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# Worlds and Times

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**Abstract** There are many parallels between the role of *possible worlds* in modal logic and that of *times* in tense logic. But the similarities only go so far, and it is important to note where the two come apart. This paper argues that even though worlds and times play similar roles in the *model theories* of modal and tense logic, there is no tense analogue of the possible-worlds *analysis* of modal operators. An important corollary of this result is that *presentism* cannot be the tense analogue of *actualism*.

## 1 Introduction

In English, tense modifiers like "always" or "sometimes" function in a similarly adverbial manner as "necessarily" and "possibly," and symbolic logic naturally represents them in terms of sentential operators.[1] But the similarities go beyond these syntactical features. The operators "always" and "sometimes" also obey the same axioms as "necessarily" and "possibly" do in the modal system S5, and many of the formal results of modal logic carry over directly to the case of a simple tense logic.[2]

A prominent example of these similarities are model theories for tense and modal logic. Suppose we are given a language of propositional modal logic with logical constants '$\Box$', '$\vee$', and '$\neg$', and the

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usual supply of atomic sentence symbols. For sake of brevity, I will not use the second modal operator '$\Diamond$', whose work is already done by '$\neg\Box\neg$'. A model for this language is a triple $\mathcal{M} = \langle \Omega, \alpha, @ \rangle$, consisting of a set of "possible worlds" $\Omega$, an element $\alpha$ of $\Omega$ chosen as "actual world," and a "true at"-relation @ between sentences of the language and elements of $\Omega$. With '$\varphi$' and '$\psi$' ranging over sentences of the language and '$v$' and '$w$' ranging over elements of $\Omega$, the relation @ is required to satisfy:

$$\begin{aligned} &@(\ulcorner \varphi \vee \psi \urcorner, w) \text{ iff either } @(\varphi, w) \text{ or } @(\psi, w). \\ &@(\ulcorner \neg\varphi \urcorner, w) \text{ iff it is not the case that } @(\varphi, w). \\ &@(\ulcorner \Box\varphi \urcorner, w) \text{ iff } @(\varphi, v) \text{ for all worlds } v \in \Omega. \end{aligned} \tag{1}$$

A sentence of our language is said to be true in a model $\mathcal{M}$ just in case it is true at its actual world:

$$\varphi \text{ is true in } \langle \Omega, \alpha, @ \rangle \text{ iff } @(\varphi, \alpha). \tag{2}$$

This definition then permits us to prove a soundness and completeness theorem for S5:[3]

$$\varphi \text{ is a theorem iff } \varphi \text{ is true in all models.} \tag{3}$$

In particular, a sentence of form $\ulcorner \Box\varphi \urcorner$ is a theorem if and only if $\varphi$ is true in all worlds in all models.

Both the language and the model theory can be adopted in their entirety for the purpose of a simple tense logic. In principle, we could just call the various elements by different names, but such notational frugality would make it hard to keep the two cases separate. Let me therefore introduce different symbols for the tense case. Instead of '$\Box$', I will use '$\boxdot$' to symbolize "always," and I will use '$\neg \boxdot \neg$' to express "sometimes." Models for tense logic will be triples $\mathcal{M} = \langle \Phi, \pi, @ \rangle$ where $\Phi$ is a set of "times," $\pi$ is an element of $\Phi$ chosen as the "present," and @ a "true at"-relation between sentences and elements of $\Phi$. This gives us the following translation manual:

| Modal Logic S5 | Formalism | Formalism | Simple Tense Logic |
|---|---|---|---|
| necessarily | $\Box$ | $\boxdot$ | always |
| possibly | $\neg\Box\neg$ | $\neg \boxdot \neg$ | sometimes |
| possible worlds | $\Omega$ | $\Theta$ | times |
| actual world | $\alpha$ | $\pi$ | the present |
| true at a world | @ | @ | true at a time |

Using this manual to rewrite the proof of (3) then yields a tense version of the soundness and completeness theorem.

These parallels have led many to believe that there are *no* significant differences between tense and modal logic, and that everything we can say about one equally applies to the other. In fact, most textbooks treat tense logic as a mere afterthought of modal logic. To the extent to which they discuss tense logic at all, it is usually to investigate the logic of additional tense operators like ‘P’ (“in the past”) or ‘F’ (“in the future”), which have no modal analogue.

However, even for ‘$\Box$’ and ‘$\boxdot$’, tense and modal logic are not as similar as they are made out to be. The two cases are only parallel as long as we use the model theory for the purpose it was originally designed for: to characterize theorems. But we often have occasion to talk about the *intended* models of tense and modal logic, and they raises quite different issues. An intended model would characterize all truths, not only the theorems.

The axiomatic systems and model theories for ‘$\Box$’ and ‘$\boxdot$’ might be the same, but they behave quite differently with regard to their intended models. In the modal case, we can give a reductive “possible worlds” analysis of ‘$\Box$’ in terms of the intended model of modal logic. My aim here is to show that this eliminability is a special feature of this particular operator that does not extend to other cases:

1. The tense operator ‘$\boxdot$’ cannot be eliminated in favor of the intended model of tense logic. That is, there is no “times” analysis that mirrors the possible-worlds analysis of ‘$\Box$’.
2. On an “actualist” construal of the intended model, the actuality operator ‘$\bigcirc$’ does not admit a possible-worlds analysis, either. (The modal realist, it turns out, is slightly better off.)

In relation to its intended model, ‘$\boxdot$’ is much more similar to ‘$\bigcirc$’ than it is to ‘$\Box$’. We shall see that it is a mistake to think of other times as analogous to the possible worlds in the intended model of modal logic. If anything, other times are more similar to the actual world.

## 2 The Possible-Worlds Analysis

The primary reason for choosing the above notion of a model is that it yields the soundness and completeness theorem (3). We *called* the elements of $\Omega$ “possible worlds,” and the elements of $\Theta$ “times,” but that was just for cuteness’ sake. As far as the logician is concerned, the worlds and times in his models could be cans of chicken soup, prime numbers, or whatever. For a satisfactory model theory, there only need to be *enough* such “worlds” around to yield (3); what *kinds* of objects they are is irrelevant since all the logical work is being done by the constraints (1) places on the relation @.

Not everybody agrees with such a modest understanding of our model theory. Many authors are impressed by the intuitive appeal of the possible-worlds picture, and argue that it provides a reductive *analysis* of the modal operators. Amongst all the models, they claim, there is one that is special: the *intended* model $\mathcal{M}^* = \langle \Omega^*, \alpha^*, @^* \rangle$ of modal logic. So far, we have only talked about what is true in a model, but this qualification can be dropped for $\mathcal{M}^*$. A statement is true simpliciter just in case it is true in the intended model:

$$\varphi \text{ is true iff } \varphi \text{ is true in } \mathcal{M}^* \tag{4}$$

With (1) and (2), this entails:

$$\ulcorner \Box\varphi \urcorner \text{ is true iff } @^*(\varphi, w) \text{ for all } w \in \Omega^* \tag{5}$$

The contention is that this biconditional constitutes a reductive analysis of the modal operator '□'. To say that a claim is necessarily true would thus mean that it is true in all possible worlds, and to say that it is possibly true would mean that it is true in some such world.

The proposal (5) would eliminate modal operators in favor of the possible worlds in the intended model, and one might worry that this makes the account hostage to ontological fortune. The truth of (5) requires that there are enough possible worlds in the intended model. If there were too few of them then we would count some claims as necessary that are merely contingent. Moreover, for it to count as an analysis, (5) need not only be true, but necessary. So we also require assurance that there would still be enough possible worlds in the intended model even if things were radically different from what they actually are.

Advocates of the possible-worlds analysis usually address these concerns in either of two ways:

1. *Actualists* [17] claim that the worlds in the intended model are abstract "ways the world might have been." Actualists disagree amongst themselves about the precise nature of these possible worlds, but the underlying strategy is the same. There are said to be enough such abstract "ways" around to ensure the material adequacy of the proposal, and since abstract objects exist necessarily, this also guarantees the necessity of (5).
2. *Modal Realists* [10] believe that ours is just one of a vast number of equally concrete worlds that are causally separated from one another. The actual world is "us and our surroundings," and to make claims about what is possible is to report what is going on in other possible worlds. The modal realist's response to the

> ontological fortune worry is to insist that he is *lucky*. He claims that there are enough of his concrete possible worlds to ensure the truth of (5). Since the number of worlds does not depend on which of the worlds is ours, and since (on the realist's view) being true according to all these concrete worlds is what makes a claim necessary, this then also guarantees the necessity of (5).

I am more sympathetic to actualism than to modal realism, but that is not a view I want to argue for here. In what follows, I will give two parallel arguments: one for the actualist, and one for the realist.

If we adopt either the actualist or the realist proposal, then the intended model of modal logic is a witness to the truth of:

$W_\Box$: There is a model $\langle \Omega, \alpha, @ \rangle$ such that, necessarily, $\Box\varphi$ iff $@(\varphi, w)$ for all $w \in \Omega$.

This is a remarkable result, but it was never the purpose of our model theory to provide such a "special" model. Model theory aims to characterize the *theorems* of a system of modal logic, and its success at doing so is the result of a joint effort of all the models. By contrast, (5) and $W_\Box$ are only concerned with what is happening in one particular model. Their aim is to characterize all *truths* of form $\ulcorner\Box\varphi\urcorner$, not only the theorems. There are many necessary truths that are not theorems of any system of logic, such as the truths of mathematics, or metaphysically necessary claims. Such claims fall outside the purview of our model theory, but they do form part of what the possible-worlds analysis of '$\Box$' is concerned with.

In the claim $W_\Box$, the existential quantifier has wide scope, and it is important to distinguish it from its narrow-scope variant:

$N_\Box$: Necessarily, there is a model $\langle \Omega, \alpha, @ \rangle$ such that $\Box\varphi$ iff $@(\varphi, w)$ for all $w \in \Omega$.

While $W_\Box$ entails $N_\Box$, the converse is not true. The latter merely says that we can always find a model such that every true sentence of form $\ulcorner\Box\varphi\urcorner$ is true in that model. But that can be achieved by choosing radically different models in different circumstances. For $W_\Box$ to be true, there needs to be one model that works under all possible circumstances—independently of what the facts are.

The reason we can move so easily from a characterization of the theorems of modal logic to a characterization of all modal truths is that—at least according to S5—claims of type $\ulcorner\Box\varphi\urcorner$ are only true if they are themselves necessary. Theorems of S5 and truths of form

$\ulcorner\Box\varphi\urcorner$ thus have the same modal status. That is not true if we use different sentential operators instead:

$$\text{Actually, } \varphi \qquad \text{Always, } \varphi \qquad \text{Sometimes, } \varphi \tag{6}$$

A claim can be actually true, or always true, or sometimes true, without being necessarily so. This makes for a significant difference. Even though we can provide a model theory for the operators in (6), it turns out that they permit no analogue of the possible-worlds analysis of the modal operator '$\Box$'.

### 3 The Actuality Operator

Let me add another sentential operator to our language: an actuality operator '$\bigcirc$' that is used to make claims not about ways the world might have been, but about the way it actually is. Such an operator is always eliminable in propositional modal logic, where the work of '$\bigcirc$' is already done by unmodalized sentences.[4] But the use of unmodalized sentences has the drawback of hiding the actuality operator in the formalism so that it is easily overlooked. The purpose of introducing '$\bigcirc$' is to make this operator more conspicuous.

Given that $\ulcorner\bigcirc\varphi\urcorner$ would just be a glorified way of saying $\varphi$, the definition (2) and the claim (4) yield:

$$\ulcorner\bigcirc\varphi\urcorner \text{ is true iff } @^*(\varphi, \alpha^*) \tag{7}$$

This is superficially similar to the possible-worlds analysis (5), but the difference is that (7) is not necessary and cannot be used as an analysis.

To prove this, we need to give two arguments: one for the actualist, and one for the realist. According to the actualist, the actual world $\alpha^*$ in the intended model is an abstract object. It must be distinguished from our world $\mathfrak{W}$, which is the mereological sum-total of all objects that actually exist. Our world $\mathfrak{W}$ *contains* the actual world along with all the other worlds in the intended model, but it also contains many concrete objects besides: sticks, stones, donkeys, etc. It is these contingent parts of $\mathfrak{W}$ that determine which of the elements of $\Omega^*$ correctly describes the way things actually are. By assumption, $\alpha^*$ is this world, but a different element of $\Omega^*$ would play this role if things in $\mathfrak{W}$ were different from what they actually are. So even though (7) might be true, it is not necessary.

According to the modal realist, the actual world $\alpha^*$ in the intended model is a concrete object that *coincides* with $\mathfrak{W}$. Unlike the actualist, the realist also believes that there are things—namely other possible worlds—that are not contained in $\mathfrak{W}$. But these differences have no impact on the status of (7). If we did not live in $\alpha^*$, but in some other possible world, then it would no longer be true that a claim is actually true if and only if it is true in $\alpha^*$. Instead, a claim would be actually true just in case it is true in the other world. Hence (7) is not necessary.

A different world would have to be chosen as actual world depending on how things are in $\mathfrak{W}$ (actualist), or where in logical space we live (realist). Hence there is no model—whether intended or not—such that, necessarily, a claim is true if and only if it is true in the actual world of that model. The following claim is *false*:

$W_\bigcirc$: There is a model $\langle \Omega, \alpha, @ \rangle$ such that, necessarily, $\bigcirc\varphi$ iff $@(\varphi, \alpha)$.

We could of course make our choice of actual world depend on what the actual facts are, but that would mean choosing a different model in different circumstances, and would only yield the narrow-scope claim:

$N_\bigcirc$: Necessarily, there is a model $\langle \Omega, \alpha, @ \rangle$ such that $\bigcirc\varphi$ iff $@(\varphi, \alpha)$.

This is an important difference between the actuality operator and the necessity operator, for it was only the wide-scope claim $W_\Box$ that opened the door to a possible-worlds analysis of '$\Box$'.

The apparent source of our problem is that we are using a proper name '$\alpha^*$' that rigidly picks out the actual world of the intended model. By doing so, we fail to appreciate that our choice of actual world should vary according to what the actual facts are. To fix this problem, we could introduce an actuality *predicate* '$A$' that applies to a possible world in $\Omega^*$ if and only if that world correctly describes $\mathfrak{W}$ (actualist), or coincides with us and our surroundings (realist). Instead of (7), we could then propose:

$$\ulcorner \bigcirc\varphi \urcorner \text{ is true iff } @^*(\varphi, w) \text{ for the } w \in \Omega^* \text{ such that } A(w). \tag{8}$$

If the intended model $\mathcal{M}^*$ is well-chosen then $\alpha^*$ is the element of $\Omega^*$ to which the actuality predicate in fact applies. But if things in $\mathfrak{W}$ were different (actualist), or if we lived in some other world (realist), then a different world would be the $A$-world. Unlike our earlier attempt (7), the revised proposal (8) is necessary.

With the aid of the actuality predicate, we thus get something quite similar to the (false) wide-scope claim $W_\bigcirc$:

$W^A_\bigcirc$: There is a model $\langle \Omega, \alpha, @ \rangle$ such that, necessarily, $\bigcirc\varphi$ iff $@(\varphi, w)$ for the $w \in \Omega$ such that $A(w)$.

But this does not bring us any closer to a possible-worlds analysis of the actuality operator. To say that a world is the $A$-world is to say that it either correctly describes $\mathfrak{W}$ (actualist), or coincides with us and our surroundings (realist). That is, according to both the actualist and the modal realist:[5]

$$A(w) \text{ if and only if } \forall\varphi(@^*(\varphi, w) \supset \bigcirc\varphi). \tag{9}$$

The trouble is that this reintroduces the very operator that we are trying to get rid of in (8). On pain of circularity, we cannot analyze claims about what is actually the case in favor of what is true in the *A*-world because we need to appeal to such claims in order to specify which possible world *is* the world to which '*A*' applies.

One might hope to do better by adopting David Lewis's "indexical" analysis of actuality,[6] but this expectation is only partially borne out. One constraint on the actuality operator is what Peter van Inwagen [18] calls the "weak theory" of indexicality. This is the condition that, according to any possible world $w$, a claim $\varphi$ is actually true if and only if it is true in $w$:

$$\text{For all } w \in \Omega^*, @^*(\bigcirc\varphi, w) \text{ iff } @^*(\varphi, w) \tag{10}$$

But even though every possible world *represents* itself as being the actual world, the vast majority of them are lying. Only one of them correctly describes $\mathfrak{W}$ (actualist) or coincides with us and our surroundings (realist), and for an analysis we need to be able to tell which world this is. What we are trying to analyze is not the notion of being actually true *according to some world*, but the notion of being actually true, and (10) tells us nothing about that.

The modal realist can do better than (10). He can claim that an utterance of $\ulcorner\bigcirc\varphi\urcorner$ expresses different propositions when uttered in different possible worlds. If uttered in world $w$, the proposition expressed by $\ulcorner\bigcirc\varphi\urcorner$ is true if and only if $@(w, \varphi)$, but it would express a different proposition if uttered in a different world.[7] This proposal yields an analysis of the actuality operator, albeit one of a rather different flavor than the possible-worlds analysis of the necessity operator.

To pick out the actual world $\alpha^*$ in the intended model, the modal realist just needs to *point* at it, because, on his view, the actual world is the concrete object made up of him and his surroundings. But if the actualist does this, he is pointing at $\mathfrak{W}$, and his view is that the actual world $\alpha^*$ is distinct from $\mathfrak{W}$. The token-reflexivity of '$\bigcirc$' is therefore of no use to the actualist. For him, the problem of determining the actual extension of '*A*' is not to identify $\mathfrak{W}$, but to specify which abstract way the world might have been correctly describes $\mathfrak{W}$, and there seems to be no way of doing that without appealing to the notion of what is actually true. Hence the actualist cannot give an indexical analysis of the actuality operator.[8]

## 4 Metaphysical Priority

At this point, the actualist might want to go back to (8) and simply propose to take '*A*' as conceptually primitive. Since he would already be assuming the notion of a way the world might have been, adding the actuality predicate seems to make little difference to the overall ideological cost of the proposal. But the actualist cannot do this. As I want to argue now, '*A*' is inadmissible as a primitive in an analysis of the actuality operator.

To make this point, I need to bring up an issue that I have so far ignored. Up to now, I have tacitly assumed that all we need for a reductive analysis is some necessary biconditional. That is not quite right. For an analysis, the proposed analysans also needs to be *metaphysically prior* to, and be what *makes it the case that*, the analysandum obtains. Admittedly, these are dark notions, but the fact that it is difficult to make them precise should not detract from their importance.

For example, an appeal to metaphysical priority is needed to distinguish between the following two proposed analyses:

A1: Person $X$ is a bachelor if and only if $X$ is an unmarried man.
A2: Agent $X$'s $\Phi$-ing at $t$ maximizes the pleasure/pain balance if and only if $X$ is morally required to $\Phi$ at $t$.

Both aim to provide an analysis of the underlined expression, but only the first succeeds. Proposal A1 is a good analysis because, in addition to the biconditional's being necessary, being an unmarried man is also what *makes* one a bachelor. Not so for the second proposal. Opponents of utilitarianism will already deny the necessity of the biconditional, but even a committed utilitarian ought to reject A2 as an analysis because it gets things backwards. If anything, it is an action's effect on the pleasure/pain balance that *makes* it morally required, not the other way around. The utilitarian's view is the converse of A2, and there seems to be no way of explaining the difference between him and the "inverse utilitarian" A2 without appealing to something like the notion of metaphysical priority.

In this sense, the actualist version of the possible-worlds account (5) is indeed a strong candidate for an analysis of '$\Box$'. We usually motivate possibility claims by sketching an alternative way the world might have been, and a convincing case can be made that these "'ways" are metaphysically prior to '$\Box$'. Being true according to some such world is arguably what makes it the case that a claim is possible. Note also that it is this feature that would distinguish the intended model of modal logic from the many other models that bear witness to the truth of $W_\Box$. For example, there are models whose worlds are prime numbers that have the required features. Since prime numbers exist necessarily, such models would yield necessary biconditional of the same form as (5). The reason they do not count as analyses is that a sentence's bearing a cleverly constructed @-relation to some numbers is not what makes it the case that it is possibly true.

To return to the case of the actuality operator, consider again the proposal (8). The underlying biconditional might be necessary, but as a proposed analysis it is as bad as inverse utilitarianism. To say that some abstract object's being *A* is what makes it the case that the contingent parts of $\mathfrak{W}$ are the way they are just gets things backwards. What is actually true is what makes it the case that a given element of $\Omega^*$ is actual. The contingent features of $\mathfrak{W}$ determine the actual extension of the actuality predicate, and not the other way around.

The actuality predicate is *less* basic than the actuality operator and cannot be used as an analysans notion in a reductive analysis. There might be interesting connections between what is actually true and what is true according to various possible worlds, but there is no reductive analysis of '$\bigcirc$' in terms of what the actualist regards as the intended model of modal logic.

This ineliminability is not much of a problem because '$\bigcirc$' is not much of a *modal* operator. Unless $\varphi$ itself already contains other modalities, to say that $\varphi$ is actually true does not say anything about merely possible cases. Unlike claims involving '$\Box$', claims involving only '$\bigcirc$' pose no challenge to the actualist's contention that nothing exists that is not actual. But while the ineliminability of the actuality operator might be a mere curiosity in the modal case, the points made here become crucial in the case of tense logic.

## 5 Tense Operators and Times

In the previous two sections, I have noted a number of differences between the necessity operator '$\Box$' and the actuality operator '$\bigcirc$'. Turning now to tense logic, one would expect similar differences between '$\boxdot$' and the present tense operator 'N' ("it is now the case that"), but one would also expect there to be an account of '$\boxdot$' that mirrors the possible-worlds analysis of '$\Box$'. This latter expectation is disappointed, and it is at this point that times and worlds part company.

Tensed discourse is clearly meant to be about what is happening in the time series, and one naturally assumes that the intended model $\langle \Theta^*, \pi^*, @^* \rangle$ of tense logic is the one constituted by all *actual times* (as opposed to what some model *calls* "times"). Suppose we take this as the intended model and let us consider the tense analogue of (5):

$$\ulcorner \boxdot\varphi \urcorner \text{ is true iff } @^*(\varphi, t) \text{ for all } t \in \Theta^* \tag{11}$$

Before even discussing this proposal, one might demand an account of what sort of things the times in the intended model are, but let me pass over this question here. On any view about the metaphysics of time, it will be a *contingent* matter what is true at any given time, and hence it will be a contingent matter what is true at all times. Past, present and future could all have been different from what they actually are, and it is possible for a claim to be always true without being necessarily so. Hence the biconditionals in (11) are true without being necessary, and cannot be taken as an analysis of the tense operator.

If the facts were different from what they actually are, we would either have to chose a different set of times $\Phi$, or a different relation $@$, or both, in order to keep (11) true. But there is no model—whether intended or not—that correctly characterizes all truths of form $\ulcorner \boxdot\varphi \urcorner$ under all possible circumstances. The following is *false*:

$W_{\boxdot}$: There is a model $\langle \Theta, \pi, @ \rangle$ such that, necessarily, $\boxdot\varphi$ iff $@(\varphi, t)$ for all $t \in \Theta$.

Of course, in any possible world, a claim is always true *there* just in case it is true at all times *in that world*. But by picking a different model in each possible world, we merely get the narrow-scope claim:

$N_{\boxdot}$: Necessarily, there is a model $\langle \Theta, \pi, @ \rangle$ such that $\boxdot\varphi$ iff $@(\varphi, t)$ for all $t \in \Theta$.

Hence the intended model of tense logic does *not* do for the tense operator '$\boxdot$' what the intended model of modal logic does for '$\Box$'.

However, one might think that a tense analogue of the possible-world analysis can be had by a slightly different route. The actualist eliminated the modal operator '$\Box$' in favor of abstract ways the world might have been, and one could propose a similar strategy in the tense case. The idea would be to eliminate the tense operator '$\boxdot$' in favor of abstract "ways the present might have been." But while every way the world might have been qualifies as a possible world for the purposes of modal logic, not every way the present might have been is a time. At best, times are ways the present *was, is,* or *will be,* and not every way the present might have been is of this kind. Some possible presents never happen. This means that if we used the set of all possible presents as the intended model in (11) then the resulting account would not even be materially adequate: a claim can be false at some possible present but true at all time. Any contingently true claim of form $\ulcorner \boxdot\varphi \urcorner$ would thus be counted as false on such a proposal.

We could try to circumvent this problem by introducing a predicate '$T$' that applies to all and only those possible presents that are *times* in a given possible world:

$$\boxdot\varphi \text{ if and only if } @(\varphi, t) \text{ for all } t \in \Theta \text{ such that } T(t) \qquad (12)$$

This would then yield something close to the (false) $\mathrm{W}_{\boxdot}$:

$\mathrm{W}_{\boxdot}^{T}$: There is a model $\langle \Theta, \pi, @ \rangle$ such that, necessarily, $\boxdot\varphi$ if and only if $@(\varphi, t)$ for all $t \in \Theta$ such that $T(t)$.

But suppose we asked what makes it the case that a possible present is a time. The obvious answer is that possible present is a time just in case anything that is true according to it is sometimes true:

$$T(t) \text{ if and only if } \forall\varphi\, (@(\varphi, t) \supset \neg \boxdot \neg\varphi) \qquad (13)$$

Provided there are enough propositions expressible in our language to distinguish amongst all possible presents, this correctly identifies all times. But it also reintroduces the very tense operator that we are trying to eliminate in (12), thus making our analysis circular.

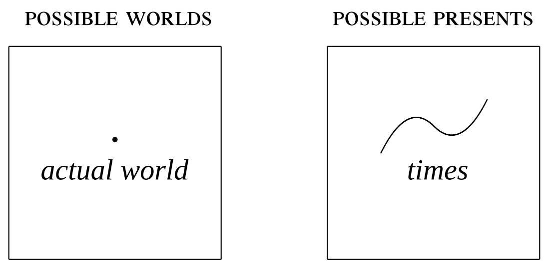
Nor are matters helped by using more fine-grained tense operators, such as the 'P', 'N', and 'F' mentioned above. In terms of these operators, we could say that a possible present is a time just in case it is a past, present, or future time:

$$T(t) \text{ iff } \forall\varphi(@(\varphi, t) \supset \mathsf{P}\varphi) \lor \forall\varphi(@(\varphi, t) \supset \mathsf{N}\varphi) \lor \forall\varphi(@(\varphi, t) \supset \mathsf{F}\varphi)$$

But if we adopted this proposal then we would be unable to get rid of 'P', 'N', and 'F'. We would be no closer to a reductive analysis of all tense operators.

The friend of (12) might propose to take '$T$' as primitive, but we cannot do that for the same reason we could not take '$A$' as primitive in the case of the actuality operator. What actually did, does, or will happen is what makes it the case that a given "possible present" is a time, and not vice versa. To say that some abstract object's being $T$ determines what is happening in the time-series gets things backwards. The predicate '$T$' is less basic than then tense operator we are trying to analyze in (12).

The present tense operator 'N' is indeed similar to the actuality operator '$\bigcirc$', but so are 'P', 'F', and '$\boxdot$'. To pick out past, present, and future times amongst all possible presents, we need to employ the very apparatus of tenses that we are trying to eliminate, and we cannot get rid of these tense operators for the very same reason that we could not get rid of the actuality operator in the modal case (when construed the actualist's way).




We are often told that the present is the tense-analogue of the actual world, and that other times are similar to merely possible worlds. That seems to be a mistake: what *is* similar to possible worlds are possible presents; other times are more similar to the actual world. It is a contingent matter which possible presents are times, just as it is a contingent matter which possible world is the actual world.

The view that the intended model of modal logic is made up of abstract ways the world might have been provides a way of accepting modal claims as true while upholding the thesis that there are no non-actual objects—that is why the view is called ‘actualism’. An important corollary of the discussion here is that there is no tense analogue of this. A prospective *presentist* cannot reconcile an acceptance of past tense claims with a rejection of non-present objects by claiming that such claims are really about presently existing abstracta. The apparent commitment to a past object in “Caesar crossed the Rubicon” cannot be eliminated in the same way in which the actualist can eliminate the apparent commitment to a non-actual objects in “There might have a talking donkey.” Those who claim that presentism is either trivial (because it merely says that past object do not exist now) or untenable (because it denies that Caesar crossed the Rubicon) thus cannot be silenced by pointing out that presentism is the temporal analogue of the non-trivial thesis of actualism. Actualism has no temporal analogue.[9]

## 6 Conclusion

The tense operator ‘$\boxdot$’ obeys the same axioms as the modal operator ‘$\Box$’, and the same model theory can be used for both of them. But important differences emerge when we turn to the *intended* models of tense and modal logic. The modal operator ‘$\Box$’ can be eliminated in favor of the possible world in the intended model of modal logic, but the tense operator ‘$\boxdot$’ cannot be eliminated in favor of the times in the intended model of tense logic: while (4) is necessary, its tense analogue (11) is not. With regard to its intended model, the tense operator

'$\boxdot$' behaves more similar to the actuality operator '$\bigcirc$', as shown by the fact that both $W_{\boxdot}$ and $W_{\bigcirc}$ are false while $W_{\Box}$ is true.

### Notes

1. Prior [14]. Evans [2] and Higginbotham [5] disagree with this adverbial view, but let me ignore their objections. I want to make a point about tense logic itself; whether tense logic is a plausible account of how tense works in ordinary language is not a question I want to address here.

2. Prior [13]. There are comparable similarities between S4 and a "Diodoran" tense logic that uses the operators "it will always be the case that" and "it will sometimes be the case that." For more details on S4 and related cases, see Prior [12], Dummett and Lemmon [1] and Hughes and Cresswell [7, ch. 7].

3. For a proof, see Hughes and Cresswell [7, ch. 6].

4. Hazen [3]. The situation is different for quantified modal logic, where the actuality operator is often ineliminable when it occurs within the scope of other modal operators, as in: "There could be something that does not actually exist." Adding an actuality operator to a system of quantified modal logic thus increases its expressive capacity. For details, see Hodes [6], Hazen [4], and Segerberg [16]. The role of the actuality operator in modal logic is similar to the case of "now" in tense logic, which is discussed in Prior [15] and Kamp [8].

5. Here I am tacitly assuming that enough propositions can be expressed in our language to distinguish between all the possible worlds in $\Omega^*$. Otherwise, there might be more than one world that satisfies the right-hand side of the biconditional in (9).

6. See Lewis [9] and [10, sec. 1.9].

7. On this view, the actuality operator is "token-reflexive" in a similar way as the temporal modifier "now." The proposition expressed by an utterance of "John is hungry now" at time $t$ is true if and only if John is hungry at $t$.

8. Stalnaker [17] argues that the indexical analysis of actuality is a semantic thesis that can be separated from the metaphysical disagreement between the actualist and the realist about the nature of possible worlds.

The considerations presented here suggest that this is only true if we think of the indexicality of actuality in terms of the weak theory (10).

9. For more on this issue, see Meyer [11].

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