Figure, Ratio, Form: Plato's Five Mathematical Studies

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At *Republic* 522b-531d Socrates recommends for the philosopher-to-be the study of 'arithmetic and calculation,'[1] plane geometry, solid geometry, astronomy, and harmonic theory. The project of this essay is to understand why Plato considered these studies so valuable as propaideutic for dialectical study of the forms. What makes them so helpful in enabling the philosopher-to-be to 'turn' from sensibles to forms and the Good? I will begin by briefly considering *each* of the five studies in order to bring to focus the common power they share, albeit each in its own way, the power, namely, to introduce the soul to the concrete work of thinking what is purely intelligible. I will then ask why Plato has Socrates put *these five* studies together, and in the *definite sequence* that he does. The primary value of the sequence, I will argue, is psychagogic. By a series of purgings, the sequence leads from figured arrays of units to the figures that such arrays express to the ratios, in turn, that such figures express; thus the philosopher-to-be is brought to the threshold of the study of forms. But the features that give the sequence its psychagogic value also suggest a striking metaphysical possibility. In essence, do the relations of ratio to figure and of figure to array serve to prepare the philosopher-to-be to understand an analogous relation of form to ratio? I will conclude by distinguishing several aspects of this

1 λογιστική τε καὶ ἀριθμητική, 525a9. Cf. ἀριθμόν τε καὶ λογισμόν, 522c6-7. On Socrates' indifference, in referring to the first of the five studies, to the distinction he elsewhere draws between these two disciplines, see Mueller (1991) and Robins.

possibility and articulating them in the form of issues and questions for a future inquiry into the forms and the Good.

The Propaideutic Value of Each Study

Though none of the five studies reaches the forms themselves, *each* of them involves a transcending of the sensible that prepares the philosopher-to-be to do so.

That none reaches the forms themselves, Socrates makes clear when he describes those who have used mathematics to climb 'up out of the cave to the sunlight' as 'still unable to look at the animals and plants [themselves] and the light of the sun'; what they are able to look at is only 'divine images in water and shadows of that which is' (532b-c).[2]

Nonetheless, it is the study of the five mathematical disciplines that enables the philosopher-to-be to first emerge from the cave, for each of the five, even while it may begin with something sense-perceptible, teaches her to treat this as, at best, an image of the purely intelligible structure to which her thought really refers. This is clear from the beginning in the cases of arithmetic and calculation, plane geometry, and solid geometry, for at the close of the earlier account of the divided line, Socrates has won Glaucon's agreement that 'students of geometry, calculation, and the like' …

> know that, although they use visible figures and talk about them, their thought isn't directed to these but to those other things that they are like. The claims they make are about the square itself and the diagonal itself,[3] not about the diagonal they draw, and similarly in the other cases (510d-e).

In the case of arithmetic and calculation, Socrates brings this distinction home by contrasting 'numbers *themselves*' (αὐτῶν τῶν ἀριθμῶν; 525d) with 'numbers attached to visible or tangible bodies.' Arithmetic and calculation study only these 'numbers themselves.' They are pure multiplicities, composed of units that are homogeneous ('each one [is] equal to every other, without the least difference') and incomposite ('having no internal parts'). Since in these ways these numbers prescind from the very conditions of the 'visible' or 'tangible,' they 'can be grasped only in thought' (526a).

In the cases of plane and solid geometry, Socrates secures the same distinction by winning Glaucon's agreement that beneath the appearance of practical dealing with the mutable that is given by the geometer's customary talk of constructive operations ('"squaring," "applying," "adding," and the like'; 527a), the real purpose of study is theoretical insight, γνῶσις (527b1, 5), and the real object is 'what always is, not what comes into being and passes away' (527b). Hence the object cannot be mistaken for anything located in place or time. Though by her 'mold[ings] and draw[ings]' (510e) the geometer makes empirical representations of the figures that she studies, these figures themselves can be grasped, again, 'only in thought.'

As for astronomy, Socrates goes out of his way to reject the empirical focus of the way it is customarily pursued. The philosopher-to-be must treat celestial phenomena analogously as the geometer treats her drawings; recognizing that nothing 'connected to body and visible' (530b), not even 'the most beautiful and exact of visible things' (529c) — that is, the motions of the planets and stars — can exhibit the absolute regularity that distinguishes being from becoming, she must 'leave the things in the sky alone' (530b), seeking instead the system of 'true motions' that these 'fall far short of' (529d). This system is a set of ratios in which the relations of the figures traced by motions are correlated with the relations of the speeds of these motions,[4] and it must be studied, Socrates says, 'as we do in geometry, by means of problems.'[5]

Socrates redirects the study of harmonic theory away from the empirical in the same way. In Pythagorean practice harmonic theory is the study of 'the numbers that are to be found in audible harmonies'; Socrates insists, however, that the philosopher-to-be turn her attention away from the relations of correspondence that link certain basic ratios (above all, 1:2, 2:3, 3:4) with harmonies of tone and seek out the strictly mathematical relations that link these ratios with one another; the key

2 For purposes of quoting, I have relied on, with minor changes, Grube and Reeve.

3 That these expressions refer not to forms but to the perfect structures that the drawn figures represent is persuasively argued by Burnyeat.

4 See Mourelatos (1981), who looks to *Laws* 893c3-d5 for exemplification.

5 On the notion of 'problems', see Mueller (1980).

question should be not which *sounds* but, rather, 'which *numbers* are harmonious and which are not, and in each case why' (531c).[6] Evidently, Plato has Socrates demand the kind of reflection that Archytas had attempted[7] — and that Plato himself would later attempt, with different results, in the *Timaeus* at 35b-36b — with his theory of the arithmetic, harmonic, and geometric means; the philosopher-to-be must seek a set of purely mathematical principles that will show how and why these ratios fit together systematically.

Thus, each of the five studies undermines the 'trust' that makes one a dreamer and a cave dweller, the unthinking assumption that sensible things are the highest order of reality; each study provides sustained experience exploring a purely intelligible order that, in the perfection of its terms, in its prescinsion from the vicissitudes of becoming, and in its systematic integrity, transcends the sensible.

The Sequence and Fit of the Five Studies

But recognizing the propaideutic value of *each* discipline is not sufficient. We must ask why Plato has Socrates prescribe study of *these five* and, as well, in *this definite sequence*. Just at the point of turning attention from mathematics to dialectic, Plato has Socrates stress that coming to understand 'the association and relatedness [of the five disciplines] with one another and ... how they are akin' (531d) is crucial to their propaideutic value for the soul's 'turn' to forms and the Good; only if studying the five leads to this synoptic understanding, Socrates says, 'does it contribute something to our goal ... — otherwise, it is labor in vain' (531c9-d4). And throughout — both by numbering them as steps in a series and by going out of his way to correct his own error of jumping from plane geometry straight to astronomy — Socrates stresses the importance of the order in which the philosopher-to-be studies the five.

From the outset, we should recognize that this order is *not* that of a series leading straightforwardly and step-by-equal-step from the simple to the complex. Rather, the middle three studies are all species of geometry whereas the first and fifth are arithmetical, focused on relations of whole numbers. Thus the five are ordered as a departure and

6 See Barker (1978).

7 See Barker (1989), 46-52, and (1994).

return, in which, it seems, we begin and end with studies of whole numbers. We may explore the 'kinship' of the five, therefore, by asking three distinct sets of questions. First, what is the 'kinship' of the middle three geometrical studies, and what is the value of their sequence? Second, why does Socrates interrupt the study of whole numbers with these three species of geometry? Or, to ask this another way, what difference does it make to the philosopher-to-be's understanding of number in harmonic theory that it is prepared for by a turn from arithmetic and calculation into geometry? And third, how do these two distinct movements — the advance through the three geometrical studies and the departure from and return to numbers — fit together?

(a) The three geometrical studies: within the ascent, reconstruction and return

What is the 'kinship' of the middle three studies? Consider, to begin with, the two passages in which Socrates, correcting his error and reordering the three, stresses their fit and sequence. '... we were wrong just now about the subject that comes next in sequence (ἑξῆς) after geometry. ... After plane surfaces,' he explains at 528a-b, 'we went on to revolving solids before dealing with solids by themselves. But the right thing to do is to take up the third dimension right after the second. And this, I suppose, consists of cubes and of whatever shares in depth.' And at 528d-e he repeats the point: 'The subject dealing with the dimension of depth was next in sequence (ἑξῆς). But ... I passed it by and after [plane] geometry I spoke of astronomy, which deals with the motion of things that have depth.'

Thus the philosopher-to-be begins with the study of figures in two dimensions — the triangles and rectangles and circles, *et cetera*, familiar to us from Euclid. She then adds the third dimension, depth, extending the methods of plane geometry to the study of solid figures. By having Glaucon protest that 'this [study] hasn't been developed yet' (528b), Plato refers obliquely to the development that solid geometry had in fact undergone in the period between the dramatic and the composition dates of the *Republic*, above all, Theaetetus' discovery of the five regular solids and of the methods for inscribing them within the sphere. What Socrates calls astronomy, in turn, is more accurately understood as kinematics,[8] for he characterizes its subject very generally as 'the motion

8 See Mourelatos (1981).

of things that have depth' (528e, just cited). Paradigmatic for this kinematics, however, would be Eudoxus' astronomy of revolving homocentric spheres. With the addition of motion, hence of time, the turn to kinematics brings a shift in the explicit focus of study. Ratios are implicit in the figures studied by plane and solid geometry; the relations between sides and diagonals, between radius and circumference, *et cetera*, are defining for two- and three-dimensional figures, and the relations between the areas and volumes of different figures are matters of ratio. Nonetheless, it is figures that plane and solid geometry take as their thematic object of study, and it is in and as figure that ratios are expressed. With kinematics, by contrast, attention shifts to relative velocities as these correlate with the relative distances of the paths traced by the bodies in motion, and with this, ratios become the primary objects of study. Figure remains essential, for the paths traced by motion are studied as the circles traced by the rotation of spheres. But the relative velocities and the correlations of time and space that kinematics seeks are given by ratios.

We began with the recognition of how each of the five studies can help with the 'turn' of the soul. We are now in position to recognize, in the sequence of the middle three, a secondary and subordinate movement of return. The additions of the third dimension and then of motion gradually reconstitute — albeit with the purity of mathematical concepts — the domain of physical-sensible things. With the shapes studied by solid geometry, the philosopher-to-be is brought to the basic sorts of spatial structure constitutive for actual bodies. And with the figures and ratios that express the paths and relative rates of the motion of bodies, she is brought to the basic sorts of relationship that the actual motions of actual bodies imply. True, as Socrates emphasizes, nothing 'connected with body and visible' can fail to 'fall far short of' and 'deviate from' the perfect structures and regularities that solid geometry and kinematics study; nonetheless, it is specifically *from these perfect structures and regularities* that actual bodies and motions 'deviate.' Even as each of the five studies leads the philosopher-to-be beyond the sensible, introducing her to a pure mathematical intelligibility, the middle three, at least, also bring her back — in the medium of this intelligibility — to the basic *structures of* the sensible.

(b) *The beginning and end of the series: from arithmetic and calculation to harmonic theory*

Recognizing the emergence of ratio in the middle three studies enables us to see one way in which the fifth study, harmonic theory, is a natural successor to them. Whereas astronomy lets ratios, implicitly present in the figures studied by plane and solid geometry, now become an explicit object along with figures, harmonic theory concentrates on ratios exclusively, studying them in and for themselves. But this raises two questions. First and most obviously, how does the first study, 'arithmetic and calculation,' fit with the sequential unity of the last four that now presents itself? Earlier we saw in the full sequence a departure from and return to number; how does this fit with the process by which the philosopher-to-be is turned from figures to ratios? (We shall consider this in sections (i) and (ii) following.) Second, how, if at all, does the exclusive focus on ratios in harmonic theory fit with the motion of return that we have just recognized in the middle three studies? Does harmonic theory break with or somehow advance the gradual recovery of the basic structures of the sensible? (We shall consider this in our closing reflections in the final section.)

(i) *Arithmetic and calculation: number as figured array.* How does Socrates envisage the study of number in arithmetic and calculation? We can give this question focus by bringing together four points that Socrates makes. First, as we have already noted, arithmetic and calculation takes each number to be a multiplicity of homogeneous and indivisible units (526a). But, second, number is not thought as such multiplicity *simpliciter*; when at 510c Socrates first points out that

> students of geometry, *calculation*, and the like hypothesize *the odd and the even*, the various figures, the three kinds of angles, and other things akin to these in each of their investigations,

he implies that number is to be ordered and studied by way of the kinds 'odd' and 'even'; these, his remark suggests, are as basic to 'calculation' as 'the various figures' and 'the three kinds of angles' are to 'geometry.' This implication he reinforces by his initial characterization of the subject matter of arithmetic and calculation as 'the one and the two and the three' (522c); while 'the one' is the paradigm unit, 'the two' (τὰ δύο) and 'the three' (τὰ τρία) are the paradigms of even- and odd-numbered multiplicities. Third, Socrates makes a point of the value of counting for the warrior, and Glaucon immediately associates this with the ability to place troops in various 'formations' (τάξεων;

522e). Socrates later endorses this association, asserting that 'calculation and arithmetic,' 'wholly concerned with number' (525a), 'are compulsory for warriors on account of formations' (δία τὰς τάξεις; 525b). Number, evidently, is to be thought in connection with the sort of spatial arrangement and order that troop formations exhibit. Fourth and finally, 'those who study calculation [and] hypothesize the odd and the even,' even while they know that the real referents of their thought are non-sensible, do nonetheless 'make use of visible figures' (τοῖς ὁρωμένοις εἴδεσι προσχρῶνται; 510d5) to represent these referents. If, now, bearing our first three points in mind — the homogeneity and indivisibility of the units, the fundamentality of the distinction of odd and even, and the connection of number and spatial arrangement — we ask what sorts of 'visible figures' would have been most appropriate as means of imagining number, we can hardly help but think of the so-called 'pebble arithmetic' of the Pythagoreans, in which numbers were represented by laying out figured arrays of pebbles. According to the compelling reconstruction offered by W.R. Knorr, it was first with Theaetetus' rethinking of Theodorus' study of 'powers,' alluded to at *Theaetetus* 147d-148b, that Greek mathematics made its Euclidean turn and came to represent number geometrically by continuous quantities, that is, by lines and areas and volumes. Before then, and continuing on in a distinct historical line that we see maintained in the Academy by Speusippus' tract 'On Pythagorean Numbers,' numbers were represented by figures composed of strings of discrete points. The basic distinction between odd and even, for instance, was represented by strings which could and could not be separated into two equal groups, to wit:

o | o
fig. i

o o o
fig. ii

Moreover, by the repeated generation of gnomons, beginning either from a single unit or from a pair, triangular, square, and oblong figures were produced to express with remarkable intuitive transparency the series of natural numbers (fig. iii), of odd numbers (fig. iv), and of even numbers (fig. v), respectively:

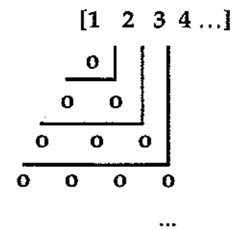

fig. iii

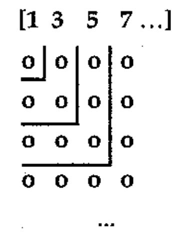

fig. iv

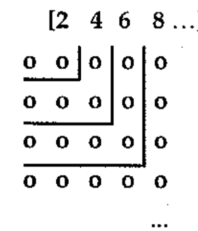

fig. v

As Knorr shows, the 'use' that students of 'calculation' could make of these 'visible figures' was powerful. Exploiting the way the pebble figures allow one to establish subgroups of odds and evens and to isolate the unit, he displays the system of multiplication and division relations essential to odd and even, and he shows the properties of Pythagorean triples. All of this, he argues, must have been background knowledge for both Theodorus and Theaetetus.[9]

(ii) *The order of the five studies as a series of purifications: the psychagogic aspect.* Recognizing in the first of the five studies the 'pebble arithmetic' of the Pythagoreans throws surprising light on the unity and the sequence of the whole series.

To begin with the turn from arithmetic and calculation to the three geometrical studies, what is crucial is that arithmetic and calculation are *already* geometrical, for the 'pebble arithmetic' studies number as a *figured* array of units. While it is the multiplicities of units that are the arithmetician's focal object, the figures these form are essential to them. It is *as these figures* that the three basic series to which these multiplicities of units belong — the series of odds, of evens, and of integers *simpliciter* — are thought: the series of odds are square numbers, the series of evens are oblong numbers, and the series of integers are triangular numbers. Thus, even while the turn to geometrical studies is a departure from arithmetic, it is also, more deeply, a bringing to explicit focus of what is essential to it. The philosopher-to-be in effect drops the units in order to let emerge, now as objects for thought in their own right, the figures to which the units belong.

9 See Knorr, 142-61.

With this emergence, moreover, she provides herself an occasion for a deeper understanding of number itself, and with this she sets the stage for the final turn, by way of astronomy, to harmonic theory. In the 'pebble arithmetic' it is the units that are conspicuous; thus, for example, the length of a side of a figured array is measured by the number of the units that make up that side, and one can just count them up. With the turn to geometry, however, this is no longer possible. Now the length of a side can only be grasped in the context of the figure to which it belongs, and the number that measures this length no longer refers to a multiplicity of prior units; rather, it expresses directly the ratio in which this length stands to other lengths in the figure — a ratio, notably, that need not be expressible in whole numbers. The figure itself, in turn, *just is the spatial expression of this ratio.* Hence, just as the turn to geometry brings to the fore what is essential to the focal object of arithmetic and calculation, so the turn, by way of astronomy, to harmonic theory brings to the fore what is essential to the focal object of geometry: as in the first turn the philosopher-to-be drops the units and lets figure emerge in its own right, so in the second turn she drops figure and lets ratio emerge in *its* own right. Nor are these two steps related merely as analogues. Since arithmetic and calculation study numbers as *figured* arrays of units and since figures, in turn, are expressions of ratio, the turn to harmonic theory in effect reaches all the way back to arithmetic and calculation, revealing what has been basic from the very beginning. The whole sequence of studies thus has the unity of a stage-by-stage process of disclosure of the essence of number as ratio.

The first fruit of this interpretation of 'the association and relatedness' (531d) of the five studies is an insight into the psychagogic value of the sequence. We noted earlier how *each* of the five, even while it begins with the sense-perceptible, teaches the philosopher-to-be that this is just an image of structures that are purely intelligible; thus each of the five contributes in its own way to preparing her for study of the forms. Now we can also see how the *series* of studies, as such, contributes to this preparation. If we focus on the way, in each phase of study, the in itself purely intelligible object is concretely represented, the process of the disclosure of ratio appears, as well, as a process of purging the sensible. This purging begins, of course, with the very turn from the sorts of 'visible or tangible bodies' (525d) that 'tradesmen and retailers' (525c) and even warriors count to the pure units that the arithmetician studies; the pebbles she uses to represent these units are well suited to remind her of the homogeneity and indivisibility that makes the units non-empirical. Nonetheless, in their dispersal and in the ensemble-character of the arrays they form, the pebbles still invite the thought of an aggregate of different parts; and, of course, they are 'visible' and 'tangible' themselves. The first of these limitations is removed with the turn to geometry, for with the dropping of the units, the figure in which they are arrayed emerges in its intrinsic unity. Now, even when we bisect, inscribe, and perform the other operations of division, what we reveal are not parts that stand prior to the divided figure but, on the contrary, new figures with their own intrinsic unities that stand in various ratios to it and to one another. Nonetheless, the figure the geometer conceives is an intrinsically spatial structure which therefore lends itself to the representations she makes of it by her 'drawings' and 'moldings' (510e), and these are still 'visible' and 'tangible.' It is only with the subsequent dropping of figure and the emergence of ratio in and for itself that thought arrives at an object that is itself, as nonspatial, 'beyond' the conditions of the visible and that which becomes. This is the psychagogic reason for the placement of harmonic theory as the fifth and final of the mathematical studies. As the discipline that most radically purges the sensible from its object of thought, it most radically prepares the philosopher-to-be for the study she really seeks, namely, dialectic, which

> without making use of anything visible at all, but only of forms themselves, move[s] on from forms to forms and end[s] in forms (511b-c).

Some Possible Metaphysical Implications

The interpretation of the sequence of the mathematical studies as, in its psychagogic aspect, a series of purifications suggests an intriguing substantive possibility as well. On the one hand, to arrive at the study of ratios in harmonic theory is to prepare for one last departure; as the study of figured arrays of units prepares the philosopher-to-be for the study of figures, and as this, in turn, prepares her for the study of ratios, so the study of ratios prepares her for the study of forms. On the other hand, each of the first two turns is, as a purification, also a return, letting emerge in its own right what was indirectly present from the beginning, expressed in and as what was purified; indeed, it is *because* figures are expressed in and as the arrays of units studied by arithmetic and calculation that purging the units lets figure as such emerge, and it is *because* ratios, in turn, are expressed in and as the figures studied by geometry that purging figure lets ratio as such emerge. How fully, we can now go on to ask, does the turn from ratios to forms conform to the pattern of the turns that prepare for it, bearing within itself, as a purification, the movement of return? That is, *is it the case that* — analogously as figures are expressed in figured arrays of units and as ratios are expressed in figures — *forms are expressed in and as ratios?*

Should we take Plato to hold this? We get some encouragement if we recall once more Socrates' allusion at 532c to the philosopher-to-be's 'ability, newly acquired [through study of the five mathematical disciplines], to look at *divine images* in water and at *shadows of the things that are.*' By 'the things that are,' τῶν ὄντων (532c2), Socrates refers to forms. Hence the objects that the five studies teach the student to 'see' will be, as 'images in water' and as 'shadows,' expressions of the forms in an alien medium; as 'divine,' however, they will be truly disclosive expressions, allowing the forms to appear for what they are to a degree or in a way that sensibles do not. But this passage can take us only so far. Plato does not have Socrates distinguish the different kinds of mathematical object, and a few speeches later he in effect leaves both this and the precise relation between forms and mathematicals as open questions when he has Socrates return to the divided line and decline to discuss 'the ratios between *the things these* [sc. the four cognitive states] *are set over* and *the division into two* of either the opinable or *the intelligible*' (534a5-7). Hence we must look to other texts to pursue his understanding of the form/ratio relation.[10] Here let me just set the stage for this work by noting three features of ratio as it emerges in *Republic* VII.

[1] *Ratio as the inner structure of the nonspatial as well as of the spatial.* If ratios were limited in their own expression to the spatial, the thesis that forms are expressed as ratios would be problematic; at the very least, it would have to be restricted to those forms that are instantiated by spatial particulars. But, of course, even while ratio is the inner structure of figure, it is not limited to the spatial. This begins to become clear in astronomy, where ratios are expressed not only by the circles traced by celestial bodies but also by the correlations between the relative lengths of, for instance, the radii of these circles and the relative speeds of the associated motions; here ratios govern the distinctly temporal as well as the spatial aspects of the sensible. In harmonic theory, in turn, ratios have no spatial expression at all; the order they establish in the sensible — namely, intervals in pitch — has nothing to do with length, width, and depth. Thus harmonic theory lets emerge a kind of structure in which, at least potentially, forms that are not instantiated by spatial particulars as well as forms that are, might be expressed.

[2] *Coimplicated with ratio, the continuum of possibilities.* Nor is ratio by itself the whole of this structure. Rather, ratio implies a continuum of possibilities, from which it selects. It is trivially the case that, e.g., 1:2 relates the relatively lesser to the relatively greater and that this relation presupposes an in principle infinite range of possible ratios that either fall short of or exceed it. As the philosopher-to-be moves through the five studies, however, she encounters a number of qualitatively different ranges of possibilities, with the result that the continuum structure as such that is common to them all becomes increasingly conspicuous. With the turn from arithmetic and calculation to geometrical studies, thought shifts from ratios of multiplicity to ratios of magnitude, and within geometry thought shifts from linear to planar (cf. ἐπίπεδον; 528a9) to solid (cf. στερεὸν; 528a9) magnitudes. In astronomy, in turn, the contrast in kind between the ratios that govern the spatial relations of figure and the correlated ratios that govern the temporal relations of speed makes it necessary to distinguish between *greater and smaller* and *faster and slower* (cf. 529d), and harmonic theory, finally, brings to the fore still another sort of continuum, not obviously spatial or temporal, that of *higher and lower*. Thus, even as the series of studies lets ratio emerge, it also brings ever more conspicuously to view, as the substructure presupposed by every set of ratios, a continuum of possibilities stretching between the maximal predominance of one and the maximal predominance of the other of the relevant pair of opposites.

[3] *Mathematically 'harmonious' ratios — expressing 'the beautiful and good'?* Finally, there is a determinateness to the way harmonic theory in particular brings ratio to the fore that may be of special relevance to the question about the possible expression of forms as ratios. Recall that it is not just ratios as such that harmonic theory studies but rather that particular set of ratios that establishes which tones are harmonious with which others — hence, most prominently, 1:2, which establishes the octave; 2:3, which establishes the fifth; and 3:4, which establishes the fourth. Moreover, Socrates insists that the philosopher-to-be shift her focus from audible to mathematical harmony, seeking out the principles on the basis of which these '*numbers* are harmonious' (531c) — an evident allusion, we noted earlier, to Archytas' theory of means. In one respect, the focus that Socrates requires for harmonic theory is strikingly narrow. With the earlier turn from arithmetic and calculation — understood as the study of units in figured arrays — to geometry, the field of number

10 See, for initial attempts, Miller (1995) and (forthcoming).

is opened up from whole number ratios alone to include, as well, ratios of numbers not sharing a common unit; to cite the key case, for the ratio of diagonal to side in a square there can be no exact expression in whole numbers, only an anthyphairetic approximation.[11] This opening of the field of number not only brings a new universality to mathematical argument, but it also allows the sense of ratio as relation to come into its own; that there can be ratio that is not expressible in whole numbers makes patent that, even where there is expression in whole numbers, the number of units in each term is a function of their relation, not vice versa. Why, we might therefore ask, does Socrates then revert, in harmonic theory, to whole number ratios — and, moreover, to the restricted set of these ratios that, in their interrelatedness as means, a purely mathematical notion of 'harmony' implies?

Socrates gives an intriguing hint at 531c. When Glaucon responds to his call for inquiry into why certain 'numbers are harmonious' by characterizing the task as 'daimonic,' Socrates replies that it is 'useful in the search for the beautiful and good (τοῦ καλοῦ τε καὶ ἀγαθοῦ) — but, pursued for any other purpose, useless' (531c6-7). The least that this suggests is that there is an elegance in the mathematical system of means that, as an exhibition of beauty and goodness, can help lead the philosopher-to-be towards the Good. But there may be something more determinate in play as well. Forms are the sources of normative structure in what partakes of them; a particular is to be judged healthy or flourishing, metaphysically speaking, just insofar as it instantiates well its defining form. In this causal function, forms reflect their own basis in the Good (cf. 509b); it is as cases of the Good, hence as 'goods' or perfections in their own right,[12] that forms are responsible for what goodness there is in particulars. This much ventured, we can pose the following heuristic questions. Does Socrates find, in the normative set of systematically interrelated ratios that is disclosed by harmonic theory, a beauty and goodness that point back to, as their proximate source, a corresponding set of forms? Does he, further, give harmonic theory the penultimate

11 The major study of anthyphairesis in connection with Plato's mathematical curriculum is by D. Fowler. I am grateful to my Vassar colleague in mathematics, John McCleary, for guidance in thinking into the relationship between incommensurability and anthyphairesis.

12 See Miller (1985).

place in the sequence that ends in dialectical study of the forms because he takes this expression of forms as the system of means to be exemplary for other, non-musical spheres of existence as well? And, finally, does he on these accounts hope that the five mathematical studies, culminating in harmonic theory, will prepare the philosopher-to-be not only for the ascent from the objects of mathematics — 'divine images ... and shadows of the things that are' (532c) — to the forms but also for the subsequent descent from the forms to what partakes of them, readying her to discern in each sphere of existence their expression as a normative set of ratios on some relevant continuum of possibilities?

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