg\mathbf{MA}p \,\&\, \neg\mathbf{MA}\neg p \,\&\, \neg\mathbf{MI}p \,\&\, \neg\mathbf{MI}\neg p.$$

So the two systems diverge here. From the standpoint of standard DWE, *complete indifference* (**IN**) and *polarity indifference* (**PI**) can be distinguished (McNamara 1990):

INp:

```
/\
|| \ all
*|   levels
|| / |p|
\/
```

PIp:

```
 <- |p|
|
*
|
 <- |p|
```

As the diagrams make clear, for **PI**p to hold in the DWE framework, all that is required is that each of p and ¬p occur among the top-ranked acceptable worlds, and among the bottom-ranked acceptable worlds. (Thus *polarity indifference*, but not *complete indifference*, depends on the upper and lower limit assumptions being satisfied.) But **IN**p requires that the same arrangement regarding p and ¬p holds at *every* acceptable level of worlds, not just the top and bottom ranked levels (if there are such). So we have established an important limit on our recast:

> Metatheorem: *In the AK-DWE framework, we can only represent that subset of standard DWE logics where E is a theorem.*

Given the nature of the indifference operator, I see no way to do better in representing indifference in an Andersonian-Kangerian framework. Indifference will need to satisfy a number of peculiar principles, such as, **IN**p ↔ **IN**¬p, which are unlike principles satisfied by ordinary necessity operators. Given the equivalence between the wffs valid on all DWE models and the wffs valid on all models in which the upper and lower limit assumptions hold, **OB**, **MA** and **MI** are each like necessity operators broadly construed: each can be thought of as applying to a wff iff the wff holds in all of an associated set of worlds. It is this fact that makes these operators ripe for a recast in an Andersonian-Kangerian framework. In contrast, **IN**, does not behave like these other operators syntactically, and its semantics is quite different as well. The best we can do, as far as I can see, is represent a weaker DWE indifference notion, that of *polarity indifference*, since this notion <u>is</u> definable in terms of the necessity-like operators, **MA** and **MI**. Even with this shortcoming we nonetheless still have *an* indifference operator in AK-DWE worthy of the name, and a large and interesting subset of the DWE logics can be recast: all those where E above holds.

3.3.3 A Natural Weakening of DWE Without Analog for AK-DWE

Finally, the standard DWE framework sketched above can be weakened. Semantically, connectivity of the ordering relation can be dropped, allowing for i-acceptable worlds that are not comparable to one another. When this is done, our first DWE axiom, A1, for the DWE "base" logic is invalid, and we replace it with the weaker:

A1': **OB**(p → q) → (*p → *q).

Then although **OB**(p → q) → (**OB**p → **OB**q), **OB**(p → q) → (**MA**p → **MA**q) and **OB**(p → q) → (**MI**p → **MI**q) are axioms (and valid), the principles, **MA**(p → q) → (**MA**p → **MA**q) and **MI**(p → q) → (**MI**p → **MI**q) are no longer deducible (nor valid for the weakened models without connectivity). Yet, **MA**(p → q) → (**MA**p → **MA**q) and **MI**(p → q) → (**MI**p → **MI**q), are unavoidable theorems in the AK-DWE framework, since they are just disguised theorems of the normal modal base logic, K. For example, **MA**(p → q) → (**MA**p → **MA**q) amounts to this theorem of K: □(x → (p → q)) → (□(x → p) → □(x → q)). So logics in the *weakened* DWE framework which lack the K principles for either **MA** or **MI** cannot be recast.

These seem to be the key contrasts and/or limits on the recast stemming from differences in the expressive and semantic resources of the standard DWE framework and those of AK-DWE framework. We turn now to our second main question.

4 EXPLORING THE AK-DWE FRAMEWORK SYSTEMATICALLY

To what extent can we explore the recast logics in a systematic way? The answer is: quite systematically, since they are essentially *normal modal logics*. (See Chellas 1980 for general background.) Notice that unlike with standard SDL, where what we have is itself a normal modal logic whose metatheory is thus as simple as that of its Andersonian-Kangerian analogues, with DWE we have logics that are not normal modal logics, and logics that are mated to richer semantic structures than those employed in normal modal logics. The result is that metatheory for DWE logics is difficult, slow, and so far, piecemeal. Thus recasting a large subset of the DWE logics in an Andersonian-Kangerian framework allows for a more systematic and sweeping study of these logics, one where we can rely on and adapt the well known results on normal modal logics. I would like to suggest that the greatest gain from recasting deontic logics as Andersonian-Kangerian modal logics will be had when the frameworks recast are frameworks that, like DWE, have more complex logics and semantic structures than do normal modal logics. The recast of SDL as AK-SDL tends to obscure this potential gain because the logics are normal modal logics in both cases. In any case, the ease with which we can generate results for the AK-DWE framework will now be demonstrated.

4.1 The Logics

For theoretical generality, we stated our semantic framework for AK-DWE with very few constraints built in. Accordingly, we define a correspondingly wide class of associated logics, with a very weak base logic, as follows:

> L is a Normal AK-DWE Logic iff
> 1) L contains all tautologous wffs,
> 2) L contains: $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$, for any wffs, p and q;
> 3) If L contains $(p \rightarrow q)$ and p then L contains q, for any wffs, p and q;
> 4) If L contains p then L contains $\Box p$, for any wffs, p and q.

Plainly, the base logic will essentially just be the normal modal system, K, but with a language that differs from that of K's in having three distinguished propositional constants. Call this system "AK-K". Consider the following nine candidate axioms:

Candidate $\Box$-$\Diamond$ Axioms:	Candidate *d*-*x*-*n* Axioms:
T: $\Box p \rightarrow p$	A1: $\Diamond d$
B: $\Diamond\Box p \rightarrow p$	A2: $\Diamond x$
S4: $\Box p \rightarrow \Box\Box p$	A3: $\Diamond n$
S5: $\Diamond p \rightarrow \Box\Diamond p$	A4: $\Box(x \rightarrow d)$
	A5: $\Box(n \rightarrow d)$

The list to the left contains four familiar modal schemata that play an important role in Andersonian-Kangerian logics in generating deontic formulae that involve embedded deontic operators, such as "$\mathbf{OB}(\mathbf{OB}p \rightarrow p)$". (See Aqvist 1984). The list on the right contains the candidates with a special deontic flavor. We will designate Normal AK-DWE Logics generated by adding combinations of the above nine candidate axioms to AK-K in a familiar way. To save space, in such designations, I will just use "K" for AK-K. We will designate other logics by appending to "K" the string of labels for any of the other nine axioms above that it contains, but we will drop the "A"s in the axioms on the right hand list in such designations. So, the result of adding schema T to system K will be called "KT" and the result of adding B, S5 and A4 to system K will be called "KBS54".

4.2 First Wave of Completeness and Soundness Results

Consider first the possible axiom systems we could get by adding any combinations of only the five *d*-*n*-*x* candidates above to K. There are 32: K, K1, K2, ... , K12345. These are not all distinct logics (theorem sets). In any of our logics, A2 and A4 together imply A1, as do A3 and A5 taken together. So these designations can be eliminated: K124, K135, K1234, K1235, K1245, K1345, K12345. (I think that the remaining 25 designators denote distinct systems, though I have not proven this.) Also, notice that there are certain symmetries. For example, the logics, K1, K2 and K3 are just notational variants of one another. Similarly for K4 and K5, as well as K24 and K35. I will rely on such symmetries later. These, then, are the *d-n-x* basic systems that will concern us:

K,	K1,	K2,	K3,	K4,
K5,	K12,	K13,	K14,	K15,
K23,	K24,	K25,	K34,	K35,
K45,	K123,	K125,	K134,	K145,
K234,	K235,	K245,	K345,	K2345.

I will adapt well-known metatheoretic results about normal modal logic to our normal AK-DWE logics. (I will only sketch proofs, focusing on just the novel components.) The crucial move is to specify a canonical model for each logic and to prove an AK-DWE analog to the Fundamental Theorem for Canonical Models for normal modal logics:

> <u>The Canonical Models for AK-DWE Logics</u>: Where L is any AK-DWE logic, the canonical model for L, designated as CM^L, is $<<W^L, R^L, DEM^L, MAX^L, MIN^L>, V^L>$, where:
> 1) W^L = the set of all L-maximal-consistent sets (of wffs);
> 2) $R^L \subseteq W^L \times W^L$ such that $R^L ij$ iff {p: □p ∈ i} is a subset of j, where $i, j \in W^L$;
> 3) $DEM^L = \{j: d \in j \ \&\ j \in W^L\}$;
> 4) $MAX^L = \{j: x \in j \ \&\ j \in W^L\}$;
> 5) $MIN^L = \{j: n \in j \ \&\ j \in W^L\}$;
> 6) $V^L(p) = \{j: p \in j \ \&\ j \in W^L\}$, where p is any propositional variable.

It is easy to see that for each logic, a unique canonical model exists and that it is an AK-DWE model. Next, we prove:

> <u>The Fundamental Theorem (FF) for Canonical Models</u>: Where L is any normal AK-DWE logic, CM^L is the canonical model for L, $i \in W^L$, and p is any AK-DWE wff:
>
> $$CM^L \models_i p \text{ iff } p \in i.$$
>
> Proof by induction on complexity of the wffs:
> <u>Base Case</u>: (p is a propositional variable or distinguished constant.)
> a) p is a propositional variable: (As with normal modal logics)
> b) p is *d*: By [*d*], $CM^L \models_i d$ iff $i \in DEM^L$. But $DEM^L = \{j: d \in j\}$.
> So $CM^L \models_i d$ iff $d \in i$.
> c) p is *x*: (Similarly).
> d) p is *n*: (Similarly).
> <u>Inductive Step</u>: (As with normal modal logics, since this step only involves wffs whose main connective is a t-functional connective or "□".)

A trivial consequence of FF is:

> <u>Corollary of FF</u>: Where L is any normal AK-DWE logic and CM^L is the canonical model for L, $CM^L \models p$ iff p is contained in every world in W^L.

Note two facts (the proofs are well-known):

> Lindenbaum's Lemma (LL): If X is an L-consistent set of wffs, then there exists an L-maximal-consistent extension of X.

> Adjunct to LL: For any logic L and wff p, p is a theorem of L iff p is in every L-maximal-consistent set.

We now draw out a consequence from our Corollary to FF and the Adjunct to LL:

> Consequence: Where L is any normal AK-DWE logic and CM^L is the canonical model for L, $|\!-_L$ p iff $CM^L \models$ p.

With this coincidence in hand, to prove completeness for a given AK-DWE logic, L, for a class of models, C, it is sufficient to show that the canonical model for L is a member of C, and thus that any wff valid in C must be a theorem of L. So AK-K is then complete for the class of all AK-DWE models. It is also sound for this class of models. This follows from well-known results for normal modal logics. So we have our first determination theorem:

> K is determined by the class of all AK-DWE models: K is sound and complete with respect to the class of all AK-DWE model.

For the remaining results, let us first list five basic constraints on frames:

5 Basic Semantic Constraints on DEM, MIN and MAX:

DEM-Seriality:	$\forall i\exists j(Rij \;\&\; j \in DEM)$.
MAX-Seriality:	$\forall i\exists j(Rij \;\&\; j \in MAX)$.
MIN-Seriality:	$\forall i\exists j(Rij \;\&\; j \in MIN)$.
MAX-DEM Subordination:	$\forall i\forall j$[if Rij then ($j \in MAX$ only if $j \in DEM$).
MIN-DEM Subordination:	$\forall i\forall j$[if Rij then ($j \in MIN$ only if $j \in DEM$).

We now show that the presence of the deontic axioms, A1, A2, ..., A5 (respectively) in any AK-DWE logic entails that the canonical model for that logic is DEM-serial, MAX-serial, MIN-serial, MAX-DEM subordinate, MIN-DEM subordinate (respectively). So the presence of these axioms in one of our logics "forces" the canonical model to satisfy the associated structural constraint:

> The 1st Forcing Theorem: If L is any normal AK-DWE logic and CM^L is its canonical model, then:
> (1) If L contains A1, then CM^L is DEM-serial;
> (2) If L contains A2, then CM^L is MAX-Serial;
> (3) If L contains A3, then CM^L is MIN-Serial;
> (4) If L contains A4, then CM^L is MAX-DEM subordinate;
> (5) If L contains A5, then CM^L is MIN-DEM subordinate.

> 1) Suppose L contains A1, so that $\Diamond d$ is a theorem of L. Then by the Adjunct to LL, $\Diamond d$ is a member of every L-maximal-consistent set, and hence by the definition of W^L in CM^L, $\Diamond d$ is a member of every world in CM^L. But then by the Fundamental

Theorem, $\Diamond d$ must also be true at every world in CM^L. Let i be any such world. Then by [$\Diamond$], $\exists$j(R^Lij & CM^L |=j d). But then by [d], CM^L |=j d iff j $\in$ DEM^L, for any j. So $\exists$j(R^Lij & j $\in$ DEM^L). And i was arbitrary. Hence CM^L is DEM^L serial.
2) (Similarly)
3) (Similarly)
4) Suppose L contains A4: $\Box(x \rightarrow d)$. Then, since A4 is a theorem of L, once again, by the Adjunct to LL, the definition of W^L in CM^L and the Fundamental Theorem, it follows that, $\Box(x \rightarrow d)$ is true at every world in CM^L. Let i be any such world. Then by [$\Box$], it follows that $\forall$j(if R^Lij then CM^L |=j $(x \rightarrow d)$. But then by [PL], it follows that
$\forall$j(if R^Lij, then CM^L |=j x only if CM^L |=j d). But by [x], CM^L |=j x iff j $\in$ MAX^L; and by [d], CM^L |=j d iff j $\in$ DEM^L. So substituting equivalents, we get $\forall$j(if R^Lij, then j $\in$ MAX^L only if j $\in$ DEM^L. So CM^L is MAX-DEM Subordinate.
5) (Similarly)

Now this 1st Forcing Theorem says that *any* canonical model for *any* logic that contains a given axiom from the list also satisfies the associated semantic constraint. So *any combination* of the five axioms in a logic will entail that the corresponding canonical model will satisfy the *combination* of associated semantic constraints. Further, we saw that for any logic, L, and any class of models, C, that contains the canonical model for L, L must be complete with respect to C. So we have our first completeness results:

<u>1st Completeness Theorem</u>: The following 25 AK-DWE logics are complete with respect to the class of AK-DWE models specified by the combination of constraints checked in the logic's row. (We repeat the minimal case, K.)

Logic:	DEM-Srl:	MAX-Srl:	MIN-Srl:	MAX-DEM-Sub:	MIN-DEM-Sub:
1. K:					
2. K1:	+				
3. K2:		+			
4. K3:			+		
5. K4:				+	
6. K5:					+
7. K12:	+	+			
8. K13:	+		+		
9. K14:	+			+	
10. K15:	+				+
11. K23:		+	+		
12. K24:		+		+	
13. K25:		+			+
14. K34:			+	+	
15. K35:			+		+
16. K45:				+	+
17. K123:	+	+	+		
18. K125:	+	+			+
19. K134:	+		+	+	
20. K145:	+			+	+
21. K234:		+	+	+	
22. K235:		+	+		+
23. K245:		+		+	+

24. K345:		+	+	+
25. K2345:	+	+	+	+

Let's turn these completeness results into determination results by showing that each logic above is also sound with respect to its associated class of models. We remind the reader of a handy fact about validity:

> <u>Fact about Validity</u>: If p_1, ..., p_n are respectively valid in the classes of models, C_1, ..., C_n, then the p_i are jointly valid in the class of models constituting the intersection of the C_i.

We will use this handy fact in a moment. The key lemma we now need is:

> <u>A1-A5 Soundness Lemma</u>:
> 1) A1 is valid in the set of DEM-serial AK-DWE models;
> 2) A2 is valid in the set of MAX-Serial AK-DWE models;
> 3) A3 is valid in the set of MIN-Serial AK-DWE models;
> 4) A4 is valid in the set of MAX-DEM subordinate AK-DWE models;
> 5) A5 is valid in the set of MIN-DEM subordinate AK-DWE models;
>
> 1) Assume *M* is any DEM-serial model. Then for every world, i, in the model, ∃j(Rij & j ∈ DEM). But by [◊], M |=i ◊*d* iff ∃j(Rij & *M* |=j *d*), and by [*d*], *M* |=j *d* iff j ∈ DEM. So given our assumption, *M* |=i ◊*d*.
> 2) (Similarly)
> 3) (Similarly)
> 4) Suppose *M* is MAX-DEM Subordinate. Then ∀j[if Rij then (j ∈ MAX only if j ∈ DEM). Select any world, i, from W in *M*. By [☐], *M* |=i ☐(*x* → *d*) iff ∀j[if Rij, then *M* |=j (*x* → *d*)]. But by [PL], this is equivalent to ∀j[if Rij, then *M* |=j *x* only if *M* |=j *d*], and by [*x*] and [*d*], this is equivalent to ∀j[if Rij, then j ∈ MAX only if j ∈ DEM], which is what our assumption gives us.
> 5) (Similarly)

Now AK-K is sound with respect to any set of AK-DWE models, not just the set of all AK-DWE models. For its axioms are true in all models, and the rule patterns, MP and Necessitation, preserve *truth* in any model. Hence they must preserve *validity* with respect to any class of AK-DWE models. This is all we need for the following result:

> <u>1st Soundness Theorem</u>: Each of the 25 logics in our chart above is sound with respect to its associated class of AK-DWE models.

For our Soundness Lemma shows that A1-A5 are respectively valid in the DEM-serial models, MAX-Serial models, MIN-Serial models, MAX-DEM subordinate models, MIN-DEM subordinate models. But given our handy fact about validity and intersection, and the fact that system K is sound with respect to any class of models, it follows that the result of adding any *combination* of A1-A5 to K will be sound with respect to the intersection of the corresponding combination of the DEM-serial, MAX-Serial, MIN-Serial, MAX-DEM subordinate and MIN-DEM subordinate models. So our completeness chart is really a determination chart:

> <u>Determination Corollary</u>: Each of the 25 logics is our chart above is determined by

its associated class of models.

4.3 Second Wave of Completeness and Soundness Results

We now turn back to our four candidate □-◊ axioms and weave those in:

T: $\Box p \rightarrow p$ S4: $\Box p \rightarrow \Box\Box p$
B: $\Diamond\Box p \rightarrow p$ S5: $\Diamond p \rightarrow \Box\Diamond p$

As is well known, of the sixteen possible axiom systems that we can generate by adding combinations of these to K, there are only 10 distinct theorem sets, and as we have already included one of them, K, we need only consider these nine:

KT, KB, KS4, KS5, KTB, KTS4, KTS5, KBS4, KS4S5.

Here are four associated constraints we can place on the accessibility relation:

4 Basic Semantic Constraints on R:
Reflexivity: $\forall i Rii$ Transitivity: $\forall i \forall j \forall k$[if Rij & Rjk then Rik]
Symmetry: $\forall i \forall j$(Rij iff Rji) Euclidean: $\forall i \forall j \forall k$[if Rij & Rik then Rjk]

Now the following forcing theorem is a trivial adaptation of well known results:

The 2nd Forcing Theorem: If L is any normal AK-DWE logic and CM^L is its canonical model, then:
1) If L contains T then R^L is Reflexive;
2) If L contains B, then R^L is Symmetrical;
3) If L contains S4, R^L is Transitive;
4) If L contains S5, R^L is Euclidean.

By parity of the reasoning used for the first 25 logics, we easily get:

2nd Determination Theorem: The following 9 AK-DWE logics are complete and sound with respect to the class of AK-DWE models specified by the combination of checked constraints in its row.

Logics:	Reflexivity:	Symmetry:	Transitivity:	Euclideanness:
1. KT	+			
2. KB		+		
3. KS4			+	
4. KS5				+
5. KTB	+	+		
6. KTS4	+		+	
7. KTS5	+			+
8. KBS4		+	+	
9. KS4S5			+	+

4.4 Merging the Two Waves of Results

Now consider the possibility of combining logics from our first list with those from our second list. I offer the following conjecture:

> Conjecture: Where L1 and L2 are any logics from our first list of 25 other than K, and L3 is any logic from the above list of 9, the intersection of L1 with L3 is distinct from the intersection of L2 with L3.

If this is right, and if I'm right in thinking that the original 25 logic designators designate 25 distinct theorem sets, then we have 241 (25 + (24 x 9)) distinct logics. If so, by parity of the reasoning employed earlier for combinations of axioms and associated constraints, we have an additional 207 derivative determination results.

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