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Counterpart Theory and the Actuality Operator

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Michael Fara and Timothy Williamson (Mind, 2005) argue that David Lewis's counterpart theory is unable to account for modal claims that use an actuality operator. This paper argues otherwise. Rather than provide a different counterpart translation of the actuality operator itself, the solution presented here starts out with a quantified modal logic in which the actuality operator is redundant, and then translates the sentences of this logic into claims of counterpart theory.

According to Michael Fara and Timothy Williamson 2005, counterpart theory must be rejected as a theory of modality because it cannot give an acceptable account of the two-dimensional actuality operator ACT. My aim in this paper is to offer a rebuttal on behalf of the counterpart theorist. To set up the discussion, I briefly review counterpart theory and the actuality operator in sections 1 and 2. Section 3 presents Fara and Williamson's argument against counterpart theory, plus a temporal variant of the argument that David McElhoes recently advanced against Theodore Sider's stage theory. My reply to both objections is developed in sections 4–8.

1. Counterpart theory

As formulated by David Lewis (1968, 1986), counterpart theory is an extensional theory of modality that employs unrestricted quantification over all possible worlds and their contents. It uses four primitive predicates:

Wx	x is a possible world
Ixy	x is in the possible world y
Cxy	x is a counterpart of y
Ax	x is actual

The theory then consists of eight postulates about worlds and counterparts:

(P1) $\forall x \forall y(Ixy \rightarrow Wy)$
(P2) $\forall x \forall y \forall z((Ixy \,\&\, Ixz) \rightarrow y = z)$
(P3) $\forall x \forall y(Cxy \rightarrow \exists z\, Ixz)$
(P4) $\forall x \forall y(Cxy \rightarrow \exists z\, Iyz)$
(P5) $\forall x \forall y \forall z((Ixy \,\&\, Izy \,\&\, Cxz) \rightarrow x = z)$
(P6) $\forall x \forall y(Ixy \rightarrow Cxx)$
(P7) $\exists x(Wx \,\&\, \forall y\, (Iyx \leftrightarrow Ay))$
(P8) $\exists x\, Ax$

The actual world @ is defined as the possible world that contains all and only actual objects. Its existence is guaranteed by (P7), and its uniqueness by (P2) and (P8).

In this theory, the counterpart relation C plays a similar role as trans-world identity does in quantified modal logic. The key difference is that the properties of C are not as restricted as those of identity. According to counterpart theory, to say that I might have been a poet is to say that there is a possible world in which there is a counterpart of me who is a poet. Since (P2) tells us that no object is in more than one possible world, my counterpart and I are different objects. And while (P5) ensures that, in the world it is in, no object has counterparts other than itself, it can have more than one counterpart in other worlds. Unlike identity, the counterpart relation can therefore be one-many. Since counterparts are not identical, counterpart theory can also admit more than one counterpart relation. The same object x can be a counterpart of y in one respect, but not in another. Due to this richer structure, there are modal claims that can be expressed in counterpart theory, but not in quantified modal logic (for examples, see Lewis 1968).

To show that the converse does not hold, and that counterpart theory can express every claim that quantified modal logic can express, Lewis 1968 offers a translation scheme that takes a claim φ of quantified modal logic and delivers a substitute $\varphi^{@}$ in counterpart theory. The quantified modal logic that Lewis employs for this purpose has two salient features. First, it contains no singular terms, which are supposed to be eliminated prior to translating. (I shall skip over the details of this term-elimination here; see Lewis 1968, sec. III, for details.) Second, the quantified modal logic uses world-relative quantifiers whose range in a possible world comprises all the objects that exist in that world. Given a formula φ of quantified modal logic and world w of counterpart theory, Lewis gives a recursive definition of φ^{w}, which is read as 'φ is true in possible world w':

φ^{w}	is	φ if φ is atomic
$(\neg\varphi)^{w}$	is	$\neg\varphi^{w}$
$(\varphi \rightarrow \psi)^{w}$	is	$\varphi^{w} \rightarrow \psi^{w}$
$(\forall x\varphi)^{w}$	is	$\forall x(Ixw \rightarrow \varphi^{w})$
$(\Box\varphi x_1 \ldots x_n)^{w}$	is	$\forall v\forall y_1 \ldots \forall y_n((Wv \mathbin{\&} Iy_1v \mathbin{\&} Cy_1x_1 \mathbin{\&} \ldots \mathbin{\&} Iy_nv \mathbin{\&} Cy_nx_n) \rightarrow (\varphi y_1 \ldots y_n)^{v})$

Here $\varphi x_1 \ldots x_n$ is the result of uniformly substituting the alphabetically ith free variable in φ with x_i for all $1 \leq i \leq n$. The proposed translation of a sentence φ of quantified modal logic is then $\varphi^{@}$, where @ is the actual world of counterpart theory. If this translation scheme works as advertised then it follows that counterpart theory has superior expressive capacity: it can express every modal claim that quantified modal logic can express, but not vice versa.

When Lewis says that the quantifiers of counterpart theory range over possible worlds and their contents, he means to exclude sets, numbers, and other mathematical objects. There is no reason to impose such a restriction on the domain of quantification in quantified modal logic. Indeed, we need the resources of such a logic to express the widely held view that mathematical objects exist necessarily. However, if we applied the above translation scheme to, say, the sentence $\Box\exists x$(x is an empty set) then we would end up with the claim that there is an empty set in each possible world of counterpart theory, $\forall v(Wv \rightarrow \exists x(Ixv \mathbin{\&} x$ is an empty set)). By (P2), all of these empty sets would be distinct from one another, and each possible world would end up containing its own copy of the hierarchy of pure sets. That is not Lewis's view. His position is that mathematical objects are not world-bound, and that they exist without being in any possible worlds (1983a, p. 40). Modal claims about mathematical objects therefore need to be given special treatment, and the account offered by Lewis is not going to be the translation of the corresponding regimentation in quantified modal logic. Whether this is a plausible view of mathematical object is a nice question, but not one that I want to discuss here. What matters for current purposes is that Lewis's translation scheme is only meant to cover modal claims about physical objects.

I also want to draw attention to the roundabout way in which Lewis's translation scheme provides a counterpart-theoretic analogue of the convention that sentences without any modal operators get evaluated in the actual world. Since atomic formulae serve as their own translation, they do not get tagged to the actual world @ by Lewis's scheme. But because the quantified modal logic assumed by Lewis does not contain any singular terms, every atomic formula contains free variables. This means that there are no *atomic* sentences, anyway. Every unmodalized sentence must be logically complex, and contain quantifiers that bind the variables. It is in translating these quantifiers, rather than in translating atomic formulae, that unmodalized sentences get restricted to objects in the actual world @ of counterpart theory.

2. The actuality operator

Ordinary modal logic uses unmodalized sentences for making claims about what is actually the case. This allows us to describe the actual world from the perspective of the actual world. For example, Fa is true in the actual world if and only if a is actually F. The purpose of the two-dimensional actuality operator ACT is to permit a description of the actual world from the perspective of *other* possible worlds. In any possible world w, ACTφ is true there if and only if the embedded sentence φ is true in the actual world. In the special case where w is itself the actual world, prefixing the actuality operator makes no difference and the biconditional $\varphi \leftrightarrow \mathrm{ACT}\varphi$ is true in all models. (Recall that to be true in a model of modal logic is to be true in its actual world.) This does not mean, however, that φ can be *substituted* for ACTφ in all contexts. Leading occurrences of ACT are always redundant, but the actuality operator can make a non-trivial contribution to the truth conditions of a sentence when it occurs within the scope of other modal operators. Consider the following example:

$$\Box(Fa \rightarrow \mathrm{ACT}Fa)$$

In evaluating this sentence in the actual world, the leading modal operator $\Box$ would direct us to look at whether the embedded conditional is true in all possible worlds. Let w be one of these worlds. In evaluating the sentence $Fa \rightarrow \mathrm{ACT}Fa$ in w, the truth of the antecedent depends on whether a is F in w, but the presence of the actuality operator ensures that the truth of the consequent depends on whether a is F in the actual world. Our sentence is therefore *false* in all models of modal logic in which Fa is contingently false. (That is, false in the actual world and true in some other possible world.) However, if we take our sentence and substitute ACTFa with Fa then we get $\Box(Fa \rightarrow Fa)$, which is a theorem of modal logic, and thus true in all models. This is a familiar feature of two-dimensional modal logic: substitution of logically equivalent subformulae is *not* a valid inference rule.[1]

Like many other logicians, Fara and Williamson believe that quantified modal logic can only express all relevant modal claims if it contains the actuality operator ACT in addition to the more familiar operators □ and ◊. For example, while the following two sentences are easily rendered in terms of the actuality operator, neither is said to possess an acceptable regimentation in terms of □ and ◊ alone (Hazen 1976, Humberstone 1977, Hodes 1984):

(A) There could be something that does not actually exist.
$\Diamond\exists x(\exists y\, x = y \ \&\ \neg\mathrm{ACT}\exists y\, x = y)$

(B) It might have been that everyone who is in fact rich was poor.
$\Diamond\forall x(\mathrm{ACT}Rx \rightarrow Px)$

If we grant this assumption then Lewis's proof of the expressive superiority of counterpart theory contains a significant lacuna. His translation scheme might provide a substitute for every modal claim that makes use of □ and ◊, but it remains to be shown that this result extends to claims that also contain ACT. Fara and Williamson argue that this challenge cannot be met.

3. The uniform substitution strategy

An obvious way in which a counterpart theorist might try to accommodate the actuality operator is by adding another clause to Lewis's translation scheme that would uniformly substitute all expressions of the form ACTφ with appropriate formulae of counterpart theory. But Allen Hazen 1979 notes that such a *uniform substitution strategy* quickly runs into difficulties. Suppose we say that $\mathrm{ACT}\varphi x_1 \ldots x_n$ is true in the possible world w of counterpart theory just in case φ is true of counterparts of $x_1, \ldots, x_n$ in the actual world @:

$$(\mathrm{ACT}\varphi x_1 \ldots x_n)^w \quad \text{is} \quad \exists y_1 \ldots \exists y_n\, (Iy_1@ \ \&\ Cy_1x_1 \ldots \&\ Iy_n@ \ \&\ Cy_nx_n \ \&\ \varphi y_1 \ldots y_n)$$

This translates $\mathrm{ACT}Fx$ as $\exists y(Iy@ \ \&\ Cyx \ \&\ Fy)$, and the *logical falsehood* $\Diamond\exists x(\mathrm{ACT}Fx \leftrightarrow \mathrm{ACT}\neg Fx)$ gets mapped to $\exists w\exists x(Ixw \ \&\ (\exists y(Iy@ \ \&\ Cyx \ \&\ Fy) \leftrightarrow \exists y(Iy@ \ \&\ Cyx \ \&\ \neg Fy)))$, which is *true* in any model of counterpart theory in which some object lacks counterparts in the actual world. Fara and Williamson review a number of other proposals for expressing ACTφ in counterpart theory and show that they all suffer from the same problem. For each proposal, they find a sentence that is the denial of a theorem of quantified modal logic with an actuality operator, but whose translation is true in some models of counterpart theory. They conclude that

[1] For details, see Segerberg 1973 and Humberstone 2004. Note that accepting ACT and the biconditional $\varphi \leftrightarrow \mathrm{ACT}\varphi$ does not commit us to *actualism*. The actuality operator permits us to refer rigidly to what is true in the actual world, but it does not presuppose that there is anything special about that world. One could easily introduce similar devices for other possible worlds; see Gabbay and Malod 2002 and the discussion of 'then' in Vlach 1973.

counterpart theory is unacceptable because it cannot account for modal claims that make use of the modal operator ACT.

David McElhoes 2010 presents a tense-variant of this argument against Theodore Sider's 2001 stage theory. Just as counterpart theory spells out claims about what might have been the case in terms of what is true of modal counterparts of actual objects, stage theory proposes an account of what was or will be the case in terms of what is true of temporal counterparts of present objects. McElhoes argues that stage theory is unable to account for claims of quantified tense logic that make use of the two-dimensional NOW operator. While the untensed sentences of tense logic make claims about the present moment from the perspective of the present, the NOW operator allows us to describe the present from the perspective of other times. At any time t, NOWφ is true then if and only if φ is true at the present time. Also in this case, the biconditional $\varphi \leftrightarrow$ NOWφ is true in all models of tense logic, but φ cannot always be substituted for NOWφ when it occurs within the scope of other tense operators. And just as ACT is said to expand the expressive capacity of quantified modal logic, NOW is said to be ineliminable in a quantified tense logic whose only other tense operators are **P** ('it was the case that') and **F** ('it will be the case that'). Hans Kamp 1971 presents the following example, which he claims to lack a NOW-free regimentation (see also Prior 1968, Vlach 1973, van Benthem 1977, and Meyer 2009):

(C) A child was born that will become ruler of the world
$\mathbf{P}\exists x(Bx \,\&\, \mathrm{NOW}\mathbf{F}Rx)$

Similar to Fara and Williamson's case against counterpart theory, McElhoes argues that stage theory cannot give an acceptable account of claims containing NOW. Any attempt at translating sentences containing NOW into stage theory is said to map some denial of a theorem of quantified tense logic with a NOW operator to a claim that is true in some models of stage theory.

4. The antecedent elimination strategy

A committed counterpart or stage theorist might want to continue the search for a better way of uniformly substituting subformulae of type ACTφ or NOWφ, but I am persuaded that there is little hope for success in this direction. Instead, I want to promote an approach that focuses on the alleged ineliminability of ACT and NOW. Since the tense and the modal case are exactly parallel in this respect, let me here restrict my attention to the actuality operator.

The actuality operator allows us to make claims about the actual world *inside* the scope of other modal operators. That is a nice feature, but describing the actual world is something we could do already, by using unmodalized sentences *outside* the scope of other operators. Offhand, one would therefore expect the actuality operator to make no difference to the expressive capacity of modal logic. Whenever a claim about the actual world is made within the scope of another modal operator, we should be able to move the claim outside that operator's scope, where we no longer need the actuality operator to talk about the actual world. However, this assumes that the rest of our logic is strong enough to permit this kind of transformation, and that is not always the case. The actuality operator does make a difference in expressive capacity when it is combined with a quantified modal logic that is too weak to permit its elimination. But there is no reason why counterpart theorists should measure the expressive capacity of their theory of modality against such a weak system, rather than against a stronger one from which ACT can be eliminated.

Instead of uniformly translating subformulae of type ACTφ into counterpart theory, I want to propose an *antecedent elimination strategy* that makes do without this operator altogether. We first eliminate all occurrences of ACT by translating the modal claims in question into a quantified modal logic in which this operator is redundant. After that, we apply a slightly modified version of Lewis's translation scheme to the resulting ACT-free sentences to generate substitutes in counterpart theory. This provides a systematic counterpart treatment of sentences involving ACT, but it does not proceed by uniformly substituting all subformulae containing actuality operators. A similar strategy works for NOW in tense logic. We first eliminate all occurrences of this operator by translating into a suitably chosen quantified tense logic, and then translate from there to stage theory.

5. ACT in propositional modal logic

To spell out the details of this proposal, I need to say a little bit more about the logic of ACT. This operator is clearly superfluous whenever it occurs in a subformula of the form ACTφ that is not within the scope of another modal operator. Prefixing a subformula φ with ACT forces φ to be evaluated in the actual world, but that is where the unadorned φ would get evaluated, anyway. All leading occurrences of ACT can therefore be erased without altering the truth conditions of the sentence in question. Similar remarks apply to subformulae that are outside the scope of other modal operators and that start with a string of $\Box$ or $\Diamond$ in any order, followed by an occurrence of ACT, followed by some formula φ. Here is an example: $\Box\Diamond\Diamond\Box\Diamond\Box$ACT$\varphi$. In such *special cases*, as I want to call them, the string of modal operators preceding φ is again redundant. No matter what world the leading boxes and diamonds take us to, ACT always sends us back to the actual world. Given such a special case, we can erase all modal operators preceding φ, including ACT.

What prevents the logic of ACT from being entirely trivial is that not all occurrences of the actuality operator are in special cases. There are instances where ACT occurs within the scope of a $\Box$ or $\Diamond$, but the two are separated by another logical constant. Here is a schematic example that uses disjunction: $\Diamond$(… $\vee$ … ACT …). Whether the actuality operator is eliminable in general therefore depends on whether the leading $\Box$ or $\Diamond$ and the nested ACT can always be moved towards each other, past the intervening logical constant, to generate a special case. In propositional modal logic, this is easily done. The actuality operator commutes with negation, $\neg$ACT$\varphi \leftrightarrow$ ACT$\neg\varphi$, and $\Diamond$ distributes over disjunctions, $\Diamond(\varphi_1 \vee \ldots \vee \varphi_n) \leftrightarrow (\Diamond\varphi_1 \vee \ldots \vee \Diamond\varphi_n)$. In general, $\Diamond$ does not distribute over conjunctions, but we do have the valid schema $\Diamond(\varphi$ & ACT$\psi) \leftrightarrow (\Diamond\varphi$ & ACT$\psi)$. Jointly, these relations between modal operators and truth functions suffice to prove that ACT is redundant in propositional modal logic. To construct an ACT-free paraphrase of a given sentence, we start by moving a leading $\Box$ or $\Diamond$ and the nearest nested occurrence of ACT towards one another until we obtain a special case. We then erase the entire string of operators and keep repeating the operation with the new sentence until we end up with a logically equivalent sentence without occurrences of ACT. (The proof can be found in Hazen 1978; Kamp 1971 employs a similar strategy to show that NOW is redundant in propositional tense logic. See also Thm. 2 in Meyer 2009.)

6. ACT in quantified modal logic

The situation is more complicated in quantified modal logic, where a quantifier can get stuck between a leading modal operator and a nested ACT. Given that $\Box$ can be taken as shorthand for $\neg\Diamond\neg$, and given that ACT commutes with negation, there are two ways in which this can happen:

$\Diamond\exists$-case: $\Diamond \ldots \exists x \ldots \text{ACT} \ldots$.
$\Diamond\forall$-case: $\Diamond \ldots \forall x \ldots \text{ACT} \ldots$

Whether our ACT-elimination strategy extends to these cases depends on what we say about the interaction between quantifiers and operators, and at this juncture the precise details of our quantified modal logic begin to matter. Let me first concentrate on $\Diamond\exists$-cases and postpone the discussion of the slightly more complicated $\Diamond\forall$-cases until section 8.

The standard arguments that purport to show that ACT is not redundant in quantified modal logic all adopt a *world-relative* account of quantification. On such a view, the range of $\exists$ and $\forall$ at a world is restricted to the objects that exist in that world. If that is how we read the quantifiers then ACT is indeed ineliminable from many sentences of quantified modal logic, including the examples (A) and (B) mentioned earlier. Similarly, NOW is ineliminable in a quantified tense logic in which the quantifiers have time-relative domains, as assumed by Kamp's treatment of (C). These are noteworthy results, but we can also do things differently. Suppose we adopt the *simplest quantified modal logic* (SQML) of Bernard Linsky and Edward Zalta 1994. In this system, quantifiers are not relativized to worlds, but always range over all possible objects. To render world-relative existence claims, SQML needs an independent, primitive *existence predicate* E! that has different extensions in different possible worlds. There is no need for such a predicate if we use world-relative quantifiers, which allow us to define an existence predicate in terms of the existential quantifier and identity:[2]

(DF) $\quad E!x \leftrightarrow \exists y\, x = y$

We cannot use this definition of E! in SQML, where quantification and existence come apart. Every object in the unrestricted domain of quantification of SQML satisfies $\exists y\, x = y$, but only those objects that exist in the world under consideration satisfy $E!x$. So the biconditional (DF) is not true in all models and cannot be used to define the existence predicate.[3] But if we have unrestricted quantifiers and an independent existence predicate at our disposal then the previously problematic (A) can easily be given an ACT-free regimentation as $\exists x(\Diamond E!x \;\&\; \neg E!x)$. Here the leading existential quantifier ranges over all possible objects, but the predicate E! gets evaluated

[2] We tacitly assumed this account when we earlier formalized (A) as $\Diamond\exists x(\exists y\, x = y \;\&\; \neg\text{ACT}\exists y\, x = y)$.
[3] Williamson (1998; 2002) views the matter differently. He accepts (DF) even for SQML and concludes that every object exists in all possible worlds, but may lack spatial and temporal location in some possible worlds. As Fara and Williamson note in 2005, p. 25, though, this is not a quantified modal logic that a counterpart theorist is likely to endorse because not every object has counterparts in every world.

in the possible world under consideration. Since the second conjunct $\neg E!x$ occurs outside the scope of the possibility operator, it gets evaluated in the actual world.

This works more generally. Since the range of quantification does not depend on what world we are in, $\exists$ and $\lozenge$ commute in SQML and the Barcan Formula $\exists x \lozenge \varphi \leftrightarrow \lozenge \exists x \varphi$ is true in all models. This allows us to push the modal operator $\lozenge$ past the quantifier $\exists$ and eliminate ACT in all $\lozenge\exists$-cases. Except for $\lozenge\forall$-cases, which we still need to deal with, the operator ACT is therefore redundant in SQML. For each sentence that contains this operator, we can find an ACT-free sentence that is true in exactly the same models. By way of illustration, here are some of the problem cases (with their original equation numbers) discussed in Fara and Williamson 2005:

	Fara and Williamson example	SQML regimentation
(10)	$Fa \mathbin{\&} \neg \mathrm{ACT}Fa$	$Fa \mathbin{\&} \neg Fa$
(12)	$\lozenge\exists x(\mathrm{ACT}Fx \leftrightarrow \mathrm{ACT}\neg Fx)$	$\exists x(Fx \leftrightarrow \neg Fx)$
(18)	$Fa \mathbin{\&} (\mathrm{ACT}\neg Fa \lor \neg\mathrm{ACT}Fa)$	$Fa \mathbin{\&} (\neg Fa \lor \neg Fa)$
(24)	$\lozenge\exists x(\mathrm{ACT}Fx \mathbin{\&} \mathrm{ACT}\neg Fx)$	$\exists x(Fx \mathbin{\&} \neg Fx)$

Note that all four sentences get converted into logical falsehoods of SQML. Similar remarks apply to a tense version of SQML in which quantifiers range over all objects that exist at some time or other. In such a quantified tense logic with untensed quantifiers, Kamp's problem case (C) can be given the NOW-free formalization $\exists x(\mathbf{P}Bx \mathbin{\&} \mathbf{F}Rx)$. (For further details and proofs, see Meyer 2009, Sec. 5.)

7. A revised translation scheme

To translate the resulting SQML sentences into counterpart theory, we need to make a few adjustments to Lewis's translation scheme. Counterpart theory employs unrestricted quantification over all possible worlds and their contents, but Lewis assumes that the quantifiers of quantified modal logic are world-relative. We can easily accommodate the unrestricted quantifiers of SQML by removing Lewis's restriction of counterpart-quantifiers to objects in w, but then we would also need to modify the way we translate atomic formulae. As noted at the end of section 1, Lewis's translation scheme ensures that unmodalized sentences get evaluated in the actual world @ of counterpart theory by the way it translates the quantifiers that bind the variables in atomic formulae. If we abolish the restriction on the quantifiers then we need to impose it on the atomic formulae to ensure the same net result. Moreover, since our existence predicate E! is primitive, and not defined in terms of quantification and identity, we also need to make allowance for atomic formulae of the form $E!x$. We do this by saying that $E!x$ is true in a possible world w just in case x is in that world:

$(E!x)^w$	is	Ixw
$(\varphi x_1 \ldots x_n)^w$	is	$Ix_1w \mathbin{\&} \ldots \mathbin{\&} Ix_nw \mathbin{\&} \varphi x_1 \ldots x_n$ if φ is an atomic formula not containing E!
$(\neg\varphi)^w$	is	$\neg\varphi^w$
$(\varphi \to \psi)^w$	is	$\varphi^w \to \psi^w$
$(\forall x\varphi)^w$	is	$\forall x\varphi^w$
$(\Box\varphi x_1 \ldots x_n)^w$	is	$\forall v \forall y_1 \ldots \forall y_n((Wv \mathbin{\&} Iy_1v \mathbin{\&} Cy_1x_1 \mathbin{\&} \ldots \mathbin{\&} Iy_nv \mathbin{\&} Cy_nx_n) \to (\varphi y_1 \ldots y_n)^v)$

In SQML, we express quantification over all actual objects as $\forall x(E!x \rightarrow \varphi)$. Our new scheme translates this as $\forall x(Ix@ \rightarrow \varphi^{@})$, which is exactly how Lewis's original translation scheme deals with quantification over actual objects. Note also that the Barcan Formula $\exists x\Diamond Fx \leftrightarrow \Diamond\exists xFx$ get translated as $\exists x\exists v\exists y(Wv \;\&\; Iyv \;\&\; Cyx \;\&\; Fy) \leftrightarrow \exists v(Wv \;\&\; \exists x(Ixv \;\&\; Fx))$, which is a theorem of counterpart theory. (To derive the right side of the biconditional from the left, we drop the x-quantifier and the conjunct Cyx; to derive the left side from the right, we use the principle (P6).)

8. Quantification over sets

Even in SQML, the modal operator $\Diamond$ does not commute with the universal quantifier $\forall$, which means that we need a different elimination strategy for $\Diamond\forall$-cases. The solution I propose supplements our quantified modal logic with the ability to quantify over *subsets* of the domain of quantification. Suppose we add the set membership relation $\in$ and set variables X, Y, Z, … to SQML. Models are the same as before, but we now also form a set hierarchy with the objects in the regular domain as *Urelemente*. Set variables range over the sets in this hierarchy. Given these stipulations, set membership is 'rigid' and both $\forall x\forall X(x\in X \rightarrow \Box x\in X)$ and $\forall x\forall X(x\notin X \rightarrow \Box x\notin X)$ are true in all models of our expanded quantified modal logic. (There are different ways in which one could extend the existence predicate to the impure sets we get in this way. One could either say that all sets exist necessarily, or that they only exist in worlds in which all of their elements exist. Since there is no need to settle this question here, let me pass over this issue.)

Theorem 4 in Meyer 2009 shows that NOW is redundant in a tense-variant of SQML with quantification over sets. For each sentence that contains NOW, the proof provides a NOW-free substitute that is true in exactly the same models. The same strategy can be used to prove that ACT is redundant in SQML with quantification over sets. Instead of repeating all the technical minutiae, let me just sketch the main idea of the elimination procedure. Consider a simple $\Diamond\forall$-case of the form $\Diamond(\ \ldots\ \forall x\ \ldots\ \mathrm{ACT}\varphi\ \ldots\ )$ that contains no quantifiers or modal operators other than the ones depicted. To eliminate this one occurrence of ACT, we first define, outside the scope of $\Diamond$, a set X that contains all objects that actually satisfy φ. Within the sentence itself, we then substitute the occurrence of ACTφ with the formula $x\in X$:

Original sentence: $\Diamond(\ \ldots\ \forall x\ \ldots\ \mathrm{ACT}\varphi\ \ldots\ )$
ACT-free paraphrase: $\exists X(\forall x(x\in X \leftrightarrow \varphi) \;\&\; \Diamond(\ \ldots\ \forall x\ \ldots\ x\in X\ \ldots\ ))$

Applied to sentence (B), which we earlier regimented as $\Diamond\forall x(\mathrm{ACT}Rx \rightarrow Px)$, our elimination method yields $\exists X(\forall x(x\in X \leftrightarrow Rx) \;\&\; \Diamond\forall x(x\in X \rightarrow Px))$ as the ACT-free regimentation of the claim that it might have been that everyone who is in fact rich was poor. The x-quantifiers range over all possible objects, but the set X gets defined outside the scope of modal operators and only contains people that are actually rich, rather than all possibly rich ones.

(At this point, it is important to remember that we restricted the ordinary quantifiers of quantified modal logic to domains without mathematical objects. Otherwise, we would now run into trouble with claims like 'Everything that is in fact an ordinal number might have been a chicken', which cannot be dealt with in the same way as (B) because there is no *set* that contains all ordinals. This problem could be avoided by using plural quantification instead of quantification over sets, as proposed in Bricker 1989. But since mathematical claims need to be treated separately, anyway, this is not a complication we need to consider here.)

Many $\Diamond\forall$-cases are of course a little bit more complicated than the example we just discussed. For example, the formula φ could contain a number of variables $x_1, \ldots, x_n$ that are bound by a mix of universal and existential quantifiers that occur between the leading $\Diamond$ and the nested ACT. In such a case, we would use φ to define a set X of n-tuples of elements of the domain outside the scope of $\Diamond$, and then substitute ACTφ with $\langle x_1, \ldots, x_n\rangle \in X$:

Original sentence: $\Diamond(\ \ldots\ \mathrm{ACT}\varphi\ \ldots\ )$
ACT-free paraphrase: $\exists X(\forall x_1 \ldots \forall x_n\ (\langle x_1, \ldots, x_n\rangle \in X \leftrightarrow \varphi)\ \&\ \Diamond(\ \ldots\ \langle x_1, \ldots, x_n\rangle \in X\ \ldots\ ))$

If the embedded formula φ itself contains further occurrences of ACT then we start by eliminating the *innermost* occurrence of ACT, and then re-apply our method until all actuality operators have been eliminated. The resulting ACT-free paraphrase would then contain more than one set definition, but because we worked from the inside out, none of these sets is defined in terms of any of the other sets. Together with the elimination strategy that we developed for $\Diamond\exists$-cases, this allows us to eliminate ACT from all sentences of SQML with quantification over sets.

There is one last problem that needs to be taken care of. After we have extended SQML with set quantifiers, it contains the resources to make claims about the hierarchy of pure sets. The question is what we should do with such claims when we translate into counterpart theory. My proposal is that we simply ignore them. How Lewis might account for modal claims about pure sets and other mathematical objects is not an issue we are concerned with here. To give ACT-free regimentations of modal claims about physical objects, we only need quantification over n-tuples of elements of the domain of ordinary quantification. This means that it suffices to provide two more clauses for our translation scheme that deal with these *specific* cases. Set quantifiers translate like ordinary quantifiers, and attributions of set membership translate like other atomic sentences:

$(\forall X\varphi)^w$ is $\forall X\, \varphi^w$
$(\langle x_1, \ldots, x_n\rangle \in X)^w$ is $Ix_1w\ \&\ \ldots\ \&\ Ix_nw\ \&\ \langle x_1, \ldots, x_n\rangle \in X$

With these final additions in place, we can now sum up the antecedent elimination strategy. We begin by regimenting all modal claims about physical objects in SQML with an existence predicate and quantification over sets. By doing so, we eliminate all occurrences of the actuality operator ACT. After that, we apply our revised translation scheme to these ACT-free sentences to produce substitutes in counterpart theory. The parallel strategy allows the stage theorist to account for claims of quantified tense logic that make use of the operator NOW. We first translate such claims into a tense-analogue of SQML with quantification over sets, and then translate these NOW-free sentences into stage theory.

These solutions are of course based on the acceptance of SQML and its tense counterpart. Actualists and presentists might well object to the 'possibilist' and 'eternalist' quantifiers that these logical theories employ, but their concerns can be ignored here. It was clear from the outset that counterpart theory is incompatible with actualism, and that stage theory is incompatible with presentism. What Fara and Williamson try to show is that counterpart theory fails on its own terms, not that it runs counter to actualism. To rebut their objection, it suffices to provide a counterpart translation of every claim in a possibilist quantified modal logic with appropriate expressive resources, and SQML fits that description.

9. Conclusion

Fara and Williamson take it for granted that any plausible response to their objection would have to provide a uniform way of substituting every subformula of type ACTφ with a claim of counterpart theory. The only other option, they suggest, is to deal with the issue case-by-case, which 'requires accepting that the semantics for natural language is radically non-compositional' (2005, p. 27). I do not think this is right. For one, it is surely not the case that the meaning of every word of English can be specified independently of the context of the sentences in which it occurs. A case-by-case treatment of ACT might be unacceptable if it was *ad hoc* or unsystematic, but that is not true of the solution proposed here. Since the elimination of ACT from SQML does not proceed by uniformly substituting every subformula of type ACTφ, the subsequent application of the translation scheme does not provide a uniform counterpart translation of the actuality operator, either. But the treatment *is* systematic, and it gives the correct account of the contribution that ACT makes to the truth-conditions of sentences of quantified modal logic. The semantic role of ACT is to make claims about what is actually the case, but that role can also be played by appropriately chosen quantificational resources.[4]

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