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Pure Logic and Higher-order Metaphysics

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Introduction

W. V. Quine famously defended two theses that have fallen rather dramatically out of fashion. The first (1956, p. 180; 1960) is that *intensions* — n-place properties (= relations, when $n \geq 2$) and propositions — are "creatures of darkness" that ultimately have no place in respectable philosophical circles, owing primarily to their lack of rigorous identity conditions. However, although he was thoroughly familiar with Carnap's (1946; 1947) foundational studies in what would become known as possible world semantics, it likely wouldn't yet have been apparent to Quine that he was fighting a losing battle against intensions, due in large measure to developments stemming from Carnap's studies and culminating in the work of Kripke (1959; 1963), Hintikka (1961; 1963), and Bayart (1958; 1959[1]). These developments undermined Quine's crusade against intensions on two fronts. First, in the context of possible world semantics, intensions could after all be given rigorous identity conditions by defining them (in the simplest case) as functions from worlds to appropriate extensions, a fact exploited to powerful and influential effect in logic and linguistics by the likes of Kaplan (1964), Montague (1970), Lewis (1970), and Cresswell (1973). Second, the rise of possible world semantics fueled

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[1]Bayart's important papers are still not very well known largely, it seems, because they remained untranslated until Max Cresswell (2015) published his translations of them, along with extensive commentary.

a strong resurgence of metaphysics in contemporary analytic philosophy that saw properties and propositions widely, fruitfully, and unabashedly adopted as ontological primitives in their own right — Plantinga 1974, Parsons 1974, Adams 1981, Bealer 1982, and Zalta 1983 are some notable examples. This resurgence — happily, in my view — continues into the present day.

For a time, at any rate, Quine experienced somewhat better success with his second thesis: that higher-order logic is, at worst, confused and, at best, a quirky notational alternative to standard first-order logic. However, Quine notwithstanding, a great deal of recent work in formal metaphysics transpires in a higher-order logical framework in which properties and propositions fall into an infinite hierarchy of types of (at least) every finite order.[2] Initially, the most philosophically compelling reason for embracing such a framework since Russell (1903) first proposed his simple theory of types was simply that it provides a relatively natural explanation of the paradoxes. However, since the seminal work of Prior (1971) there has been a growing trend to consider higher-order logic to be the most philosophically *natural* framework for metaphysical inquiry, many of the contributors to this volume being among the most important and influential advocates of this view. Indeed, this is now quite arguably the dominant view among formal metaphysicians.

In this paper, and against the current tide, I will argue in §1 that there are still good reasons to think that Quine's second battle is not yet lost and that the correct framework for logic is first-order and *type-free* — properties and propositions, logically speaking, are just individuals among others in a single domain of quantification — and that it arises naturally out of our most basic logical and semantical intuitions. The data I will draw upon are not new and are well-known to contemporary higher-order metaphysicians. However, I will try to defend my thesis in what I believe is a novel way by suggesting that these basic intuitions ground a reasonable distinction between "pure" logic and non-logical theory, and that Russell-style semantic paradoxes of truth and exemplification arise only when we move beyond the purely logical and, hence, do not of themselves provide any strong objection to a type-free conception of properties and propositions.

Most of my arguments in §1 are largely independent of any specific account of the nature of properties and propositions beyond their type-freedom. However, I will in addition argue that there are good reasons to take propositions, at least, to be very fine-grained. My arguments are thus bolstered significantly if it can be shown that there are in fact well-defined examples of logics that are not only type-

[2]See Skiba 2021 for a helpful overview.

free but which comport with such a conception of propositions. It is the purpose of §2 to lay out a logic of this sort in some detail, drawing especially upon work by George Bealer (1982; 1989; 1998) and related work of my own (1993). With the logic in place, it will be possible to generalize the line of argument noted above regarding Russell-style paradoxes and, in §3, apply it to two propositional paradoxes — the Prior-Kaplan paradox and the Russell-Myhill paradox — that are often taken to threaten the sort of account developed here.

In brief, then, my goal here is to outline a broad defense of a somewhat neglected type-free alternative to the ascendent higher-order conception of properties and propositions in contemporary metaphysics. The goal is ambitious — perhaps overly so for a volume of this sort — and my arguments are at points, I fear, more compressed than the issues deserve. My hope, however, is that the overall picture is sufficiently coherent and compelling to warrant a place in the ongoing philosophical conversation.

1 The Natural Ontology of Pure Logic

1.1 Pure Logic

The question of what distinguishes logic from more general philosophical and scientific disciplines is a disputed matter. And, as with pretty much all deep disputes in philosophy, there is a spectrum of reasonable answers.[3] Nonetheless, there does seem to be an overarching idea — call it *pure logic* — that is common to the majority of these answers, namely, that logical systems — systems of formal reasoning, broadly understood — should be *universal* and *topic neutral* (Ryle 1954, ch. 8). The idea of universality is reasonably clear: the general patterns of representation and reasoning that a logic is capable of expressing should be applicable to most any conceivable domain. Topic neutrality is somewhat vaguer. A first crack at it might be that logic functions as "a neutral arbiter of metaphysical disputes, whose proper function is compromised by any metaphysical commitments of its own".[4] Logic should thus be "independent of metaphysical assumptions" (Chwistek 1967, p. 359). However, not only is it difficult to spell

[3]Notably, at opposing ends of the spectrum are semantic (e.g., Sher 1991) and proof-theoretic (e.g., Hacking 1979) approaches to the question. For comprehensive overviews of the issues, see MacFarlane 2017 and Beall, Restall, and Sagi 2019.

[4]The characterization is Williamson's (2013, p. 146) but is assuredly not a view that he himself espouses.

this idea out in a non-question-begging way,[5] there is no intuitive warrant for dismissing *a priori* the prospect that logic might naturally entail some substantive metaphysics. Another common characterization that gets a bit closer to the mark is that logic should "lack ... a distinctive subject matter" (Hodes 1984, p. 123). As Boolos (1975, p. 517) has pointed out,[6] this characterization threatens to rule out logic itself, insofar as it seems that any logic can be thought to have its logical constants as its subject matter. I think Russell (1920, p. 196) gets us closer to the mark:[7]

> [W]e do not, in this subject, deal with particular things or particular properties: we deal formally with what can be said about *any* thing or *any* property .... It is not open to us, as pure ... logicians, to mention anything at all, because, if we do so, we introduce something irrelevant and not formal.

That is, according to Russell, what distinguishes pure logic, in addition to its universality, is its freedom from *specific* ontological commitments: while a logic might well have a distinctive subject matter in its logical constants and may well involve commitment to general ontological *categories* like *thing* and *property*, a logic shouldn't of itself entail the existence of any *particular things* or *particular properties*; it should be *indifferent* to the particular identities and natures of things.[8] Such commitments arise only through the introduction of specific non-logical expressions.

Although it will need a bit of qualification below, this conception of pure logic is a good starting point. I will argue that basic pre-theoretic syntactic and semantic phenomena of ordinary discourse in large measure warrant this conception and lead us naturally to the type-free, first-order, hyperintensional logic that I will develop here.

1.2 Talking about Things

The most basic semantic phenomenon is that we talk about things. We *describe* or *characterize* them, most fundamentally by means of acts of *predication*. In such an act, a thing denoted by a name or other singular noun phrase is characterized by a verb phrase, as in

[5]Williamson (2014, pp. 229-30) argues this cogently against Kaplan (1995) and Etchemendy (1990).

[6]Haack (1978, pp. 5-6) raises related concerns.

[7]See also, e.g., Peacocke (1976), p. 229.

[8]Cf. MacFarlane 2017, §4. Similar ideas have been advocated by a number of other philosophers, of course, e.g., Peacocke 1976 and Sainsbury 2001, p. 314. See also Russell (1984, p. 98).

(1) Annemiek is a cyclist

or

(2) Chelsea is wise.

When a verb is transitive, such characterizations can involve more than one thing, as in

(3) Chelsea loves David.

Predication is of course familiarly represented in first-order logic by conjoining a predicate symbol with one or more individual constants: for the above cases, Ca, Wc, and Lcd. Importantly, though, in keeping with the thesis of topic neutrality, the *specific* constants and predicates chosen and their intended meanings are not a matter of pure logic but of its application. From the standpoint of pure logic, the observation that we characterize things by means of predication only entails commitment to the general syntactic categories *individual constant* and *n-place predicate* and the general semantic category of *things*.

That the semantic function of names, and singular terms more generally, is to denote is largely uncontroversial. The semantics of *predicates* has been a more disputed matter. Quine (1948, p. 30) famously argued that *meaningfulness* does not require *meanings*. One can acknowledge (so says Quine) the meaningfulness of 'is a cyclist' or 'loves' — that is, the fact that they can be used in sentences that convey information — without being committed to the existence of any *things* that they mean. Quine's ontological austerity, however, is unsustainable. For it is undercut by the ubiquitous phenomenon of *nominalization* that enables us to create singular noun phrases (typically, gerunds and infinitives) from predicates, e.g.: from 'is a cyclist', we have 'being a cyclist' and 'to be a cyclist'; from 'loves', 'loving' and 'to love'; and from 'is wise', in addition to 'being wise' and 'to be wise', the abstract noun 'wisdom'. And, as singular noun phrases, nominalizations *denote* and their denotations can themselves be subjects of predication, as in

(4) To be a cyclist is all that David desires.

(5) Some people seek wisdom

and

(6) Being a gymnast is more challenging than being a cyclist.

Moreover, intuitively, what a nominalized predicate denotes is exactly what the predicate itself means, or *expresses* — what is attributed to Chelsea in (2) is exactly what some people are said to seek in (5). So, Quine notwithstanding, natural grammatical and semantic intuitions lead to the conclusion that there are after all *things* that meaningful predicates mean — and hence that, among the categories of things that pure logic is committed to are the semantic values of predicates and their nominalizations, i.e., *properties*.[9]

These observations suggest that, in order to represent the phenomenon of nominalization adequately in a logical language, corresponding to each predicate symbol π, there should be a corresponding term τ_π representing its nominalization that denotes exactly the same property that π expresses. However, given that π and τ_π denote the same thing, there is no need formally to introduce any explicit term-forming operator to distinguish a predicate symbol from its nominalized counterpart, as context alone suffices: we can simply allow predicate symbols to occur as arguments to other predicate symbols and understand any such occurrence of a predicate symbol to serve as its nominalized counterpart. This enables to us formalize (4), (5), and (6) simply as AdC, $\exists \mathsf{x}(\mathsf{Px} \wedge \mathsf{SxW})$, and MGC, respectively.

1.3 Nominalization and Type-freedom

Note, however, that there are complications if we take properties to be typed and we take the denotation of a nominalization τ_π to be the property expressed by the corresponding predicate π. For if we do, then it appears that we are forced to acknowledge ambiguities where, intuitively, none exist. For example, in a typed framework, the occurrence of 'loves' in

(7) Chelsea loves David,

must mean something entirely different than does its occurrence in, say,

(8) Chelsea loves wisdom

or

(9) Annemiek loves being a cyclist.

In a classical (typed) possible worlds framework, for example, in the first of these the predicate 'loves' signifies a first-order intension l_0 taking worlds to sets of pairs

[9] This argument admittedly passes hastily over a host of subtle and complex issues. See, e.g., MacBride 2006 for a more comprehensive overview.

of individuals, whereas in the latter two it signifies a second-order intension l_1 taking worlds to sets of pairs of the form $\langle a, f_0 \rangle$ where a is an individual and f_0 a first-order intension. No doubt, psychologically, loving a person is quite different from loving a virtue or a quality. But there is no obvious reason to deny that there is (perhaps over and above more specialized subproperties) a most general *loving* property that might well be signified in (written or spoken instances of) all three occurrences of 'loves'; at any rate, the existence of such a property doesn't seem to be the sort of thing that *logic* should rule out *a priori*.

The issue is even more pronounced in sentences like

(10) Chelsea loves loving.

A legitimate reading of (10) is that it is a completely general claim about Chelsea's feelings for *loving* of any sort. But this reading cannot even be captured on a typed conception. For the assignment of an intension of any specific logical type to 'loves' in (10) will only apply to *loving* properties of lower type and, in particular, will not include the instance expressed in (10) itself. The best one can do is take (10) to be a sort of schematic claim expressing infinitely many different propositions involving distinct *loving* properties of increasingly higher type. On the face of it, though, there need be no such ambiguity; (10) can be read as a simple and perfectly univocal claim about Chelsea's attitude toward a single general property. Its division into a hierarchy of distinct intensions of increasingly higher type, and the corresponding division of distinct but homonymous 'loves' predicates into a hierarchy of syntactic types, appears to be much more of a theoretical artifact than a natural reflection of ordinary semantic intuitions.

It is important to note as well that this sort of self-exemplification is not essentially tied to intentional predicates like 'loves'. For many theoretical properties like *being a property*, *having no mass*, and *being self-identical* are all intuitively self-exemplifying and, hence, also require a linguistic framework in which predicates can apply to their own nominalizations. Indeed, the entire framework of type theory teeters rather perilously on the edge of self-refutation. For instance, it is a metatheoretic truth of the semantics of higher-order logic that

(11) *Every* property belongs to *some* type.

It is difficult to see any other way of understanding (11) than as a general, untyped proposition of the form $\forall \mathsf{x}(\mathsf{Px} \rightarrow \exists \mathsf{y}(\mathsf{Ty} \wedge \mathsf{Bxy}))$ whose quantifiers range unrestrictedly over a universe that contains all properties and all types.

All of these awkward consequences appear to cut against a typed conception of properties.[10] It is much simpler just to follow the above phenomena where they seem to lead, namely, to the view that properties are not only *things*, but *things among all other things* that we talk about and quantify over within a single universe of discourse. Our reflections have thus led us naturally to a conception of pure logic that is first-order and whose nominalized predicate symbols are of the same syntactic type as individual constants.[11]

Hyperintensionality. The central motivation for introducing intensions is of course to explain apparent failures of classical substitutivity principles. Possible world intensions — functions from worlds to appropriate extensions — explain well-known failures in modal contexts effectively but, notoriously, such intensions are identical if necessarily coextensional, and this leads to equally well-known substitutivity failures in so-called *hyperintensional* contexts:[12] to use a standard example, while *triangularity* and *trilaterality* are necessarily true of the same geometrical objects, they are intuitively distinct, as one concerns the number of angles of a geometric figure and the other the number of sides. Hence, from the fact that

(12) Chelsea is pondering triangularity

it doesn't seem to follow that

(13) Chelsea is pondering trilaterality.

Type-freedom alone does not preclude identifying necessarily coextensional properties like *triangularity* and *trilaterality* and, hence, cannot of itself explain such apparent failures of substitutivity. But if we are taking our clues from data like this, a proper framework for pure logic should at the least provide an explicit mechanism that can. A simple enhancement to our framework will do for purposes here: we simply take properties to be unstructured elements of the domain, each with its own corresponding possible world intension — albeit one that it

[10]The brevity of my presentation here is not meant to suggest that these consequences are clearly decisive. See, notably, Williamson 2013, ch. 5, where considerable space is devoted to the challenge of interpreting higher-order logic.

[11]Cocchiarella's (1973) second-order system T* also treats properties as logical individuals and, accordingly, classifies predicates as terms. As Cocchiarella (1985) also documents, the idea clearly traces back to Frege's (1960) notion of concept correlates (a.k.a., value-ranges, *Wertverläufe*), although of course Frege (1951) postulated an inviolable ontological divide between concepts and objects, and hence between the semantic values of predicates and the semantic values of terms. He would thus have vociferously objected in particular to our identification of the semantic value of a predicate with that of its nominalization.

[12]See Perry 1998 and Berto and Nolan 2021 for informative overviews.

can share with other properties. In this way, necessarily coextensional properties — *triangularity* and *trilaterality*, in particular — can nonetheless be distinct, and intuitive substitutivity failures like the one above can be explained accordingly.

1.4 Predicate Quantifiers and the Specter of Paradox

A natural concern might arise at this point. On a type-free reading, (10) says that *loving* holds between an individual and *loving* itself. More generally, in a type-free setting, where properties are both individuals and, at each world, can be *true of*, or *hold between*, individuals, the prospect of this sort of self-exemplification is unavoidable. But does this not open the door to an intensional version of Russell's paradox? For consider the property N of *non-self-exemplification* that is true of a property P exactly when P is not true of itself. By introducing a dedicated predicate N we can axiomatize N schematically as follows in the type-free framework we are developing here:

(14) $\mathsf{N}\pi \leftrightarrow \neg\pi\pi$, for monadic predicates π.

The contradictory instance $\mathsf{NN} \leftrightarrow \neg\mathsf{NN}$ is of course immediate.

However, it is important not to stray beyond what we are about here, namely, the construction of a general logical framework that is rooted in equally general syntactic and semantic phenomena. While our observations concerning reference, predication, and nominalization have led us to a framework that is committed to a very general ontological subcategory of things — *properties* — that serve as the semantic values of predicates and their nominalizations, nothing in those observations has forced us to include any *specific* predicates in our language or to adopt any principle that commits us to the existence of any specific properties satisfying any specific principles *a priori*, the property *non-self-exemplification* in particular.

But perhaps this response is too facile. For one might well argue that, in fact, our observations so far *do* after all set us up for paradox. For, first of all, since, as just noted, we have acknowledged the general category (*n-place*) *property*, the introduction of predicate quantifiers ranging over them would seem to be warranted. Second, our assumption that meaningful predicates express properties would seem to be captured by something like the following informal *comprehension principle*:

(15) For any meaningful predicate φ, there is a property F_φ such that, necessarily, F_φ is true of something x just in case x is φ.

And given predicate quantifiers, the natural formalization of (15) is simply the standard second-order comprehension principle:[13]

$\mathbf{C^2}$ $\exists F\forall x(Fx \leftrightarrow \varphi)$, for any formula φ in which F does not occur free.

However, in a type-free setting where properties are individuals and (hence) predicate variables (or nominalizations thereof) are terms, $\mathbf{C^2}$ leads quickly to disaster. For, letting φ be the purely logical formula $\exists G(x = G \wedge \neg Gx)$, $\mathbf{C^2}$ explicitly yields the existence of a *non-self-exemplification* property. And, assuming standard principles of identity and classical quantifier elimination and introduction rules, a Russellian paradox quickly ensues:

R1[14]	$\exists F\forall x(Fx \leftrightarrow \exists G(x = G \wedge \neg Gx))$	$\mathbf{C^2}$
R2	$\forall x(Rx \leftrightarrow \exists G(x = G \wedge \neg Gx)))$	Assumption (for **∃Elim**)
R3	$RR \leftrightarrow \exists G(R = G \wedge \neg GR)$	R2, **∀Elim**
R4	$\forall F(\neg FF \leftrightarrow \exists G(F = G \wedge \neg GF))$	logic
R5	$\neg RR \leftrightarrow \exists G(R = G \wedge \neg GR)$	R4, **∀Elim**
R6	$RR \leftrightarrow \neg RR$	R3, R5
R7	$\exists F(FF \leftrightarrow \neg FF)$	R6, **∃Intr**
R8	$\exists F(FF \leftrightarrow \neg FF)$	R1, R2-R7, **∃Elim**

But now it appears we have *not* after all strayed beyond what we are about and have still generated an inconsistency directly from within our general, purely logical framework.

This important objection deserves a careful response that bears directly on the general thesis of the primacy of first-order logic. As noted, our methodology here is to design a framework for logic based upon fundamental grammatical and semantic intuitions. Our initial reflections on reference, predication, and nominalization have led us to our type-free logical framework. And further reflection on these fundamental intuitions suggests that predicate quantifiers have no place here. To make things more concrete, suppose we were to infer

[13]See Enderton 2001, ch. 4 for a thorough introduction to basic second-order logic and Shapiro 1991 for an advanced mathematical and philosophical exploration.

[14]Since it includes second-order comprehension, R1 is also a theorem of Cocchiarella's (1973) system T* (see note 11 above) but the argument here cannot be replicated in T* because Cocchiarella defines identity as indiscernibility (Ibid., note 22), from which the right-to-left direction of the embedded biconditional in R4 cannot be proved, preventing thereby the derivation of R4 itself. As a consequence, it is a theorem of T* that there there are indiscernible properties that are not coextensive. (One might reasonably think that if two properties are not coextensive, then there must be properties each has that the other lacks, but the existence of such properties is not provable from Cocchiarella's form of Comprehension. See Cocchiarella 1972, pp. 169ff, for his justification of (his form of) the principle.)

from the fact that Socrates and Plato are both wise that there is something that they both are, and that we formalize this inference as second-order, $\exists F(Fs \wedge Fp)$. Consider now a notorious (and, in the literature on higher-order metaphysics, much discussed) passage from Quine (1986, pp. 66-67):

> To put the predicate letter 'F' in a quantifier...is to treat predicate positions suddenly as name positions....The quantifier '$\exists F$' or '$\forall F$' says...that some or all entities of the sort named by predicates are thus and so.

In standard model theoretic semantics, of course, the entities named by predicates are sets. Hence, so understood, second-order quantifiers just range over yet more objects — albeit of a certain distinctive sort — and so are really just restricted first-order quantifiers in disguise. Hence, it is much more ontologically transparent — and philosophically more on the level — to rewrite second-order statements (and higher-order statements generally) in the language of applied first-order set theory: instead of $\exists F(Fs \wedge Fp)$, $\exists x(s \in x \wedge p \in x)$. Of course, modern-day proponents of higher-order metaphysics take their higher-order quantifiers to be ranging, not over sets, but over intensions — properties and propositions. But Quine's argument still seems to cut:[15] instead of saying that some or all sets are thus and so, the higher-order quantifiers say that some or all intensions (of some type or other) are thus and so. But if, as we have found strong reason to believe above, intensions, too, are objects, yet more *things* among other things, then the higher-order quantifiers are still just restricted first-order quantifiers in disguise and, hence, in particular, our inference from the shared wisdom of Socrates and Plato is more transparently (and more honestly) rendered in first-order terms: instead of $\exists F(Fs \wedge Fp)$, $\exists x(E^2sx \wedge E^2px)$, where E^2 expresses the (binary) *exemplification* relation.[16]

The obvious — and by far the most common — strategy for avoiding the Quinean argument is to identify some reason why higher-order quantifiers are not after all reducible to first-order. This gambit was famously played by George Boolos (1984; 1985), who first pointed out that second-order quantifiers can be understood to be *plural* quantifiers. As plural quantification is (provably) essentially

[15]Indeed, Quine himself (Ibid., p. 67) generalizes his argument to "attributes", i.e., properties.

[16]As argued, e.g., in Menzel 1993 and van Inwagen 2004. It is important to note that the suggestion is *not* that atomic predications $\pi\tau_1...\tau_n$ are *really* of the form $E^{n+1}\pi\tau_1...\tau_n$; that way lies a vicious variety of the Third Man regress. The proposition expressed by an n-place predication involve only the property predicated and the n objects of which it is predicated — and this is reflected rigorously in the model theory of §2. Rather, the idea is that when one moves from the general commitments of pure logic to *theorizing* about properties *per se*, one needs to introduce exemplification relations.

different from singular quantification, second-order quantifiers, so understood, are irreducible to first-order, *contra* Quine,[17] but they are no more ontologically committing: they range over the same domain as ordinary first-order singular quantifiers, they just do so *plurally*.[18] Alternatively, one might pursue a nominalist strategy and — following Sellars (1960),[19] Prior (1971), and, more recently, Rayo and Yablo (2001)[20] — point out that natural English renderings of second-order quantification are in fact non-nominal and, hence, once again, are not ontologically committing. Indeed, this is illustrated in our own inference above: from the fact that Socrates and Plato are both wise, it follows only that there is something that both of them are, namely, *wise* — as Prior (p. 37) points out, "'something' here is quite clearly adjectival rather than nominal in force." And if one is antecedently uncommitted to realism about intensions, the irreducibility of higher-order quantifiers to first-order follows — the first-order reduction incurs ontological commitments that the second-order rendering does not of itself require.

However, this argument to irreducibility is not available to most contemporary higher-order metaphysicians, as they are *robustly* committed to an extraordinarily rich hierarchy of typed intensions. And here these metaphysicians — notably, Williamson (2013, §5.7) — just seem to brazen it out: robust ontological commitment notwithstanding, higher-order quantifiers are nonetheless irreducible to first-order. Williamson's express concern is to avoid the machinery of set theoretic model theory to interpret higher-order logic: sets are just individuals of a certain sort; hence, if higher-order quantifiers simply range over a hierarchy of increasingly complex sets, then quantifiers of every type are simply restricted first-order quantifiers over individuals (of which sets are just a certain sort). But his more general concern is directed toward any understanding on which higher-order quantifiers are not "doing anything genuinely new, semantically" (Ibid.,

[17] Boolos (1984, pp. 432-3) cites the Geach-Kaplan sentence in this regard: *Some critics admire only one another*. Assuming a domain consisting of all the critics, the natural second-order formalization is $\exists X(\forall x\forall y((Xx \wedge Axy) \rightarrow (x \neq y \wedge Xy)))$. Kaplan (apparently in private correspondence with Quine) pointed out that, substituting $x = 0 \vee x = y + 1$ for A yields a sentence true in all and only non-standard models of arithmetic. Its negation is therefore true in exactly the standard models and, hence, cannot be expressible in the language of first-order arithmetic, since no first-order theory with infinite models is categorical.

[18] Our second-order generalization $\exists F(Fs \wedge Fp)$ from the observation that they are both wise is thus to be rendered in English as: there are some things — notably, wise people — such that Socrates and Plato are among them.

[19] Responding to Geach's contribution to Geach, Ayer, and W. V. Quine 1951. Sellars' prescient arguments are largely neglected in the recent literature on higher-order metaphysics, which as far as I have seen uniformly attributes the origin of this line of argument to Prior (1971).

[20] See also, more recently, Dunaway 2013, Jones 2018, Cameron 2019, and Liggins forthcoming.

p. 236), and that would apply to any approach that (like the one defended here) would assimilate them to first-order quantifiers. Accordingly, Williamson argues that higher-order quantification is *sui generis* and, hence, that a proper semantics for higher-order logic can only "be formulated in a higher-order metalanguage, with unrestricted first-order quantifiers and higher-order quantifiers irreducible to first-order quantifiers over sets" (Ibid., p. 236).[21]

I confess I am not entirely confident I understand the view but there are obviously strongly Fregean overtones:[22] properties are of such a fundamentally different nature than individuals that, just as reference to concepts was impossible for Frege, it is likewise impossible for a single quantifier to range over properties and individuals alike. On such a view (problems with *expressing* the view of the sort noted above notwithstanding), a typed conception of properties and an irreducibly higher-order logic seem inevitable.

But this line of reasoning depends very much on the metaphysical significance one attributes to the sort of data we've identified above that have led us to a type-free conception of properties and, consequently, to our Quinean take on higher-order quantification. Many of those data involve intentional relations like *aspiring*, *loving*, and *pondering* that give rise to hyperintensional contexts and, according to Williamson (p. 266), hyperintensionality "arises at the level of thought and linguistic meaning and should be explained at that level, not at the level of anything like a general theory of properties and relations". Hence, for Williamson, data like the above that have led us to embrace type-freedom (and more) are irrelevant to "[t]he logic of metaphysical modality" (p. 217) and are justifiably dismissed. However, first, as noted above, some of the data we cited supporting type-freedom have nothing essentially to do with thought and meaning at all. Second, contrary to what Williamson appears to imply, ontological commitments that arise "at the level of thought and linguistic meaning" are entirely pertinent to the logic of metaphysical modality. On a realist view of thought and meaning, *there are* the things thought and meant. And, as they have distinctly modal properties and our reasoning with and about them embodies distinctly logical principles, they would seem to warrant full inclusion in a logic of metaphysical modality. So there is a concern here that, by ignoring the data assembled above in arguing for a Fregean conception of properties that purportedly explains the irreducibility of higher-order quantification, Williamson "arbitrarily restricts the evidence that we are allowed to use" in constructing or evaluating

[21] Williamson (2013) sketches such a semantics on pp. 236-8. See also Jones 2018, Trueman 2020, and the contribution of Button and Trueman to this volume.

[22] As Williamson (2003, pp. 458-9) himself suggests.

a logical theory (Sullivan 2014, p. 735). Here we are taking those data seriously. And we take them to indicate that the proper framework of pure logic is not only hyperintensional but type-free and, hence, first-order.

Returning to our Russellian paradox, then, from this perspective, the introduction of predicate quantifiers *masks* a subtle but (with respect to pure logic) methodologically illegitimate shift from the commitments of that very general logical framework to *specific* commitments that, in turn, are responsible for the specter of paradox. The first step in this shift is the move from the mere recognition of the general *phenomenon* of predication to an extra-logical, theoretical commitment to the existence of a new, specific *thing* — the relation *exemplification*. However, commitment to this reified entity is hidden in the syntax of predicate quantification. Only when we heed the reflections above and avail ourselves of completely general first-order quantifiers alone is the commitment made clear by the explicit introduction of the dyadic exemplification predicate E^2 (hence also its nominalized counterpart).

Of course, in our type-free framework, while the introduction of this new predicate symbol itself is ontologically committing, it is not the heart of the problem. Rather, it is the concomitant obligation to fix its intended meaning. Second-order logic fulfills this obligation — albeit obliquely since the commitment to the exemplification relation is only implicit in second-order syntax — via the introduction of the comprehension principle $\mathbf{C^2}$ which, even on a coarse-grained conception of properties, represents a huge increase in specific ontological commitments. In the context of standard (i.e., typed) second-order logic, those commitments might seem innocent or, at least, "thin", as every property generated by $\mathbf{C^2}$ is simply a logical construct "built up" ultimately from basic non-logical properties and relations to which one might be committed independent of $\mathbf{C^2}$. All the more so for those properties emerging simply in the pure logic of identity — for a property whose existence emerges out of such an austere context might arguably be a justifiable ontological commitment of pure logic. But the pretense of pure logicality cannot be maintained in a first-order framework, where the commitments of predicate quantification that are obscured in $\mathbf{C^2}$ are made manifest in its more ontologically transparent first-order counterpart:

$\mathbf{C^1}$ $\exists \mathsf{y} \forall \mathsf{x}(\mathsf{E}^2\mathsf{xy} \leftrightarrow \varphi)$, for any formula φ in which y does not occur free.

Expressing comprehension in the form of $\mathbf{C^1}$, with its overt postulation of an explicit exemplification relation, makes its role as the source of paradox clear. For, unlike $\mathbf{C^2}$, whose inconsistency depends on the additional (albeit, as we've seen, entirely warranted) assumption of type-freedom, $\mathbf{C^1}$ is unmistakably recognizable as an intensional analogue of the set theoretic principle of naive comprehension, whose inconsistency is immediate in any standard, classical first-order logic: just as in Russell's original paradox, letting $\varphi = \neg \mathsf{E}^2 xx$, the existence of a *non-self-exemplification* property, and inconsistency, follow apace.

Once identified, of course, the paradox can be avoided. One might, for example, consider restricting the permissible values of φ in $\mathbf{C^1}$ to formulas not containing the exemplification predicate;[23] or, following Bealer (1982, ch. 5, §29) and Jubien (1989), one might develop intensional analogues of the Zermelo-Fraenkel axioms, albeit for E^2 rather than the membership predicate $\in$. Regardless of the theoretical path one chooses, *that* one must choose one path or another to rein in the ontological profligacy of $\mathbf{C^1}$ makes it abundantly clear that we have left the domain of pure logic *per se* behind in pursuit of a robust, specialized theory of a distinguished relation — *exemplification* — whose properties are largely irrelevant to most areas of inquiry. If we mask this fact via the use of predicate quantifiers and "purely logical" (if restricted) comprehension principles, we are, as Quine would rightly insist, simply doing property theory in sheep's clothing.

1.5 Propositions

Of course, nominalization applies not only to verb phrases but also — and more significantly for our purposes here — to (declarative) sentences. Nominalized sentences come in two common forms: that-clauses and gerunds. Thus, corresponding to (1) we have the clause

(16) that Annemiek is a cyclist

and to (7), the gerund

(17) Chelsea's loving David.

As with predicates, it is natural to take the thing denoted by a nominalized sentence *qua* singular term — viz., a *proposition* — to be exactly what is meant, or expressed, by the sentence. And, as with the properties denoted by nominalized predicates, the propositions denoted by nominalized sentences can themselves be subjects of predication, as in, e.g.,

(18) That Annemiek is a cyclist surprises no one

and

[23] Zalta (1983, pp. 158-160) makes a move in this vein to avoid paradox in his object theory.

(19) Hortense doesn't approve of Chelsea's loving David.

As for representing nominalized sentences formally, in principle, as with predicates, we could classify sentences also as singular terms alongside individual constants and let context determine whether a sentence is playing a declarative role, in which it stands alone as an assertion, or a nominalized role, in which it is an argument to a predicate in an atomic sentence. However, facts about standard logical syntax would engender confusion that is easily avoided if we introduce a simple term-forming operator [...] that, applied to a sentence φ yields its nominalization $[\varphi]$. Thus, we represent the two nominalized sentences (16) and (17) above as $[\mathsf{Ca}]$ and $[\mathsf{Lcd}]$, respectively, and (18) and (19) as $\neg\exists \mathsf{x}\mathsf{Sx}[\mathsf{Ca}]$ and $\neg\mathsf{Ah}[\mathsf{Lcd}]$, respectively.

Intuitively, propositions, so represented, exhibit a level of richness and complexity that, in the framework at hand, properties do not; the latter are taken to be structurally simple (and will be so represented in the model theory of §2) and hence are only represented by syntactically simple predicate symbols in our language.[24] For instance, given just a single adjective or noun phrase α and an individual b, we immediately have the existence of a large number of intuitively distinct propositions: that b is α ($[\mathsf{Ab}]$, where A formalizes $\ulcorner$is $\alpha\urcorner$), that b is α but might not have been ($[\mathsf{Ab} \wedge \Diamond\neg\mathsf{Ab}]$), that b is α if anything is ($[\exists\mathsf{x}\mathsf{Ax} \rightarrow \mathsf{Ab}]$), and so on. However, the introduction of a general logical category of complex propositions is fully warranted on exactly the same grounds as the introduction of a general category of n-place properties — they are the semantic values of a grammatically indispensable class of meaningful expressions with corresponding nominalizations, in this case, sentences.

That said, unlike the category of properties, the introduction of a general category of propositions does appear to entail specific ontological commitments, and this might seem to violate the conception of pure logic that has been driving the development of our framework. For even in the logic of pure identity with no

[24]The framework developed here generalizes quite naturally to one that contains complex n-place predicates. I took significant steps toward that end in Menzel 1993, but it is no longer so clear to me that such a generalization is forced upon us as unavoidably by nominalization as complex propositional terms are. Perhaps the strongest argument that they are is that complex properties can also be the objects of intentional attitudes: thus, on this view, 'Joan desires *to be a wealthy entrepreneur*' is to analyzed as a relation between Joan and the complex property $[\lambda\mathsf{x}\,\mathsf{Wx} \wedge \mathsf{Ex}]$ of being wealthy and an entrepreneur and, hence, has the logical form $\mathsf{Dj}[\lambda\mathsf{x}\,\mathsf{Wx} \wedge \mathsf{Ex}]$. However, arguably, these constructions are in fact just somewhat streamlined alternatives to *de se* propositional attitudes: the grammatical form of the above report notwithstanding, Joan's desire is directed, not toward a complex property, but toward the proposition *that she be a wealthy entrepreneur*. Hence, its actual logical form is $\mathsf{Dj}[\mathsf{Wj} \wedge \mathsf{Ej}]$. (*De se* belief generates further puzzles of its own that would need to be addressed, of course.)

non-logical constants or predicates we have, *a priori*, such general propositions as that something is self-identical ($[\exists x\, x = x]$), that there are at least two things ($[\exists x \exists y\, x \neq y]$), that, necessarily, there are things that might not have existed, ($[\Box \exists x \Diamond \forall y\, x \neq y]$), and so on. As those are quite specific propositions, we appear to be at odds with our guiding idea that pure logic is free of specific ontological commitments.

A radical response here would be that even identity does not properly belong to pure logic. But that seems Draconian. A more measured response is that we can justifiably loosen our idea of pure logic. The absence of specific commitments was just our first take on the idea of topic-neutrality. But topic-neutrality does not rule out specific ontological commitments *per se*; rather, it suggests only that such commitments should not extend beyond *those needed to think and reason within any conceivable domain*.[25] And whether or not one thing is identical to another seems as fundamental to thinking about and reasoning upon the information in a given domain as predication. Hence, a dedicated symbol for identity is fully warranted as a part of pure logic, no less than the basic predicative syntax of atomic formulas. Given that, since we have also found the phenomenon of sentence nominalization to warrant the introduction of a semantic category of propositions, any proposition expressed solely in terms of identity and the rest of our logical apparatus constitutes no violation of topic-neutrality and is justifiably deemed purely logical.

1.6 Propositions and Hyperintensionality

Nominalized sentences provide the best known and most dramatic examples of hyperintensional contexts. For example, the proposition

(20) that alligators don't exist

is necessarily equivalent to the proposition

(21) that, if alligators exist, there is a largest prime.

It follows in standard possible world semantics as well as on a number of recent, more sophisticated accounts of propositions that (20) and (21) are identical.[26] But, intuitively, from the fact that

[25]A similar view is argued by Leitgeb, Nodelman, and Zalta (ms). The general connection between logic and ontology is, of course, a major issue in philosophy and is obviously at the heart of many of the issues discussed in this volume. Hofweber 2008 is an excellent survey of the broader issues.

[26]See, notably, Stalnaker 2012, ch. 2, and Williamson 2013, pp 102-4, 140-1.

(22) Sasha believes that alligators don't exist,

we cannot reasonably infer that

(23) Sasha believes that, if alligators exist, there is a largest prime.

For, not only do the propositions in question differ considerably in their logical forms, viz.,

(24) $[\neg\exists x A x]$

and

(25) $[\exists x A x \rightarrow \exists x(P x \wedge \forall y((P y \wedge y \neq x) \rightarrow x > y))]$,

the mere belief that alligators don't exist reported in (22) has nothing whatever to do with, and certainly does not presuppose any knowledge of, advanced number theory, contrary to what (23) appears to imply.

The explanation we will offer for these substitutivity failures is that propositions exhibit a fine-grained structure similar to (but by no means isomorphic to) the grammatical structure of the sentences that express them. As noted above, hyperintensionality for properties in the framework we are developing here is easy to represent: since we are taking properties to be unstructured semantic primitives, we can simply allow that distinct properties can be assigned the same intension. However, if we take the referents of nominalized sentences to be propositions, we can't glibly assign different propositions to the likes of (20) and (21) and leave it at that. Rather, if we are to explain such failures as the invalid inference from (22) to (23), (21) must, at the least, be connected to the property of being prime in a way that (20) is not. And the most natural way to make this connection is to appeal to some notion of *structure*. On such an approach, a proposition *p*'s identity is determined, not *extrinsically* by its truth value across possible worlds, but *intrinsically* by both its bare logical form — (20) is a *negation* and (21) a more complex *conditional* — and its structural "components". Thus, the reason that it is possible for Sasha to believe (20) without believing (21) is that the property of being a prime number is (in some reasonably rigorous sense) "involved" in the latter's logical structure but not the former's. That structure, in turn, will determine *p*'s truth value across possible worlds. These ideas are made more precise in the following section.

2 Pure Formal Logic

The purpose of this section is to undergird the conception of pure logic at play in the arguments of the preceding section through the development of a type-free first-order logic with structured propositions.

2.1 Languages

A language $\mathscr{L}$ is built up from a standard lexicon of variables $\mathsf{x}_0, \mathsf{x}_1, \ldots$, and logical constants $\neg, \rightarrow, \forall, \Box, =$ along with a sentential operator $[\ldots]$; $=$ is also categorized as a binary predicate. In applied contexts, $\mathscr{L}$ can also contain non-logical primitives: zero or more constants $\mathsf{c}_0, \mathsf{c}_1, \ldots$, and, for each n, zero or more non-logical n-place predicates $\mathsf{P}^n_0, \mathsf{P}^n_1, \ldots$ In practice I'll continue to use lowercase letters from the upper end of the alphabet to stand for arbitrary variables, lowercase letters from the lower end of the alphabet to stand for constants, and uppercase letters to stand for predicates of $\mathscr{L}$, with the understanding that distinct letters stand for distinct elements of the lexicon.

The grammar of a language $\mathscr{L}$ is as follows:

1. Every variable, constant, and predicate is a *term* (*of* $\mathscr{L}$).
2. If π is a n-place predicate and $\tau_1, \ldots, \tau_n$ are terms, then $\pi\tau_1 \ldots \tau_n$ is an (atomic) formula.
3. If φ is a formula, $[\varphi]$ is a term.[27]
4. If ψ and θ are formulas, then so are $\neg\psi$, $(\psi \rightarrow \theta)$, and $\Box\psi$.
5. If ψ is a formula and ν is any variable, then $\forall\nu\psi$ is a formula.
6. Nothing else is a formula or term.

The classification of predicates as terms and the introduction of propositional terms $[\varphi]$ obviously enables us to express all of the examples discussed in §1. As usual, $= \tau\tau'$ can (and typically will) be rewritten as $\tau = \tau'$.

[27] In a broader framework that included complex predicates, we would define $[\varphi]$ to be a 0-place predicate and we would have a general λ-conversion schema:

$\mathbf{C}_n$ $[\lambda\nu_1 \ldots \nu_n\, \varphi]\tau_1 \ldots \tau_n \leftrightarrow \varphi^{\nu_1 \ldots \nu_n}_{\tau_1 \ldots \tau_n}$, where τ_i is free for ν_i in φ

with the special case $n = 0$:

$\mathbf{C}_0$ $[\varphi] \leftrightarrow \varphi$,

where $[\varphi] =_{df} [\lambda\, \varphi]$. But for purposes here we can just take nominalized sentences $[\varphi]$ to be terms only.

2.2 Proposition Structures

The model theory for a language $\mathscr{L}$ is the heart of our hyperintensional logic. Specifically, in our framework, properties and propositions are not set theoretic constructions out of worlds and *possibilia* but, instead, are semantic primitives. The semantics is *untyped* and *algebraic*.[28] That is, there is a single, unstratified semantic domain D that includes all of the properties and propositions and which is closed under a collection of operations that, intuitively, "construct" propositions logically from properties, individuals, and other propositions. So, for instance, given a binary relation r and individuals a and b, the *predication* operation on r, a, and b yields the proposition $Pred(r, a, b)$ that a bears r to b. Negation, conditionalization, necessitation, and generalization operators then yield more complex propositions from less complex, more or less as one would expect. Conditions on these operations are then provided to ensure that the resulting propositions are indeed hyperintensional. The *logical structure* of a proposition will thus be understood in terms of the particular compositions of the logical operations that "generate" it; and that structure, in turn, will determine its identity.

To spell out this idea rigorously, we first define a *proposition structure* $\mathcal{P}$ to be a triple $\langle D, P, Op\rangle$ such that $P \subseteq D$ and $P = \bigcup_{n\in\omega} P_n$ is the union of countably many pairwise disjoint sets $P_0, P_1, ...$ such that each P_i, for $i > 0$, is possibly empty except P_2, which contains at least a distinguished element Id. Intuitively, P_0 is the set of propositions and, for $n > 0$, P_n is the set of n-place properties and D is the set of *things*, or *objects*, of $\mathcal{P}$, including the properties and propositions in P. $D \setminus P$ is thus the set of *individuals* of $\mathcal{P}$, i.e., things that are not properties or propositions. Accordingly, henceforth, I will usually write '*Ind* ' for $D \setminus P$, and we stipulate that *Ind* is at least countably infinite.[29] The third element $Op = \{Pred, Neg, Cond, Nec, UGen\}$ is a set of logical operations — *predication, negation*, *conditionalization*, *necessitation*, and *universal generalization* — that, intuitively, yield propositions of the corresponding logical categories:[30]

[28]Bealer 1979 (subsequently, 1982) is the *locus classicus* of the use of algebraic semantic methods to define first-order hyperintensional properties and propositions, although he draws heavily upon Quine's (1960) method of variable elimination (reprinted in Quine 1966). McMichael and Zalta (1980) independently developed methods similar to Bealer's, although their properties and propositions are not first-order objects in their semantics.

[29]The stipulation that there are at least countably many *individuals* is avoidable; it just makes it easier to define the semantics of generalized propositions below. The existence of at least countable many *things* — in particular, infinitely many propositions — will in any case follow immediately from the existence of *Id* and the logical operations in *Op*.

[30]I am using a minimal set of primitive logical operations here only to simplify the metatheory.

- $Pred : \bigcup_{0<n<\omega} (P_n \times D^n) \xrightarrow{1-1} P_0$,
- $Neg : P_0 \xrightarrow{1-1} P_0$
- $Cond : P_0 \times P_0 \xrightarrow{1-1} P_0$
- $Nec : P_0 \xrightarrow{1-1} P_0$
- $UGen : Ind \times P_0 \longrightarrow P_0$.

Thus, for example, the proposition *If Bernie is a Democrat, he is not a Marxist*, $[\mathsf{Db} \rightarrow \neg \mathsf{Mb}]$, is $Cond(Pred(d,b), Neg(Pred(m,b)))$, where b, d, and m are Bernie and the properties *being a Democrat* and *being a Marxist*, respectively. We assume that these operations are total functions and, hence, closed on D.

Note that, all of the operations except for *UGen* are one-to-one. We stipulate two further conditions:

(**D**) The ranges of the logical operations are pairwise disjoint.[31]

(**R**) Every proposition is in the range of one of the logical operations.

By condition (**D**), no proposition is of more than one of the given logical types, and by condition (**R**), every proposition is of one of the given logical types, from which it follows that there are no logically simple, unstructured propositions. Every proposition, then, as we might put it, is a logical construction from the above operations applied to the elements of D. For the remainder of this subsection, we assume that we are given an arbitrary proposition structure $\mathcal{P} = \langle D, P, Op \rangle$.

It is important to note that, while propositions are *structured* in this framework, they are not *structures*, that is, metaphysically complex entities with literal parts standing in constitutive relations of some sort to one another.[32] As far as the notion of a proposition structure tells us, propositions are metaphysically simple; the relations that a proposition bears to the individuals, properties, and propositions that it is "built up" from via the operations in *Op* are *logical*, not constitutive.

There is no warrant in a fine-grained account of propositions to privilege these particular logical operations over others, e.g., *conjunction*, *possibilization*, etc.

[31] To be clear, we are taking the *range* of a function $f : A \longrightarrow B$ to be the subset of B consisting of exactly the values of f, i.e., the set $\{f(a) : a \in A\}$.

[32] There are several well-known accounts of metaphysically structured fine-grained propositions in the marketplace, notably, Salmon 1986 (Appendix C) Soames 1987, and King 2007. Such accounts have fallen under sharp criticism by, e.g., Keller (2013; 2019), L. J. Keller and J. A. Keller (2013), and Pickel (2020).

Notably, atomic propositions are not "Russellian"; the proposition *Pred*(*being a Democrat*, Bernie) that Bernie Sanders is a Democrat is not a metaphysical complex consisting of the property of being a Democrat and Bernie Sanders — the ordered pair ⟨*being a Democrat*, Bernie⟩, say. As far as the semantics goes, it is simply the value of the *Pred* operation on those two arguments.[33]

The various constraints on the logical operations provide us with obvious identity conditions for some propositions. Since their ranges are pairwise disjoint, we only have to consider conditions within each logical category. Since *Pred* is one-to-one, predications $Pred(r, a_1, ..., a_n)$ and $Pred(s, b_1, ..., b_m)$ are identical if and only if $m = n$, $r = s$, and, for positive $i \leq n$, $a_i = b_i$. For the same reasons, negations are identical if and only if they are negations of the same proposition; likewise necessitations. And conditionalizations are identical if they have the same "antecedent" and "consequent". However, because *UGen* is not one-to-one, we could have $UGen(a, q) = UGen(b, r)$ even though $a \neq b$ or $q \neq r$. The reason for this is pretty clear. The intuitive idea underlying *UGen* is that, in a generalization, something that has been said about an individual a in a singular proposition q that "is about", or "involves", a is said of everything — generalizing upon Socrates in the proposition *Socrates is wise if a philosopher*, for example, yields *Everything is wise if a philosopher*; the particularity of the former gives way to the generality of the latter. And, intuitively, in virtue of its structural similarity to *Socrates is wise if a philosopher*, generalizing upon Plato in *Plato is wise if a philosopher* will likewise yield the *same* generalization. However, nothing in the semantics as it stands guarantees this. And, given the compositional nature of logical structure, this reverberates into the identity conditions for every other category of proposition — since, e.g., *Pred* is one-to-one, we know that $Pred(f, UGen(a, q)) = Pred(f, UGen(b, r))$ *if* $UGen(a, q) = UGen(b, r)$ but we need to know *whether* $UGen(a, q) = UGen(b, r)$ to settle the former identity.

To provide general identity conditions, we need a more rigorous characterization of our conception of logical structure. This can be done by means of a certain type of algebraic structure, namely, that of an *ordered tree*:[34]

Definition 1. Let $\langle V, \rightarrow \rangle$ be a (directed) tree, and let $\prec$ be a strict partial ordering on V. Then $T = \langle V, \rightarrow, \prec \rangle$ is an *ordered tree* if, for all $v, v' \in V$, v and v' are

[33]Of course, atomic propositions might be *modeled* as such in an application of the semantics or *postulated* as such in a philosophical *take* on the semantics (see, e.g., Salmon 1986 ch. 2).

[34]A (directed) tree T, recall, is a triple $\langle V, \rightarrow \rangle$ where V is a nonempty set and $\rightarrow$ is a binary relation on V such that, for some node $v^* \in V$ — the *root* of T — there is a unique path from v^* to every other element of V. If $v \rightarrow v'$, v' is said to be a *child of* v and if, in addition $v \rightarrow v''$ ($v'' \neq v'$), v' and v'' are *siblings*.

$\prec$-comparable if and only if they are siblings.[35]

An ordered tree will serve as a representation of the structure of a proposition just in case we can "label" the nodes of the tree in such a way that it corresponds precisely to the manner in which the proposition is "built up" by means of the logical operations.

Definition 2. A *labeling* ℓ for an ordered tree $T = \langle V, \rightarrow, \prec \rangle$ is a total function on V, i.e., an assignment of objects ("labels") to the nodes of T. ℓ is *cyclic* if there is a path $v_1, ..., v_n$ in T such that $\ell(v_1) = \ell(v_n)$. We let T_ℓ indicate T together with the labeling ℓ.

Definition 3. Let $T = \langle V, \rightarrow, \prec \rangle$ be an ordered tree, $\ell : V \longrightarrow D$ a labeling for T, and $p \in P_0$ a proposition. T_ℓ is a *structure tree for* p — $T_\ell(p)$, for short — if, for any node $v \in V$,

(a) if v is the root node v^* of T, $\ell(v) = p$;

(b) if $\ell(v)$ is a proposition $q \in P_0$, then

- if $q = Pred(f, a_1, ..., a_n)$, then, (i) v has (exactly) children $v_0 \prec v_1 \prec ... \prec v_n$, (ii) $\ell(v_0) = f$, and (iii) for positive $i \leq n$, $\ell(v_i) = a_i$;
- if $p = Neg(q)$ or $Nec(q)$, then v has exactly one child v' and $\ell(v') = q$;
- if $q = Cond(r, s)$, then v has exactly two children $v_1 \prec v_2$ such that $\ell(v_1) = r$ and $\ell(v_2) = s$;
- if $q = UGen(a, r)$, then v has exactly two children $v_1 \prec v_2$ such that $\ell(v_1) = a$ and $\ell(v_2) = r$.

(c) $\ell(v) \in D \setminus P_0$ (i.e., if $\ell(v)$ is an individual or a property), v has no children.

Given the fine-grainedness conditions on the logical operations, if $T_\ell(p)$ and $T'_{\ell'}(p)$, the ordered trees T and T' are isomorphic. However, intuitively, many different labelings of the same ordered tree can yield the same proposition — generalizing on either Fa or Fb,for example, yields the proposition [∀xFx] but a structure tree for [Fa] will obviously have a childless node labeled with a and a structure tree for [Fb] a childless node labeled with b. Accordingly, we can define structure trees T_ℓ and $T'_{\ell'}$ to be *individual variants* of one another just in case T and T' are isomorphic and (very roughly put) ℓ and ℓ' differ only in the individuals in

[35]Otherwise put, if a node has two or more children, $\prec$ linearly orders them; and it orders no nodes that are not siblings. (v' and v'' are *siblings*, of course, if they have the same "parent", i.e., if there is a (unique) node v of the tree such that $v \rightarrow v'$ and $v \rightarrow v''$.)

their ranges that are generalized upon (obviously, a propositional analog of the syntactic notion of *alphabetic variance*). We can then stipulate that propositions p and q in general are identical if and only if there are structure trees T_ℓ and $T'_{\ell'}$ for p and q, respectively, that are individual variants of one another.[36]

A Note on Non-well-founded Propositions. One might think that it is a fairly natural corollary of the conception of propositions that is emerging here that every proposition is *well-founded* in the sense that it is ultimately "built up" via our logical operations from logically *simpler* elements and hence, ultimately, from logically simple objects and relations. But our conditions on the logical operations do not guarantee this — nothing rules out the existence of, say, a predication $p = \mathit{Pred}(f, p)$ or a conditional $q = \mathit{Cond}(r, \mathit{Neg}(q))$. Nor is it at all clear that we would want to rule out non-well-founded propositions in a fully general account.[37] Another virtue of the type-free algebraic approach here is that it permits them — a non-well-founded proposition will be such that any structure tree for it will have a cyclic labeling (and, hence, will have an infinite branch).

The prospect of such propositions is of course highly relevant to the general question of the nature of propositions. However, permitting them adds significant complications to our semantics. Given the limited goals of this paper, it is convenient to rule them out by stipulating that, if T is a structure tree for a proposition p, T has no infinite branches.

[36]Fine-grained accounts are sometimes charged with "over-generating", i.e., with distinguishing propositions that should be considered identitical, e.g., in frameworks with a conjunction operator *Conj* it is reasonable to think that, e.g., $[\mathsf{Db} \wedge \mathsf{Mb}]$ and $[\mathsf{Mb} \wedge \mathsf{Db}]$ pick out the same proposition. More relaxed identity conditions can be introduced to accommodate such cases at the cost of added complexity.

[37]Barwise (1988, p. 194), for instance, suggests that Descartes' *cogito* is best understood as a situation that "comprehends itself", viz., Descartes' comprehending that he is comprehending that very situation. Taking situations to be (or at least to be correlated with) true propositions, we have the proposition $p = \mathit{Pred}(c, d, p) = \textit{Descartes comprehends } p$. Non-well-founded propositions might also be useful in the analysis of various semantic paradoxes. For instance, various types of infinite descending application chains are also not ruled out by our operations, e.g., where $p_0 = \mathit{Pred}(f, p_1)$, $p_1 = \mathit{Pred}(f, p_2)$, ... The propositions in Yablo's Paradox have a similar structure: $p_n = \mathit{UGen}(a, \mathit{Cond}(m > n, \mathit{Neg}(\mathit{True}(p_m))))$. Again, by building on a paraconsistent or other non-classical foundation, the Liar paradox might be analyzed in terms of a proposition $p = \mathit{Neg}(p)$ that is identical to its own negation. (Conditions on the intensions assigned to propositions introduced below will prevent the existence of such a proposition in our classical framework even if non-well-founded propositions are allowed.)

2.3 Model Structures, Models, and Logical Consequence

The above identity conditions ensure that, e.g., (20) and (21) are different propositions and, hence, explain the failure of the inference from (22) to (23). However, in order for our framework to illustrate that this is a genuine case of hyperintensionality, it must also be demonstrable that (20) and (21) share the same intension. Adding the necessary ingredients to the notion of a proposition structure to yield this result gives us a *model structure*.

Specifically, a *model structure* $\mathcal{S}$ for an arbitrary language $\mathscr{L}$ is a 6-tuple $\langle D, P, Op, W, w^*, int\rangle$, where $\langle D, P, Op\rangle$ is a proposition structure, W is a nonempty set (intuitively, the set of possible worlds of $\mathcal{S}$), $w^* \in W$ (intuitively, the *actual* world), and *int* is a (total) function on P that assigns a traditional possible worlds *intension* — that is, a function from worlds to extensions of the appropriate sort – to each property or proposition $p \in P$. For monadic properties $f \in P_1$, $int_f(w) \subseteq D$ is the set of things f is true of at w, and for relations $r \in P_n$, for $n > 1$, $int_r(w) \subseteq D^n$ is the set of n-tuples of objects that stand in r at w. In particular, $int_{Id}(w) = \{\langle a, a\rangle : a \in D\}$, for all $w \in W$. For propositions $p \in P_0$, $int_p(w) \in \{\top, \bot\}$, that is, the truth value of p at w, subject to a number of conditions corresponding naturally to p's logical type. Since different properties/propositions can be assigned the same intension, they are obviously hyperintensional.

The intension assigned to a proposition must track its logical form. The conditions for *Pred*, *Neg*, *Cond*, and *Nec* are obvious:

C1 If $p = Pred(f, a_1, ..., a_n)$, then $int_p(w) = \top$ iff $\langle a_1, ..., a_n\rangle \in int_f(w)$;

C2 If $p = Neg(q)$, then $int_p(w) = \top$ iff $int_q(w) = \bot$;

C3 If $p = Cond(q, r)$, then $int_p(w) = \top$ iff either $int_q(w) = \bot$ or $int_r(w) = \top$;

C4 If $p = Nec(q)$, then $int_p(w) = \top$ iff, for all $w \in W$, $int_q(w) = \top$.

UGen generalizes on individuals in singular propositions in a manner analogous to the way the universal quantifier generalizes on variables. Consequently, expressing the condition on intensions for generalizations requires notions for structure trees corresponding to the syntactic notions of *free occurrence* and *substitutability*. Very briefly, suppose $T_\ell(p)$, where $T = \langle V, \rightarrow, <\rangle$. An *occurrence* of an individual a in T_ℓ is a pair $\langle v, a\rangle$ such that $\ell(v) = a$; that occurrence is *predicative* if, in addition, where $v' \rightarrow v$, $\ell(v') = Pred(a_1, ..., a_n)$ and $a = a_i$, for some $1 \leq i \leq n$. A predicative occurrence $\langle v, a\rangle$ of a is *free* in T_ℓ if there is no ancestor[38] v' of v in

[38] v' is an ancestor of v if there is a path from v' to v.

T such that v_1 and v_2 are the children of v' in T, $\ell(v') = UGen(\ell(v_1), \ell(v_2))$, and $\ell(v_1) = a$ (that is, roughly put, no ancestor of v labeled by a proposition that that results from generalizing on a). A singular proposition p is *about* a, then, if some occurrence of a in a structure tree T_ℓ for p is free in T_ℓ.

Given a structure tree T_ℓ for a proposition q, if q is about an individual $a \in D$ then, for *any* object b, we can define what it is for b to be *substitutable for* a in T_ℓ. Specifically, first, if b is a property, it is substitutable straightaway for a in T_ℓ. If b is an individual, b is substitutable for a in T_ℓ just in case, for any predicative occurrence $\langle v, a\rangle$ of a in T_ℓ, there is no ancestor v' of v in T such that v_1 and v_2 are the children of v' in T, $\ell(v') = UGen(\ell(v_1), \ell(v_2))$ and $\ell(v_1) = b$ and $\ell(v_2)$ is about b.[39] And if b is a proposition, it is substitutable for a in T_ℓ just in case every individual it is about is substitutable for a in T_ℓ.

Given these notions, it is possible to define the idea of the proposition q' that "results" when an arbitrary object (individual, property, or proposition) b is substituted for an individual a in a proposition q about a. The intuitive idea should be clear: q' will be "built up" by means of the logical operations exactly as q is, but for the fact that the construction involves b rather than a. A bit more carefully expressed, suppose $T_\ell(q)$ and that q is about a. We can assume without any loss of generality that b is substitutable for a in T_ℓ.[40] If b is an individual, we can define a labeling ℓ' for T in which the predicative occurrences of a in $T_{\ell'}$ are replaced by occurrences of b and the effects are propagated upwards through T. If b is a *proposition* with structure tree S_{ℓ_S}, every node v of T such that $\langle v, a\rangle$ is a free occurrence of a in T_ℓ is replaced with a copy of S and a new labeling is defined for the resulting ordered tree in terms of both ℓ and ℓ_S. We write $q \approx_{a/b} q'$ to indicate that q' results from substituting b for a in q.

Given this apparatus, we have:

C5 If $p = UGen(a, q)$, then $int_p(w) = \top$ iff, for all $b \in D$ and $q' \in P_0$ such that $q \approx_{a/b} q'$, $int_{q'}(w) = \top$.[41]

[39]There would of course be no need to look beyond T to determine if $\ell(v_2)$ is about b. For if $p = \ell(v_2)$ is about b, then, where T' is the subtree of T determined by v_2 and $\ell' = \ell \upharpoonright T'$, $T'_{\ell'}(p)$ and some occurrence of b in T' will be free in $T'_{\ell'}$.

[40]The reason for this is that, if b is not substitutable for a in T_ℓ, there will always be another labeling ℓ' for T such that $T_{\ell'}(q)$ in which b *is* substitutable for a. Such a structure tree $T_{\ell'}$ is the semantic analog of choosing an alphabetic variant of a formula to avoid variable collisions. If b is an individual, the new labeling would simply relabel non-predicative occurrences of b with a "new" individual; and if b is a proposition, it would replace the non-predicative occurrences of the individuals b is about with "new" individuals.

[41]This *might* seem ill-defined at first sight because p itself could be b and, hence p could be included in the logical structure of q'. But p's *intension* has no role in determining the intension of q'. For example, let $p = UGen(a, Pred(f, a))$. $q' = Pred(f, p)$ is an "instance" of p, but its truth

We now define an $\mathscr{L}$*-model* to be a pair $\langle \mathcal{M}, V \rangle$ (alternatively, $\mathcal{M}_V$) where $\mathcal{M}$ is a model structure for $\mathscr{L}$ and V is a function on the terms of $\mathscr{L}$ such that

- for variables ν, $\nu^V \in Ind$;
- for individual constants τ, $\tau^V \in D$;
- $=^V$ is Id;
- for non-logical n-place predicates π, $\pi^V \in P_n$; and
- for propositional terms $\tau = [\varphi]$,
  - if φ is $\pi\tau_1...\tau_n$, $\tau^V = Pred(\pi^V, \tau_1^V, ..., \tau_n^V)$;
  - if φ is $\neg\psi$, $\tau^V = Neg([\psi]^V)$;
  - if φ is $(\psi \rightarrow \theta)$, $\tau^V = Cond([\psi]^V, [\theta]^V)$;
  - if φ is $\Box\psi$, $\tau^V = Nec([\psi]^V)$;
  - if φ is $\forall\nu\psi$, $\tau^V = UGen(\nu^V, [\psi]^V)$.[42]

Truth for formulas can now be defined simply in terms of the truth of the propositions they express: a formula φ of $\mathscr{L}$ is *true at w in* an $\mathscr{L}$-model $\mathcal{M}_V$ just in case $int_{[\varphi]^V}(w) = \top$. φ is *true in* $\mathcal{M}_V$ just in case it is true in $\mathcal{M}_V$ at the actual world w^* of $\mathcal{M}$. $\mathcal{M}_V$ is a *model of* a set Σ of formulas of $\mathscr{L}$ just in case every member of Σ is true in $\mathcal{M}_V$. And a formula φ of $\mathscr{L}$ is a *logical consequence of* Σ just in case φ is true in every model of Σ.

Call the logic defined by the above semantics $\mathfrak{P}$.

2.4 Proof Theory

The proof theory for $\mathfrak{P}$ will be a rather straightforward extension of any standard axiomatization of classical predicate logic with identity built over the propositional modal logic S5. The only axioms that need to be added are those reflecting the structural details of an $\mathscr{L}$-model, in particular, the fine-grained identity conditions for propositions.

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value at a given world w will depend only on the intension of f and, in particular, on whether or not p, *qua object*, is in $int_f(w)$. That question is entirely independent of p's intension.

[42]It is important to note that, although we only *generalize* on individuals in propositions (hence the restriction on the values of variables to *Ind* above), our condition **C5** on the intension assigned to a universally generalized proposition guarantees that $UGen(\nu^V, [\psi]^V)$ is *true* in $\mathcal{M}_V$ only if $[\psi^\nu_\tau]^V$ is, for *any* term τ that is free for ν in ψ and, hence, that universal instantiation is valid. The purpose of the restriction on the first argument of *UGen* to individuals is just to simplify the semantics, especially the "tree surgery" required to define the $\approx_{a/b}$ relation rigorously.

A1 $\pi \neq \tau$, for predicates π, if τ is either a propositional term or a predicate of different arity than π

A2 $[\varphi] \neq [\psi]$, if φ and ψ are not of the same syntactic category[43]

A3 $[\pi\tau_1...\tau_n] \neq [\rho\sigma_1...\sigma_m]$, if $m \neq n$

A4 $[\pi\tau_1...\tau_n] = [\rho\sigma_1...\sigma_n] \leftrightarrow (\pi = \rho \wedge \tau_1 = \sigma_1 \wedge ... \wedge \tau_n = \sigma_n)$

A5 $[\neg\psi] = [\neg\theta] \leftrightarrow [\psi] = [\theta]$

A6 $[\psi \to \theta] = [\psi' \to \theta'] \leftrightarrow ([\psi] = [\psi'] \wedge [\theta] = [\theta'])$

A7 $[\Box\psi] = [\Box\theta] \leftrightarrow [\psi] = [\theta]$

A8 $[\forall\nu\psi] = [\forall\mu\theta] \to [\psi^{\nu}_{\tau}] = [\theta^{\mu}_{\tau}]$, where τ is free for ν in ψ and for μ in θ

A9 $[\forall\nu\psi] = [\forall\mu\theta]$ if $\forall\nu\psi$ and $\forall\mu\theta$ are alphabetic variants

Well-foundedness (should it be imposed in the semantics) can be axiomatized as follows:

A10 $\forall\nu\, \nu \neq [\varphi]$, where ν occurs free in φ

Proving the soundness of the proof theory is straightforward. Its consistency can be demonstrated by means of a simple term model, where we take every predicate, constant, and variable of $\mathscr{L}$ to denote itself and every propositional term to denote the class of its alphabetic variants. The completeness of the system at this point has not been fully investigated, though the way forward to a Henkin-style proof seems clear. Complete or not as it stands, there is no ground to doubt that a complete axiomatization is possible.

3 Propositions and Paradox

In this final section I examine two important paradoxes involving propositions. The first, the Prior-Kaplan paradox, has been the subject of much recent discussion and I believe the take on higher-order logic in §1.4 sheds some new light on how it should be understood. The second, another paradox due originally to Russell (1903) and brought back into prominence by Myhill (1958), is often said to show that the sort of fine-grained conception of propositions that we have developed here is not viable, so it clearly demands a close look.

[43] I.e., if they are not both predications, or both negations, etc.

3.1 The Prior-Kaplan Paradox

In a 1995 paper, David Kaplan noted that the following statement of standard quantified propositional modal logic is *logically false*, where $\mathcal{A}$ is a monadic sentential operator:

(**A**) $\forall p \Diamond \forall q(\mathcal{A}q \leftrightarrow q = p)$.[44]

That is, informally put, it is logically impossible for there to be a property $\mathcal{A}$ that, for any proposition p, could be true of p alone. As it is perfectly consistent with first-order modal logic that there be a property that, for every *individual* x, could be true of x alone, Kaplan found the logical falsity (**A**) deeply paradoxical.

As the title of his paper indicates, Kaplan laid the blame on possible world semantics and, in particular, the standard identification of propositions with sets of worlds.[45] However, some three decades earlier, Prior had shown that the problem is more deeply entrenched in the foundations of quantified propositional logic. In his 1960 paper "On a Family of Paradoxes", Prior analyzes an informal paradox discovered by Church that follows from the paradox of the Cretan, a familiar variant of the Liar paradox. Epimenides, being a Cretan, cannot truly assert that everything asserted by a Cretan — on the day and at the time of the assertion, say — is not the case. For if he were to make that assertion, it would itself be asserted by a Cretan on the day and at the time in question and, hence, if the assertion were true, it would be false; hence, it is false. But, as Church observed, that isn't the end of matter. For if Epimenides' assertion — that nothing asserted by a Cretan is true — itself had to have been false, then it must be that something asserted by a Cretan is true. Hence, contrary to all intuition, it is *logically impossible* for Epimenides' assertion to have been the only one made by a Cretan on that particular day at that particular time;[46] paradoxically, by logic alone, although we have (let us assume) only witnessed Epimenides' assertion, we have deduced that some other Cretan *also* had to have asserted something simultaneously and, moreover, that what they asserted had to have been true.

[44]Identity can be replaced by necessary equivalence here. See Anderson 2009 and Bueno, Menzel, and Zalta 2014 for further discussion.

[45]Suppose $\mathcal{A}$ is the only nonlogical constant of the language and that (**A**) is true in some interpretation $\langle W, V\rangle$ of the language, where W is a set of "worlds" and V assigns a property $\mathcal{A}^V$ of propositions to $\mathcal{A}$, i.e., a function from worlds to sets of propositions. Then the relation $\{\langle w, p\rangle \in W \times \wp(W) : \mathcal{A}^V(w) = \{p\}\}$ maps a subset of W onto the set of propositions, i.e., onto $\wp(W)$, which is impossible by Cantor's theorem.

[46]As Bacon, Hawthorne, and Uzquiano (2016, §2) point out, paradoxes that are even more puzzling can be generated by considering natural language operators other than "Epimenides asserts that ..."

Prior's formalization of the argument enhances the sense of a genuine logical paradox here. Prior renders the argument in classical quantified propositional logic using a single sentential operator $\mathcal{A}$ standing intuitively for "it is asserted by a Cretan that ...". The proposition *Everything asserted by a Cretan is not the case* is then represented as $\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p})$. As for the logical framework needed, over and above basic propositional logic the argument relies only upon the classical axiom schema for propositional universal instantiation. To express this schema rigorously, let φ^{π}_{ψ} stand for the result of replacing every free occurrence of the propositional variable π in the formula φ with an occurrence of the formula ψ, and say that ψ is *free for π in φ* just in case no free variable occurrence in ψ becomes bound in φ^{π}_{ψ}. Then we have:

UI $\forall\pi\varphi \to \varphi^{\pi}_{\psi}$, where ψ is free for π in φ.

Given **UI**, of course, we can derive the corresponding existential generalization schema, which it is also useful to have in formalizing the argument:

EG $\varphi^{\pi}_{\psi} \to \exists\pi\varphi$, where ψ is free for π in φ.

With this apparatus in place, Prior renders the argument as follows:[47]

T1	$\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p}) \to (\mathcal{A}\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p}) \to \neg\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p}))$	**UI**
T2	$\mathcal{A}\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p}) \to (\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p}) \to \neg\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p}))$	T1
T3	$\mathcal{A}\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p}) \to \neg\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p})$	T2
T4	$\mathcal{A}\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p}) \to \exists \mathsf{p}(\mathcal{A}\mathsf{p} \wedge \mathsf{p})$	T3
T5	$\mathcal{A}\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p}) \to (\mathcal{A}\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p}) \wedge \neg\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p}))$	T3
T6	$(\mathcal{A}\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p}) \wedge \neg\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p})) \to \exists \mathsf{q}(\mathcal{A}\mathsf{q} \wedge \neg\mathsf{q})$	**EG**[48]
T7	$\mathcal{A}\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p}) \to \exists \mathsf{q}(\mathcal{A}\mathsf{q} \wedge \neg\mathsf{q})$	T5, T6
T8	$\mathcal{A}\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p}) \to (\exists \mathsf{p}(\mathcal{A}\mathsf{p} \wedge \mathsf{p}) \wedge \exists \mathsf{q}(\mathcal{A}\mathsf{q} \wedge \neg\mathsf{q}))$	T4, T7

Thus, on the intended reading of $\mathcal{A}$, it is a *theorem* of classical, quantified propositional logic that a Cretan can manage to (falsely) assert the proposition that everything asserted by a Cretan is false only if some other proposition is truly asserted by a Cretan. Assuming the principle of necessitation, the falsity of Kaplan's (**A**) can be shown to follow directly.[49]

[47] In the interest of complete clarity here: In T1, the antecedent $\forall p(\mathcal{A}p \to \neg p)$ is itself substituted for p in $\mathcal{A}p \to \neg p$. T2 follows from T1 via the equivalence of $\varphi \to (\psi \to \theta)$ and $\psi \to (\varphi \to \theta)$; T3 from T2 by that of $\psi \to \neg\psi$ and $\neg\psi$. T4 follows from T3 by quantifier exchange and some basic propositional logic; and T5 from T3 by the equivalence of $\varphi \to \psi$ and $\varphi \to (\varphi \wedge \psi)$. T7 follows from T5 and T6 by an application of hypothetical syllogism; and the derivation of T8 from T4 and T7 is an instance of the valid pattern $\varphi \to \psi, \varphi \to \theta \vdash \varphi \to (\psi \wedge \theta)$.

[48] Where ψ is $\forall \mathsf{p}(\mathcal{A}\mathsf{p} \to \neg\mathsf{p})$ here.

[49] Specifically: assuming Necessitation (**Nec**) and basic principles of identity, we have:

Prior's theorem is typically viewed as a *logical* paradox because it imposes no non-logical restrictions on the operator $\mathcal{A}$ and a solution to the paradox appears to require the abandonment or modification of some classical logical principle used in the above deduction. From the perspective advanced in this paper, however, since quantified propositional logic is essentially higher-order — it is second-order logic with 0-adic predicate variables only — the theorem, although certainly paradoxical (on the intended reading of $\mathcal{A}$), is not a *logical* paradox but, instead, emerges from principles that take us beyond pure logic into *theories* of assertion (or some related semantic notion) and truth. Translating the theorem into a first-order framework, propositional quantifiers are understood simply as first-order quantifiers and the operator $\mathcal{A}$ as an ordinary monadic predicate A. And, as propositions are individuals alongside others, we need a dedicated sortal predicate P^0 true of all and only propositions, and the predicates A and T are needed to express that a given proposition is asserted or true. So understood, the statement $\forall p(\mathcal{A}p \rightarrow \neg p)$ unpacks to $\forall x(\mathsf{P}^0 x \rightarrow (\mathsf{A}x \rightarrow \neg \mathsf{T}x))$. However, we can streamline this formula to appear more like Prior's original if we assume natural axioms expressing that only propositions can be either asserted or true:

AP $\mathsf{A}x \rightarrow \mathsf{P}^0 x$

TP $\mathsf{T}x \rightarrow \mathsf{P}^0 x$

This yields the following restatement of Prior's T1 and T2 in our logic $\mathfrak{P}$ (where **UI** and **EG** are now just the usual first-order inference rules):

S1	$\forall x(\mathsf{A}x \rightarrow \neg \mathsf{T}x) \rightarrow (\mathsf{A}[\forall x(\mathsf{A}x \rightarrow \neg \mathsf{T}x)] \rightarrow \neg \mathsf{T}[\forall x(\mathsf{A}x \rightarrow \neg \mathsf{T}x)])$	**UI**
S2	$\mathsf{A}[\forall x(\mathcal{A}x \rightarrow \neg \mathsf{T}x)] \rightarrow (\forall x(\mathsf{A}x \rightarrow \neg \mathsf{T}x) \rightarrow \neg \mathsf{T}[\forall x(\mathsf{A}x \rightarrow \neg \mathsf{T}x)])$	S1

But now how do we proceed? How do we get from S2 to our version of T3, viz.,

S3 $\mathsf{A}[\forall x(\mathsf{A}x \rightarrow \neg \mathsf{T}x)] \rightarrow \neg\forall x(\mathsf{A}x \rightarrow \neg \mathsf{T}x)$?

K1	$\mathcal{A}\forall p(\mathcal{A}p \rightarrow \neg p) \rightarrow \exists r \exists q(\mathcal{A}r \wedge \mathcal{A}q \wedge r \neq q)$	T8, **UI**
K2	$\Box(\mathcal{A}\forall p(\mathcal{A}p \rightarrow \neg p) \rightarrow \exists r \exists q(\mathcal{A}r \wedge \mathcal{A}q \wedge r \neq q))$	K1, **Nec**
K3	$\exists p \Box(\mathcal{A}p \rightarrow \exists r \exists q(\mathcal{A}r \wedge \mathcal{A}q \wedge r \neq q))$	K2, **EG**
K4	$\exists p \Box(\mathcal{A}p \rightarrow \exists q(\mathcal{A}q \wedge q \neq p))$	K3
K5	$\neg\forall p \Diamond(\mathcal{A}p \wedge \forall q(\mathcal{A}q \rightarrow q = p))$	K4
¬(**A**)	$\neg\forall p \Diamond \forall q(\mathcal{A}q \leftrightarrow q = p)$	K5

Kaplan raised the Epimenides paradox in his paper (p. 45) and saw that the assumption that the proposition (expressed by) $\forall p(\mathcal{A}p \rightarrow \neg p)$ is the only $\mathcal{A}$ is inconsistent with (**A**) but apparently missed the implications for his own paradox. Note identity is not essential here, as the paradox goes through if identity is replaced by necessary equivalence. In the context of possible world semantics, of course, identity just *is* necessary equivalence, but the logical falsity of (**A**) is no less paradoxical — indeed it is arguably moreso — if propositions are taken to be hyperintensional. See Bueno, Menzel, and Zalta 2014, §5, esp. note 34.

Clearly, Prior's move from T2 to T3 is justified simply by the fact that the consequent of T2 is of the form $\varphi \to \neg\varphi$ and, hence, logically equivalent to $\neg\varphi$. To make an analogous move we need $\neg\mathsf{T}[\forall x(\mathsf{A}x \to \neg\mathsf{T}x)]$ to entail $\neg\forall x(\mathsf{A}x \to \neg\mathsf{T}x)$. But this purported entailment, of course, is exactly (the contrapositive of) an instance of the problematic right-to-left direction of the usual Tarskian T-schema:

(**T**) $\mathsf{T}[\varphi] \leftrightarrow \varphi$.

The missing steps from S2 to S3, and similar steps in the corresponding inference from T5 to T6, and thence to the paradoxical conclusion are filled in as follows:

S2′	$\mathsf{A}[\forall x(\mathsf{A}x \to \neg\mathsf{T}x)] \to (\forall x(\mathsf{A}x \to \neg\mathsf{T}x) \to \neg\forall x(\mathsf{A}x \to \neg\mathsf{T}x))$	S2, (**T**)
S3	$\mathsf{A}[\forall x(\mathsf{A}x \to \neg\mathsf{T}x)] \to \neg\forall x(\mathsf{A}x \to \neg\mathsf{T}x)$	S2′
S4	$\mathsf{A}[\forall x(\mathsf{A}x \to \neg\mathsf{T}x)] \to \exists x(\mathsf{A}x \wedge \mathsf{T}x)$	S3
S5	$\mathsf{A}[\forall x(\mathsf{A}x \to \neg\mathsf{T}x)] \to (\mathsf{A}[\forall x(\mathsf{A}x \to \neg\mathsf{T}x)] \wedge \neg\forall x(\mathsf{A}x \to \neg\mathsf{T}x))$	S3
S5′	$\mathsf{A}[\forall x(\mathsf{A}x \to \neg\mathsf{T}x)] \to (\mathsf{A}[\forall x(\mathsf{A}x \to \neg\mathsf{T}x)] \wedge \neg\mathsf{T}[\forall x(\mathsf{A}x \to \neg\mathsf{T}x))]$	S5, (**T**)
S6	$(\mathsf{A}[\forall x(\mathsf{A}x \to \neg\mathsf{T}x)] \wedge \neg\mathsf{T}[\forall x(\mathsf{A}x \to \neg\mathsf{T}x)]) \to \exists x(\mathsf{A}x \wedge \neg\mathsf{T}x)$	**EG**
S7	$\mathsf{A}[\forall x(\mathsf{A}x \to \neg\mathsf{T}x)] \to \exists x(\mathsf{A}x \wedge \neg\mathsf{T}x)$	S5′, S6
S8	$\mathsf{A}[\forall x(\mathsf{A}x \to \neg\mathsf{T}x)] \to (\exists x(\mathsf{A}x \wedge \mathsf{T}x) \wedge \exists x(\mathsf{A}x \wedge \neg\mathsf{T}x))$	S4, S7

So the paradox can be generated in our framework as well but its nature as a *semantic* paradox — specifically a paradox of *truth* (i.e., the 1-place case of *exemplification*) — as opposed to a paradox of pure logic, is manifest. I have no general solutions to offer, just as I had no solutions to offer for avoiding the Russellian paradox of (binary) exemplification in §1.4 — I am only arguing for the proper framework of pure logic. Prior's theorem is thus no more a problem for our logic $\mathfrak{P}$ than it is for any variety of classical first-order logic in which one might try to formalize a first-order theory of truth — theories of truth are difficult and prone to paradox. However, as noted (and as Williamson (2016, p. 542) acknowledges), it *is* a problem for extensions of quantified propositional logic (like Williamson's own) in which it falls out as a *logical* theorem given only a unary sentential operator.

3.2 The Russell-Myhill Paradox

Russell (1903) announced a very different paradox involving propositions in Appendix B of *The Principles of Mathematics* that many higher-order metaphysicians and philosophers of logic take to show that a fine-grained conception of propositions of the sort on offer here is simply not tenable (e.g., Uzquiano (2015), Dorr (2016), Goodman (2017), and Fritz (2019)). Like the better known paradox that bears his name, this paradox also involves sets, specifically, the assumption that a certain set of singular propositions about sets exists; paired with the fine-grainedness principle that propositions about distinct sets are themselves distinct, a contradiction follows. Russell's own presentation of the paradox involves the notion of truth: his paradoxical set consists of all propositions of the form *Every member of m is true*, for sets *m*. But the particular form here is unnecessary; *any* singular propositions about sets would do just as well as long as they are all of the same form. Indeed, even the stipulation that *m* is a set is unnecessary; we will just use propositions of the form $x = x$.[50] A bit more formally, then, in our version of the paradox — call it *RPP* — we let r consist of exactly those propositions of that form that are not members of the object they are about:

R1 $\forall y(y \in r \leftrightarrow \exists x(y = [x = x] \wedge y \notin x))$.

It is an immediate consequence of our fine-grainedness axiom schema **A4** that propositions of the given form are identical only if they are "about" the same thing:[51]

R2 $\forall x \forall y([x = x] = [y = y] \rightarrow x = y)$.

The contradiction $[r = r] \in r \leftrightarrow [r = r] \notin r$ follows quickly from **R1** and **R2** with a bit of elementary reasoning in first-order logic.

Our logic $\mathfrak{P}$, of course, does not presuppose the existence of sets, but we should surely hope that it would be compatible with their existence and, in particular, compatible with Zermelo-Fraenkel set theory with urelements (ZFU), given its fundamental role in contemporary logic, mathematics, and philosophy. So what are we to say? We surely do not want to give up our schema **A4,** which axiomatizes what is arguably the most fundamental principle of fine-granularity. So, for friends of fine-granularity, the paradox must turn on the existence of the set r defined in **R1**. As it stands, **R1** is best seen as an instance of the inconsistent naive Comprehension principle, and we already knew that that spells trouble. The easiest way to derive **R1** from legitimate set theoretic principles is to assume that there is a set s of all propositions; one can then derive $r = \{y \in s : \exists x(y = [x = x] \wedge y \notin x)\}$ by Separation. However, given **R2**, there are obviously at least as many such propositions as there are sets, as, for each set a, there is the proposition $[a = a]$ and, by **R2**, for any set $b \neq a$, $[a = a] \neq [b = b]$.

[50] Russell's initial statement of the paradox also seems to involve the use of the Powerset axiom but his more careful version in the second paragraph of §500 avoids it.

[51] Recall that '=' is a binary predicate in our framework.

Hence, as with the sets themselves, there is an "absolute infinity" of such propositions,[52] so there can be no such set as s in ZFU. Given ZFU, then, the friend of fine-granularity can simply reject **R1** with a clean conscience.

But RPP isn't dismissed quite so easily. For, although there are at least as many propositions as sets, they are not themselves sets on our account and, hence, set theoretically, they are urelements. And, as shown in Menzel 2014, it is possible to modify the axioms of Replacement and Powerset such that, in the modified theory ZFCU*, the urelements constitute a set u even if there is an absolute infinity of them. By Separation, then, we again have our paradoxical Russell set $\mathsf{r} = \{\mathsf{y} \in \mathsf{u} : \exists \mathsf{x}(\mathsf{y} = [\mathsf{x} = \mathsf{x}] \wedge \mathsf{y} \notin \mathsf{x})\}$.

But an answer in the spirit of the preceding response to the Prior-Kaplan paradox seems equally available here: interesting and important as RPP is, it is not a paradox of pure logic, as it must presuppose not only a distinctly non-logical relation — *membership* — but the highly non-logical assumption that there are sets and, moreover, among them, a set of all urelements. It thus no more exposes a flaw in our logic $\mathfrak{P}$ and its fine-grained conception of propositions than Russell's more famous paradox exposes one in standard first-order logic; it is at root a paradox of set theory.

However, a plural version of RPP — call it PRPP — that doesn't presuppose the existence of sets arguably poses a more significant challenge. It is widely (albeit not universally) accepted that plural reference and plural quantification are ontologically innocent (Boolos 1985; Linnebo 2022, §5); they are simply *modes* of reference and quantification that are irreducible to their singular counterparts. If so, they are justly considered parts of pure logic alongside singular names and singular quantifiers, and the *among* or *one of* relation — signified by $\prec$ — is justly considered a purely logical relation alongside identity. So understood, unlike set theoretic comprehension principles, the *plural* comprehension principle

PC $\exists \mathsf{y}\varphi \rightarrow \exists \mathsf{xx}\forall \mathsf{y}(\mathsf{y} \prec \mathsf{xx} \leftrightarrow \varphi)$

is arguably both ontologically innocuous and logically self-evident: it simply says that, if anything at all satisfies a description φ, then there are the *things* that satisfy it. So in augmenting $\mathfrak{P}$ with the principles of plural quantification (with plural identity) and **PC** and generalizing its fine-granularity axioms accordingly — call the result $\mathfrak{P}^*$ — we justifiably remain within the bounds of pure logic.

Now, it will be easy to show in $\mathfrak{P}^*$ that there are pluralities zz such that the

[52]There is an absolute infinity of Xs just in case, for every cardinal number κ, there is a set of Xs of at least cardinality κ.

proposition $[zz = zz] \nprec zz$, i.e., that $\exists zz(y = [zz = zz] \land y \nprec zz)$.[53] So, by **PC**, there are some things — call them rr — comprising exactly those propositions of the form $[zz = zz]$ that are not among the things zz they are about:

P1 $\forall y(y \prec rr \leftrightarrow \exists zz(y = [zz = zz] \land y \nprec zz))$.

Furthermore, generalizing the fine-granularity axiom **A4** to plurals, we have as an instance:

P2 $\forall xx \forall yy([xx = xx] = [yy = yy] \to xx = yy)$.

Paralleling, in the logic of plurals, the set theoretic reasoning in RPP, the contradiction $[rr = rr] \prec rr \leftrightarrow [rr = rr] \nprec rr$ quickly follows but an answer parallel to the one we gave in response RPP seems unavailable. As we've seen, RPP presents no threat to $\mathfrak{P}$ because premise **R1** depends on very strong and distinctly non-logical assumptions about the existence of sets. In stark contrast to RPP, assuming the ontological innocence of plural quantification and the logicality of the *one of* relation, PRPP appears to be a paradox of our pure plural logic $\mathfrak{P}^*$. So something has got to go; and the preponderance of contemporary opinion seems to be that the culprit is fine-granularity, in particular, schema **A4**.[54]

There is unfortunately no time to pursue the deep issues that PRPP raises in detail. So I will simply note that there are two further culprits in the derivation of PRPP on which we might equally well hang the chief responsibility for the paradox. The first is the plural comprehension principle **PC**. The modern trend in favor of its validity notwithstanding, powerful challenges have been laid down questioning the logicality of **PC**, particularly with regard to the purported ontological innocence of plural quantification (see Resnik 1988, Hazen 1993, Rouilhan 2002, Linnebo 2003, and Florio and Linnebo 2021, ch. 12). The second culprit, especially in the context of our algebraic model of propositions, is the propositional comprehension principle implicit in the grammar of the languages of $\mathfrak{P}^*$ that entails the existence of a proposition $[\varphi]$ for any well-formed formula φ. On this model, well-founded propositions are "built up" structurally via the logical functions ultimately from an initial collection of individuals and n-place properties in a manner analogous to the way that sets are ultimately "built up" from urelements. Thus, like sets, propositions — hence, pluralities of them — fall into

[53]For example, consider the propositions p and q such that $p = [\forall x\, x = x]$ and $q = [\neg\forall x \forall y\, x = y]$. Let zz be the plurality comprising exactly p and q and let y be the proposition $[zz = zz]$. Since y is an identity, it is provably distinct from p and q by (a plural extension of) axiom **A2**, and thus it is not among the plurality zz. Hence, $y \prec rr$.

[54]See Fritz, Lederman, and Uzquiano (forthcoming) for perhaps the most recent example.

an iterative hierarchy in which the propositions of a given level presuppose the objects in preceding levels. Identity propositions in particular, whether singular or plural, are paradigms; a proposition of the form $[\tau = \tau']$ can only be a value of the *Pred* function (suitably generalized to take plural arguments) at a given level if the objects signified by τ and τ' are available in a preceding level. But there is, in particular, no level of the hierarchy at which *all* propositions of the form $[\mathsf{zz} = \mathsf{zz}]$ that are not among the objects zz they are about are available — further propositions of that form that meet that condition "arise" at every (successor) level. So, while there are all of the propositions of the form in question — which (assuming the ontological innocence of plurals) is just to say that there is the plurality rr — there are no *propositions* about rr, in particular, no proposition of the form $[\mathsf{rr} = \mathsf{rr}]$, just as there is no *set* $\{\mathsf{x} : \mathsf{x} \notin \mathsf{x}\}$ "about" all the non-self-membered sets.[55] Thus, extending $\mathfrak{P}$ to accommodate pluralities will arguably require justifiable restrictions on the formation of propositional terms that will undercut PRPP.[56]

References

Adams, Robert (1981). "Actualism and Thisness". In: *Synthese* 49, pp. 3–41.

Anderson, C. Anthony (2009). "The Lesson of Kaplan's Paradox About Possible World Semantics". In: *The Philosophy of David Kaplan*. Ed. by Joseph Almog and Paolo Leonardi. New York: Oxford University Press, pp. 85–92.

Bacon, Andrew, John Hawthorne, and Gabriel Uzquiano (2016). "Higher-Order Free Logic and the Prior-Kaplan Paradox". In: *Canadian Journal of Philosophy* 46.4-5, pp. 493–541.

Barwise, Jon (1988). "Situations, Sets, and the Axiom of Foundation". In: *The Situation in Logic*. Stanford, CA: CSLI Publications, pp. 177–200.

Bayart, Arnould (1958). "Correction de La Logique Modale Du Premier Et Du Second Ordre S5". In: *Logique Et Analyse* 1.1, pp. 28–45.

[55]In §7.1 of his contribution to this volume, Linnebo suggests, alternatively, that, for any well-defined plurality xx, there should always be propositions about xx, e.g., $[\mathsf{xx} = \mathsf{xx}]$, but that, on a hierarchical view like the one proposed, the existence of such "unbounded" pluralities as rr needn't be conceded.

[56]Yu (2017) develops an iterative conception of plural propositions similar to the one sketched here and suggests a potentialist solution to RPPP analogous to Linnebo's (2010; 2013) solution to Russell's set theoretic paradox. In fact, I think that, ultimately, a potentialist account of the hierarchy of propositions along the lines Yu suggests will be needed to fend off objections like the one Williamson raises in the challenging comments on this paper that he supplied for this volume.

Bayart, Arnould (1959). "Quasi-Adéquation de La Logique Modale Du Second Ordre S5 et Adéquation de La Logique Modale Du Premier Ordre S5". In: *Logique et Analyse* 2.6-7, pp. 99–121.

Bealer, George (1979). "Theories of Properties, Relations, and Propositions". In: *The Journal of Philosophy* 76, pp. 634–648.

— (1982). *Quality and Concept*. Oxford University Press.

— (1989). "On the Identification of Properties and Propositional Functions". In: *Linguistics and Philosophy* 12.1, pp. 1–14.

— (1998). "Propositions". In: *Mind* 107.425, pp. 1–32.

Beall, J. C., Greg Restall, and Gil Sagi (2019). "Logical Consequence". In: *Stanford Encyclopedia of Philosophy* (Spring 2019 Edition). Ed. by Edward N. Zalta. URL: https://plato.stanford.edu/archives/spr2019/entries/logical-consequence.

Berto, Francesco and Daniel Nolan (2021). "Hyperintensionality". In: *Stanford Encyclopedia of Philosophy*.

Boolos, George (1984). "To Be Is To Be the Value of a Variable (or To Be Some Values of Some Variables)". In: *Journal of Philosophy* 81, pp. 430–449.

— (1985). "Nominalist Platonism". In: *The Philosophical Review* 94.3, pp. 327–344.

Boolos, George S. (1975). "On Second-Order Logic". In: *The Journal of Philosophy* 72.16, pp. 509–527.

Bueno, Otavio, Christopher Menzel, and Edward Zalta (2014). "Worlds and Propositions Set Free". In: *Erkenntnis* 79.4, pp. 797–820.

Cameron, Ross P. (2019). "Truthmaking, Second-Order Quantification, and Ontological Commitment". In: *Analytic Philosophy* 60.4, pp. 336–360. DOI: 10.1111/phib.12162.

Carnap, Rudolf (1946). "Modalities and Quantification". In: *Journal of Symbolic Logic* 11.2, pp. 33–64. DOI: 10.2307/2268610.

— (1947). *Meaning and Necessity*. Chicago: The University of Chicago Press.

Chwistek, Leon (1967). "Antinomies of Formal Logic". In: *Polish Logic 1920-1939*. Ed. by Storrs McCall. Trans. by Z. Jordan. Oxford: Oxford University Press, pp. 338–345.

Cocchiarella, Nino (1972). "Properties as Individuals in Formal Ontology". In: *Nous* 6.2, pp. 165–187.

— (1973). "Whither Russell's Paradox of Predication?" In: *Logic and Ontology*. Ed. by Milton Karl Munitz. New York: New York University Press, pp. 133–158.

— (1985). "Frege's Double Correlation Thesis and Quine's Set Theories NF and ML". In: *Journal of Philosophical Logic* 14.1, pp. 1–39.

Cresswell, M. J. (2015). “Arnoud Bayart’s Modal Completeness Theorems — Translated with an Introduction and Commentary”. In: *Logique Et Analyse* 229.1, pp. 89–142.

Cresswell, Max (1973). *Logics and Languages*. London: Methuen [Distributed in the U.S.A. By Harper & Row.

Dorr, Cian (2016). “To Be F Is To Be G”. In: *Philosophical Perspectives* 30.1, pp. 39–134. DOI: 10.1111/phpe.12079.

Dunaway, Billy (2013). “Modal Quantification Without Worlds”. In: *Oxford Studies in Metaphysics* 8.

Enderton, Herbert (2001). *A Mathematical Introduction to Logic*. 2nd ed. San Diego: Academic Press. 317 pp.

Etchemendy, John (1990). *The Concept of Logical Consequence*. Cambridge, MA: Harvard University Press.

Florio, Salvatore and Øystein Linnebo (2021). *The Many and the One: A Philosophical Study of Plural Logic*. Oxford University Press.

Frege, Gottlob (1951). “On Concept and Object”. Trans. by P. T. Geach and Max Black. In: *Mind* 60.238, pp. 168–180. ISSN: 0026-4423. JSTOR: 2251430.

— (1960). “Function and Concept”. In: *Translations from the Philosophical Writings of Gottlob Frege*. Ed. by P. Geach and M. Black. 2nd ed. Oxford: Oxford: Blackwell, pp. 22–41.

Fritz, Peter (2019). “Structure by Proxy, with an Application to Grounding”. In: *Synthese* 198.7, pp. 6045–6063. DOI: 10.1007/s11229-019-02450-z.

Fritz, Peter, Harvey Lederman, and Gabriel Uzquiano (forthcoming). “Closed Structure”. In: *Journal of Philosophical Logic*, pp. 1–43. DOI: 10.1007/s10992-021-09598-5.

Geach, P. T., A. J. Ayer, and W. V. Quine (1951). “Symposium: On What There Is”. In: *Proceedings of the Aristotelian Society, Supplementary Volumes* 25, pp. 125–160. ISSN: 0309-7013. JSTOR: 4106673.

Goodman, Jeremy (2017). “Reality Is Not Structured”. In: *Analysis* 77.1, pp. 43–53.

Haack, Susan (1978). *Philosophy of Logics*. Cambridge University Press.

Hacking, Ian (1979). “What Is Logic?” In: *Journal of Philosophy* 76.6, pp. 285–319. DOI: 10.2307/2025471.

Hazen, A. P. (1993). “Against Pluralism”. In: *Australasian Journal of Philosophy* 71.2, pp. 132–144. DOI: 10.1080/00048409312345142.

Hintikka, Jaakko (1961). “Modality and Quantification”. In: *Theoria* 27.3, pp. 119–128. DOI: 10.1111/j.1755-2567.1961.tb00020.x.

— (1963). “The Modes of Modality”. In: *Acta Philosophica Fennica* 16, pp. 65–82.

Hodes, Harold (1984). "Logicism and the Ontological Commitments of Arithmetic". In: *Journal of Philosophy* 81.3, pp. 123–149.
Hofweber, Thomas (2008). "Logic and Ontology". In: *Stanford Encyclopedia of Philosophy*.
Jones, Nicholas K. (2018). "Nominalist Realism". In: *Noûs*, pp. 808–835.
Jubien, Michael (1989). "Straight Talk About Sets". In: *Philosophical Topics* 17.2, pp. 91–107. DOI: `10.5840/philtopics19891725`. URL: `https://doi.org/10.5840/philtopics19891725`.
Kaplan, David (1964). "Foundations of Intensional Logic". PhD thesis. Monograph Collection (Matt - Pseudo).
— (1995). "A Problem in Possible World Semantics". In: *Modality, Morality and Belief: Essays in Honor of Ruth Barcan Marcus*. Ed. by W. Sinnott-Armstrong, N. Asher, and D. Raffman. Cambridge University Press, pp. 41–52.
Keller, Lorraine (2013). "The Metaphysics of Propositional Constituency". In: *Canadian Journal of Philosophy* 43.5-6, pp. 655–678.
— (2019). "What Propositional Structure Could Not Be". In: *Synthese* 196.4, pp. 1529–1553.
Keller, Lorraine Juliano and John A. Keller (2013). "Compositionality and Structured Propositions". In: *Thought: A Journal of Philosophy* 2.4, pp. 313–323.
King, Jeffrey (2007). *The Nature and Structure of Propositions*. Oxford: Oxford University Press.
Kripke, Saul (1959). "A Completeness Theorem in Modal Logic". In: *Journal of Symbolic Logic* 24.1, pp. 1–14.
— (1963). "Semantical Considerations on Modal Logic". In: *Acta Philosophica Fennica* 24, pp. 83–94.
Leitgeb, Hannes, Uri Nodelman, and Edward Zalta (manuscript). "A Defense of Logicism".
Lewis, David (1970). "General Semantics". In: *Synthese* 22.1/2, pp. 18–67.
Liggins, David (forthcoming). "Should a Higher-Order Metaphysician Believe in Properties?" In: *Synthese*, pp. 1–21.
Linnebo, Øystein (2003). "Plural Quantification Exposed". In: *Noûs* 37.1, pp. 71–92.
— (2010). "Pluralities and Sets". In: *The Journal of Philosophy* 107.3, pp. 144–164.
— (2013). "The Potential Hierarchy of Sets". In: *The Review of Symbolic Logic* 6.2, pp. 205–228.
— (2022). "Plural Quantification". In: *Stanford Encyclopedia of Philosophy (Spring 2022 Edition)*. Ed. by Edward N. Zalta. Stanford University.

MacBride, Fraser (2006). "Predicate Reference". In: *The Oxford Handbook of Philosophy of Language*. Ed. by Barry C. Smith. Oxford University Press, pp. 422–475.

MacFarlane, John (2017). "Logical Constants". In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Winter 2017. Metaphysics Research Lab, Stanford University. URL: `https://plato.stanford.edu/archives/win2017/entries/logical-constants/`.

McMichael, Alan and Ed Zalta (1980). "An Alternative Theory of Nonexistent Objects". In: *Journal of Philosophical Logic* 9.2, pp. 297–313.

Menzel, Christopher (1993). "Singular Propositions and Modal Logic". In: *Philosophical Topics* 21.2, pp. 113–148.

— (2014). "Wide Sets, ZFCU, and the Iterative Conception". In: *Journal of Philosophy* 111.2, pp. 57–83.

Montague, Richard (1970). "English as a Formal Language". In: *Linguaggi Nella Societ{á} e Nella Tecnica*. Ed. by Bruno Visentini Et al. Milan: Edizioni di Communit{à}, pp. 189–224.

Myhill, John (1958). "Problems Arising in the Formalization of Intensional Logic". In: *Logique et Analyse* 1.1, pp. 78–83.

Parsons, Terence (1974). "A Prolegomenon to Meinongian Semantics". In: *Journal of Philosophy* 71.16, pp. 561–580.

Peacocke, Christopher (1976). "What Is a Logical Constant?" In: *Journal of Philosophy* 73.9, pp. 221–240. DOI: `10.2307/2025420`.

Perry, John R. (1998). "Semantics, Possible Worlds". In: *Routledge Encyclopedia of Philosophy*. URL: `https://doi.org/gx93` (visited on 10/04/2021).

Pickel, Bryan (2020). "Structured Propositions and Trivial Composition". In: *Synthese* 197.7, pp. 2991–3006. DOI: `10.1007/s11229-018-1853-1`.

Plantinga, Alvin (1974). *The Nature of Necessity*. New York: Oxford University Press.

Prior, A. N. (1971a). *Objects of Thought*. Ed. by P. T. Geach and A. J. P. Kenny. Oxford: Clarendon Press.

— (1971b). "Platonism and Quantification". In: *Objects of Thought*. Ed. by P. T. Geach and A. J. P. Kenny. Oxford: Clarendon Press, pp. 31–47.

Prior, Arthur (1960). "On a Family of Paradoxes". In: *Notre {D}ame Journal of Formal Logic* 2, pp. 16–32.

Quine, W. V. (1948). "On What There Is". In: *The Review of Metaphyiscs* 2.5, pp. 21–38.

— (1956). "Quantifiers and Propositional Attitudes". In: *Journal of Philosophy* 53.5, pp. 177–187.

Quine, W. V. (1966). *Selected Logic Papers*. Harvard University Press.
— (1986). *Philosophy of Logic*. Second Edition. Cambridge, MA: Harvard University Press.
Quine, W. V. O. (1960). *Word and Object*. Harvard University Press.
Quine, Willard V. (1960). "Variables Explained Away". In: *Proceedings of the American Academy Philosophical Society* 104.3, pp. 343–347.
Rayo, Agustin and Stephen Yablo (2001). "Nominalism Through De-Nominalization". In: *Nous* 35.1, pp. 74–92.
Resnik, Michael D. (1988). "Second-Order Logic Still Wild". In: *Journal of Philosophy* 85.2, pp. 75–87. DOI: 10.2307/2026993.
Rouilhan, Philippe De (2002). "On What There Are". In: *Proceedings of the Aristotelian Society* 102, pp. 183–200.
Russell, Bertrand (1903). *The Principles of Mathematics*. 1st. Cambridge: Cambridge University Press.
— (1920). *Introduction to Mathematical Philosophy*. 2nd ed. London: George Allen & Unwin.
— (1984). *Theory of Knowledge: The 1913 Manuscript*. Routledge.
Ryle, Gilbert (1954). *Dilemmas*. Cambridge University Press.
Sainsbury, R. M. (2001). *Logical Forms: An Introduction to Philosophical Logic*. Blackwell.
Salmon, Nathan (1986). *Frege's Puzzle*. Cambridge, Massachusetts: MIT Press.
Sellars, Wilfrid (1960). "Grammar and Existence: A Preface to Ontology". In: *Mind* 69.276, pp. 499–533. DOI: 10.1093/mind/LXIX.276.499.
Shapiro, Stewart (1991). *Foundations without Foundationalism: A Case for Second-Order Logic*. Oxford: Oxford University Press.
Sher, Gila (1991). *The Bounds of Logic: A Generalized Viewpoint*. MIT Press.
Skiba, Lukas (2021). "Higher-Order Metaphysics". In: *Philosophy Compass* forthcoming. DOI: 10.1111/phc3.12756.
Soames, Scott (1987). "Direct Reference, Propositional Attitudes, and Semantic Content". In: XV.1, pp. 47–87.
Stalnaker, Robert (2012). *Mere Possibilities: Metaphysical Foundations of Modal Semantics*. Princeton, NJ: Princeton University Press.
Sullivan, Meghan (2014). "Modal Logic as Methodology". In: *Philosophy and Phenomenological Research* 88.3, pp. 734–743. DOI: 10.1111/phpr.12102.
Trueman, Robert (2020). *Properties and Propositions: The Metaphysics of Higher-Order Logic*. Cambridge: Cambridge University Press.
Uzquiano, Gabriel (2015). "A Neglected Resolution of Russell's Paradox of Propositions". In: *Review of Symbolic Logic* 8.2, pp. 328–344.

Van Inwagen, Peter (2004). “A Theory of Properties”. In: *Oxford Studies in Metaphysics, Volume 1*. Ed. by Dean Zimmerman. Clarendon Press, pp. 107–138.

Williamson, Timothy (2003). “Everything”. In: *Philosophical Perspectives: Language and Philosophical Linguistics* 17, pp. 415–465.

— (2013). *Modal Logic as Metaphysics*. Oxford: Oxford University Press.

— (2014). “Logic, Metalogic and Neutrality”. In: *Erkenntnis* 79 (Suppl 2), pp. 211–231. DOI: 10.1007/s10670-013-9474-z.

— (2016). “Reply to Bacon, Hawthorne and Uzquiano”. In: *Canadian Journal of Philosophy* 46.4-5, pp. 542–547. DOI: 10.1080/00455091.2016.1205852. URL: https://doi.org/10.1080/00455091.2016.1205852.

Yu, Andy Demfree (2017). “Logic for Alethic Pluralists”. In: *Journal of Philosophy* 114.6, pp. 277–302. DOI: 10.5840/jphil2017114621. URL: https://doi.org/10.5840/jphil2017114621.

Zalta, Edward N. (1983). *Abstract Objects: An Introduction to Axiomatic Metaphysics*. Dordrecht: D. Reidel.