The Marriage of Metaphysics and Geometry in Kant's *Prolegomena*

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I. Introduction

As various commentators have noted, Kant was engaged in a lifelong struggle to achieve what he calls in the 1756 *Physical Monadology* a "marriage" of metaphysics and geometry (1:475).[1] On one hand, this involved showing that metaphysics and geometry are complementary, despite the seemingly irreconcilable conflicts between these disciplines and between their respective advocates, the Leibnizian-Wolffians and the Newtonians. On the other hand, it involved defining the terms of their union, which meant among other things, articulating their respective roles in grounding Newtonian natural science, whose crowning achievement was the inverse-square law of gravitation. While Kant changed his mind between the pre-Critical and Critical periods about the nature of metaphysics and geometry, he continued to believe in the importance of showing that and how, despite their substantial differences, they could be unified.

Michael Friedman has argued that the *Prolegomena* can be fruitfully read in light of this lifelong struggle.[2] I agree. In this paper, I consider how, generally, Kant's project of marrying metaphysics and geometry evolves from the pre-Critical to the Critical period and how, specifically, key discussions in the *Prolegomena* are related to the Critical marriage project. At

[1] See, e.g., Friedman (1992), Schönfeld (2000), and Holden (2004).
[2] Friedman (1992, 298ff.). Despite our agreement on this point, our interpretations differ in substantive ways (some of which are highlighted in section 5).

the same time, I highlight the similarities and differences between the account of the marriage contained in the *Prolegomena* and the account given in other texts. My interpretation has implications for some key, contested points in Kant's philosophy of science, including the role that Kant sees for geometric construction in the grounding of some specific causal laws (like the inverse-square law of gravitation), and relatedly, the service Kant sees geometric construction as playing in the "special metaphysics of nature" of the 1786 *Metaphysical Foundations of Natural Science.*

I begin in §2 with a general account of the marriage project in the pre-Critical period. In §3, I lay the groundwork for my interpretation of the account of the marriage in the *Prolegomena* by first presenting Kant's Critical reconceptualization of metaphysics and geometry and then explaining what an account of their marriage entails in the context of the Critical philosophy. In §4, I show how the marriage issue is connected to *Prolegomena*'s search for a "common origin" of pure mathematics and pure natural science. In §5, I consider the implications of the *Prolegomena* for the role of geometric construction in natural philosophy (including its bearing on specific causal laws). In §6, I consider some ways in which geometry and metaphysics serve one another. In §7, I conclude.

II. The Pre-Critical Marriage Project

Kant's interest in the relationship between geometry and metaphysics is evident in a number of pre-Critical works. I will here focus on only those texts and discussions that I take to be particularly relevant for understanding the account of that relationship in the *Prolegomena*, noting in parentheses key points that show up again in the *Prolegomena* (and which I will explore further in later sections).

As the full title indicates, Kant's 1756 *The Employment in Natural Philosophy of Metaphysics combined with Geometry, of which Sample I contains the Physical Monadology* attempts to show that and how geometry and metaphysics are to be combined in the pursuit of a broadly Newtonian natural philosophy. That they could be combined was not obvious, in part because the foremost advocates of "metaphysics," the Leibnizian-Wolffians, took its results to be at odds in various ways with geometry. A key example was the infinite divisibility of space, which it was generally believed could be geometrically demonstrated. The Leibnizian-Wolffians posited indivisible simple substances (monads) as in some sense the foundation of all matter.[3] The Wolffians among them took this to mean that matter and physical space (the space filled by matter and its parts) is only divisible to the point where one reaches the monads from whose aggregation they arise.[4] While they admitted that geometric space was infinitely divisible, they took this space to be imaginary and distinct from physical space. They thus abandoned what Kant would call the "objective validity of geometry" in the face of metaphysical considerations about substances.

As Kant understands them, the Leibnizian-Wolffians also took metaphysics to be at odds in other ways with "geometry," as advocated by the Newtonians: they differ not just on the existence of monads, but also on the ontological status of space, the existence of empty space,

[3] For helpful discussion of the arguments of Leibniz and the Wolffians for monads, see Watkins (2006). For background on the debate between the Leibnizian-Wolffians and Newtonians and its cultural significance, see Friedman (1992) and Schönfeld (2000).
[4] One needs to distinguish between Leibniz's view and that of Wolff and his followers. Leibniz is committed to monads *and* to the infinite divisibility of bodies (see [Levey 1998]). It is worth noting, though, that Leibniz arguably shares with the Wolffians a (from Kant's standpoint) problematic commitment to a disconnect between the space of geometry (which Leibniz regards as a continuum) and reality (which Leibniz takes to consist of discrete parts).

and the possibility of action-at-a-distance.[5] It is in light of these apparently insuperable difficulties that Kant in the *Physical Monadology* likens a marriage of geometry and metaphysics to a mating of griffins and horses:

> Metaphysics, therefore, which many say may be properly absent from physics is, in fact, its only support; it alone provides illumination. For bodies consist of parts; it is certainly of no little importance that it be clearly established of which parts, and in what way they are combined together, and whether they fill space merely by the co-presence of their primitive parts or by the reciprocal conflict of their forces. But how, in this business, can metaphysics be married to geometry, when it seems easier to mate griffins with horses than to unite transcendental philosophy with geometry? For the former peremptorily denies that space is infinitely divisible, while the latter, with its usual certainty, asserts that it is infinitely divisible. Geometry contends that empty space is necessary for free motion, while metaphysics hisses the idea off the stage. Geometry holds universal attraction or gravitation to be hardly explicable by mechanical causes but shows that it derives from the forces which are inherent in bodies at rest and which act at a distance, whereas metaphysics dismisses the notion as an empty delusion of the imagination. (1:475-6)

In contrast to some who would abandon all metaphysics, understood as, among other things, an investigation of the fundamental nature and constituents of bodies and the basis of physical laws, Kant maintains that it is "the only support" of physics (4:175). Defying the apparent absurdity of marrying metaphysics and geometry, Kant struggles to unify them in the *Physical Monadology*.

His first step, I take it, is to show that apparent incompatibilities are merely apparent. To do this, Kant provides a revised monadology, according to which matter has its foundation in "physical monads" endowed with both repulsive and attractive force. Physical space, which Kant sees as arising out of the interaction of monads, is infinitely divisible, something which can be established by geometric proof (1:478-9). But Kant argues that this does not entail the divisibility of the monads themselves that are the elements of matter and physical space. Physical space is

[5] Kant sees the Newtonians/geometers as rejecting monads and as accepting absolute space, empty space, and gravitational action-at-a-distance. This last point reveals that by the "geometers" Kant is evidently thinking not so much of Newton, who was famously diffident on the issue of action at a distance, but of some of his followers, like John Keill, who were not.

infinitely divisible insofar as the dynamic relations between them are infinitely divisible, but the relata that stand in these relations, the monads themselves, are not divisible. Put differently, a monad's "sphere of activity" (which constitutes the physical space it fills) is infinitely divisible, but the monad itself is not (1:480-482). In this way, Kant maintains that matter is ultimately composed of indivisible monads (something he thinks is required by metaphysics) while also upholding, at least in this instance, the non-fictional nature of geometric knowledge – its direct bearing on physical space and matter.[6] (The Critical Kant will go on to revisit his earlier solution in the *Prolegomena* [4:288].)[7]

While the Physical *Monadology*'s first step is to show that metaphysics and geometry are not irreconcilable, the second step is to show how they should function together in natural philosophy – something which has received less attention than Kant's approach to the divisibility problem. Kant turns to this second step in the scholium to proposition 10, upon concluding his argument that for matter to fill a determinate volume the monads that constitute it have to be endowed with attractive and repulsive force. The scholium considers the question of the mathematical laws governing the diffusion of the forces. Kant offers what appear to be geometric proofs that attractive force diffuses in accordance with an inverse-square law while repulsive force does so in accordance with an inverse-cube law. To focus on the former, the idea is that the intensity of attraction will decrease inversely with the square of the distance to a center of force because attractive force is dispersed uniformly across the surfaces of concentric spheres (whose surface areas are directly proportional to the square of their radii) (1:484). The derivation is of particular interest for what it suggests about the division of labor between metaphysics and

[6] For further discussion, see Friedman (1992), Schönfeld (2000), and Watkins (2006).
[7] He also alludes to his earlier solution in the Second Antinomy of the first *Critique* (at 4:441/B469) and explicitly critiques it in the *Metaphysical Foundations* (4:504-5).

geometry: the former shows various fundamental forces to exist and to be essential to matter, while (among other things) the latter explains – at least partly – why the mathematical character of their force laws is just so and not otherwise. (This geometric derivation shows up again in the *Prolegomena* [4:321].[8])

The *Physical Monadology* does not provide a complete account of the relationship between geometry and metaphysics. While it removes some impediments to the marriage – including the issue of infinite divisibility, and while Kant clearly expects that geometry and metaphysics will always be in agreement, he does not explain why they will always be. That is, he does not explain why laws and forces of matter (the object of physics as well as metaphysics) must always complement the laws and properties of geometric objects. Indeed, he does not explain why either kind of object exhibits laws at all, much less the sorts of harmonious, fruitful laws he takes there to be in both domains.

Along with various other goals, I think the 1763 *Only Possible Argument* (henceforth *OPA*) tries to fill in these explanatory gaps, thereby providing a more complete account of the marriage of geometry and metaphysics.[9] The need for a kind of guarantee of compatibility was especially pressing for Kant at this time as, in other work from the period (especially the 1764 *Inquiry*), he emphasizes the substantial methodological differences between geometry and metaphysics. He is insistent that metaphysics cannot be carried out *more geometrico*, and that mathematics cannot be pursued in a metaphysical manner. So why should one think they will always deliver consistent, complementary, convenient results regarding the physical world?

[8] It also occurs in the *Metaphysical Foundations* (4:519ff.).
[9] Other commentators have not, to my knowledge, explicitly connected the *OPA* to the marriage project laid out in the *Physical Monadology*.

In the first part of the *OPA*, Kant argues a priori that God must exist in order to ground the possibility of things. In the second part, which has received less attention,[10] he points to the extensive harmony, beauty, and fruitfulness evident in the realms of both geometry and matter to provide a posteriori confirmation of his a priori argument. It is here that Kant can be seen as trying to advance the marriage project of the *Physical Monadology*.

In the case of geometry, Kant enthuses that a "seemingly straightforward and simple thing such as a circle" contains a "wondrous unity of the manifold subject to such fruitful rules" (2:94). One of his examples is Proposition 35 of Book Three of Euclid's *Elements:* the rectangles formed by the intersecting chords of a circle are equal (2:94).[11] (The same proposition shows up again in the *Prolegomena* [4:320].) One respect in which the geometric rules of circles are "fruitful" is that they provide surprisingly simple solutions to what might seem to be complex *physical* problems. Here's one of Kant's examples: "Inclined surfaces of varying gradients are to be constructed, with the inclined surfaces of such a length that bodies freely rolling down them shall all take the same time to reach the bottom." Kant notes that the solution is contained, as if by design, in the circle: "free fall through all the chords that meet at the vertical diameter of the circle takes the same time" (2:94).

[10] One exception here is Schönfeld (2000). However, he focuses primarily on the question of how to make sense of Kant's apparently offering two distinct arguments for the existence of God. Laywine (1993), (2003), and (2014) is another exception. She rightly calls attention to the larger question Kant is asking about why and how geometry could be such an effective guide to kinematics. She also notes the overlap between these discussions and *Prolegomena* §38. While I learned much from her treatment, one way we differ is that I think Kant is particularly interested in explaining why as a matter of necessity the laws of geometry and physics synch up with each other so nicely. I take this to be part of Kant's larger marriage project. Laywine does not, to my knowledge, link this particular discussion to the marriage project in *Physical Monadology*, and she does not emphasize the *explanatory* role that Kant accords to geometry. Moreover, in describing God's role in the grounding of the laws of physics and geometry, she suggests that Kant thinks God could have made matter with no, or with a very different, geometric structure and as subject to other basic laws of motion than those that actually obtain (Laywine 1993, 126-127). By contrast, I take Kant in the *OPA* to be rejecting such contingency.

[11] Friedman (1992, 182) explains it as follows: "If two straight lines intersect one another within a circle at point E, and meet the circle at A, C, B, and D respectively, then AE x EC=BE x ED." Laywine (2014, 721) calls this Kant's "favorite Euclidean theorem".

This is one numerous examples of the beneficial and seemingly purposive alignment between geometry and laws of motion. Kant underscores how surprisingly relevant geometric properties and laws are not just to *discovering* what the laws of motion and force in fact are, but to *explaining* (at least partly) why they are as they are. For Kant, the distinctive explanatory power of geometry lies in the special character of spatial relations: "Spatial relations can also enable us to recognize, from the simplest and most universal principles, the rules of perfection present in naturally necessary causal laws, in so far as they depend upon relations" (2:134). Kant thinks that geometric considerations about space – specifically what he describes as a "necessary equality" inherent in the structure of space – explain at least partly why, for example, nature is subject to Maupertuis' principle of least action, as well as why there is a necessary equality of action and reaction (2:134). Though not directly mentioned in the *OPA*, the geometric derivation of the inverse-square law in the *Physical Monadology* should be considered in this light, as another instance of the way that spatial relations can explain – at least partly – the character of causal laws (in that case the inverse-square law).

So far, we have considered the harmony and fruitfulness evident in geometry. In the *OPA* Kant also offers various examples of the way that the laws and forces of matter exhibit these same properties. Kant repeatedly points, for example, to the large number of "useful" and "harmonious" effects that result from the inverse-square law of gravitation (2:106-7; 2:149; 2:152). (He does this again in the *Prolegomena* [4:321]). He argues that such harmony, unity, and beauty in geometry and physics, and in their relation to one another, is more remarkable insofar as it is necessary rather than contingent. He thinks this can only be explained by the fact that the laws and properties of matter and geometry have a common ground of possibility in God, whom he regards as the legislator of laws. By tracing these laws and properties back to a common, divine origin, Kant explains why geometric objects and matter admit of necessary laws, and why these laws are consistent, complementary, and convenient.[12] The fruitful marriage between metaphysics and geometry was, Kant believed at the time, made in heaven.

III. A Marriage on New Terms

It was not to last, at least not on the terms upon which it was conceived in the pre-Critical Period. Various changes in Kant's philosophy around 1770 and thereafter, such as his acceptance of the subjectivity of space and time, and his denial of cognition of things-in-themselves, precluded him from holding on to all aspects of the account described above. These changes meant, among other things, that Kant could not deal with the divisibility problem in the manner that he had previously and that he could not claim theoretical cognition of a common divine origin for the laws of matter and geometry. These changes went together with a reconceptualization of the nature of metaphysics and geometry. In this section, I lay the groundwork for an exploration of the *Prolegomena*'s account of the marriage by first explaining this reconceptualization and then showing what an account of their marriage entails in the context of the Critical philosophy.

3.1 Metaphysics and Geometry in the *Prolegomena*

In the preamble to the *Prolegomena*, Kant touches on the distinction between metaphysical cognition and mathematical cognition, referring the reader to the first *Critique*'s

[12] Kant's account in the *OPA* assigns a role both to the divine essence (which is the source of all possibility) and to God's will. While one can grant that there would be no laws for the pre-Critical Kant without the existence of things (which requires God's will), I take it that for Kant God's essence is the more important factor in explaining the necessity of laws.

Doctrine of Method for a more detailed treatment. Both types of cognition are a priori but that of metaphysics "must therefore be denominated pure philosophical cognition; but concerning the meaning of this expression I refer to the Critique of Pure Reason, pp. 712 f." (4:266). By contrast, "the essential feature of mathematical cognition, differentiating it from all other a priori cognition, is that it must throughout proceed not from concepts, but always and only through the construction of concepts (Critique, p.713) (4:272). Mathematical cognition is a matter of construction in pure intuition (4:281). To attain geometric cognition, for example, we construct, using our pure intuition of space, a particular figure in accordance with a prescribed procedure (a "schema") in such a way that the properties of the constructed figure can be generalized to all figures of that type.[13] The "pure philosophical cognition" involved in metaphysics is not like this: it is "rational cognition from concepts" (A713/B741). The concepts that Kant has foremost in mind here are the categories[14], and paradigmatic metaphysical judgments are synthetic a priori judgments involving the categories like "all that is substance in things persists" (4:272). We cannot arrive at cognition of such synthetic judgments through construction in pure intuition.

In order to understand why that is, and to better understand what is distinctive about the method and object of metaphysical cognition, it is helpful to consider Kant's Critical view of metaphysics as a discipline, so far as this can be gleaned from texts like the Architectonic of Pure Reason[15] section of the Doctrine of Method in the first *Critique* and the Preface to the 1786 *Metaphysical Foundations*.[16] This will be further useful as Kant here draws distinctions between

[13] For discussion, see Shabel (2012).
[14] See, e.g., 4:260 and 4:472.
[15] In making sense of Kant's conception of metaphysics of nature, I have found especially helpful Plaass (1965), Dahlstrom (1991), and Haag (2012).
[16] Following a number of commentators (such as Friedman [1992], [2012], and [2013]), I take the *Prolegomena* and the *Metaphysical Foundations* to be in important respects mutually illuminating. As we will see, there is considerable overlap between these texts on a number of points, including the marriage issue.

types of metaphysics (true vs. deceptive; general vs. special) that are highly relevant for the *Prolegomena*.

A common theme in the texts mentioned above is that, as Kant says in the *Metaphysical Foundations*, "proper natural science presupposes metaphysics of nature" (4:469).[17] The metaphysics of nature that Kant envisages as a basis for natural science – what we might think of as the "true," (4:472), non-transcendent metaphysics of nature – is not the traditional dogmatic metaphysics that strives after cognition of God, the world as a totality, and the soul. Nor is it the underlying disposition of the soul that fuels this pursuit and gives rise to illusions that Kant punctures in the Critique's Transcendental Dialectic and, in abbreviated form, in the Third Part of the *Prolegomena* (which deals with the question, "how is metaphysics in general possible?"). Instead, it is a reconstructed, critically purified metaphysics that yields real rather than apparent theoretical cognition.[18] The "nature" upon which it focuses consists of objects of experience. I take it that this is the metaphysics whose future coming as a science is heralded in the full title of the *Prolegomena to any Future Metaphysics That Will Be able to Come Forward as Science.*[19]

Within this true metaphysics, Kant distinguishes a "transcendental" or general part, which "treat[s] the laws that make possible the concept of a nature in general, even without relation to any determinate object of experience, and thus [is] undetermined with respect to the nature of this or that thing in the sensible world" and a special metaphysics (4:469). This latter

[17] For other usages of the phrase 'metaphysics of nature,' see A841/B869, A845/B873, Axxi, and Bxliii
[18] Some recent commentators who emphasize this distinction are Pollock (2001), Haag (2012), and Mohr (2012).
[19] If, as I suggest below, the metaphysics of nature includes the transcendental part (whose essentials are delivered in the *Critique*) and the special part, why is Kant (as Haag [2012, 258] points out) enlisting the help of his readers in the problem of bringing about a scientific metaphysics in the *Prolegomena*? Relatedly, why in the B-version of the Critique (written after Kant had completed to his satisfaction the special part of metaphysics) does he speak of the metaphysics of nature as something that he has still not provided (Bxliii)? The answer has to do with Kant's view of what transcendental philosophy requires to be a science or complete system. Namely, it requires the complete analysis of the categories and the systematic assemblage of all the predicables (A81-82/B107-8; 4:366; 4:273-4. This is evidently what Kant had not completed by 1788 (or apparently ever). See in this regard Plaass (1965, 18-21, 67) and Haag (2012, 257-8).

“concern[s] itself with a particular nature of this or that kind of things, for which an empirical concept is given” (4:469-70). The transcendental part of metaphysics of nature includes the categories and the system of categorical principles as presented in the *Critique*’s Transcendental Analytic and recapitulated, in the Second Part of the *Prolegomena* (which deals with the question “how is pure natural science possible?”). These include, of course, the dynamical categories (including substance, causation, and mutual interaction) and the principles of experience involving them, such as the causal principle, and the principle that substance persists. Kant regards these as transcendental laws of nature (A216/B263; 4:307), the most fundamental laws of the order of nature, so far as we can experience it. The special metaphysics of nature, by contrast, deals with a particular empirical nature, namely that of *matter*. Drawing on the transcendental part of metaphysics, as well as considerations about the mathematization of material objects, the “special metaphysics of corporeal nature” tries to determine a priori the more specific forces and laws (including laws of mechanics, like the law of inertia and the law of the equality of action and reaction) that are specifically necessary to matter as such. These more “specific” a priori causal laws of special metaphysics “stand under” the dynamical categories and principles, as do other specific empirical causal laws, to use Kant’s phraseology from the B-Deduction (B165).

What Kant calls “pure natural science” in *Prolegomena* §15 corresponds to precisely these two parts of the true metaphysics of nature. The completely pure and universal portion of pure natural science corresponds to the transcendental/general part; the impure (but still a priori)[20] portion that involves empirical concepts connected to matter like motion,

[20] In the introduction to the B-*Critique*, Kant distinguishes between two types of a priori judgments: pure and impure, where only the latter contain empirical concepts (B3). Evidently, Kant thinks that judgments that contain empirical concepts can nevertheless be justified through some non-empirical process and so count as a priori.

impenetrability, and inertia corresponds to the special metaphysics of corporeal nature (4:295). In much of the Second Part of the *Prolegomena*, Kant focuses on the possibility of the former. However, in *Prolegomena* §38, he treats a specific law – the inverse-square law of universal gravitation – which figures centrally in the special metaphysics of the *Metaphysical Foundations,* and in doing so, descends into the impure part of universal natural science. In subsequent sections, I will have more to say about §38.

Let's return to the differences between metaphysical cognition and geometrical cognition. While both yield genuine synthetic a priori cognition, we do not arrive at cognition of metaphysical claims through construction in pure intuition. Instead, Kant thinks we must prove them discursively, by means of a transcendental proof. In the case of the transcendental part of the metaphysics of nature, I must consider, as Kant says in the *Prolegomena* §14, "my understanding, and the conditions under which alone it can connect things in their existence" (4:294). These "synthetic conditions" are the concepts and principles necessary for perceiving spatiotemporal objects and experiencing them as connected together in an objective space and time. Such principles of the possibility of experience are simultaneously the fundamental conditions of nature in general, defined as "the existence of things, insofar as that existence is determined by universal laws" (4:294; cf. 4:474). Here again Kant has in mind primarily the dynamical categories and dynamical principles.[21]

We are now in a position to explain why metaphysics cannot proceed by construction in Kant's technical sense. It cannot since neither the dynamical concepts pertaining to nature in general nor the specific dynamical concepts pertaining to corporeal nature can be constructed.

[21] See in this regard Friedman (1992, 180ff.)

That is, we are unable to construct in pure intuition general concepts like causality, force, or substance (A770/B798; A720-2/B748-750), or specific concepts of the attractive and repulsive forces characteristic of matter (4:525). More generally, we cannot construct the existence of such things in pure intuition but must be given the data for their existence in empirical intuition. In contrast to metaphysics, mathematics can proceed by construction in pure intuition because it does not concern itself with existence (A719/B747). This is closely related to the fact that the objects of pure geometry fall under the mathematical categories but not under dynamical categories – they are causally inert (5:366n).[22] It is for this reason that, as Kant says in the *Metaphysical Foundations*, "one can attribute only an essence to geometrical figures, but not a nature (since in their concept nothing is thought that would express an existence)" (4:467n).[23]

3.2. Why an Account of the Marriage is Necessary and What Questions it Needs to Answer

It is with these stark differences in mind that Kant describes judgments of mathematics and metaphysics in the *Prolegomena* as "worlds apart" (4:370-1). Nevertheless, Kant continues to think that metaphysics and geometry can and must be combined for the purposes of natural philosophy. A remark in the first *Critique*'s Doctrine of Method is representative: "mathematics and philosophy are two entirely different things, although they each offer the other their hand in natural science" (A726/B754).[24] Moreover, it is clear that Kant takes the marriage between

[22] As noted, e.g., by Plaass (1965, 32ff.)
[23] The first sentence of *Prolegomena* §38 *seems* to be in conflict with the *Metaphysical Foundations* on this point. A number of explanations of this passage are possible. One is that it reflects a temporary oversight on Kant's part (this seems to be the view of Plaass [1965, 31]). Another is that, as Friedman [1992, 190, 194] argues, Kant is directing our attention not to the objects of pure geometry but rather to particular concrete physical instantiations of such objects, such as orbits, which as actually existing items do have natures. The explanation I find most plausible is that Kant is describing a natural (but deceptive) metaphysical tendency that is fueled by geometry. See Messina (2018).
[24] Though Kant uses the term 'philosophy' here rather than 'metaphysics', he defines metaphysics as philosophical cognition and in various places he indicates that pure philosophy and metaphysics are synonyms (e.g. 4:469).

mathematics and metaphysics (understood as the true metaphysics of nature described above) to be necessary not just for the existence and progress of natural science but also for their own possibility. On the last point, Kant says in the preface to the *Metaphysical Foundations* that special metaphysics is "only possible by means of mathematics" (4:470). At the same time, Kant insists that mathematics – particularly geometry – requires metaphysics (evidently general/transcendental as well as special) (4:478-9).

The Critical Kant needs a way to account for this marriage, as he attempted to do in the pre-Critical period. This requires answering various questions. First, as he tried to do in the pre-Critical period (especially in the *OPA*), Kant needs to explain how it can be that geometry, on the one side, and metaphysics and physics, on the other side, will always agree with each other. More exactly, he needs to explain how the truths of geometry are able to harmonize with the categories and laws of nature (where this includes the transcendental laws of nature as well as the other causal laws, such as those of physics, that stand under them). This is a general "how is x possible question" – with the x here being the complementary character of geometry and metaphysics. Its answer is not obvious because it is not immediately obvious why or how geometry, as concerned with the realm of *essence*, and metaphysics as concerned with *nature,* should be in agreement. It includes, as a specific instance, the question, which had also exercised Kant in the pre-Critical period, of how it is possible for the infinite divisibility that is provable in geometry to be applicable to physical space, and indeed, how it is simultaneously possible *for it to be applicable and for there to be substances in space*. This question was particularly pressing for the Critical Kant since his previous, pre-Critical solution was no longer available to him.[25] Call these two question the general and specific “how is complementarity possible?” questions.

Second, Kant needs to explain what metaphysics and geometry do for each other. As with the previous question, we can distinguish both a general version of the question – how generally is each necessary for the other – and a specific version of the question regarding their respective roles in accounting for specific laws of nature, like the inverse-square law and the laws of mechanics. (Call these the general and specific “respective roles” questions.) In the next sections, I show how the *Prolegomena* bears on these questions.

IV. Kant’s Accounts of the Common Origin and the “How is Complementarity Possible?” Questions.

The first two parts of the *Prolegomena* are explicitly concerned with the following questions:

(1) How is pure mathematics possible? (2) How is pure natural science possible?

Kant answers these questions not just by trying to identify the origin (or ground of possibility) of each; he seeks a “common origin” (*gemeinschaftlichen Ursprung*) (4:280).[26] The search for a common origin is apparently identical with the “deduction” that Kant describes himself as having completed by §40 (4:327). As we will see in this section, there are strong indications that

[25] As we will see, one reason it is unavailable to him is that now takes Kant space as a form of intuition to underlie physical space. Another reason is that the Critical Kant thinks that it can be proven that substance in space is itself infinitely divisible (pace his earlier claim that physical space is infinite divisible but *not* the substances in space).
[26] Consider in this regard the following remark: “But what obscured the fundamental idea of metaphysics from yet another side was that, as a priori cognition, it shows a certain homogeneity with mathematics, to *which, as far as a priori origin is concerned, it is no doubt related* (A844/B872; my emphasis)

Kant views finding a common origin for these sciences as important for the same reason that it was important in the *OPA* to search for a common source of geometric laws and laws of nature. Namely, Kant is pursuing the common origin of both sciences so that he can answer the general and specific "how is complementarity possible?" questions. Indeed, as we will see, at key places, Kant's account(s) of the common origin in the *Prolegomena* invokes key ideas and examples from the *Physical Monadology* and *OPA* – sometimes to straightforwardly criticize them, but sometimes to re-appropriate them in subtle ways.

Unsurprisingly, Kant thinks the origin of pure mathematics and pure natural science lies in our cognitive faculties. Kant offers in the *Prolegomena* what might be thought of as an initial, oversimplified account of the way the faculties of sensibility and understanding underlie geometry and pure natural science, followed by a later, more sophisticated account.[27]

According to the oversimplified account, pure mathematical cognition, including geometry, is made possible by the fact that we have space and time as our a priori forms of sensibility. We can do geometry a priori because it concerns the forms of intuition that make experience possible for us. We can be confident that the results will necessarily apply to those objects because those objects are mere appearances dependent on our forms of intuition. This holds, inter alia, for the infinite divisibility of physical space, which the Wolffians had sacrificed on the altar of metaphysics. As Kant writes in Note 1 to *Prolegomena* §13:

> It will forever remain a remarkable phenomenon in the history of philosophy that there was a time when even mathematicians who were at the same time philosophers began to doubt, not, indeed, the correctness of their geometric propositions insofar as they related merely to space, but the objectivity validity of and application to nature of this concept itself and all its geometrical determinations, since they were concerned that a line in nature might indeed be composed of physical points, consequently that true space in objects might be composed of simple parts, notwithstanding that the space which the

[27] The *Prolegomena* is similar in this regard to the first *Critique*.

> geometer holds in thought can by no means be composed of such things. They did not realize that this space in thought makes possible physical space, i.e. the extension of matter; that is this space is by no means a property of things in themselves but representations of our sensory intuition; and that since space as the geometer thinks it is precisely the form of sensory intuition which we find in ourselves a priori and which contains the ground of the possibility of all outer appearances (with respect to their form), these appearances must of necessity and with the greatest precision harmonize with the propositions of the geometer…. (4:287-8)

Because geometry has its basis in the form of outer intuition, and this space "in thought," makes possible physical space, geometry is not imaginary as the Wolffians claim; rather, the results of geometry apply directly to physical space, so that if the former is infinitely divisible then the latter is as well. In addition to criticizing the Wolffians, Kant is implicitly criticizing his earlier position in the *Physical Monadology*.[28] While he had there attempted to maintain that geometry applies to physical space, he did not claim that space was a form of intuition and that this space in thought made possible physical space. Instead, he there took physical space to be the result of dynamic relations among physical monads.

On the side of pure natural science, Kant's initial oversimplified account runs as follows. The a priori cognition of pure natural science is possible because the categories and principles are a priori conditions of the possibility of experience: the categories are forms of the understanding, just as space and time are forms of sensibility. The common basis of pure mathematics and pure natural science is then this: both have to do with forms of our mind (intellectual and sensible, respectively) that make possible experience, and both concern appearances rather than things-in-themselves.

This account is oversimplified in various respects. It is misleading in giving the impression that pure mathematics depends purely on sensibility, while pure natural science

[28] As noted, e.g., by Lyre (2012, 93-94).

depends solely the understanding. In fact, it is Kant's official view that pure mathematics also depends on categories and synthesis, while pure natural science for its part also depends on the conditions of sensibility. On this last point, Kant holds that understanding can only prescribe laws to nature in conjunction with sensibility, and that the categories have to be defined in sensible terms (in particular, in terms of space and time) to apply to nature, per the schematism doctrine.[29] This is absolutely crucial for a satisfying answer to the question of how space could be infinitely divisible and yet allow for substances within it (this is the specific "how is complementarity possible?" question). If, for example, one defines a substance in what Kant would regard as purely intellectual terms as a *simple* item – as the Leibnizian-Wolffians do – and if from the infinite divisibility of space it can be shown that everything in space is infinitely divisible and lacks any simple element (something the Critical Kant thinks can be demonstrated[30]), it would follow that nothing in space is a substance.

A more sophisticated account of the "common origin" of pure mathematics and pure natural science, one that is better suited to answering the general and specific "how is complementarity possible?" questions, comes into view in *Prolegomena* §38. Kant gives two examples of laws concerning conic sections, the first of which (Proposition 35 of Book III of Euclid's *Elements*) he had invoked in the *OPA*. Kant also references the inverse-square law of gravitation, along with the same geometric derivation from properties of concentric spherical surfaces that he had earlier presented in *Physical Monadology*. Throughout his discussion, Kant is concerned to emphasize, just as he did in the *OPA*, the remarkable character of both the geometric laws and the inverse-square law. One way they are remarkable is that they involve

[29] Consider Kant's (more careful) formulation in the Appendix to the *Prolegomena*: "space and time (in combination with the pure concepts of the understanding) prescribe their law a priori to all possible experience" (4:375).
[30] See *Metaphysical Foundations* (4:504-50.

surprising patterns among things that might initially seem unrelated to, respectively, conic sections and gravitation. (For example, Proposition 35 involves a pattern among the intersecting chords of a circle, while the inverse-square law is responsible for the elliptical pattern of orbits.) In this way, the laws unify the phenomena. But they are also themselves systematically unified[31] in remarkable ways. Proposition 35 is, as Kant points out, an instance of a more general law extending to other types of conic sections, and the inverse-square law has its "sources" in geometric laws (4:321). Related to this last point, Kant is calling our attention to a kind of harmony between these different types of laws; he is emphasizing just how useful geometry is in accounting for specific causal laws.[32] Finally, in the closing sentence of the first paragraph Kant notes that "No other law of attraction save that of the inverse square of the distances can be conceived as suitable for a system of the world" (4:321). The inverse-square law is peculiarly suited to systems of the world (solar systems) insofar as it renders them hospitable to our purposes.[33] The implication of the first paragraph of §38 is that some explanation is required of the remarkable unity, harmony, and purposiveness of the laws (something he had also emphasized in the *OPA*, and indeed, with some of the very same examples).[34]

But in contrast to the *OPA*, Kant doesn't invoke God, at least not directly. Instead, he holds that geometry as well as specific causal laws and the dynamical principles that they stand under have a common basis in "the understanding and in the way it determines space in accordance with the conditions of the synthetic unity towards which its concepts are one and all directed" (4:321). Geometry, insofar as it involves geometric schemata (to which Kant is clearly

[31] Laywine (2014) places particular emphasis on the systematic character of these laws.
[32] In section 5, I consider *how* it is supposed to account for them.
[33] In a discussion of geometry in the *Critique of Judgement* that echoes various points from §38, Kant highlights the suitability and purposiveness of geometry itself (5:362ff.). For discussion of this passage, see Fugate (2014).
[34] Laywine (1993), (2003), and (2014) has called attention to the recurrence in the *Prolegomena* of examples and themes – especially regarding systematicity and purposiveness -- from the *OPA*.

alluding in the second paragraph, at 4:321-2), rests on a mathematical synthesis and mathematical categories. In fact, Kant had made explicit that geometric judgments stand under the mathematical categories and principles earlier at *Prolegomena* §20 (4:301). But he is now calling attention to another important fact. The mathematical synthesis associated with the mathematical principles and mathematical categories and the dynamical synthesis associated with the dynamical principles and dynamical categories are really two aspects of a single procedure for determining space and time so as to make experience possible.[35] The concepts involved in this procedure, the categories, are "all directed" towards this end (4:321).

One thing this means is that the categories are limited in their application to space as well as time and have to be defined (for purposes of cognition) in terms of them.[36] This is absolutely crucial for answering the specific "how is complementarity possible?" question. If substance is to be defined not in purely intellectual terms as a simple item, but instead in sensible terms – for example, as permanently existing thing (as Kant defines it in the Transcendental Analytic), or as "the movable in space" (as Kant defines it in the *Metaphysical Foundations* [4:502-3]) – then it is possible to uphold both the geometrically demonstrable claim that physical space is infinitely divisible, as well as the metaphysical claim that there are substances in space.

Another thing it means is that, because the mathematical and dynamical syntheses underlying experience are aspects of an integrated cognitive process that makes experience possible, geometry will be generally consistent with and complementary of metaphysics and physics. The fact that the laws of geometry, transcendental laws, as well as the causal laws that

[35] My reading here is influenced by Friedman (1992, 134n, 201ff).
[36] In this regard, I take §38 to be correcting the Schematism Chapter's misleading impression that the categories can be given objective reality just through time (as does, too, the General Note on the System of Principles in the B-version of the *Critique*.) In this respect, my reading differs from that of Guyer (2012), which takes the absence of considerations connected to the doctrine of schematism to be essential to the *Prolegomena*'s method.

fall under them, have an ultimate common source in the understanding's determination of space and time – in the application of the understanding to sensibility – both explains how they are each possible and in principle explains their unity, harmonious interplay, and purposiveness.[37] Kant's story is the Critical successor to the divine origin story given in the *OPA.* Where the generally complementary character of geometry and metaphysics/physics was earlier explained in terms of a common origin in a unitary God, it is now explained in terms of a common origin in our seamlessly integrated cognitive faculties.

V. The Specific "Respective Roles" Question

Prolegomena §38 contains at least a partial account of the manner in which geometry and metaphysics function together in natural philosophy, in the grounding of our knowledge of specific causal laws (most obviously, the inverse-square law, but implicitly at least some of the laws of mechanics as well). That is, it bears on what I called above the specific "respective roles" question.

Let's consider one sophisticated interpretation of how it does so. According to Friedman, Kant is, initial appearances to the contrary, alluding in §38 to the details of Newton's own empirically based "deduction from the phenomena" of the inverse-square law. As described by Friedman, this procedure begins with empirical data, namely Kepler's laws. In subsequent steps, it uses geometric considerations involving conic sections as well as the definition of acceleration

[37] I will briefly register two worries about this account. First, Kant gives the impression here that the systematicity and purposiveness of laws is guaranteed by the understanding operating in tandem with sensibility, whereas elsewhere he regards these things as regulative principles dependent on reason and/or reflective judgment. Second, Kant leaves unexplained why there happens to be such a seamless fit between our sensibility (with its a priori forms)) and the understanding. Kant himself calls attention to this explanatory gap in the *Critique of Judgment* and speculates about a "supersensible ground" for the fitted-ness of the faculties (5:364).

to show that any body moving in conic sections and satisfying Kepler's law of areas with respect to a focus of the conic will have an acceleration directed at the focus that is inversely proportional to the square of the distance. In further steps, Newton uses his laws of motion to show that there is an inverse-square *force* directed towards the focus in these cases, that it is the same as terrestrial gravity, that it is mutual between all bodies, and that it is directly proportional to their masses.[38]

One possible objection to Friedman's claim – which he anticipates – is that Kant makes no mention of Kepler in §38 and the initial geometric examples are not initially marked as related to Newton's argument. But that is not a fatal problem, since as Friedman and others have noted, Newton relies in his deduction of the phenomena on some of those laws of conics that Kant mentions.[39]

Now, I think it is quite plausible that Kant knew that Newton used geometry in this way and approved of this sort of role of geometry in natural science – its application to empirical facts so as to discover specific laws, as it were a posteriori.[40] What I want to suggest, though, is that this a posteriori use of geometry doesn't exhaust the role that Kant thought geometry can and should play in natural science in accounting for specific laws. One place where we see Kant assigning an additional role to geometry is in his discussion of the "sources" of the inverse-square law in the properties of concentric spheres. This is an idea we encountered in the *Physical Monadology* and it also recurs again in the *Metaphysical Foundations*, where Kant offers it as a "perhaps possible construction" of the inverse-square law (4:518ff.). In contrast to Friedman,

[38] Friedman (1992, 170, 192) and Friedman (2012, 313)
[39] Friedman (1992, 192); Friedman (2012, 315-316); Laywine (2014, 746).
[40] Though I will note that I am not entirely convinced that Kant meant to call attention to this procedure in §38.

who denies that Kant accepts this as a geometric derivation of the inverse-square law[41] I and others have argued that Kant does accept it, and that it implies that the law is for him in a sense a priori, something strongly suggested by the language of §§36-38.[42]

Assuming the latter view is correct, it means that geometry is not just relevant in the process of *discovering* what the laws are but also that geometry also serves to at least partly *explain* (as it were a priori) why certain mathematical laws (having to do with force and motion) are as they are. This was the view in the *Physical Monadology* and the *OPA*, where Kant claimed that geometry and considerations about spatial relations more generally can at least partly explain some causal laws. In this context, Kant gave the example of the law of the equality of action and reaction (2:134). In fact, there is reason to think that Kant continues to believe in the Critical period that construction, and considerations about spatial relations more generally, can serve to at least partly explain some of the laws of mechanics. In the *Metaphysical Foundations*, Kant's proof of the law of the equality of action and reaction crucially involves what he refers to as a "construction" (4:546). And in a 1791 letter discussing the Mechanics Chapter of the book, he speaks, in language reminiscent of the *OPA* (2:134), of this law, as well as the law of inertia, as having their "general and sole sufficient ground in character of space, viz., that spatial relationships are reciprocal and equal" (11:247).[43] For this reason, I think that when Kant speaks in the *Prolegomena* §38 of "laws that the understanding cognizes a priori, and chiefly from universal principles of the determination of space," this includes not just laws of geometry, and transcendental laws of nature, but also the inverse-square law, and at least some laws of mechanics (4:321). As specific causal laws, the "latter stand" under the transcendental laws of

[41] Friedman (1992, 195ff, 204) and Friedman (2013, 222-224).
[42] Warren (2017); Messina (2018); Messina (manuscript). Plaass (1965, 122ff.) seems to regard the law as in important respects a priori as well.
[43] For discussion, see Messina (manuscript).

nature and to this extent they require general metaphysics. However, the specific mathematical content of these laws – why it is this way and no other – admits of at least partial explanation through spatial relations via a construction in pure intuition.[44]

VI. General "Respective Roles" Question

I will now briefly consider how the *Prolegomena* bears on the general "respective roles question," leaving it to other occasions to further develop the points in this section.

The account of the role of space in grounding the laws of motion and the inverse-square law that emerged in our consideration of §38 suggests a certain account of how construction is necessary for *special* metaphysics. Namely, it suggests that the forces posited in special metaphysics, though they cannot themselves be constructed, have to be such that they allow for laws that can be constructed – lest special metaphysics lose its scientific status. As we have seen, Kant thinks that such construction partly explains the content of the laws. There are indications that Kant also believes that one cannot have a proper science concerning motion and forces in the absence of mathematically constructible laws to go along with them.[45]

The *Prolegomena* also has implications for the way that *general* metaphysics serves geometry. One way that it serves geometry is by providing a philosophical account of geometry's applicability to nature. Another way that it does so is by dispelling certain metaphysical illusions of the sort that the traditional advocates of geometry (the Newtonians)

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[44] Admittedly, this leaves open the question of what exactly "standing under" amounts to.

[45] Kant denies that chemistry is as yet a science precisely because it has not yet managed to "construct" a "law of approach and withdrawal" and present it a priori in pure intuition (4:470-1). For further discussion see Messina (manuscript).

have fallen prey to, like absolute space. I have argued elsewhere that *Prolegomena* §38 contains a critique of such a metaphysics of space, while also acknowledging that geometry produces a natural temptation for it.[46]

In fact, the *Metaphysical Foundation* suggests that *special metaphysics* is intended to serve geometry (and mathematics more generally) in ways similar to those mentioned. One way is by explaining the applicability of mathematics, not to objects of nature in general, but specifically to motion and movable objects[47], as well as to other key concepts of Newtonian physics, like mass.[48] Another way it serves geometry is by dispelling further metaphysical illusions in matter theory – like the notions of absolute impenetrability and empty space – to which the "mathematical-mechanical" natural philosophers (a label that evidently encompasses the Newtonians/geometers) are peculiarly subject. Here, again, there are suggestions that geometry itself fuels these dubious metaphysical notions (4:524-5; 4:532ff.).

VII. Conclusion

The *Prolegomena* is an important chapter in Kant's lifelong struggle to achieve a marriage of metaphysics and geometry. For both the pre-Critical and Critical Kant, this involved grappling with (versions of) the "how is complementarity possible?" and "respective roles" questions. I have tried to show how the First and Second Parts of the *Prolegomena* contain at least partial answers to these questions. I have tried to show further that Kant's answers are continuous in some respects with his pre-Critical answers and discontinuous in others. One

[46] See Messina (2018).
[47] As Dahlstrom (1991, 278) suggests.
[48] Friedman (2013) emphasizes this point.

respect in which there is considerable continuity is in the idea that answering the complementarity questions requires a "common origin" story. Another respect is in the idea that geometric construction (and considerations about spatial relations more generally) provides a partial explanation of certain specific a priori laws concerning force and motion.[49]

[49] I am grateful to Peter Thielke and Eric Watkins for comments on an earlier draft.

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