Relational Blockworld: A Path Integral Based Interpretation of Quantum Field Theory

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Abstract

We propose a new path integral based interpretation of quantum field theory (QFT). In our interpretation, QFT is the continuous approximation of a more fundamental, discrete graph theory (theory X) whereby the transition amplitude Z is not viewed as a sum over all paths in configuration space, but measures the symmetry of the differential operator and source vector of the discrete graphical action. We propose that the differential operator and source vector of theory X are related via a self-consistency criterion (SCC) based on the identity that underwrites divergence-free sources in classical field theory, i.e., the boundary of a boundary principle. In this approach, the SCC ensures the source vector is divergence-free and resides in the row space of the differential operator. Accordingly, the differential operator will necessarily have a non-trivial eigenvector with eigenvalue zero, so the SCC is the origin of gauge invariance. Factors of infinity associated with gauge groups of infinite volume are excluded in our approach, since Z is restricted to the row space of the differential operator and source vector. We show it is possible that the underlying theory X, despite being discrete, is the basis for exact Poincaré invariance. Using this formalism, we obtain the two-source transition amplitude over a (1+1)-dimensional graph with N vertices fundamental to the scalar Gaussian theory and interpret it in the context of the twin-slit experiment to provide a unified account of the Aharonov-Bohm effect and quantum non-separability (superposition and entanglement) that illustrates our ontic structural realist alternative to problematic particle and field ontologies. Our account also explains the need for regularization and renormalization, explains gauge invariance and largely discharges the problems of inequivalent representations and Haag's theorem. This view suggests corrections to general relativity via modifications to its graphical counterpart, Regge calculus. We conclude by presenting the results of our modified Regge calculus approach to Einstein-de Sitter cosmology where we produced a fit to the Union2 Compilation data for type Ia supernovae rivaling that of the concordance model (ΛCDM), but without having to invoke dark energy or accelerated expansion.

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1. INTRODUCTION

1.1 Foundational Problems of Quantum Field Theory. When it comes to quantum field theory (QFT) some have stressed that the conceptual problems besetting non-relativistic quantum mechanics (NRQM) remain the central concerns[(1)], while others stress that QFT exacerbates some of the interpretive problems of NRQM and possesses foundational problems all its own[(2)]. Some (especially physicists) have stressed that QFT is the greatest and most explanatory intellectual achievement of modern science[(3)], while others believe QFT is "much more a set of formal strategies and mathematical tools than a closed theory[(4)]." Of course on both counts, both sides are right. In addition to the problems of NRQM, an interpretation must address concerns unique to QFT, e.g., notorious problems with particle and field ontologies and renormalization, how to interpret gauge invariance and the Aharonov-Bohm effect (AB effect), the problem of inequivalent representations, and explaining the effectiveness of the interaction picture and perturbation theory in light of Haag's theorem. As for progress in this area, Healey notes[(5)], "no consensus has yet emerged, even on how to interpret the theory of a free, quantized, real scalar field." And[(6)], "There is no agreement as to what object or objects a quantum field theory purports to describe, let alone what their basic properties would be."

Those who emphasize the incompleteness of QFT over its successes often focus on the many *ad hoc* and, for some, troubling "fixes" involved in the practice of QFT[1]. For example, since QFT is independent of overall factors in the transition amplitude, such factors are simply "thrown away" even when these factors are infinity as is the case when the volume of the gauge symmetry group in Fadeev-Popov gauge fixing is infinite[(7)]. And, in the process of renormalization one must "tweak" parameters in the Lagrangian so they remain finite under regularization[(8)]. QFT has triumphed empirically, but virtually all agree that it is not a fundamental theory because it does have a limited domain of applicability, viz., it does not deal with particle interactions at ranges where gravity becomes important. It might be that the Standard Model plus the gravitational field is fundamental[(9)], but most physicists assume there exists an underlying, unified theory

[1] We are focusing on the "textbook variant of QFT." Fraser, D.: Quantum Field Theory: Underdetermination, Inconsistency, and Idealization. Philosophy of Science 74, 536-565 (October 2009). In particular, we are concerned primarily with QFT as applied to particle physics.

called quantum gravity which would naturally justify the *ad hoc* fixes employed in QFT and tell us how to handle the particle interactions where gravity is deemed relevant(10).

Clearly, QFT is in need of more philosophical attention. As Glashow stated(11), "in a sense it really is a time for people like you, philosophers, to contemplate not where we're going, because we don't really know and you hear all kinds of strange views, but where we are. And maybe the time has come for you to tell us where we are." Rovelli goes further stating(12), "As a physicist involved in this effort, I wish that the philosophers who are interested in the scientific description of the world would not confine themselves to commenting and polishing the present fragmentary physical theories, but would take the risk of trying to look *ahead*." Consequently, we propose a new ontology and commensurate path integral account of "theory X" underlying QFT[2].

1.2 Ontic Structural Realism in a Blockworld: The Graphical, Quantum and Classical. Our account of spacetime and matter is very much in keeping with Rovelli's intuition that(13):

> General relativity (GR) altered the classical understanding of the concepts of space and time in a way which...is far from being fully understood yet. QM challenged the classical account of matter and causality, to a degree which is still the subject of controversies. After the discovery of GR we are no longer sure of what is spacetime and after the discovery of QM we are no longer sure of what matter is. *The very distinction between space-time and matter is likely to be ill-founded*....I think it is fair to say that today we do not have a consistent picture of the physical world. [italics added]

Our ontological account of quantum physics is conceptually challenging but, succinctly, it is a form of ontic structural realism in a blockworld setting (4D)[3] with a co-determining amalgam of space, time and matter that we call "spacetimematter." As with GR, topological and geometric properties are fundamental, but on our view matter cannot be separated at all from spacetime (unlike GR with its vacuum solutions), so matter also gets a geometric treatment. We will briefly unpack this description in the remainder of this

[2] Here we follow the possibility articulated by Wallace (p 45) that, "QFTs as a whole are to be regarded only as approximate descriptions of some as-yet-unknown deeper theory," which he calls "theory X." Wallace, D.: In defence of naiveté: The conceptual status of Lagrangian quantum field theory. Synthese 151, 33-80 (2006).

[3] For the reader with an aversion to 4Dism (blockworld), we are simply saying topological and geometric facts that encompass the entire history of physical systems are deeper than dynamical or mechanical facts.

subsection, but we don't expect the reader will fully appreciate or totally understand this ontology until reading the commensurate formalism in sections 2 & 3, because our account is metaphysically and methodologically perverse by the lights of what we will call dynamism or the dynamical bias. Fundamental theories of physics (M-theory, loop quantum gravity, causets, etc.) may deviate from the norm by employing radical new fundamental entities (branes, loops, ordered sets, etc.), but the game is always dynamical, broadly construed (vibrating branes, geometrodynamics, sequential growth process, etc.). As Healey puts it[(14)]:

> Physics proceeds by first analyzing the phenomena with which it deals into various kinds of systems, and them ascribing states to such systems. To classify an object as a certain kind of physical system is to ascribe certain, relatively stable, qualitative intrinsic properties; and to further specify the state of a physical system is to ascribe to it additional, more transitory [time dependent], qualitative intrinsic properties….A physical property of an object will then be both qualitative and intrinsic just in case its possession by that object is wholly determined by the underlying physical states and physical relations of all the basic systems that compose that object.

If one takes it on faith that dynamical explanation is fundamental (however far from ordinary experience and classical physics it might be), it may be impossible to take us seriously, maybe even impossible to clearly envision what we are suggesting. Our ontology and our fundamental methodology violate every tenet of dynamism. Indeed, we will argue that the incompatibility of quantum physics and general relativity is really pointing to the relative failure of dynamism at more fundamental "levels."

Our violation of dynamism is in accord with ontic structural realism[(15)] (OSR):

> Ontic structural realists argue that what we have learned from contemporary physics is that the nature of space, time and matter are not compatible with standard metaphysical views about the ontological relationship between individuals, intrinsic properties and relations. On the broadest construal OSR is any form of structural realism based on an ontological or metaphysical thesis that inflates the ontological priority of structure and relations.

More specifically, our version of OSR (called Relational Blockworld—RBW [(16)]) claims that[(17)] "The relata of a given relation always turn out to be relational structures themselves on further analysis." Note that OSR does not claim there are relations without relata, just that the relata are not individuals (e.g., things with primitive thisness and intrinsic properties), but always ultimately analyzable as relations as well. As will be apparent in section 3, there is no infinite regress of relata and relations in our graphical approach, because a boundary operator on the vector of links (fundamental relations) produces a very intuitive, but not tautological, characterization for the vector of nodes (relata for the links). OSR already violates the dynamical bias by rejecting *things* with intrinsic properties and their dynamics as fundamental *building blocks* of reality—the world isn't fundamentally *compositional*—the deepest conception of reality is not one in which we decompose things into other things at ever smaller length and time scales. Unfortunately for dynamism, we must further exacerbate this violation by applying OSR to a blockworld.

The blockworld perspective (the reality of all events past, present and future including the outcomes of quantum experiments) is suggested by the relativity of simultaneity in special relativity or, more generally, the lack of a preferred spatial foliation of spacetime in GR, and even by quantum entanglement according to some of us(18). Geroch writes(19):

> There is no dynamics within space-time itself: nothing ever moves therein; nothing happens; nothing changes. In particular, one does not think of particles as moving through space-time, or as following along their world-lines. Rather, particles are just in space-time, once and for all, and the world-line represents, all at once, the complete life history of the particle.

When Geroch says that "there is no dynamics within space-time itself," he is not denying that the mosaic of the blockworld possesses patterns that can be described with dynamical laws. Nor is he denying the predictive and explanatory value of such laws. Rather, given the reality of all events in a blockworld, dynamics are not "event factories" that bring heretofore non-existent events (such as measurement outcomes) into being; fundamental dynamical laws that are allegedly responsible for discharging fundamental "why" questions in physics are not brute unexplained explainers that "produce" events on our view. Geroch is advocating for what philosophers call Humeanism about laws. Namely, the claim is that relatively fundamental dynamical laws are *descriptions of regularities* and not the *brute explanation* for such regularities. His point is that in a blockworld, Humeanism about laws is an obvious position to take because everything is just "there" from a "God's eye" (Archimedean) point of view. There is a caveat, however. In the relational reality of RBW, there can be no "God's eye" point of view because "observers" have to be part of that which they observe—themselves relations in a relational network. Consequently, in section 2, we argue for an OSR blockworld characterized as spacetimematter, as opposed to the spacetime + matter picture of current physics.

To formalize spacetimematter and provide a basis for quantum physics we will use graphical relations to self-consistently[4] co-construct space, time and sources[5] (matter) in a graphical fashion (theory X). There are two immediate conceptual consequences to spacetimematter. First, there is no "empty spacetime" so GR, which contains vacuum solutions, cannot be a fundamental theory of physics per theory X. We will speculate briefly on how GR must be "corrected" in section 5. In essence we claim that GR phenomena are only *approximately separable* in a statistical sense to be specified, and therefore GR is applicable only when its *approximation* of "separability" holds. As Healey notes[(20)], "By contrast, classical general relativity is separable, since all the qualitative intrinsic physical properties it ascribes on a loop do supervene on qualitative intrinsic physical properties assigned on (infinitesimal neighborhoods of) space-time points on that loop." On the spacetime + matter picture it is common to try and square quantum non-separability with the separability of GR. This has proven to be problematic thus far. We resolve this problem with our spacetimematter theory wherein the non-separability of quantum states in Hilbert space and Healey's characterization of non-separability in terms of the relations between spacetime points (such as EPR correlations) get a unified explanation.

Second, there are no "quantum entities" with "quantum states" (of any sort) emitted by the Source, moving through the various pieces of experimental equipment (e.g., beam splitters, mirrors) and impinging on the detector(s) to cause experimental outcomes in quantum experiments. Space, time and sources are co-constructed (a fusion or unity) to represent the relevant relationships comprising the various pieces of experimental equipment (OSR) from an experiment's initiation to its termination

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[4] Our form of self-consistency is topological, i.e., it is characterized via boundary operators in the spacetime chain complex of our spacetimematter graph.

[5] We use the word "source" as in QFT, i.e., to mean "particle sources" or "particle sinks" (creation or annihilation events, respectively). When we want to specify "a source of particles" we will use "Source."

(blockworld); past, present and future are co-constructed as well, there are no dynamical entities or dynamical laws in our *fundamental* formalism. As we shall soon explain, spacetimematter underwrites quantum non-separability (superposition and entanglement) in a kinematical fashion. Accordingly, all dynamical explanation supervenes on, and is secondary to, non-dynamical topological facts about the graph world.

Consequently, fundamental explanation is in terms of a global, adynamical organizing principle. Thus, ultimate explanation in physics is not in terms of some *thing* or dynamical entity (obeying a new dynamical equation) "at the bottom" conceived at higher energies and smaller spatiotemporal scales, begging for justification from some*thing* at some yet "deeper" scale, but self-consistency writ large for the explanatory "process" as a whole. As we shall see, this goes well beyond consistency as typically conceived by physicists. Self-consistency writ large is *extremum* thinking writ large, which truly transcends and underwrites the dynamical perspective. Mathematically speaking, the topological characterization of self-consistent spacetimematter at the graphical level is mirrored by the resulting geometric classical field theory at the classical level.

In short, distributions of spatiotemporal geometric relations over the (topological) graphical realm (Figure 1 → Figure 2) are averaged to obtain the spacetime geometry for the unity of spacetimematter of the classical realm (Figure 3). Classical equations of motion are given in terms of this "average spacetime geometry" for the unity of spacetimematter such that the standard spacetime + matter picture obtains as a statistical approximation. A graph (Figure 1) overlaid with a particular spatiotemporal geometric distribution (Figure 2) can result in geometrically localizable subsets, which we call "Clusters" in Figure 2. There are several different geometric versions of a Cluster that are consistent with a particular classical Object (Figure 3), analogous to the many different velocity distributions for the molecules of a gas that give rise to the same temperature and pressure per statistical mechanics. If one wants to explore *specific* spatiotemporal geometric relations (specific line segments in Figure 2) in a *particular* distribution over the graph (specific trial in the experiment), one is doing quantum physics (Figure 4).

This ontology of spatiotemporal geometry over spacetimematter graphs can be described in terms of the distributions of individual geometric relations (quantum physics) or it can be described approximately in terms of the *averages* of the distributions of individual geometric relations (classical physics). Obviously, the (average) spacetime + matter approximation becomes more accurate as the number of geometric relations increases.

Probably the most important aspect of the RBW ontology for the interpretation of quantum physics is that there are no "quantum Clusters," so there are no "quantum Objects," i.e., all Objects are classical and quantum physics is an exploration of their relational "composition" (Figure 4). This is in stark contrast to those interpretations of quantum physics which employ dynamical ontological *constituents* of the *essentially* quantum realm (particles, waves, wave-functions, fields, etc.) with their strange non-commutative properties and struggle to somehow compose or realize the *essentially* classical realm of dynamical ontological constituents with commutative properties. Thus, there simply is no possibility of a measurement problem(21) on our view (a problem driven by taking quantum dynamics realistically), and quantum non-separability is ultimately explained kinematically by the unity of spacetimematter. In section 3, we will show how the fusion of spacetimematter in this approach explains the interference pattern of the twin-slit experiment without invoking "quantum entities" moving through space as a function of time to "cause" detector events. But, before jumping into the formalism, we want to provide a conceptual primer.

Methodologically, we start with a graph and use boundary operators in its spacetime chain complex to provide a topological representation of the relations under investigation in a particular experiment. We use this topological characterization to create a partition function for the ensemble of possible geometric relations over the spacetimematter graph. Essentially, this partition function provides a measure of the graph's ability to accommodate various spacetime geometries for its unity of spacetimematter[6]. So, the equipment in a particular quantum experiment, understood in

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[6] Technically speaking, we use a discrete path integral over graphs with a Wick-rotated action in the transition amplitude. All this will be explained in sections 2 & 3.

the context of an “average spacetime geometry” (Figure 3), is idealized as the instantiation by the graphical spacetimematter (Figure 1) of some particular spatiotemporal geometric distribution in the ensemble (Figure 2), where one can have a different distribution for each trial of the experiment (again, there can be many different spatiotemporal geometric distributions consistent with a particular average spacetime + matter experimental configuration). The experimental outcome then reflects a *specific* spatiotemporal geometric relation in the distribution of that particular trial (Figure 4). Thus, the partition function is used to compute the probability of finding a specific spatiotemporal geometric relation (representing a particular experimental outcome) in the conduct of the experiment. The most probable of these specific outcomes is given by the extremum of the probability function and, since the most probable value is the average value in our Gaussian distribution, we recover classical equations of motion in terms of the “average spacetime geometry” for the unity of spacetimematter. As will be seen, the manner by which the boundary operators in the spacetime chain complex of the graph give rise to its partition function is mirrored precisely in the classical equations of motion. As we explain in section 5, the classical result is a sort of modified Regge calculus[7], which obviously suggests a bridge from theory X for spacetimematter to its continuous, separable, statistical approximation of GR for spacetime + matter.

Given Figures 1-4 and the explanation immediately above, it should be clear how the ontology of spacetimematter gives rise to quantum non-separability. The unity of spacetimematter gives the separable spacetime + matter *on average* as an approximation to situations involving large numbers of geometric relations. But, it is possible to construct (quantum) experiments that reveal individual relations between classical Objects which then appear as non-separable outcomes in the context of the “average spacetime geometry” over spacetime + matter. Thus, quantum non-separability will be “mysterious” if one believes (erroneously) that the separable spacetime + matter is fundamental, rather than recognizing it as a mere statistical approximation to what is truly fundamental, i.e., the unity of spacetimematter. [More on this in section 5.]

[7] Regge calculus is a discrete approximation to general relativity where the discrete counterpart to Einstein’s equations is obtained from the least action principal on a 4D graph. This generates a rule for constructing a discrete approximation to the spacetime manifold of GR using 4D graphical “tetrahedra” called “simplices” (Figure 10). For more information, see Chap 42 of Misner, C.W., Thorne, K.S., Wheeler, J.A.: Gravitation. W.H. Freeman, San Francisco (1973).

Thus, the payoff for an OSR blockworld ontology (with its commensurate methodology) that violates the dynamical bias is a unified picture of physics (theory X) that resolves the conceptual, foundational and technical issues of quantum physics. This is in accord with Smolin's prediction[(22)] that, "The problem of quantum mechanics is unlikely to be solved in isolation; instead, the solution will probably emerge as we make progress on the greater effort to unify physics." Unfortunately, the mathematical counterpart to this extremely counterintuitive ontology is equally obscure, i.e., a discrete path integral over graphs.

1.3 Overview of Paper. We understand the reader may not be familiar with the path integral formalism, as Healey puts it[(23)], "While many contemporary physics texts present the path-integral quantization of gauge field theories, and the mathematics of this technique have been intensively studied, I know of no sustained critical discussions of its conceptual foundations." Therefore, we begin in section 2 with an overview and interpretation of the path integral formalism. Immediately after we introduce and interpret the path integral formalism, we motivate our use of a discrete path integral approach to theory X to include the self-consistency criterion (SCC) responsible for the co-construction of space, time and matter. The SCC is based on the boundary of a boundary equals zero ($\partial\partial = 0$), responsible for the divergence-free nature of the stress-energy tensor in classical physics[8]. The SCC provides the rule by which boundary operators in the spacetime chain complex of the graph "provide a topological representation of the relations under investigation in a particular experiment."

In section 3, we provide the mathematical details of theory X via our discrete path integral formalism over graphs, explaining how it yields quantum physics and classical physics in its continuum wake. Using this formalism, we obtain the two-source transition amplitude over a (1+1)-dimensional graph with N vertices fundamental to the scalar Gaussian theory, and interpret it in the context of the twin-slit experiment. Having formally composed our OSR blockworld, we address various conceptual and technical issues associated with QFT in section 4. Specifically, we provide an OSR alternative to problematic particle and field ontologies that also explains the need for regularization and

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[8] A divergence-free stress-energy tensor characterizes the conservation of momentum and energy in classical physics.

renormalization, explain gauge invariance, provide a unified account of the Aharonov-Bohm effect and quantum non-separability, and largely discharge the problem of inequivalent representations. We will also speculate on how our graphical theory X might provide a basis for exact Poincaré invariance, which includes Lorentz invariance – typically a problem for discrete lattice theories[9]. We conclude section 4 with a brief explanation of why Haag's theorem creates problems for the interaction picture according to theory X. In section 5, we provide a summary using the Maxwell and Einstein-Hilbert actions as examples, and present the results of our modified Regge calculus approach to Einstein-de Sitter cosmology which produced a sum of squares error (SSE) in fitting the Union2 Compilation data for type Ia supernovae of 1.77. This result rivals the best fit (SSE = 1.79) of this same data by the concordance model (ΛCDM), but without having to invoke dark energy or accelerated expansion (Figure 11).

2. THE DISCRETE PATH INTEGRAL FORMALISM AND RBW

In this section we provide an overview and interpretation of the path integral approach, showing explicitly how we intend to use "its conceptual foundations." We employ the discrete path integral formalism because it embodies a 4Dism of the sort outlined above that allows us to model spacetimematter. For example, the path integral approach is based on the fact that[(24)] "the [S]ource will emit and the detector receive[10]," i.e., the formalism deals with Sources and sinks as a unity while requiring a description of the experimental process from initiation to termination (blockworld). By assuming the discrete path integral is fundamental to the (conventional) continuum path integral, we have a graphical basis for the co-construction of time, space and quantum sources via a self-consistency criterion (SCC). We will show in section 3 how the graphical amalgam of spacetimematter is the basis for quantum and classical physics.

2.1 Path Integral in Quantum Physics. In the conventional path integral formalism[(25)] for NRQM one starts with the amplitude for the propagation from the initial point in configuration space q_I to the final point in configuration space q_F in time T via the unitary

[9] In lattice gauge theory, spacetime is modeled as a hypercubic lattice in 4-dimensional Euclidean space. One obtains rotationally invariant QFT in the limit as the lattice spacing goes to zero, and this gives Lorentz invariance after Wick rotation. However, one does not have the full rotational invariance on the discrete lattice, so lattice theories which are to remain discrete typically have problems with exact Lorentz invariance.

[10] The path integral formalism requires both an emission event and a reception event; the formalism was motivated by the idea of treating advanced and retarded potentials equally.

operator e^{-iHT}, i.e., $\langle q_F | e^{-iHT} | q_I \rangle$. Breaking the time *T* into *N* pieces *δt* and inserting the identity between each pair of operators $e^{-iH\delta t}$ via the complete set $\int dq |q\rangle\langle q| = 1$ we have

$$\langle q_F | e^{-iHT} | q_I \rangle = \left[\prod_{j=1}^{N-1} \int dq_j \right] \langle q_F | e^{-iH\delta t} | q_{N-1} \rangle \langle q_{N-1} | e^{-iH\delta t} | q_{N-2} \rangle \ldots \langle q_2 | e^{-iH\delta t} | q_1 \rangle \langle q_1 | e^{-iH\delta t} | q_I \rangle$$

With $H = \frac{\hat{p}^2}{2m} + V(\hat{q})$ and *δt* → 0 one can then show that the amplitude is given by

$$\langle q_F | e^{-iHT} | q_I \rangle = \int Dq(t) \exp\left[i \int_0^T dt L(\dot{q}, q) \right] \tag{1}$$

where $L(\dot{q}, q) = \frac{1}{2} m \dot{q}^2 - V(q)$. If *q* is the spatial coordinate on a detector transverse to the line joining Source and detector, then $\prod_{j=1}^{N-1}$ can be thought of as *N*-1 "intermediate" detector surfaces interposed between the Source and the final (real) detector, and $\int dq_j$ can be thought of all possible detection sites on the j^{th} intermediate detector surface. In the continuum limit, these become $\int Dq(t)$ which is therefore viewed as a "sum over all possible paths" from the Source to a particular point on the (real) detector, thus the term "path integral formalism" for conventional NRQM is typically understood as a sum over "all paths through space."

To obtain the path integral approach to QFT one associates *q* with the oscillator displacement at a *particular point* in space ($V(q) = \mathrm{k}q^2/2$). In QFT, one takes the limit *δx* → 0 so that space is filled with oscillators and the resulting spatial continuity is accounted for mathematically via $q_i(t) \rightarrow q(t,x)$, which is denoted $\varphi(t,x)$ and called a "field." The QFT amplitude (denoted "*Z*") then looks like

$$Z = \int D\varphi \exp\left[i \int d^4 x L(\dot{\varphi}, \varphi) \right] \tag{2}$$

where $L(\dot{\varphi}, \varphi) = \frac{1}{2} (d\varphi)^2 - V(\varphi)$. Impulses *J* are located in the field to account for particle creation and annihilation; these *J* are called "sources" in QFT and we have $L(\dot{\varphi}, \varphi) = \frac{1}{2} (d\varphi)^2 - V(\varphi) + J(t,x)\varphi(t,x)$, which can be rewritten as

$L(\dot{\varphi},\varphi)=\frac{1}{2}\varphi D\varphi+J(t,x)\varphi(t,x)$, where *D* is a differential operator. In its discrete form (typically, but not necessarily, a hypercubic spacetime lattice), $D \rightarrow \vec{\vec{K}}$ (a difference matrix), $J(t,x) \rightarrow \vec{J}$ (each component of which is associated with a point on the spacetime lattice[11]) and $\varphi \rightarrow \vec{Q}$ (each component of which is associated with a point on the spacetime lattice). The discrete counterpart to Eq. (2) is then[(26)]

$$Z=\int\ldots\int dQ_1\ldots dQ_N \exp\left[\frac{i}{2}\vec{Q}\cdot\vec{\vec{K}}\cdot\vec{Q}+i\vec{J}\cdot\vec{Q}\right] \qquad (3).$$

In conventional quantum physics, NRQM is understood as (0+1)-dimensional QFT.

2.2 Our Interpretation of the Path Integral in Quantum Physics. We agree that NRQM is to be understood as (0+1)-dimensional QFT, but point out this is at conceptual odds with our derivation of Eq. (1) when $\int Dq(t)$ represented a sum over all paths in space, i.e., when *q* was understood as a location in space (specifically, a location along a detector surface). If NRQM is (0+1)-dimensional QFT, then *q* is a *field displacement at a single location in space*. In that case, $\int Dq(t)$ must represent a sum over all field values at a particular point on the detector, not a sum over all paths through space from the Source to a particular point on the detector. So, how *do* we relate a point on the detector (sink) to the Source?

In answering this question, we now explain a formal difference between conventional path integral NRQM and our proposed approach: roughly we are connecting discrete sources $\vec{J}$, where one part of $\vec{J}$ is used for the Source and the other part of $\vec{J}$ is used for the detector click (sink). Instead of $\delta x \rightarrow 0$, as in QFT, we assume δx is measureable for (such) NRQM phenomenon. More specifically, we propose starting with Eq. (3)[12] whence (roughly) NRQM obtains in the limit $\delta t \rightarrow 0$, as in deriving Eq. (1), and QFT obtains in the additional limit $\delta x \rightarrow 0$, as in deriving Eq. (2). The QFT limit is well

[11] Part of $\vec{J}$ represents particle Sources the other part represents particle sinks in the conventional view of path integral QFT so that field disturbances emanate from one source location (Source) and are absorbed at another source location (sink). In particle physics, these field disturbances are the particles. We will keep the partition of $\vec{J}$ into Sources and sinks in our theory X, but there will be no disturbance (or any "thing" else) propagating between them because, as we shall show, there will be no medium (field) to be "disturbed" between the discrete set of sources.

[12] Actually, we're going to start with the Euclidean path integral version of Eq. (3), as we'll explain later.

understood as it is the basis for lattice gauge theory and regularization techniques, so one might argue that we are simply *clarifying* the NRQM limit where the path integral formalism is not widely employed. However, again, we are proposing a discrete starting point[13] for theory X, as in Eq. (3).

2.3 Discrete Path Integral is Fundamental. The version of theory X we propose is a discrete path integral over graphs, so *Eq. (3) is not a discrete approximation of Eqs. (1) & (2)*, but rather *Eqs. (1) & (2) are continuous approximations of Eq. (3)*. In the arena of quantum gravity it is not unusual to find discrete theories[(27)] that are in some way underneath spacetime theory and theories of "matter" involving dynamical entities such as QFT, e.g., causal dynamical triangulations[(28)], quantum graphity[(29)] and causets[(30)]. While these approaches are interesting and promising, the approach taken here for theory X will look more like Regge calculus quantum gravity (see Bahr & Dittrich [(31)] and references therein for recent work along these lines).

Placing a discrete path integral at bottom introduces conceptual and analytical deviations from the conventional, continuum path integral approach. Conceptually, Eq. (1) of NRQM represents a sum over all field values at a particular point on the detector, while Eq. (3) of theory X is a mathematical machine that measures the "symmetry" (strength of stationary points) contained in the core of the discrete action

$$\frac{1}{2}\vec{\vec{K}} + \vec{J} \qquad (4).$$

This core or *actional* yields the discrete action after operating on a particular vector $\vec{Q}$ (field). The actional represents a *fundamental, 4D description of the experimental arrangement* and Z is a measure of its symmetry[14]. For this reason, and because *transition* amplitude connotes a *dynamical* process, we prefer to call Z the *symmetry amplitude* of the 4D experimental configuration. Since $\vec{Q}$ is only an integration variable, fields have no ontic significance at this fundamental level – they are merely part of the computational device for measuring the symmetry of the actional (representing what *is* ontically significant at the fundamental level). Analytically, because we are *starting* with a discrete

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[13] That discrete spacetime is fundamental while "the usual continuum theory is very likely only an approximation" is, of course, an old idea. See, for example, arguments in Feinberg, G., Friedberg, R., Lee, T.D., and Ren, H.C.: Lattice Gravity Near the Continuum Limit. Nuclear Physics B245, 343-368 (1984).

[14] In its Euclidean form, which is the form we will use, Z is a partition function.

formalism, we are in position to mathematically explicate trans-temporal identity, whereas this process is unarticulated elsewhere in physics (as elaborated immediately below). As we will now see, this leads to our proposed self-consistency criterion (SCC) underlying *Z*.

2.4 Time, Space & Discrete Quantum Sources $\vec{J}$. The NRQM limit $\delta t \rightarrow 0$ of Eq. (3) results in a spatially discrete distribution of "interacting" sources $J_i(t)$ and illustrates a key aspect of the RBW ontology, i.e., what is typically understood as "interaction" in quantum physics is modeled without mediating waves, particles, etc., traveling through intervening space (in fact, there is no medium either, i.e., field, between sources $J_i(t)$). The spatiotemporally discrete formalism also illustrates nicely how NRQM tacitly assumes an *a priori* process of trans-temporal identification, $\vec{J} \rightarrow J_i(t)$ as $\delta t \rightarrow 0$. Indeed, there is no principle which dictates the construct of diachronic entities fundamental to the formalism of dynamics in general – these objects are "put in by hand" throughout physics. When Albrecht and Iglesias[32] allowed time to be an "internal variable" after quantization, as in the Wheeler-DeWitt equation, they found "there is no one set of laws, but a whole library of different cosmic law books[33]." They called this the "clock ambiguity." In order to circumvent this "arbitrariness in the predictions of the theory" they proposed that "the principle behind the regularities that govern the interaction of entities is … the idea that individual entities exist at all[34]." Albrecht and Iglesias characterize this as "the central role of quasiseparability."

Similarly, the RBW approach requires a fundamental principle ($\partial\partial = 0$) whence the trans-temporal identity employed tacitly in NRQM and all dynamical theories. Our discrete (graphical) starting point provides a topological basis for sources $\vec{J}$, space and time. Clearly, the process $\vec{J} \rightarrow J_i(t)$ is an organization of the set $\vec{J}$ on two levels – there is the split of the set into *i* subsets, one for each source, and there is the ordering *t* over each subset. The split represents space, the ordering represents time and the result is (trans-temporal) objecthood. In this sense, space, time and sources $\vec{J}$ are relationally co-constructed in our formalism. Consequently, we believe the articulation of the otherwise tacit construct of dynamical entities has a mathematical counterpart fundamental to the action, viz., the boundary of a boundary principle, $\partial\partial = 0$, at the fundamental level[15]. This is in accord with Toffoli's belief that there exists a mathematical tautology fundamental to the action[(35)]:

> Rather, the motivation is that principles of great generality must be by their very nature *trivial*, that is, expressions of broad tautological identities. If the principle of least action, which is so general, still looks somewhat mysterious, that means we still do not understand what it is really an expression of—what it is trying to tell us.

2.5 Self-Consistency Criterion. Our use of a self-consistency criterion is not without precedent, as we already have an ideal example in Einstein's equations of GR. Momentum, force and energy all depend on spatiotemporal measurements (tacit or explicit), so the stress-energy tensor cannot be constructed without tacit or explicit knowledge of the spacetime metric (technically, the stress-energy tensor can be written as the functional derivative of the matter-energy Lagrangian with respect to the metric). But, if one wants a "dynamic spacetime" in the parlance of GR, the spacetime metric must depend on the matter-energy distribution in spacetime. GR solves this dilemma by demanding the stress-energy tensor be "consistent" with the spacetime metric per Einstein's equations[16]. This self-consistency hinges on divergence-free sources, which finds a mathematical counterpart in $\partial\partial = 0$, i.e., the boundary of a boundary principle[(36)]. So, Einstein's equations of GR are a mathematical articulation of the boundary of a boundary principle at the classical level, i.e., they constitute a self-consistency criterion at the classical level. In fact, our SCC will be based on the same topological maxim ($\partial\partial = 0$) for the same reason[17], as are quantum and classical electromagnetism[(37)]. In section 5, we will show that the same structure obtains in the Maxwell action and weak field expansion of the Einstein-Hilbert action.

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[15] Miller showed $\partial\partial = 0$ applies to Regge's discrete spacetime in Miller, W.A.: The Geometrodynamic Content of the Regge Equations as Illuminated by the Boundary of a Boundary Principle. Foundations of Physics 16, 143-169 (1986).

[16] Concerning the stress-energy tensor, Hamber and Williams write, "In general its covariant divergence is not zero, but consistency of the Einstein field equations demands $\nabla^{\alpha} T_{\alpha\beta} = 0$," Hamber, H.W. and Williams, R.: Nonlocal Effective Gravitational Field Equations and the Running of Newton's G. arXiv: hep-th/0507017 (2005).

[17] Einstein's equations of GR are the continuous, separable, statistical approximation to the SCC of theory X.

In order to illustrate the discrete mathematical co-definition of space, time and sources $\vec{J}$, we will use graph theory *a la* Wise[(38)] and find $\partial_1\partial_1^T$, where ∂_1 is a boundary operator in the spacetime chain complex of our graph satisfying $\partial_1\partial_2 = 0$, has precisely the same form as the matrix operator in the discrete action for coupled harmonic oscillators. Therefore, we are led to speculate that $\vec{\vec{K}} \propto \partial_1\partial_1^T$. Defining the source vector $\vec{J}$ relationally via $\vec{J} \propto \partial_1\vec{e}$ then gives tautologically (per the boundary of a boundary principle) both a divergence-free $\vec{J}$ and $\vec{\vec{K}}\vec{v} \propto \vec{J}$, where $\vec{e}$ is the vector of links and $\vec{v}$ is the vector of vertices. $\vec{\vec{K}}\vec{v} \propto \vec{J}$ is our SCC following from the graphical counterpart to $\partial\partial = 0$, i.e., $\partial_1\partial_2 = 0$, and it defines what is meant by a self-consistent co-construction of space, time and divergence-free sources $\vec{J}$, thereby constraining $\vec{\vec{K}}$ and $\vec{J}$ in *Z*. Thus, our SCC provides a basis for the discrete action[18] in accord with Toffoli and supports our view that Eq. (3) is fundamental to Eqs. (1) & (2), rather than the converse. Conceptually, that is the basis of our discrete, graphical path integral approach to theory X. We now provide the details.

3. THE FORMALISM

3.1 The General Approach. Again, in theory X, the symmetry amplitude *Z* contains a discrete action constructed per a self-consistency criterion (SCC) for space, time and divergence-free sources $\vec{J}$. As introduced in section 2 and argued later in this section, we will codify the SCC using $\vec{\vec{K}}$ and $\vec{J}$; these elements are germane to the transition amplitude *Z* in the Central Identity of Quantum Field Theory[(39)],

$$Z = \int D\vec{\varphi}\exp\left[-\frac{1}{2}\vec{\varphi}\cdot\vec{\vec{K}}\cdot\vec{\varphi} - V(\varphi) + \vec{J}\cdot\vec{\varphi}\right] = \exp\left[-V\left(\frac{\delta}{\delta J}\right)\right]\exp\left[\frac{1}{2}\vec{J}\cdot\vec{\vec{K}}^{-1}\cdot\vec{J}\right] \qquad (5).$$

While the field is a mere integration variable used to produce *Z*, it must reappear at the level of classical field theory (FT). To see how the field makes it appearance in theory X, consider Eq. (5) for the simple Gaussian theory ($V(\varphi) = 0$). On a graph with *N* nodes/vertices, Eq. (5) is

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[18] This replaces the use of classical fields to motivate the construct of QFT, as is the case in Lagrangian QFT. Wallace, D.: In defence of naiveté: The conceptual status of Lagrangian quantum field theory. Synthese 151, 33-80 (2006).

$$Z = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} dQ_1 \ldots dQ_N \exp\left[-\frac{1}{2} \vec{Q} \cdot \vec{\vec{K}} \cdot \vec{Q} + \vec{J} \cdot \vec{Q} \right] \tag{6}$$

with a solution of

$$Z = \left(\frac{(2\pi)^N}{\det(K)} \right)^{1/2} \exp\left[\frac{1}{2} \vec{J} \cdot \vec{\vec{K}}^{-1} \cdot \vec{J} \right] \tag{7}.$$

It is easiest to work in the eigenbasis of $\vec{\vec{K}}$ and (as will argue later) we restrict the path integral to the row space of $\vec{\vec{K}}$, this gives

$$Z = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} d\widetilde{Q}_1 \ldots d\widetilde{Q}_{N-1} \exp\left[\sum_{j=1}^{N-1} \left(-\frac{1}{2} \widetilde{Q}_j^2 a_j + \widetilde{J}_j \widetilde{Q}_j \right) \right] \tag{8}$$

where $\widetilde{Q}_j$ are the coordinates associated with the eigenbasis of $\vec{\vec{K}}$ and $\widetilde{Q}_N$ is associated with eigenvalue zero, a_j is the eigenvalue of $\vec{\vec{K}}$ corresponding to $\widetilde{Q}_j$, and $\widetilde{J}_j$ are the components of $\vec{J}$ in the eigenbasis of $\vec{\vec{K}}$. The solution of Eq. (8) is

$$Z = \left(\frac{(2\pi)^{N-1}}{\prod_{j=1}^{N-1} a_j} \right)^{1/2} \prod_{j=1}^{N-1} \exp\left[\frac{\widetilde{J}_j^2}{2a_j} \right] \tag{9}.$$

On our view, the experiment is described topologically at the fundamental (graphical) level by $\vec{\vec{K}}$ and $\vec{J}$. Again, per Eq. (9), there is no field $\widetilde{Q}$ appearing in *Z* at this level, i.e., $\widetilde{Q}$ is only an integration variable. $\widetilde{Q}$ makes its first appearance as something more than an integration variable when we produce probabilities from *Z*. That is, since we are working with a Euclidean path integral, *Z* is a partition function and the probability of measuring $\widetilde{Q}_k = \widetilde{Q}_o$ is found by computing the fraction of *Z* which contains $\widetilde{Q}_o$ at the k$^{\text{th}}$ graphical element[40]. We have

$$\mathrm{P}\left(\widetilde{Q}_k = \widetilde{Q}_o\right) = \frac{Z\left(\widetilde{Q}_k = \widetilde{Q}_o\right)}{Z} = \sqrt{\frac{a_k}{2\pi}} \exp\left[-\frac{1}{2} \widetilde{Q}_o^2 a_k + \widetilde{J}_k \widetilde{Q}_o - \frac{\widetilde{J}_k^2}{2a_k} \right] \tag{10}$$

as the part of theory X approximated in the continuum by QFT. The most probable value of $\tilde{Q}_o$ at the k[th] graphical element is then given by

$$\delta \mathrm{P}\left(\tilde{Q}_k = \tilde{Q}_o\right) = 0 \Rightarrow \delta\left[-\frac{1}{2}\tilde{Q}_o^2 a_k + \tilde{J}_k \tilde{Q}_o - \frac{\tilde{J}_k^2}{2a_k}\right] = 0 \Rightarrow a_k \tilde{Q}_o = \tilde{J}_k \qquad (11).$$

That is, $\vec{\vec{K}} \cdot \vec{Q}_o = \vec{J}$ is the part of theory X approximated in the continuum by classical FT. We note, of course, that $\vec{\vec{K}} \cdot \vec{Q}_o = \vec{J}$ is in accord with acquiring classical FT from QFT via the stationary phase method [(41)]. [The sign of the second derivative evaluated at $\tilde{Q}_o = \dfrac{\tilde{J}_k}{a_k}$ goes as $-a_k$, so this extremum is a relative maximum for positive a_k (all those for $\vec{\vec{K}}$ in section 4 are positive, for example).] In summary:

1. Z is a partition function for an experiment described topologically (graphically) by $\frac{1}{2}\vec{\vec{K}} + \vec{J}$ (Figure 1).
2. Theory X gives us the probability, $\mathrm{P}\left(\tilde{Q}_k = \tilde{Q}_o\right) = \dfrac{Z\left(\tilde{Q}_k = \tilde{Q}_o\right)}{Z}$, for a particular outcome (geometric relationship under investigation) in that experiment (Figure 4).
3. $\vec{\vec{K}} \cdot \vec{Q}_o = \vec{J}$ gives us the most probable values of the experimental outcomes (Figure 3), i.e., the average geometry relationally constituting the experimental equipment as it relates to the experimental procedure.
4. $\mathrm{P}\left(\tilde{Q}_k = \tilde{Q}_o\right) = \dfrac{Z\left(\tilde{Q}_k = \tilde{Q}_o\right)}{Z}$ and $\vec{\vec{K}} \cdot \vec{Q}_o = \vec{J}$ are the parts of theory X approximated in the continuum by QFT and classical FT, respectively.

3.2 The Two-Source Symmetry Amplitude/Partition Function. In order to motivate our general method, we will first consider a simple graph with six vertices, seven links and two plaquettes for our (1+1)-dimensional spacetime model (Figure 5). Our goal with this simple model is to seek relevant structure that might be used to infer an SCC. We begin by constructing the boundary operators over our graph.

The boundary of $\mathbf{p}_1$ is $\mathbf{e}_4 + \mathbf{e}_5 - \mathbf{e}_2 - \mathbf{e}_1$, which also provides an orientation. The boundary of $\mathbf{e}_1$ is $\mathbf{v}_2 - \mathbf{v}_1$, which likewise provides an orientation. Using these conventions for the orientations of links and plaquettes we have the following boundary operator for $C_2 \rightarrow C_1$, i.e., space of plaquettes mapped to space of links in the spacetime chain complex:

$$\partial_2 = \begin{bmatrix} -1 & 0 \\ -1 & 1 \\ 0 & -1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \tag{12}$$

Notice the first column is simply the links for the boundary of $\mathbf{p}_1$ and the second column is simply the links for the boundary of $\mathbf{p}_2$. We have the following boundary operator for $C_1 \rightarrow C_0$, i.e., space of links mapped to space of vertices in the spacetime chain complex:

$$\partial_1 = \begin{bmatrix} -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{13}$$

which completes the spacetime chain complex, $C_0 \xleftarrow{\partial_1} C_1 \xleftarrow{\partial_2} C_2$. Notice the columns are simply the vertices for the boundaries of the edges. These boundary operators satisfy $\partial_1\partial_2 = 0$ as required by the boundary of a boundary principle.

The potential for coupled oscillators can be written

$$V(q_1, q_2) = \sum_{a,b} \frac{1}{2} k_{ab} q_a q_b = \frac{1}{2} k q_1^2 + \frac{1}{2} k q_2^2 + k_{12} q_1 q_2 \tag{14}$$

where $k_{11} = k_{22} = k$ (positive) and $k_{12} = k_{21}$ (negative) per the classical analogue

(Figure 6) with $k = k_1 + k_3 = k_2 + k_3$ and $k_{12} = -k_3$ to recover the form in Eq. (14). The Lagrangian is then

$$L = \frac{1}{2}m\dot{q}_1^2 + \frac{1}{2}m\dot{q}_2^2 - \frac{1}{2}kq_1^2 - \frac{1}{2}kq_2^2 - k_{12}q_1q_2 \tag{15}$$

so our NRQM symmetry amplitude is

$$Z = \int Dq(t)\exp\left[-\int_0^T dt\left[\frac{1}{2}m\dot{q}_1^2 + \frac{1}{2}m\dot{q}_2^2 + V(q_1,q_2) - J_1q_1 - J_2q_2\right]\right] \tag{16}$$

after Wick rotation. This gives

$$\vec{\vec{K}} = \begin{bmatrix} \left(\frac{m}{\Delta t}+k\Delta t\right) & \frac{-m}{\Delta t} & 0 & k_{12}\Delta t & 0 & 0 \\ \frac{-m}{\Delta t} & \left(\frac{2m}{\Delta t}+k\Delta t\right) & \frac{-m}{\Delta t} & 0 & k_{12}\Delta t & 0 \\ 0 & \frac{-m}{\Delta t} & \left(\frac{m}{\Delta t}+k\Delta t\right) & 0 & 0 & k_{12}\Delta t \\ k_{12}\Delta t & 0 & 0 & \left(\frac{m}{\Delta t}+k\Delta t\right) & \frac{-m}{\Delta t} & 0 \\ 0 & k_{12}\Delta t & 0 & \frac{-m}{\Delta t} & \left(\frac{2m}{\Delta t}+k\Delta t\right) & \frac{-m}{\Delta t} \\ 0 & 0 & k_{12}\Delta t & 0 & \frac{-m}{\Delta t} & \left(\frac{m}{\Delta t}+k\Delta t\right) \end{bmatrix} \tag{17}$$

on our graph. Thus, we borrow (loosely) from Wise[42] and suggest $\vec{\vec{K}} \propto \partial_1\partial_1^T$ since

$$\partial_1\partial_1^T = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix} \tag{18}$$

produces precisely the same form as Eq. (17) and quantum theory is known to be "rooted in this harmonic paradigm[43]." [In fact, these matrices will continue to have the same form as one increases the number of vertices in Figure 5.] Now we construct a suitable candidate for $\vec{J}$, relate it to $\vec{\vec{K}}$ and infer our SCC.

Recall that $\vec{J}$ has a component associated with each node so here it has components, J_n, n = 1, 2, …, 6; J_n for n = 1, 2, 3 represents one source and J_n for n = 4, 5, 6 represents the second source. We propose $\vec{J} \propto \partial_1 \vec{e}$, where e_i are the links of our graph, since

$$\partial_1 \vec{e} = \begin{bmatrix} -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \end{bmatrix} = \begin{bmatrix} -e_1 - e_4 \\ e_1 - e_2 - e_3 \\ e_3 - e_7 \\ e_4 - e_5 \\ e_2 + e_5 - e_6 \\ e_6 + e_7 \end{bmatrix} \tag{19}$$

automatically makes $\vec{J}$ divergence-free, i.e., $\sum_i J_i = 0$,and relationally defined, e.g., vertex 1 is the origin of both links 1 and 4, and the first entry of $\partial_1 \vec{e}$ is $-e_1 - e_4$ (negative/positive means the link starts/ends at that vertex). Since J_n are associated with the vertices to represent sources, $\vec{J} \propto \partial_1 \vec{e}$ is a graphical representation of "relata from relations." [Note: $\partial_1 \vec{e}$, which we denote $\vec{v}\,*$ and associate with $\vec{v}$, is not equal to $\vec{v}$ proper[19].]

With these definitions of $\vec{\vec{K}}$ and $\vec{J}$ we have, *ipso facto*, $\vec{\vec{K}}\vec{v} \propto \vec{J}$ as the basis of our SCC since

$$\partial_1 \partial_1^T \vec{v} = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 2v_1 - v_2 - v_4 \\ -v_1 + 3v_2 - v_3 - v_5 \\ -v_2 + 2v_3 - v_6 \\ -v_1 + 2v_4 - v_5 \\ -v_2 - v_4 + 3v_5 - v_6 \\ -v_3 - v_5 + 2v_6 \end{bmatrix} = \begin{bmatrix} -e_1 - e_4 \\ e_1 - e_2 - e_3 \\ e_3 - e_7 \\ e_4 - e_5 \\ e_2 + e_5 - e_6 \\ e_6 + e_7 \end{bmatrix} = \partial_1 \vec{e} = \vec{v}\,* \tag{20}$$

where we have used $e_1 = v_2 - v_1$ (etc.) to obtain the last column. You can see that the boundary of a boundary principle holds by the definition of "boundary" and from the fact that the links are directed and connect one vertex to another, i.e., they do not start or end

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[19] Thus, we have characterized nodes (relata of links) in terms of the links (fundamental relations) in a non-tautological fashion as alluded to in section 1.

"off the graph." Likewise, this fact and our definition of $\vec{J}$ imply $\sum_i J_i = 0$, which is our graphical equivalent of a divergence-free, relationally defined source (every link leaving one vertex goes into another vertex). Thus, the SCC $\vec{\vec{K}}\vec{v} \propto \vec{J}$ and divergence-free sources $\sum_i J_i = 0$ obtain tautologically via the boundary of a boundary principle. The SCC also guarantees that $\vec{J}$ resides in the row space of $\vec{\vec{K}}$ so, as will be shown, we can avoid having to "throw away infinities" associated with gauge groups of infinite volume as in Fadeev-Popov gauge fixing. Since $\vec{\vec{K}}$ has at least one eigenvector with zero eigenvalue which is responsible for gauge invariance, *the self-consistent co-construction of space, time and divergence-free sources entails gauge invariance*.

Moving now to *N* dimensions, the Wick rotated version of Eq. (3) is Eq. (6)

$$Z = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} dQ_1 \ldots dQ_N \exp\left[-\frac{1}{2} \vec{Q} \cdot \vec{\vec{K}} \cdot \vec{Q} + \vec{J} \cdot \vec{Q} \right]$$

and the solution is Eq. (7)

$$Z = \left(\frac{(2\pi)^N}{\det(K)} \right)^{1/2} \exp\left[\frac{1}{2} \vec{J} \cdot \vec{\vec{K}}^{-1} \cdot \vec{J} \right]$$

Using $\vec{J} = \alpha \partial_1 \vec{e}$ and $\vec{\vec{K}} = \beta \partial_1 \partial_1^T$ (*α, β* ϵ reals) with the SCC gives $\vec{\vec{K}}\vec{v} = \frac{\beta}{\alpha}\vec{J}$, so that $\vec{v} = \frac{\beta}{\alpha}\vec{\vec{K}}^{-1}\vec{J}$. However, $\vec{\vec{K}}^{-1}$ does not exist because $\vec{\vec{K}}$ has a null space, therefore the row space of $\vec{\vec{K}}$ is an (*N*-1)-dimensional subspace of the *N*-dimensional vector space[20]. The eigenvector with eigenvalue of zero, i.e., normal to this hyperplane, is $[1,1,1,\ldots,1]^T$, which follows from the SCC as shown supra. Since $\vec{J}$ resides in the row space of $\vec{\vec{K}}$ and, on our view, *Z* does not reflect a "sum over all paths in configuration space" but is a functional of $\vec{\vec{K}}$ and $\vec{J}$ which produces a partition function for the various $\frac{1}{2}\vec{\vec{K}} + \vec{J}$ associated with different 4D experimental configurations, we restrict the path integral of Eq. (6) to the row space of $\vec{\vec{K}}$, i.e., Eq. (8)

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[20] This assumes the number of degenerate eigenvalues always equals the dimensionality of the subspace spanned by their eigenvectors, which we will see is true for $\vec{\vec{K}}$ in this example.

$$Z = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} d\widetilde{Q}_1 \ldots d\widetilde{Q}_{N-1} \exp\left[\sum_{j=1}^{N-1}\left(-\frac{1}{2}\widetilde{Q}_j^2 a_j + \widetilde{J}_j \widetilde{Q}_j\right)\right]$$

where $\widetilde{Q}_j$ are the coordinates associated with the eigenbasis of $\bar{\bar{K}}$ and $\widetilde{Q}_N$ is associated with eigenvalue zero, a_j is the eigenvalue of $\bar{\bar{K}}$ corresponding to $\widetilde{Q}_j$, and $\widetilde{J}_j$ are the components of $\vec{J}$ in the eigenbasis of $\bar{\bar{K}}$. Thus, our gauge independent approach revises Eq. (7) to give Eq. (9)

$$Z = \left(\frac{(2\pi)^{N-1}}{\prod_{j=1}^{N-1} a_j}\right)^{1/2} \prod_{j=1}^{N-1} \exp\left[\frac{\widetilde{J}_j^2}{2a_j}\right]$$

Since $\vec{J}$ is defined via links we have characterized the symmetry amplitude in terms of relations and the non-zero eigenvalues of $\bar{\bar{K}}$, which is also relational in nature.

Caveat: we chose $\bar{\bar{K}} = \beta\partial_1\partial_1^T$ because it reproduced the action for coupled harmonic oscillators and therein V is quadratic in q. However, keep in mind that q is *not* the spatial location x of a particle in the potential V as we explained above is standard in conventional NRQM, but q is the *field value at a point in space* per our interpretation of the path integral formalism. Thus, one must distinguish between V in the propagator of QFT's free (Gaussian) theory and V in NRQM. We will use the free-particle propagator of QFT, which employs quadratic V per coupled harmonic oscillators, to model the twin-slit experiment since therein the NRQM 'particle' is free and V in the Schrödinger equation is zero. Further, at our proposed fundamental level, it is $\frac{1}{2}\bar{\bar{K}} + \vec{J}$ that provides the basic 4D ontological depiction of the experiment and Q is merely part of the mathematical machinery used to provide a partition function Z for $\frac{1}{2}\bar{\bar{K}} + \vec{J}$.

Returning to Eq. (9), we find in general that half the eigenvectors of $\bar{\bar{K}}$ are of the form $\begin{bmatrix} \vec{x} \\ \vec{x} \end{bmatrix}$ and half are of the form $\begin{bmatrix} \vec{x} \\ -\vec{x} \end{bmatrix}$. The eigenvalues are given by $\lambda \pm 1$ where $\lambda - 1$

is the eigenvalue for $\begin{bmatrix}\vec{x}\\ \vec{x}\end{bmatrix}$, $\lambda + 1$ is the eigenvalue for $\begin{bmatrix}\vec{x}\\ -\vec{x}\end{bmatrix}$, and

$\lambda_j = 3 - 2\cos\left(\frac{j2\pi}{N}\right),\ \ j = 0,\ldots,\left(\frac{N}{2}-1\right)$. The k components of $\vec{x}$ for a given λ_j are

$x_{jk} = \sqrt{\frac{2}{N}}\cos\left(\frac{j(2k-1)\pi}{N}\right),\ \ k = 1,\ldots,\frac{N}{2}$ for $j > 0$ and $x_{0k} = \sqrt{\frac{1}{N}},\ \ k = 1,\ldots,\frac{N}{2}$ for $j = 0$

($j = 0 \rightarrow$ eigenvalues of $\vec{\vec{K}}$ are 0 and 2). As you can see, there are no degeneracies within the $\begin{bmatrix}\vec{x}\\ \vec{x}\end{bmatrix}$ subspace or the $\begin{bmatrix}\vec{x}\\ -\vec{x}\end{bmatrix}$ subspace. Therefore, the only degeneracies occur between subspaces, so we know all degenerate eigenvalues are associated with unique eigenvectors, as alluded to in a previous footnote.

We have N nodes and $(3N/2 - 2)$ links. Define the temporal (vertical) links e_i in terms of vertices v_i in the following fashion:

$$e_i = v_{i+1} - v_i \qquad \text{i = 1 to } N/2 - 1$$

and

$$e_{\frac{N}{2}+i-1} = v_{\frac{N}{2}+i+1} - v_{\frac{N}{2}+i} \qquad \text{i = 1 to } N/2 - 1.$$

Define the spatial (horizontal) links via:

$$e_{N+i-2} = v_{\frac{N}{2}+i} - v_i \quad \text{i = 1 to } N/2.$$

This gives

$$\vec{J} = \begin{bmatrix} -e_1 - e_{N-1} & \\ -e_i + e_{i-1} - e_{N+i-2} & i = 2,\ldots\frac{N}{2}-1 \\ e_{\frac{N}{2}-1} - e_{N+\frac{N}{2}-2} & \\ e_{N-1} - e_{\frac{N}{2}} & \\ e_{\frac{N}{2}+i-2} + e_{N+i-2} - e_{\frac{N}{2}+i-1} & i = 2,\ldots\frac{N}{2}-1 \\ e_{N+\frac{N}{2}-2} + e_{N-2} & \end{bmatrix} \qquad (21).$$

We then need to find the projection of $\vec{J}$ on each of the orthonormal eigenvectors of $\vec{\vec{K}}$ that have non-zero eigenvalues. Call each projection $\tilde{J}_i = \langle i | J \rangle$, where $\langle i |$ is the i$^{\text{th}}$ orthonormal eigenvector. Let a_i ($i = 1, N$-1) be the non-zero eigenvalues of $\vec{\vec{K}}$ associated with the eigenvectors $\langle i |$, ($i = 1, N$-1), respectively. To complete the two-source symmetry amplitude we need to compute the exponent

$$\Phi = \sum_{i=1}^{N-1} \frac{\left(\tilde{J}_i\right)^2}{2a_i \hbar \beta} \tag{22}$$

where $\hbar$ is viewed as a fundamental scaling factor with the dimensions of action. We find $\Phi = (\Phi_S + \Phi_T + \Phi_{ST})/(2\hbar\beta)$, where

$$\Phi_S = \frac{2\alpha^2}{N}\left[\sum_{k=1}^{\frac{N}{2}} e_{k+N-2}\right]^2 \tag{23}$$

involves only spatial links

$$\Phi_T = \frac{2\alpha^2}{N}\sum_{j=1}^{\frac{N}{2}-1}\left[\sum_{k=1}^{\frac{N}{2}-1}\left(e_k + e_{k+\frac{N}{2}-1}\right)\sin\left(\frac{jk2\pi}{N}\right)\right]^2 \tag{24}$$

involves only temporal links and

$$\Phi_{ST} = \sum_{j=1}^{\frac{N}{2}-1}\frac{4\alpha^2}{N\left(1+2\sin^2\frac{j\pi}{N}\right)}\left[\sin\frac{j\pi}{N}\sum_{k=1}^{\frac{N}{2}-1}\left(e_k - e_{k+\frac{N}{2}-1}\right)\sin\left(\frac{jk2\pi}{N}\right) + \sum_{k=1}^{\frac{N}{2}}\left(e_{k+N-2}\right)\cos\left(\frac{(2k-1)j\pi}{N}\right)\right]^2 \tag{25}$$

involves a mix of spatial and temporal links. Eq. (23) comes from the eigenvalue 2 associated with $\begin{bmatrix} \vec{x} \\ -\vec{x} \end{bmatrix}$, which exists for all N under consideration. Eq. (25) comes from the remaining eigenvalues associated with $\begin{bmatrix} \vec{x} \\ -\vec{x} \end{bmatrix}$. Eq. (24) comes from the eigenvalues associated with $\begin{bmatrix} \vec{x} \\ \vec{x} \end{bmatrix}$ having omitted zero, which exists for all N under consideration.

3.3 Theory X. To summarize theory X mathematically:

$$\bar{\bar{K}}\vec{v} \propto \vec{J} \rightarrow \frac{1}{2}\bar{\bar{K}} + \vec{J} \rightarrow Z \rightarrow \mathrm{P}\left(\widetilde{Q}_k = \widetilde{Q}_o\right) = \frac{Z\left(\widetilde{Q}_k = \widetilde{Q}_o\right)}{Z} \rightarrow \bar{\bar{K}} \cdot \vec{Q}_o = \vec{J}\,.$$

In words, the self-consistency criterion $\bar{\bar{K}}\vec{v} \propto \vec{J}$ gives the actional $\frac{1}{2}\bar{\bar{K}} + \vec{J}$ serving as a graphical model of the relations under experimental investigation, and the actional gives the partition function Z for the graph (topological level). The partition function Z gives the probability of a particular experimental outcome $\mathrm{P}\left(\widetilde{Q}_k = \widetilde{Q}_o\right) = \frac{Z\left(\widetilde{Q}_k = \widetilde{Q}_o\right)}{Z}$, i.e., a specific geometric relationship under investigation that comprises, in part, the experimental arrangement (Figure 4), and the most probable of all the possible outcomes renders an average relational description (geometric level) of the experimental arrangement $\bar{\bar{K}} \cdot \vec{Q}_o = \vec{J}$, in so far as it concerns the experiment (Figure 3). Keep in mind there are relations responsible for the experimental equipment presumably not under investigation, e.g., those relations between various pieces of equipment and Mars, the experimentalist, the wall, etc. The art of good experimental procedure is to isolate the relevant results and "screen off" the irrelevant ones.

3.4 The Twin-Slit Experiment. The simple twin-slit experiment is used for a preliminary study of our two-source amplitude since our analysis reproduces the interference pattern without the use of mediating entities, such as waves or particles. We point out again that conventional NRQM uses the free-particle propagator for this case while our two-source amplitude is obtained via the discrete, free (Gaussian) theory fundamental to QFT – those are the two formalisms we relate here in order to gain insight into both. We begin with what we already know of this idealized situation per NRQM, then we make inferences concerning our graph structure via the analytic continuation of Eqs. (23) – (25).

For a free particle of mass *m* we have from NRQM[44]

$$\psi = A\sqrt{\frac{m}{2\pi\hbar it}}\exp\left[\frac{imx^2}{2\hbar t}\right] \propto \exp\left[\frac{imx^2t}{2\hbar t^2}\right] = \exp\left[\frac{imv^2t}{2\hbar}\right] = \exp\left[\frac{ipvt}{2\hbar}\right] = \exp\left[\frac{iv_\varphi t}{\lambda}2\pi\right] \quad (26)$$

where v_φ is the phase velocity and equal to half the particle velocity[45] and $\psi(x,0) = A\delta(x = 0)$. [The conventional NRQM path integral produces a propagator and Eq. (26) is obtained from it by connecting a point Source to a point at the detector, each of these points is understood to be half of our source vector $\vec{J}$, thus our use of the two-source symmetry amplitude.] Using Eq. (26), the twin-slit interference pattern is given by

$$|\psi_1 + \psi_2|^2 \propto \left|\exp\left[\frac{iv_\varphi t_1}{\lambda}2\pi\right] + \exp\left[\frac{iv_\varphi t_2}{\lambda}2\pi\right]\right|^2 = 2 + 2\cos\left[\frac{v_\varphi(t_1 - t_2)}{\lambda}2\pi\right] \quad (27)$$

and therefore maxima occur at angles where

$$v_\varphi(t_1 - t_2) = n\lambda \quad n \in \text{integers} \quad (28).$$

For photons[46]

$$\psi \propto \exp\left[\frac{iE(t_1 - t_2)}{\hbar}\right] = \exp\left[\frac{ihf(t_1 - t_2)}{\hbar}\right] = \exp\left[\frac{ic(t_1 - t_2)}{\lambda}2\pi\right] \quad (29)$$

so maxima occur at angles where

$$c(t_1 - t_2) = n\lambda \quad n \in \text{integers} \quad (30).$$

Since a photon yields a single click (not a series of clicks whence a trajectory), *c* cannot be directly measured for a photon just as v_φ cannot be directly measured for a massive particle, so Eqs. (26) & (29) do not differ structurally. Since the experimental outcome (interference pattern) is time-independent and does not involve successive clicks linked temporally (explicit trajectories), the theoretical description of the interference pattern is purely kinematical (involves concepts of length, time and velocity, but not mass, momentum, force, energy, etc.).

In order to make correspondence with NRQM in this case, we obtain the oscillatory analytic continuation of our Wick rotated result[47]. Since dτ → -i˙dt to obtain the Euclidean action, Eq. (6), we have $\vec{J} \rightarrow i\vec{J}$ in Eq. (7) since J in the continuous action absorbs a discrete time interval to become $\vec{J}$ in the discrete action. Now letting $\Delta t \rightarrow i\Delta\tau$ in Eq. (17) we find

$$\bar{\bar{K}} \to -i\begin{bmatrix} 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 1 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix} \equiv -i\bar{\bar{K}}_M \qquad (31).$$

Plug this back into Eq. (6) and one obtains the analytic continuation, i.e., Eq. (3):

$$Z = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} dQ_1 \ldots dQ_N \exp\left[\frac{i}{2}\vec{Q}\cdot\bar{\bar{K}}_M\cdot\vec{Q} + i\vec{J}\cdot\vec{Q}\right]$$

with solution

$$Z = \left(\frac{(2\pi i)^N}{\det(K_M)}\right)^{1/2} \exp\left[\frac{-i}{2}\vec{J}\cdot\bar{\bar{K}}_M^{-1}\cdot\vec{J}\right] \qquad (32)$$

where $\vec{J}$ has not changed from Eq. (9) (except possibly by the addition of a minus sign due to its association with a force, i.e., a Δt^2 in the denominator, but this does not change the solution). Noting that $\bar{\bar{K}}_M + 2\begin{bmatrix} I & -I \\ -I & I \end{bmatrix} = \bar{\bar{K}}$, we see that the eigenvectors of $\bar{\bar{K}}_M$ are the same as the eigenvectors of $\bar{\bar{K}}$ which have the form $\begin{bmatrix} x \\ x \end{bmatrix}$ and $\begin{bmatrix} x \\ -x \end{bmatrix}$. The $\bar{\bar{K}}_M$ eigenvalues for $\begin{bmatrix} x \\ x \end{bmatrix}$ are the same as for $\bar{\bar{K}}$ and those for $\begin{bmatrix} x \\ -x \end{bmatrix}$ are related to the eigenvalues Λ of $\bar{\bar{K}}$ by $\Lambda - 4$. Since $\vec{J}$, the eigenvectors and half the eigenvalues are the same, there is very little change in Eq. (24) and the phase $\Phi = (\Phi_S + \Phi_T + \Phi_{ST})/(2\hbar\beta)$ as in Eqs. (23 – 25) except that Eq. (23) acquires an overall minus sign (it is obtained from the eigenvalue 2 for an eigenvector of the form $\begin{bmatrix} x \\ -x \end{bmatrix}$ in the Euclidean regime, so this eigenvalue becomes -2 under analytic continuation) and Eq. (25) now acquires a minus sign in the denominator of the first term and becomes

$$\Phi_{ST} = \sum_{j=1}^{\frac{N}{2}-1} \frac{4\alpha^2}{N\left(-1+2\sin^2\frac{j\pi}{N}\right)} \left[\sin\frac{j\pi}{N} \sum_{k=1}^{\frac{N}{2}-1} \left(e_k - e_{k+\frac{N}{2}-1}\right) \sin\left(\frac{jk2\pi}{N}\right) + \sum_{k=1}^{\frac{N}{2}} \left(e_{k+N-2}\right) \cos\left(\frac{(2k-1)j\pi}{N}\right) \right]^2$$

We note that Z is not always well-defined because $\bar{\bar{K}}_M$ has an additional zero eigenvalue when $\bar{\bar{K}}$ has an eigenvalue of 4 associated with $\begin{bmatrix} x \\ -x \end{bmatrix}$. Thus, the magnitude of Z is not defined, as well as Φ_{ST} if $\vec{J}$ has a non-zero projection onto this eigenvector. This is reflected in the fact that $\left(-1+2\sin^2\frac{j\pi}{N}\right)=0$ when $j=\frac{N}{4}$, yet the numerator of Φ_{ST} obtained from the projection of $\vec{J}$ onto this eigenvector may not be zero. Thus, $\vec{J}$ does not necessarily reside in the row space of $\bar{\bar{K}}_M$ so an argument to restrict path integral to the row space of $\bar{\bar{K}}_M$ is significantly weaker than in the Euclidean sector. For this reason, we believe analyses are best conducted in the Euclidean sector, however we will see that, in our simplified case here, $\vec{J}$ does reside in the row space of $\bar{\bar{K}}_M$ so we restrict the path integral to the row space of $\bar{\bar{K}}_M$ rendering the magnitude of Z and its phase Φ_{ST} well-behaved.

Continuing, given the conventional NRQM result above, we must have

$$\Phi = \left[\frac{vt}{\lambda}2\pi\right] \tag{33}$$

where v is simply a scaling factor between space and time in this purely geometric result. In the twin-slit experiment this means

$$\Phi_1 - \Phi_2 = \frac{v(t_1 - t_2)}{\lambda} \quad (34).$$

Again, Z is now a propagator and to render this a wave function (complex probability amplitude) we must assume a delta function Source and sink, as in obtaining

Eq. (26). Strictly speaking, this requires we fix values for $\vec{Q}$ at two nodes, one on $\vec{J}$ for the Source and one on $\vec{J}$ for the click (sink). However, we can't assign a value for $\vec{Q}$ on $\vec{J}$ for the Source because we don't know when that event occurs; we only know when we get a click, so let us assign the click event $\widetilde{Q}_k = \widetilde{Q}_o$. Let e_i be the links of graph 1 (whence Φ_1) and $\widetilde{e}_i$ the links of graph 2 (whence Φ_2) with the two graphs ultimately combined as depicted in Figure 7 to model the twin-slit apparatus. We expect the temporal links of the source representing the click to be equal between graphs since these sources in both graphs represent one and the same click. We also expect the temporal links of the sources representing each slit to be equal since these sources are presumed coherent in the twin-slit experiment. Suppose further that all temporal links of either graph are equal to one another (nothing intrinsic to the experimental configuration requires variable clock rates), so we have $\widetilde{e}_i = e_i = e_T$ for $i = 1$ to $N - 2$, i.e., for the temporal links. We do expect the spatial links to differ between graphs, reflecting the different distances from each slit to a particular click location. Let us assume all spatial links of each graph are equal to one another (static situation) so we have $e_{N+i-2} = e_x$ and $\widetilde{e}_{N+i-2} = \widetilde{e}_x$ for $i = 1$ to $N/2$, i.e., for the spatial links (Figure 7). In this simplified case for Φ_{ST} we have $e_k = e_{k+\frac{N}{2}-1}$ for the temporal links, so $\sum_{k=1}^{\frac{N}{2}-1}\left(e_k - e_{k+\frac{N}{2}-1}\right)\sin\left(\frac{jk2\pi}{N}\right) = 0$, and $\sum_{k=1}^{\frac{N}{2}}\cos\left(\frac{(2k-1)j\pi}{N}\right) = 0$ for any j from 1 to $N/2 - 1$. When $j = \frac{N}{4}$ we have $\left(-1 + 2\sin^2\frac{j\pi}{N}\right) = 0$, so the numerator of Φ_{ST} is $4\alpha^2 e_x^2 (0)^2 = 0$ which means $\vec{J}$ has no projection on the eigenvector with eigenvalue zero, so it resides in the row space of $\bar{\bar{K}}_M$ for this graph. Thus, Φ_{ST} equals zero and our restriction of the path integral to the row space of $\bar{\bar{K}}_M$ is justified for this graph. Eqs. (23) (with minus sign) & (24) are not affected by our restriction to the row space of $\bar{\bar{K}}_M$ since Eq. (23) comes from the eigenvalue 2 associated with $\begin{bmatrix} \vec{x} \\ -\vec{x} \end{bmatrix}$ and Eq. (24) comes from the eigenvalues associated with $\begin{bmatrix} \vec{x} \\ \vec{x} \end{bmatrix}$ having omitted zero. Thus, we have

$$\Phi_S + \Phi_T = -\frac{N}{2}\alpha^2 e_x^2 + (N-2)\alpha^2 e_T^2 \tag{35}$$

since $\sum_{j=1}^{\frac{N}{2}-1}\left[\sum_{k=1}^{\frac{N}{2}-1}\sin\left(\frac{jk2\pi}{N}\right)\right]^2 = \left(\frac{N-2}{4}\right)\left(\frac{N}{2}\right)$, which follows from two results, i.e.,

$$\sum_{k=1}^{\frac{N}{2}-1}\sin\left(\frac{jk2\pi}{N}\right) = \frac{\cos\left(\frac{j\pi}{N}\right) - \cos\left(j\pi\left(1-\frac{1}{N}\right)\right)}{2\sin\left(\frac{j\pi}{N}\right)} = \begin{Bmatrix} 0 & j \quad even \\ \cot\left(\frac{j\pi}{N}\right) & j \quad odd \end{Bmatrix} \text{ and}$$

$\sum_{k=1}^{n}\cot^2\left(\frac{\pi}{2}\frac{2k-1}{2n}\right) = 2n^2 - n$. The remainder of the phase for Z is determined by having fixed $\tilde{Q}_k = \tilde{Q}_o$ which gives

$$Z\left(\tilde{Q}_k = \tilde{Q}_o\right) = \sqrt{\frac{(2\pi i)^{N-2}}{\prod_{j=1, j\neq k}^{N-1} a_j}}\exp\left[-i\left[\frac{1}{2}\tilde{Q}_o^2 a_k + \tilde{J}_k\tilde{Q}_o + \frac{\Phi_S + \Phi_T}{2\beta}\right]\right] \tag{36}$$

When we compute $\Phi_1 - \Phi_2$ the only part of the phase that remains is Φ_S so we have

$$\frac{v(t_1 - t_2)}{\lambda\!\!\!^{-}} = \frac{N\alpha^2}{4\hbar\beta}\left(e_x^2 - \tilde{e}_x^2\right) \tag{37}$$

The numerator of the LHS of Eq. (37) reflects the belief in NRQM that we know when the "particle" was emitted from the Source. Typically, we know only when and where the "particle" hits the screen (spacetime location of the click) and indeed what we ultimately measure is a distance, $\Delta\ell := v(t_1 - t_2)$. Thus, we rewrite Eq. (37) to read

$$\frac{\Delta\ell}{\lambda\!\!\!^{-}} = \frac{N\alpha^2}{4\hbar\beta}\left(e_x^2 - \tilde{e}_x^2\right) \tag{38}$$

which is a purely geometric result in spatial quantities, in accord with what is known in the twin-slit experiment. An interference pattern then allows us to deduce λ, since in those cases $\lambda = n\Delta\ell$ per Eqs. (28) or (30).

Let us therefore suppose that λ is the fundamental, relational unit of length for this particular pair of graphs. We have $[\alpha]$ = (momentum) and $[\beta]$ = (momentum)/(length), and Eqs. (28), (30) and (38) give us

$$\frac{N\alpha^2}{4\hbar\beta}\left(e_x^2 - \tilde{e}_x^2\right) = 2\pi n \qquad n \in \text{integers} \tag{39}$$

With $h = 2\pi\hbar$ the fundamental unit of action we infer $\alpha = h/\lambda$ and $\beta = h/\lambda^2$, so Eq. (39) gives us

$$\left(\frac{N}{2}\right)\frac{\left(e_x^2 - \tilde{e}_x^2\right)}{2} = n \qquad n \in \text{integers} \tag{40}$$

for interference maxima. Eq. (40) implies $Ne_x^2/4$ can be thought of as the number of fundamental, relational length units (let us call them "waves" since $\Delta\ell := v(t_1 - t_2)$ is a "phase distance") represented by the spatial part of the graph. In that case, since $N/2$ is the number of spatial links, $e_x^2/2$ is the number of waves represented by each spatial link[21].

While this analysis is highly heuristic given the underdetermination of variables at this point, it is a reasonable start and does suggest a basis for wave-particle duality and quantum non-separability in our proposed discrete formalism fundamental to quantum physics. Of course, Eqs. (23) – (25) are far more complex than the RHS of Eq. (26), resulting from the less fundamental, continuous formalism of NRQM, and we are not suggesting they be used in place of Eqs. (27) – (30). [Analogously, NRQM did not replace Newtonian mechanics in describing the trajectory of a baseball.] Rather, in this context, we are leaning on the established continuous result to provide analytical guidance for what we believe is the more fundamental discrete approach; in return, the more fundamental discrete result provides conceptual clarity to the established continuous approximation. As we make progress analytically, we expect to move beyond

[21] We then expect e_T^2 to be proportional to the number of fundamental, relational units of time ($T = \lambda/v$) represented by the temporal links. Notice this reflects a particular spatial foliation of spacetime as is customary in NRQM.

providing conceptual clarity to already established formal results and bring our analytic technique to bear on unresolved formal issues.

4. RBW INTERPRETATION OF QFT

Now that we have the ground work for a discrete, graphical path integral account of theory X underlying quantum physics, we focus on the interpretative issues of QFT, i.e., the ontological status of particles and fields, regularization and renormalization, gauge invariance, the AB effect, inequivalent representations, and Haag's theorem. We will also speculate on the basis of Poincaré invariance in theory X.

4.1 Ontological Status of Particles and Fields. From the formalism we see the role of the 'field' (if it can still be called that) in theory X is restricted to the mere designation of the relative spatiotemporal locations of discrete experimental outcomes. *There is no graphical counterpart to "quantum systems" traveling through space as a function of time from Source to sink to "cause" detector clicks.* There are only space, time and sources $\vec{J}$ co-constructed from graphical relations representing the roles of Sources and sinks played by beam splitters, mirrors, particle sources, detectors, etc., in the given experiment from initiation to termination. This implies the empirical goal at the fundamental level is to tell a unified story about detector events to include individual clicks – how they are distributed in space (e.g., interference patterns, interferometer outcomes, spin measurements), how they are distributed in time (e.g., click rates, coincidence counts), how they are distributed in space and time (e.g., particle trajectories), and how they generate more complex phenomena (e.g., photoelectric effect, superconductivity). Thus, particle physics per QFT is in the business of characterizing large sets of detector events. As was eminently apparent from our graphical solution to the discrete scalar, two-source Gaussian amplitude, which gives Z for one Source and one click (sink), it is practically impossible to compute Z (in theory X) for all possible spatiotemporally relative click locations in a particle physics "event," which contains "approximately 100,000 individual measurements of either energy or spatial information[48]." However, we know from theory[49] and experiment that, with overwhelming probability, detector clicks will trace classical paths[22], so it makes sense to

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[22] Individual detector clicks (called "hits in the tracking chamber") are first localized spatially (called "preprocessing"), then associated with a particular track (called "pattern recognition"). The tracks must

partition large click distributions into individual trajectories and treat these as the fundamental constituents of high energy physics experiments[23]. This is exactly what QFT does for particle physics according to our interpretation. Since the individual trajectories are themselves continuous, QFT uses propagators in continuous spacetime which entails an indenumerably infinite number of locations for both clicks and interaction vertices. *Thus, issues of regularization and renormalization are simply consequences of the continuum approximation necessary to deal with very large click distributions, having decided to parse the click distributions into continuum trajectories*[24].

So, the RBW ontology for QFT is neither particles nor fields, both of which have multiple problems in their own right[(50)], but rather the alternative ontology of OSR. While many might consider OSR radical, this is certainly not the first OSR-type account of QFT[(51)]. And, as Kuhlmann notes[(52)]:

> On the background of various problems with particle as well as field interpretations of QFT there are a number of proposals for alternative ontological approaches. In Auyang[(53)] 1995 and Dieks[(54)] 2002 different versions of event ontologies are proposed. Seibt[(55)] 2002 and Hättich[(56)] 2004 defend process-ontological accounts of QFT.

Given the notorious problems with the ontology of particles and fields in QFT, the complete lack of consensus about the right alternative ontology and the foundational importance of OSR-type views in the history and interpretation of QFT via symmetry

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then be parameterized to obtain dynamical characteristics (called “geometrical fitting”). See Fernow, R.C.: Introduction to experimental particle physics. Cambridge University Press, Cambridge (1986), sections 1.7.1, 1.7.2 & 1.7.3, respectively.

[23] Some assumptions are required, e.g., “Sometimes it is necessary to know the identity (i.e., the mass) of at least some of the particles resulting from an interaction” (Fernow, 1986, p 17), “Within the errors [for track measurements], tracks may appear to come from more than one vertex. Thus, the physics questions under study may influence how the tracks are assigned to vertices” (Fernow, 1986, p 25), and “Now there must be some minimum requirements for what constitutes a track. Chambers may have spurious noise hits, while the chambers closest to the target may have many closely spaced hits. The position of each hit is only known to the accuracy of the chamber resolution. This makes it difficult to determine whether possible short track combinations are really tracks” (Fernow, 1986, p 22). Despite these assumptions, no one disputes the inference. While we do not subscribe to the existence of “click-causing entities,” we agree that clicks trace classical paths. Indeed, we believe this is the basis for the appearance of classical reality and consequently, the results of particle physics experiments characterize the transition from quantum phenomena to classical phenomena.

[24] There is also the issue of infinities which arise because of infinite spacetime volume (so-called infrared/IR divergences). In our version of theory X, infinite spacetime intervals between a finite number of sources are not a problem, but the jury is out for situations with an infinite number of sources. Obviously, this also bears on IR-inequivalences as discussed below.

groups, we ask the reader to set aside prejudice and *a priori* metaphysical reservations and rather judge our enterprise on what problems we can solve with our brand of OSR.

4.2 Gauge Invariance. Of the various interpretations of gauge invariance in the literature(57), ours is closest to the view of non-separable, as opposed to non-local (non-localized but no action-at-a-distance), *gauge potential properties* encoded by holonomies. As Drieschner *et al*. put it(58), "since holonomies do not uniquely correspond to regions of space, they render gauge theories non-separable." As Healey says(59), "holonomy properties may act locally even if they are not 'locally possessed'." While an underlying discrete structure is at odds with the continuum basis necessary to support parallel transport as defined on differentiable manifolds, one can easily discuss holonomy in the context of graphs using paths constructed from links(60). In fact, it is in this vein that we have chosen the structural form of the discrete source vector $\vec{J}$ and discrete differential operator $\vec{\vec{K}}$ for the SCC $\vec{\vec{K}}\vec{v} \propto \vec{J}$, which guarantees that $\vec{J}$ is divergence-free and resides in the row space of $\vec{\vec{K}}$. Since we have demanded that $\vec{J}$ be relationally defined per our brand of OSR, we expect[25] $\sum_i J_i = 0$, i.e., any given element of $\vec{J}$, call it J_i, is constructed using relations to other elements of $\vec{J}$, so those other elements must contain the exact opposite relations to J_i in their co-definitions, e.g., if you're standing in front of me in a line of people, then I'm standing behind you. This is what we mean by divergence-free $\vec{J}$ in theory X. Therefore, it is always the case that $\vec{\vec{K}}$ will have a non-trivial eigenvector with eigenvalue zero, i.e., $[1,1,\ldots,1]^T$. Of course, that $\vec{\vec{K}}$ has a non-trivial null space is responsible for gauge invariance[26]. Thus, *gauge invariance is a consequence of the SCC per our brand of OSR[27]*, which leads us to associate our interpretation of gauge invariance with that of non-separable gauge potential properties encoded by holonomies. Now, again, the symmetry amplitude Z is not a sum over all

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[25] See also section IV.B of Sorkin, R.: The electromagnetic field on a simplicial net. Journal of Mathematical Physics 16, 2432-2440 (1975).

[26] Only non-trivial eigenvectors with eigenvalues of zero will lead to gauge invariance, so we are only interested in non-trivial null spaces. Thus, we drop the qualifier "non-trivial" hereafter.

[27] Such a relationship on discrete spacetime lattices is not without precedent. For example, Sorkin showed that charge conservation follows from gauge invariance for the electromagnetic field on a simplicial net. See section IV.C of Sorkin, R.: The electromagnetic field on a simplicial net. Journal of Mathematical Physics 16, 2432-2440 (1975).

paths in configuration space but a partition function for $\frac{1}{2}\bar{\bar{K}} + \bar{J}$, so it makes perfect sense for us to restrict the path integral to the row space of $\bar{\bar{K}}$ and $\bar{J}$. *This automatically removes infinities associated with gauge groups of infinite volume*, which one must otherwise "throw away" in Fadeev-Popov gauge fixing. And, since there is no gauge fixing, there are no commensurate ghost fields.

That QFT on discrete lattices deals successfully with gauge invariance is already well known, e.g., Wilson's formulation(61) of Yang-Mills theory, but we bring a new interpretation which improves the outlook for this approach. First, as a fundamentally discrete path integral formalism where sources $\bar{J}$, space and time are self-consistently co-constructed, we deal pragmatically and empirically with finite spacetime regions of non-zero graphical spacing so we have no concerns about infrared and ultraviolet divergences associated with regularization and renormalization. In particular, as implied *supra*, the analysis of particle physics experiments should be concerned with the spatiotemporal distributions of clicks at the pixels of the detector[28]. Thus, the possible discrete spatial locations for the sinks of $\bar{J}$ are the pixels of the detector and the discrete time intervals would be bounded from below by the temporal resolution of the detector. Second, our interpretation allows us to deal with the primary concern of discrete lattice theories, i.e., its perceived problem with Poincaré invariance.

4.3 Poincaré Invariance. Theory X provides a basis for the invariance of physics under spacetime translations and spatial rotations because these transformations depend on the worldtube segment (graph) depicting a system undergoing some process (experiment from initiation to termination) being embedded in a "surrounding" empty M4. For example, if one imagines a worldtube segment embedded vertically in (2+1)-dimensional spacetime (time is up and down, space is the horizontal plane), the geometric structure of the tube does not change if the tube is moved up or down, side to side or back and forth (spacetime translations), since time and space are homogeneous. Spatial isotropy dictates that rotating the tube about the vertical axis (spatial rotation) does not change the geometric structure of the tube. The graph of theory X for the worldtube is the basis of

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[28] This illustrates an important distinction between our discrete approach and lattice gauge theory, i.e., all spacetime relations are used to construct sources, so *we have no empty spacetime* – there is nothing "happening" between pixels. This was evident in our treatment of the twin-slit experiment in section 3.

the 4D counterparts to these invariances because there is no 4D "empty spacetime" surrounding the graph in which to translate or rotate it and this invariance is not based on a deformation of the graph proper. That is, invariance under spacetime translations and spatial rotations in the formalism of spacetime + matter must be explicitly checked, but these invariances are not even questionable in spacetimematter. However, the remaining Poincaré invariance, the Lorentz transformation or boost, is based on a deformation of the graph proper.

Because boosts are "rotations" whereby vectors in the spatial plane acquire a temporal component (and vice-versa), one has to consider the anisotropy between space and time represented by the Lorentz signature of the metric in M4 (one can no longer simply visualize a tube segment rotated in 3-dimensional Euclidean space). Where does this difference originate with our graph? Until we link the formalism to Regge calculus, we can only speculate as to how this will happen but the speculation is consistent with conventional path integral approach to GR, as we will now see.

Consider a simple (1+1)-dimensional graph shaped like a ladder (Figure 5). Recall, we start with a graphical rendering of spacetimematter in which spatial and temporal dimensions are germane to the construct of sources $\vec{J}$, so there is no ambiguity about the (n+1) structure, where n is the dimensionality of space used in the construct of $\vec{J}$, i.e., $\vec{J} \propto \partial_1 \vec{e}$ (as argued in section 2). Essentially, space is used to differentiate various temporal segments of $\vec{J}$ and there are two such temporal segments in our ladder graph, the right and left sides of the graph. Thus, shifting one side of the ladder relative to the opposite side can have consequence if the deformation goes "too far" and obscures the spatial and temporal distinction, thereby changing the number of temporal segments in $\vec{J}$. So, the orientation of spatial/temporal links must remain spatial/temporal under the deformation. As long as that rule is obeyed, the structure of the graph, as it concerns $\vec{J}$, is invariant. Now we argue that the geometric consequence of this topological invariance is Lorentz invariance.

In the modified Regge calculus version of our graphical approach[(62)], the field is the square of link lengths and the source is the stress-energy tensor on those links (Figure 1). Recall, a classical Object is actually composed of many sources (Figure 3), so Q_o is

the most probable value of Q between sources in two *different* classical Objects. Thus, when we solve $\bar{\bar{K}} \cdot \vec{Q}_o = \vec{J}$ for Q_o, we're finding the average value of Q_o between the sources of distinct classical Objects. Since Q takes on all values between $+\infty$ and $-\infty$ in the computation of Z, we simply let positive values of Q_o denote space-like relationships, negative values denote time-like relationships and zero denote null relationships (demarcation between temporal and spatial orientations) in order that Q be consistent with the differentiated structure of space and time in the graphical co-construction of sources $\vec{J}$. Thus, Q_o is a Lorentz-invariant quantity, so while the graph provides a partition function Z *a la* Euclidean path integrals, it may be possible to have our discrete spacetimematter graph provide a basis for exact Poincaré invariance in a natural fashion.
4.4 The Aharonov-Bohm Effect. Unsurprisingly, given our Healey-friendly account of gauge invariance, like Healey(63) we interpret the Aharonov-Bohm effect (AB effect) in terms of non-separability rather than causal non-locality. As he notes(64), "There need be no action at a distance if the behavior both of the charged particles and of electromagnetism are non-separable processes." However, unlike Healey(65) and others(66), quantum non-separabilility and the AB effect have precisely the same origin in our approach, i.e., both result from the SCC so that detector clicks evidence the non-separable nature of the devices in the experiment (recall the non-separability of J_1, J_2 and J_3 in our explanation of the twin-slit experiment).

Given the analysis of the twin-slit experiment, the spatial links alone are responsible for the interference pattern, so let us focus on the spatial projection of our graphical model of the twin-slit experiment with the scalar Sources (slits) being labeled J_1 and J_2, and the detector event being labeled J_3 (Figure 8). Now we want to couple these three sources to a directional source J_S (Figure 9). Since this source is not in phase with J_1 and J_2, it is clear that the resulting analysis would be far more complex than that for Eqs. (39) & (40). However, heuristically, we expect the directional nature of J_S to result in two different forms for its coupling to J_1, J_2, and J_3 so that two different locations of J_3 would be required to satisfy the interference criterion for the two different directions of J_S. In a sense, the direction of J_S adds an orientation to the otherwise non-oriented triangle J_1-J_3-J_2.

4.5 IR and UV-inequivalences. We now want to say something about the inequivalent representations which exist in QFT due to the failure of the Stone-von Neumann theorem to apply to the infinite degrees of freedom generated by the underlying (associated) spacetime manifold. As Wallace[(67)] notes, there are two ways in which inequivalent representations can occur in QFT, one associated with the short distance and high-energy (ultraviolet) degrees of freedom (UV-inequivalence) and one associated with the long-distance (infrared) degrees of freedom (IR-inequivalence). As Wallace points out[(68)], discrete QFT "has only finitely many degrees of freedom per space-time point, and hence no UV inequivalent representations." Obviously, this applies to our theory X even though it differs from discrete QFT on lattices as explained *supra*. [One can dismiss this problem nearly as neatly with QFT if one *already* subscribes to QFT as an effective theory[(69)].] As for the "global" or IR-inequivalences there is the potential for ontological ambiguity if theory X is applied to an infinite spacetime region, i.e., a finite number of sources with infinite spatiotemporal separations or an infinite number of sources at finite separations. Our spacetimematter view of theory X suggests that it is well behaved for a finite number of sources as their spatiotemporal separations become infinite since there is no infinite spacetime distribution of fields *surrounding* sources $\vec{J}$, and *Z* is well behaved as Q_o goes to infinity *between/among* $\vec{J}$ in that case. It remains to be seen if theory X is well behaved in the case of an infinite number of sources where one would have to deal with infinite-dimensional matrices, but heuristically, we expect outcomes in some finite graphical subset of such a distribution to be affected less by sources at increasing values of Q_o, as is the case for a finite number of sources with increasing Q_o. If this does not hold, spurious experimental outcomes could always be blamed on events outside the experiment's causal horizon. Certainly the unity ontology we are proposing suggests this possibility, but it's not a concern as long as the probabilities for such events are vanishingly small. Thus, we do not believe the ontological ambiguity associated with infinitely large Q_o and/or $\vec{J}$ will ever be of practical or empirical consequence where, pragmatically, one is only dealing with detectors of finite size and, empirically, the amount of the spacetime manifold accessible to observation is finite[(70)]. This view of theory X resonates with Jackiw's sentiment[(71)], "the consensus is that infrared divergences do not arise from any intrinsic defect of the theory, but rather from illegitimate attempts at forcing the theory to address unphysical questions." And, it is also in accord with Wallace's belief that, within its domain of applicability, QFT has no foundational problems with IR or UV-inequivalences.

4.6 Haag's Theorem. Closely related to representational inequivalances is Haag's theorem[(72)]. A proper treatment of this topic would constitute a paper in and of itself, so we cannot do it justice in a mere subsection. However, our version of theory X and the RBW ontology suggest a novel resolution to the problem Haag's theorem creates for the interaction picture of QFT, so we offer a short version here. To briefly summarize the problem[(73)]:

> … the interaction picture presupposes all of the assumptions needed to prove [Haag's] theorem; but this theorem shows that the interaction picture cannot be used to represent a non-trivial interaction. And yet the interaction picture and perturbation theory work. Some explanation of why they work is called for.

Ideally, one would like to negate an offending assumption in the proof of Haag's theorem in order to explain the effectiveness of the interaction picture. Clearly for us, Poincaré invariance is not the offending assumption, since theory X underwrites Poincaré invariance per subsection 4.3. We believe the offending assumption is not articulated in the proof of Haag's theorem but is a tacit assumption of QFT in general – the fundamentality of the field quanta, i.e., the normal modes of the classical fields quantized by QFT which supply a basis for Fock space. Per theory X, the field is an approximation needed to solve problems with large numbers of detector events (again, no one in their right mind would try to use theory X to compute probabilities for the distributions of a hundred thousand detector clicks). However, when using this approximation with the interaction picture the precision breaks down as pointed out by Haag's theorem. While we have certainly not mapped theory X to the interaction picture, we can speculate as to why this happens.

In theory X, there are only Sources and sinks co-constructed with space and time per spacetimematter and one calculates the probability of a specific configuration relative to others in the set of all configurations relevant to a particular experimental arrangement from initiation to termination (that is, in 4D). Contrast this with the interaction picture whereby one has a set of incoming fields that interact and produce outgoing fields. Accordingly, one is to calculate the relative probabilities of the various possible configurations of outgoing fields and these configurations manifest large scale patterns of detector events, i.e., the field quanta manifest trajectories in the bubble chambers and calorimeters of particle physics detectors. Thus, according to theory X, the quanta of the incoming fields (supplied by the accelerator) and outgoing fields (measured by the detectors) are modeled by a set of Sources and sinks in one (enormous) 4D graph.

So, essentially, the 4D graph for a particular particle physics experiment is composed of two parts – incoming and outgoing field quanta where quanta are composed of sources. The manner by which detector clicks (which evidence sources) lend themselves to the construct of trajectories (which evidence field quanta) was pointed out in footnotes 22 and 23 of section 4.1. At this stage, one is dealing with "free" outgoing fields in that the spatiotemporal distribution of the sources giving rise to a particular outgoing trajectory is affected far more by the external electro- and magnetostatic fields in the detector than the sources associated with the other trajectories in the detector, so we can ignore relations between sources in different trajectories as was done between J_1 and J_2 in Figure 8. One likewise assumes the sources of the accelerator beam are composed of "free" incoming field quanta in that the spatiotemporal distribution of the sources giving rise to a particular incoming trajectory is affected far more by the electromagnetic field of the accelerator than the sources associated with the other trajectories in the accelerator beam. These two aspects of the experiment are modeled by the free field terms of the Lagrangian. The question is then, how probable are the various possible outcomes (sets of outgoing quanta) for a particular set of incoming quanta? Answering this question in terms of theory X requires marrying up the two parts of the graph and in this process one cannot ignore source-source relations (as between J_1, J_2 and J_3 in Figure 8). In the path integral approach of Lagrangian QFT, one uses interaction terms between the free fields satisfying certain symmetry constraints to answer this question.

So what do the interaction terms in the Lagrangian of the interaction picture correspond to in theory X? Unlike Figure 8, in particle physics experiments we would need many thousands of sources representing the accelerator beam and detector trajectories. Just as there are many different Clusters consistent with a particular classical Object, there are many spatiotemporal geometric distributions over a graph consistent with a particular set of detector trajectories, since a trajectory can be comprised of clicks in many different ways so as to satisfy the symmetry constraints. Thus, all of these different distributions contribute to the probability of a particular particle physics outcome. Specifically, the sources composing each incoming/outgoing field quantum can serve as Sources and sinks in the overall collection of quanta in various ways, and each of these various ways constitutes a different spatiotemporal geometric distribution over the graph. Thus, we speculate that this graphical ambiguity corresponds to summing terms in the perturbative expansion of the interaction Lagrangian with interaction vertices given by the various Source-to-sink relationships (as in Figure 8). In essence, that means the incoming and outgoing "free" field quanta can be co-constructed in various ways via the free fields themselves. Just as in Figure 8 where there are only Sources (J_1 and J_2) and sinks (J_3) – that is, there are no "quantum entities" leaving the Sources and moving through space to impinge on the detector to cause a click – in theory X underlying interaction QFT there are no "interaction fields" that change incoming "free" field quanta into outgoing "free" field quanta, there are only incoming and outgoing quanta modeled by various spatiotemporal geometric distributions over the graph. Certainly it is necessary to treat field quanta as fundamental when dealing with enormous data sets, but decomposing the trajectories (incoming/outgoing quanta) into individual detector events (sources) provides a computational precision that QFT can only approximate indirectly via its perturbative formalism. Thus, as with regularization and renormalization, Haag's theorem simply elucidates part of the price one pays for making the approximations needed to do the "messy" business of particle physics.

5. CONCLUSION

5.1 Summary. We have introduced a new interpretation of QFT that assumes the existence of a discrete theory (X) fundamental to quantum physics, the characteristics of which we articulated and explored via a path integral formalism over graphs. Ontologically, the classical Objects described by classical field theory are composed of field quanta per QFT and the field quanta are composed of discrete sources per theory X. Decomposing field quanta into 4D discrete sources finishes the (historical) progression from the dynamical characterization of phenomena in classical physics to the adynamical characterization of phenomena in theory X. Mathematically, one can summarize our version of theory X as follows:

$$\bar{\bar{K}}\vec{v} \propto \vec{J} \rightarrow \frac{1}{2}\bar{\bar{K}} + \vec{J} \rightarrow Z \rightarrow \mathrm{P}\left(\widetilde{Q}_k = \widetilde{Q}_o\right) = \frac{Z\left(\widetilde{Q}_k = \widetilde{Q}_o\right)}{Z} \rightarrow \bar{\bar{K}} \cdot \vec{Q}_o = \vec{J}\ .$$

While the mathematical details of theory X provided herein are too simplistic to unify physics formally, they do provide a mathematical articulation of a radical new ontology – an OSR blockworld called Relational Blockworld (RBW) – with the potential to unify physics conceptually and suggest a new approach to quantum gravity formally. According to this ontology, the most fundamental "level" of reality is not to be described via some fundamental entity or entities evolving in time according to dynamical laws against a spacetime background per certain boundary conditions, i.e., not via "spacetime + matter." Instead, the most fundamental "level" of reality is topologically based on the self-consistency and unity of space, time and divergence-free sources $\vec{J}$. We call this fusion "spacetimematter" and it is used to describe relationships among the constituents of the experimental apparatus which are under investigation in an experiment. We used this ontology and commensurate (simplified) formalism to address various conceptual and technical issues associated with QFT, specifically we provided an OSR alternative to problematic particle and field ontologies that also explains the need for regularization and renormalization, explained gauge invariance, provided a unified account of the Aharonov-Bohm effect and quantum non-separability, and largely discharged the problems of inequivalent representations and Haag's theorem. We also showed how a discrete, graphical theory X might provide a basis for exact Poincaré invariance, which includes Lorentz invariance, a known problem for discrete lattice theories in general.

5.2 Implications for unification and quantum gravity. While the formalism we presented is simplistic, we believe it contains valuable insights for a new direction in quantum gravity, i.e., the unification of quantum physics with GR. In our approach, for example, gauge invariance is a consequence of the SCC and one can naturally avoid problems associated with gauge fixing. This follows from the fact that the SCC guarantees $\vec{J}$ is divergence-free and resides in the row space of $\bar{\bar{K}}$, where by divergence-free $\vec{J}$ we mean

$\sum_i J_i = 0$. Therefore, it is always the case that $\bar{\bar{K}}$ will have a non-trivial eigenvector with eigenvalue zero, i.e., $[1,1,\ldots,1]^T$; and, that $\bar{\bar{K}}$ has a non-trivial null space is responsible for gauge invariance. Since the transition (or "symmetry") amplitude Z is a partition function for $\frac{1}{2}\bar{\bar{K}} + \vec{J}$, we restrict Z to the row space of $\bar{\bar{K}}$ and $\vec{J}$, which automatically avoids infinities associated with gauge groups of infinite volume. Thus, there is no gauge fixing and there are no commensurate ghost fields. As with the rest of our proposed theory X, this notion of gauge invariance is not *truly* new to the formalism of physics, as evidenced by the following textbook examples.

For example[74], in the Maxwell action one has the operator $K_{\alpha\beta} = \partial^2\eta_{\alpha\beta} - \partial_\alpha\partial_\beta$ and the vector field A^β ($\eta_{\alpha\beta}$ is the metric in M4). Maxwell's equation $\partial_\alpha F^{\alpha\beta} = J^\beta$ results from $K_{\alpha\beta}A^\beta \propto J_\alpha$, which guarantees a divergence-free source, i.e., $\partial_\alpha J^\alpha = 0$, and is invariant under the gauge transformation $A^\beta \rightarrow A^\beta - \partial^\beta\Lambda$ since $K_{\alpha\beta}\partial^\beta\Lambda = 0$, i.e., $\partial^\beta\Lambda$ is an eigenvector of $K_{\alpha\beta}$ with eigenvalue zero. The operator $-\eta_{\alpha\beta}\partial^2$ is then inverted to find the propagator in the Feynman gauge. In our approach to theory X, we are proposing an SCC underneath quantum physics whence $\bar{\bar{K}} \cdot \vec{Q}_o = \vec{J}$ (the continuum approximation is classical FT), or formally $\bar{\bar{K}}\vec{v} \propto \vec{J} \rightarrow K_{\alpha\beta}A^\beta \propto J_\alpha \rightarrow$ $\partial_\alpha F^{\alpha\beta} = J^\beta$, where $\sum_i J_i = 0 \rightarrow \partial_\alpha J^\alpha = 0$.

As another example, in the weak field expansion of the Einstein-Hilbert action[75] one has the operator $K_{\mu\nu;\lambda\sigma} = \frac{1}{2}\partial^2\left(\eta_{\mu\lambda}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\sigma}\right) - \partial^2\eta_{\mu\lambda}\eta_{\nu\sigma} + \frac{1}{2}\partial_\alpha\partial_\beta\eta_{\lambda\sigma}$ and the tensor field $h_{\lambda\sigma}$. The QFT counterpart to our SCC $\bar{\bar{K}}\vec{v} \propto \vec{J}$ is then $K_{\mu\nu;\lambda\sigma}h^{\lambda\sigma} \propto T_{\mu\nu}$ which gives linearized gravity in M4, i.e., $\partial^2 h_{\mu\nu} \propto \left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T^\alpha_\alpha\right)$ in the harmonic gauge, and ensures a divergence-free source, i.e., we are proposing $\sum_i J_i = 0 \rightarrow \partial_\alpha T^{\alpha\beta} = 0$. $K_{\mu\nu;\lambda\sigma}h^{\lambda\sigma} \propto T_{\mu\nu}$ is invariant under $h^{\lambda\sigma} \rightarrow h^{\lambda\sigma} + \partial^\lambda e^\sigma + \partial^\sigma e^\lambda$ since $K_{\mu\nu;\lambda\sigma}\left(\partial^\lambda e^\sigma + \partial^\sigma e^\lambda\right) = 0$, i.e., $\partial^\lambda e^\sigma + \partial^\sigma e^\lambda$ is an eigenvector of $K_{\mu\nu;\lambda\sigma}$ with eigenvalue zero.

The operator $-\frac{1}{2}\partial^2\left(\eta_{\mu\lambda}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\lambda}-\eta_{\mu\nu}\eta_{\lambda\sigma}\right)$ is then inverted to find the propagator in the harmonic gauge. Thus, we are proposing $\bar{\bar{K}}\vec{v} \propto \vec{J} \rightarrow K_{\mu\nu;\lambda\sigma}h^{\lambda\sigma} \propto T_{\mu\nu}$ should bear on GR as we explain below.

With these examples and the RBW ontology we can now speculate on how our theory X might impact unification and the program of quantum gravity. First, given our fundamental OSR graphical ontology for QFT, empirical evidence for the fundamental basis of physics is probably not going to be found in the collision debris of high energy particle accelerators. That is, in our theory X, high energy particle physics is not a study of what is truly fundamental about reality since the spatiotemporal relationships between *individual clicks* reveal the fundamental structure while particle physics assumes the spatiotemporal relationships between *individual trajectories* reveal the fundamental structure. Second, theory X should have something novel to say about how quantum physics is to be carried out in curved spacetime, a point we want to elaborate on.

Essentially, to understand how quantum physics is to be married up with curved spacetime, we need to know how GR is to be "corrected" by theory X. It seems to us that the most glaring deviation from GR phenomena posed by directly connected sources per theory X would be found in the exchange of photons on cosmological scales. To explore this situation, we studied large redshift photon exchange in the Einstein-de Sitter cosmology (EdS) using modified Regge calculus (MORC)[76]. Specifically, we constructed a Regge differential equation for the time evolution of the scale factor *a*(t) in EdS. We then introduced two modifications to the Regge calculus approach: 1) we allowed the graphical links on spatial hypersurfaces to be large, as when the interacting sources reside in different galaxies, and 2) we assumed luminosity distance D_L is related to graphical proper distance D_p by the equation $D_L = (1+z)\sqrt{\vec{D}_p \cdot \vec{D}_p}$ where z is redshift and the inner product was allowed to differ from its usual trivial form. There are two reasons we made this second assumption. First, in our view, space, time and sources are co-constructed, yet D_p is found without taking into account EM sources responsible for D_L. That is to say, in Regge EdS (as in EdS) we assume that pressureless dust dominates the stress-energy tensor and is exclusively responsible for the graphical notion of spatial distance D_p. However, even though the EM contribution to the stress-energy tensor is negligible, EM sources are being used to measure the spatial distance D_L. Second, in the continuous, GR view of photon exchange, the expansion of space orthogonal to the photons' paths decreases their numeric intensity exactly as if they had been emitted at a distance d_p without expansion. The loss of energy per photon due to redshift then gives $D_L = (1+z)d_p$. In our view, there are no "photon paths being stretched transversely by expanding space," so we cannot simply assume $D_L = (1+z)D_p$ as in EdS. The form for that inner product that we employed was borrowed from linearized gravity in the harmonic gauge above. In the case here however, $h_{\alpha\beta}$ corrects the graphical inner product $\eta_{\alpha\beta}$ in the inter-nodal region between the worldlines of photon emitter and receiver ($ds^2 = -c^2dt^2 + dD_p^2$), where $\eta_{\alpha\beta}$ is obtained via a matter-only stress-energy tensor. Since the EM sources are negligible in the matter-dominated solution and we're only considering a classical deviation from a classical background, we used $\partial^2 h_{\alpha\beta} = 0$ and solved for h_{11}, as we need $\vec{D}_p \cdot \vec{D}_p = (1+h_{11})D_p^2$. Obviously, $h_{11} = 0$ is the solution that gives the trivial relationship of EdS, but allowing for h_{11} to be a function of D_p allows for the possibility that D_L and D_p are not trivially related. We have $h_{11} = \mathrm{A}D_p + \mathrm{B}$ where A and B are constants and, if the inner product is to reduce to $\eta_{\alpha\beta}$ for small D_p, we have B = 0. Presumably, A should follow from the corresponding theory of quantum gravity, so an experimental determination of its value provides a guide to quantum gravity per our view of classical gravity. Of course, we expect A to be small and indeed (as explained below) we found A^{-1} = 8.38 Gcy, so the correction to η_{11} is negligible except at cosmological distances.

MORC, EdS and the concordance model ΛCDM (EdS plus a cosmological constant to account for dark energy) were compared using the data from the Union2 Compilation[77], i.e., distance moduli μ and redshifts z for type Ia supernovae (Figure 11). We found that a best fit line through $\log\left(\frac{D_L}{Gpc}\right)$ versus $\log(z)$ gives a correlation of 0.9955 and a sum of squares error (SSE) of 1.95. By comparison, the best fit ΛCDM gives SSE = 1.79 using H_o = 69.2 km/s/Mpc, $\Omega_M = 0.29$ and $\Omega_\Lambda = 0.71$. The parameters for ΛCDM yielding the most robust fit to "the Wilkinson Microwave Anisotropy Probe data with the latest distance measurements from the Baryon Acoustic Oscillations in the distribution of galaxies and the Hubble constant measurement[78]" are $H_o = 70.3$ km/s/Mpc, $\Omega_M = 0.27$ and $\Omega_\Lambda = 0.73$, which are consistent with the parameters we found for its Union2 Compilation fit. The best fit EdS gives SSE = 2.68 using $H_o = 60.9$ km/s/Mpc. The best fit MORC gives SSE = 1.77 and $H_o = 73.9$ km/s/Mpc using $R = A^{-1} = 8.38$ Gcy and $m = 1.71 \times 10^{52}$ kg, where R is the current graphical proper distance between nodes and *m* is the nodal mass. A current (2011) "best estimate" for the Hubble constant is $H_o = (73.8 \pm 2.4)$ km/s/Mpc[79]. Thus, MORC improves EdS as well as ΛCDM in accounting for distance moduli μ and redshifts z for type Ia supernovae without having to invoke accelerated expansion, i.e., there is no dark energy and the universe is always decelerating.

This is but one test of the RBW approach and MORC must pass more stringent tests in the context of the Schwarzschild solution where GR is well confirmed. However, MORC's empirical success in dealing with dark energy gives us reason to believe this formal approach to classical gravity may provide creative new techniques for solving other long-standing problems, e.g., quantum gravity, unification, and dark matter. In particular, if MORC passes empirical muster in the context of the Schwarzschild solution, then information such as A^{-1} might provide guidance to a theory of quantum gravity underlying a graphical classical theory of gravity. Of course, whether this conditional can be satisfied is highly uncertain, so we will not speculate further here.

ACKNOWLEDGEMENT

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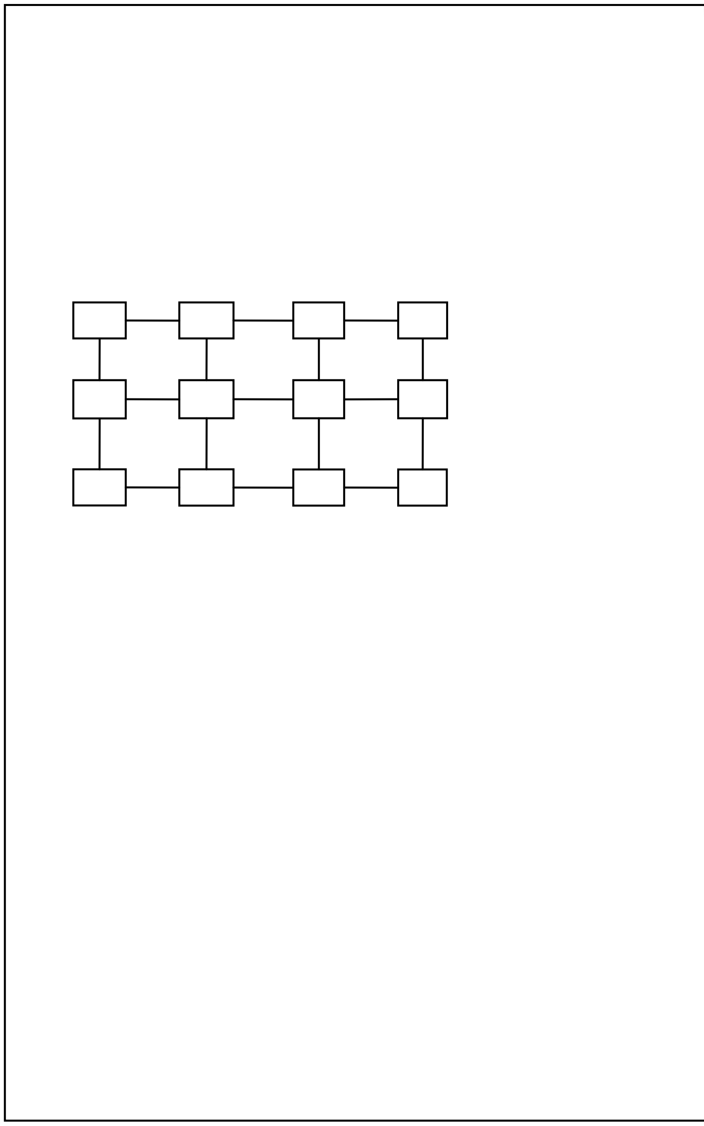

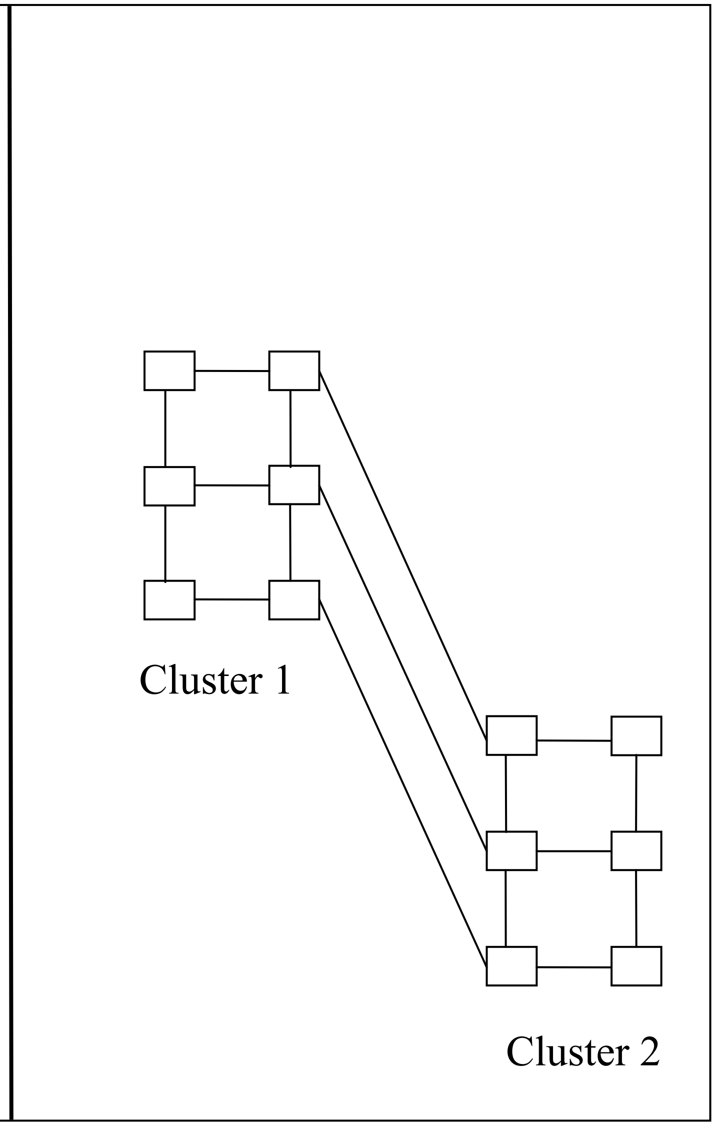

Figure 1. Topological Graph (Left) – This spacetimematter graph depicts four sources, i.e., the columns of squares. The graph's actional $\frac{1}{2}\vec{\vec{K}} + \vec{J}$, such that $\vec{\vec{K}}\vec{v} \propto \vec{J}$, characterizes the graphical topology, which underwrites a partition function Z for spatiotemporal geometries over the graph.

Figure 2. Geometric Graph (Right) – The topological graph of Figure 1 is endowed with a particular distribution of spatiotemporal geometric relations (link lengths). The short links in Clusters 1 & 2 reflect the various 'smaller' values of Q (metric) between sources (squares) in those Clusters. The long links between sources of Clusters 1 & 2 reflect the various 'larger' values of Q between sources in the two different Clusters.

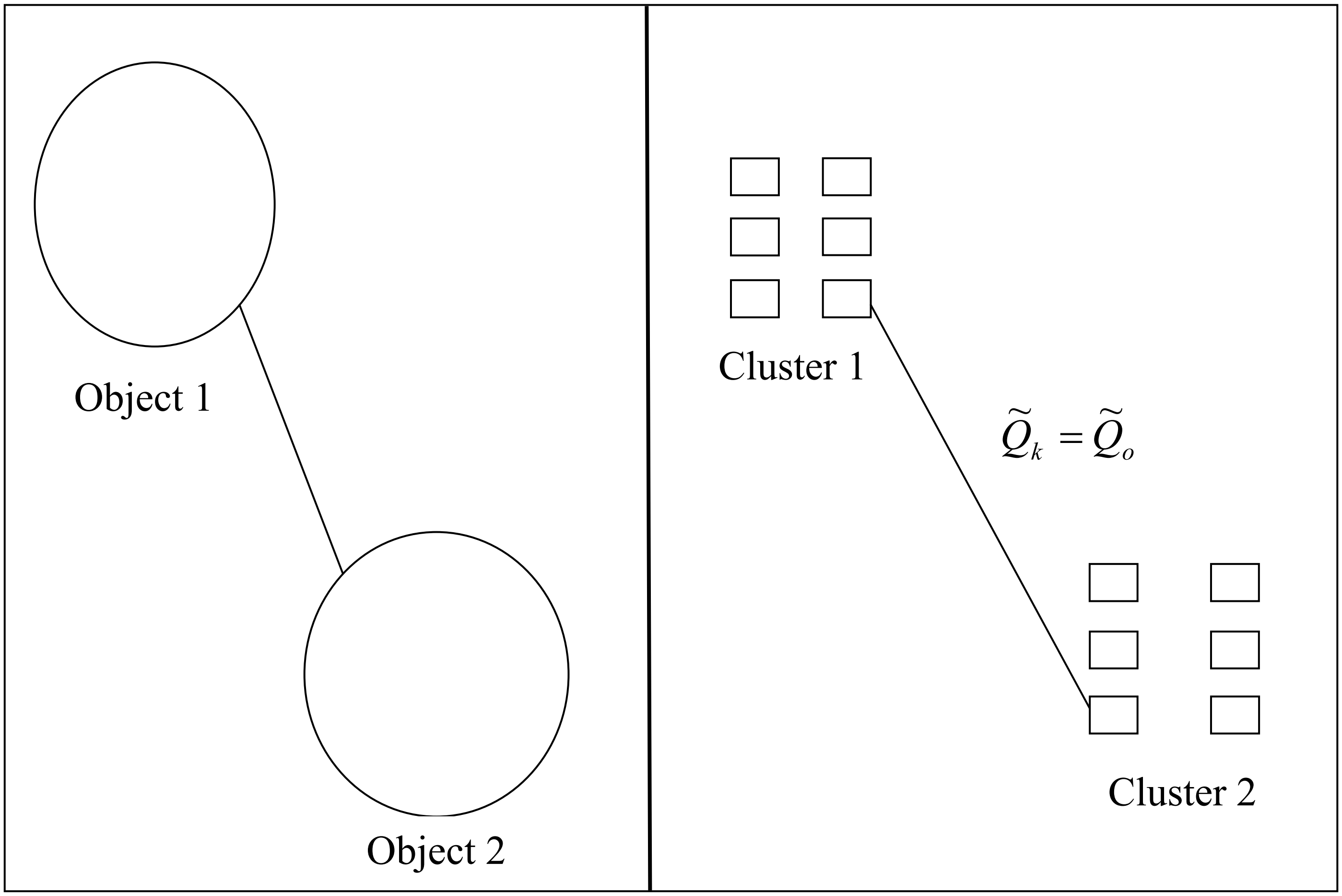

Figure 3. Classical Physics (Left) – The spatiotemporally localized geometric relations of Clusters 1 & 2 in Figure 2 allow them to be treated as individual classical Objects 1 & 2, respectively. The lone link in this figure represents the average of the various 'larger' values of Q between sources in the two different Clusters of Figure 2. The average over the most probable values Q_o between sources in the clusters is the spacetime interval of classical physics.

Figure 4. Quantum Physics (Right) – The outcome $\tilde{Q}_o$ of a quantum physics experiment reveals the k^th spatiotemporal geometric relation $\tilde{Q}_k$ of the geometric graph in the context of the classical Objects comprising the experiment, e.g., Source, beam splitters, mirrors, and detectors. The partition function provides the probability of this particular outcome, i.e., $\mathrm{P}\left(\tilde{Q}_k = \tilde{Q}_o\right) = \dfrac{Z\left(\tilde{Q}_k = \tilde{Q}_o\right)}{Z}$.

Figure 5

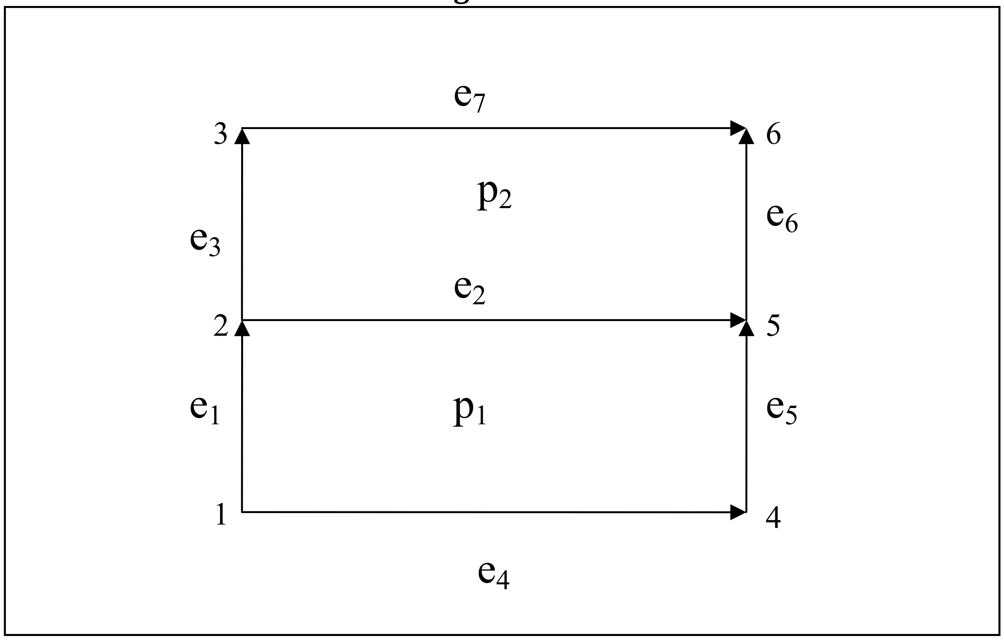

Figure 6

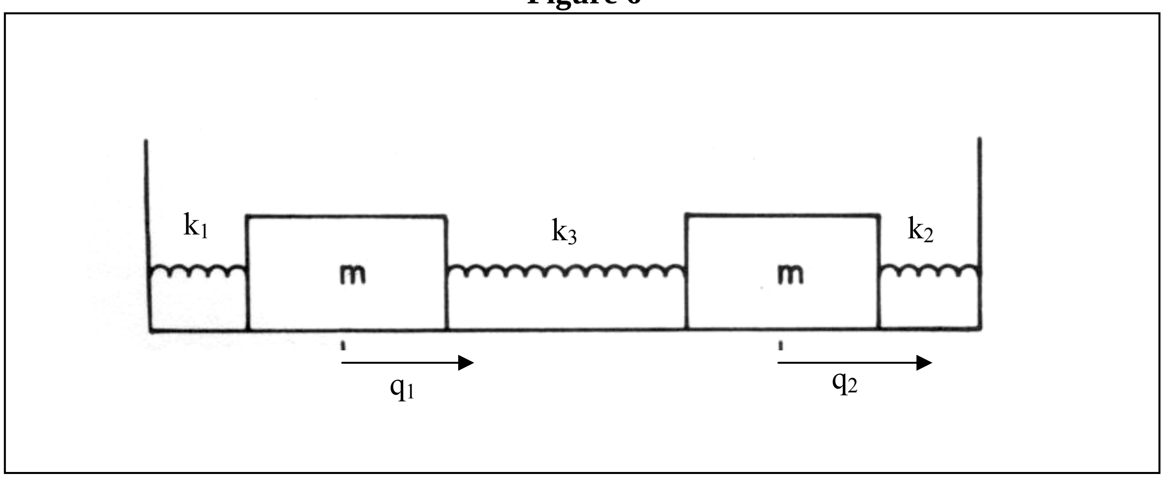

Figure 7

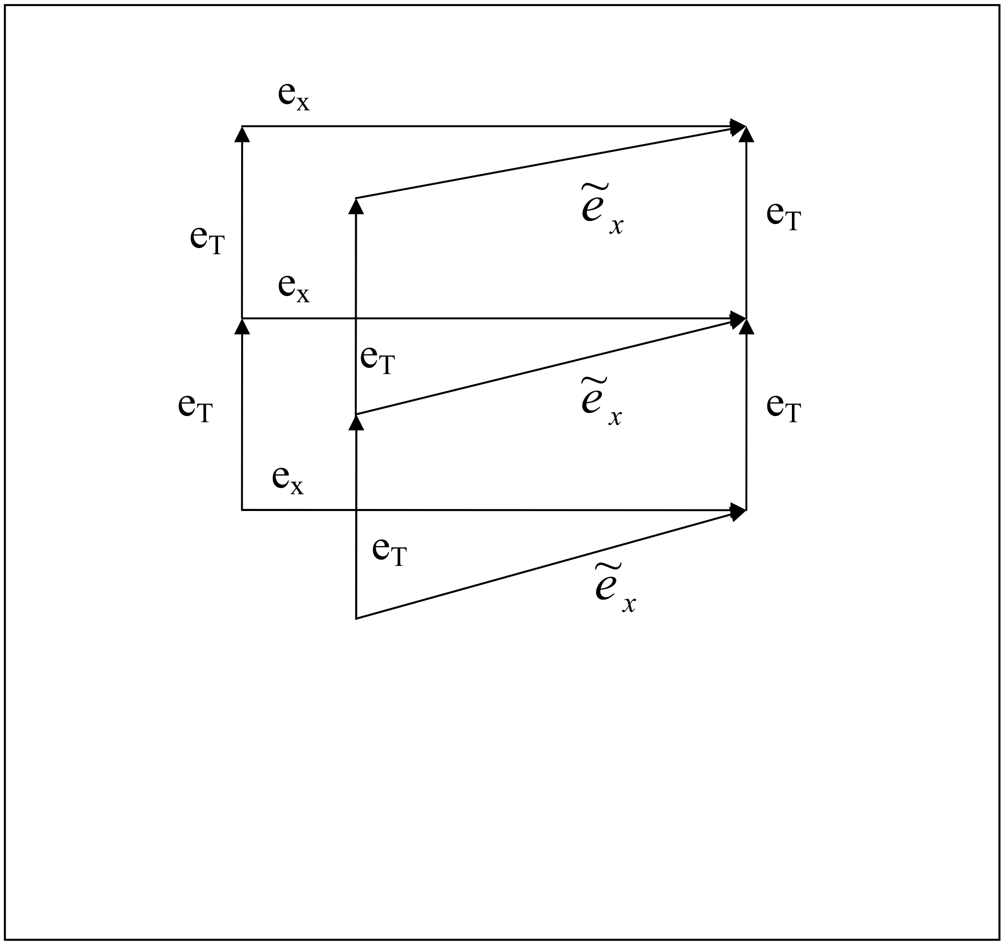

Figure 8

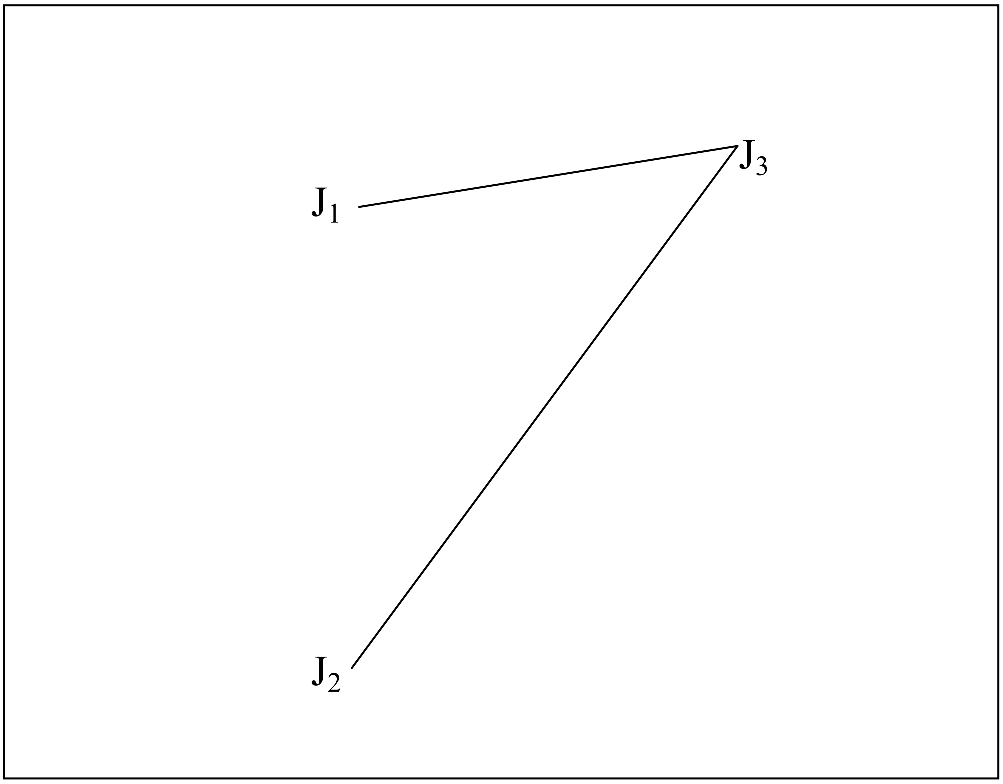

Figure 9

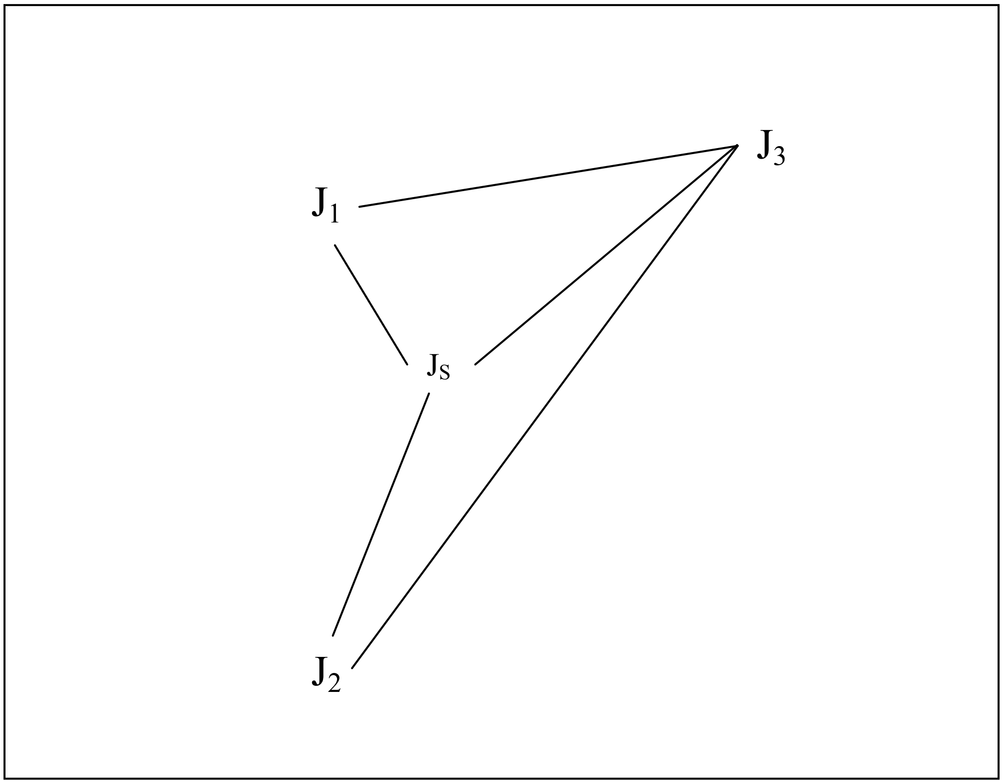

Figure 10

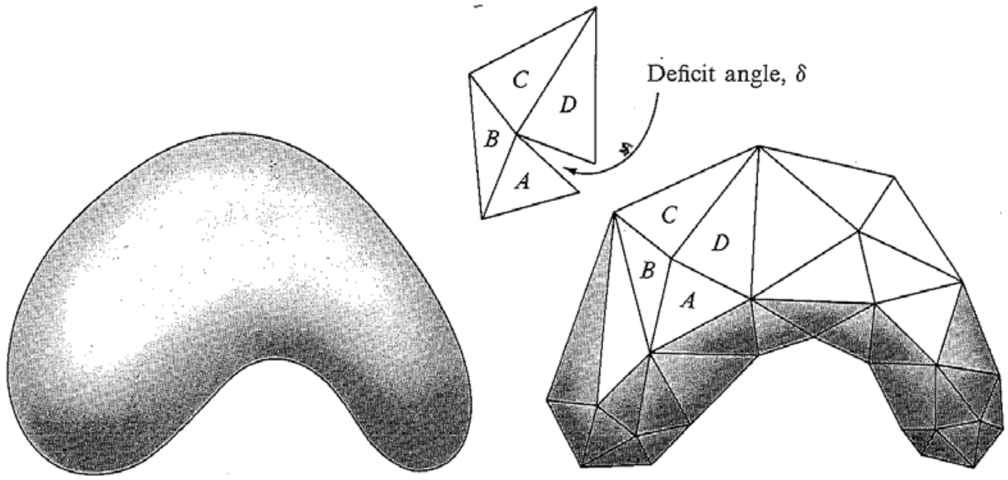

Figure 42.1.

A 2-geometry with continuously varying curvature can be approximated arbitrarily closely by a polyhedron built of triangles, provided only that the number of triangles is made sufficiently great and the size of each sufficiently small. The geometry in each triangle is Euclidean. The curvature of the surface shows up in the amount of deficit angle at each vertex (portion *ABCD* of polyhedron laid out above on a flat surface).

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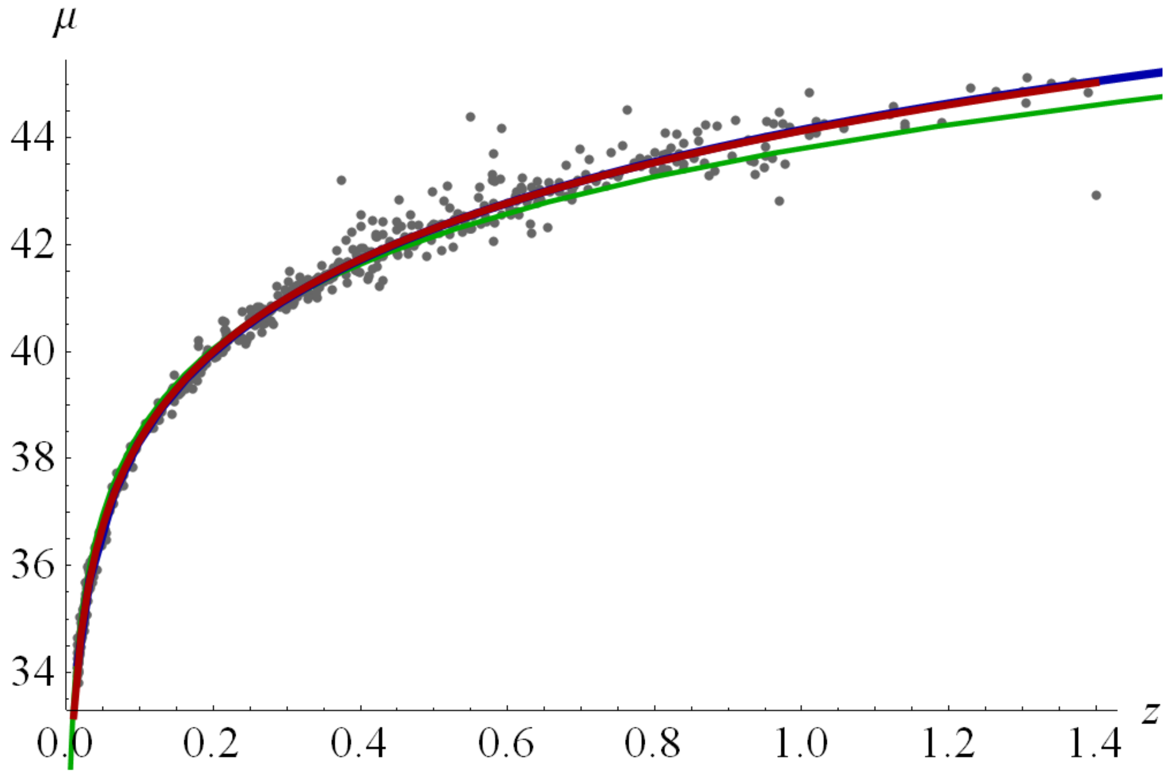

Figure 11. Plot of Union2 Compilation data of distance moduli μ versus redshifts z for type Ia supernovae. Superimposed are the best fits for EdS (green), ΛCDM (blue), and MORC (red). The MORC curve is terminated at z = 1.4 in this figure so that the ΛCDM curve is visible underneath.

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