Hans Reichenbach's Debt to David Hilbert and Bertrand Russell

Abstract

Despite of the fact that Reichenbach clearly acknowledged his indebtedness to Hilbert, the influence of this leading mathematician of the time on him is grossly neglected. The present paper demonstrates that the decisive years of the development of Reichenbach as a philosopher of science coincide with, and also partly followed the "philosophical" turn of Hilbert's mathematics after 1917 that was fixed in the so called "Hilbert's program".

The paper specifically addresses the fact that after 1917, Hilbert saw the axiomatic method as an instrument for providing the foundations not only of mathematics but also of other sciences. In particular, Hilbert's axiomatic program was closely connected with the theoretical physics and arguably helped Einstein to discover the general theory of relativity. In this context, one can see Reichenbach's project to axiomatize Einstein's theory of relativity as a continuation of this project.

Under Russell's influence, after 1914 Hilbert also developed an interest in mathematical logic. Reichenbach experienced similar transition from axiomatics to logic which went together with turn of his interest from Hilbert to Russell. Reichenbach's rapprochement to Russell also supported the transition of his interests from epistemology to ontology.

Keywords

axiomatics, logic, philosophy of science, Hilbert, Reichenbach, Russell

1. Opening

Reichenbach's studies often follow one-sided venues. Most discussed is the close relatedness of his philosophy of nature (*Naturphilosophie*) with the Vienna Circle, in particular with Moritz Schlick. In fact, however, there was a considerable difference between them (Milkov 2013). In the 1920s, when Moritz Schlick and the Vienna Circle were oriented towards the logical analysis of science that had its roots in the philosophy of language (Wittgenstein's *Tractatus* that they studied and tried to follow was nothing but a treatise in philosophy of language), Reichenbach considered as more important another method: that of axiomatization of sciences started by David Hilbert. Unfortunately, this alignment is often blurred by the fact that Reichenbach used terminology that was difficult to distinguish from that of the Vienna Circle, meaning with it, however, something significantly different (Milkov 2011: xxi).

Thomas Ryckman fights another one-sidedness of Reichenbach studies. He deplores the fact that “most contemporary readers of Hans Reichenbach’s works on the philosophy of space and time have not considered them in the scientific context of their origin” (2005: 77). Against this aberration, Ryckman explores Reichenbach in connection with Arthur Eddington’s and Hermann Weyl’s philosophy of space and time.[1]

In this paper we are aiming at what can be called a broad-brush *historical-philosophical approach*. It concentrates on the figure under analysis—on Hans Reichenbach—in its historical connection with scholars of different orientations. To be more explicit, we are going to concentrate not only on the influence of David Hilbert but also on that of Bertrand Russell (in § 8) on Hans Reichenbach. Unfortunately, despite the fact that Reichenbach clearly acknowledges his indebtedness to Hilbert (and to Russell), in particular to Hilbert’s “program of a complete formalization of the object language and of a proof of consistency” (1947: viii), his debt to this leading mathematician of the time is completely overlooked. An important point in this connection is that so far, practically nobody has explored the fact that the development of Reichenbach as a philosopher of science, from 1920 to1928, coincided with the “philosophical” turn of Hilbert’s mathematics after 1917 it as was evident in the so-called “Hilbert’s program” (see § 2.2). In order to conclusively demonstrate this, we shall first to say more about Hilbert’s intellectual development.

2. David Hilbert

At the beginning of the last century, David Hilbert was widely recognized as the leading mathematician of his time. His mathematical colloquium in Göttingen attracted colleagues from all over Europe and North America. Hilbert worked in many directions including mathematical physics. In this paper we are going to concentrate, however, on three stages of Hilbert’s development that are of special importance to our analysis.

2.1. The First Stage of Hilbert’s Axiomatic

According to Hilbert (1899), axiomatics is a system built up with logical strength and precision. One cannot change part of it without changing the whole. It is achieved via demonstrations (proofs) and exact definitions. An axiomatic system starts with:

[1] In § 7 we are going to see, however, that Ryckman’s analysis poses problems of its own.

(i) *indefinable terms*. As the name of these terms suggests, they have not real but apparent *definitions*;

(ii) *undemonstrated propositions*, usually called “axioms”.

Importantly enough, a term is indefinable and a proposition is undemonstrated in a system, but not absolutely. To put the first point in other words, in a well formed axiomatic system, the *explicit definitions* are to be discriminated from *implicit* (apparent) *definitions*. The meaning of the latter is determined by their use in the axioms, or hypotheses that state the logical relations between them.

Deductive demonstrations in an axiomatic system are to be absolutely rigorous. They are to be purified and made free from intuition: the intuitions are to be *dissolved* into logic.[2] To this end, everything in an axiomatic system is to be made explicit; nothing is to be presupposed. The *meaning* of the terms plays no role in it. Only relevant are the purely logical relations between them. Theories that use intuitive (material) terms are at the pre-axiomatic stage. In short, Hilbert looked for real proofs that had nothing to do with intuition.

This conception of Hilbert’s strongly supported the program of the logical empiricists both in Vienna and in Berlin. Together with Einstein’s theory of relativity, it worked against Kant’s understanding that human knowledge was impossible without the help of a priori but intuitive concepts.

2.2. Hilbert’s Philosophical Turn

After November 1917 Hilbert launched his proof theory based on strict finitism. This development was preceded by an increased influence of Russell’s and Whitehead’s mathematical logic in Göttingen. Starting in 1914, Hilbert’s mathematical colloquium organized a series of discussions. Soon after, this development bore fruit. In the winter term of 1917–18, Hilbert delivered his first lectures on mathematical logic. In parallel, he developed his axiomatics further. This, of course, was not surprising since logical theory was closely related to axiomatics from the very beginning. Reichenbach understood this quite well. He was explicit that axiomatics is “the only method that will reveal the logical structure of the theory with perfect clarity” (Reichenbach 1924: xii–xiii). Hilbert’s programmatic paper “Axiomatic Thought” (1918) was written in this context.

[2] Hilbert’s claim was related to that of other mathematicians of the time, for example, to Gottlob Frege who fought against the psychologism in logic, for “cleaning” (*Reinigen*) logical theory from psychology.

Important for our study is the fact that in these years Hermann Weyl argued against Hilbert's axiomatic method pleading, instead, for "a constructive development of mathematics". (Mancosu 2003: 72). Following Edmund Husserl, Weyl maintained that mathematics was to first create its objects. One can fight hidden and problematic mathematical entities only this way.[3]

In these months, Hilbert actively looked for professional philosophers who would be willing to join his project in mathematics. When Husserl left Göttingen for Freiburg in 1916, Hilbert invested much of his effort in replacing the open professorial position with an (extraordinary) professorship for philosophy of exact science. His preferred applicant for this position was Leonard Nelson. Eventually, Hilbert's plan succeeded (Peckhaus 2001). Unfortunately, after the Great War, Nelson dedicated all of his time and energy to fighting social injustice in Germany. Meanwhile, Hilbert started working in philosophy of mathematics with one of Nelson's collaborators (Müller 1976: xiv), Paul Bernays, who actually became the architect of Hilbert's proof theory and of his deeper understanding of axiomatics.

In order to face the crisis in studying the foundations of mathematics triggered by the discovery of paradoxes, Hilbert introduced the so called "Hilbert's program" which had a clearly philosophical undertone (Bernays 1922: 98 f.; Peckhaus 2003: 148; Sieg 1999: 1). The crux of this program was the presentation of the logical core of the axiomatic system through symbols or signs put on paper. In this way, Hilbert turned back to the visible evidence, to be more explicit, "to immediate intuitions which no one calls in question" (Blanché 1962: 56).[4] The final objective was to demonstrate the *consistency* (*die Widerspruchsfreiheit*) of axiomatic systems with finite methods, which was the core of the so called "finitism".

Hilbert's axiomatic system was radically formalized: its terms had no meaning.[5] In this way he set up a new discipline which did not explore mathematical entities but formulas that had no content at all. It is pure calculus.[6] Instead of reasoning, one can speak in it about calculating with signs. And since the number of the signs in it is limited, one cannot make mistakes in it. Hilbert called this new discipline somehow misleadingly "meta-mathematics".

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[3] We are going to return to Hilbert's disagreements with Weyl in § 7.

[4] In § 6 we shall see that in his philosophy of physics Reichenbach paralleled this idea of Hilbert's.

[5] Hilbert's opponents (above all Brouwer) called him "formalist" holding that in Hilbert's hands, mathematics turned to a kind of game. Hilbert and his acolytes repudiated this implication.

[6] In somehow similar sense, later Wittgenstein also spoke about "pure calculi" (Milkov 2020, p. 144),

In fact, one could also name it “a general theory of forms”[7] (Bernays 1922: 98). Its problems (completeness, decidability) are not exclusively mathematical.

There are a number of disciplines that are close to this second stage of development of axiomatics despite the fact that they are not identical with it. One of them is the *theory of groups* which is cleared of substances that are reduced to their pure forms. The same is true about the *theory of probabilities*, *general topology,* and the *measure theory* (Blanché 1955: 79).

Reichenbach engaged himself with the theory of probability from the very beginning and, in parallel to following the main tenets of the Hilbert’s program, in the early 1920s he simply added to it the theory of space and time (*die Raum-Zeit-Lehre*) (see § 5).[8] Accordingly, he axiomatized both the theory of space and time (in 1924) and the theory of probability (in 1932, 1935). This claim can be supported by a manuscript note of Reichenbach in which he was explicit that along with the theory of knowledge and ethics, and also set theory, mathematical logic and the axiomatics of the conception of space and time were to be seen as philosophical disciplines (N 23-33-06; italics added).

Another implication of Hilbert’s program was that it helped mathematical theories to get closely related to logical theories. To be sure, Hilbert’s proof theory is generally considered to be a branch of mathematical logic. In contrast to Frege and Russell, Hilbert did not believe that mathematics could not be reduced to logic. He simply maintained that its consistency could be proved by logic. Other formal disciplines also got close to logic, for example the theory of probability to some many-valued logics and topology to modal logic (Blanché 1955: 81). Reichenbach followed Hilbert’s program also in this direction. The theory of probability brought him to the many-valued logic in (1935) and his topological ontology, to modal logic in (1954).[9]

2.3. Hilbert as a Philosopher of Science

In order to show the full dimension of David Hilbert’s influence on Hans Reichenbach, we will next address the fact that after 1917, Hilbert saw the axiomatic method as an instrument

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[7] Husserl, who was Hilbert’s fellow Professor in Göttingen between 1901 and 1916, also spoke about “a general theory of forms”, or theory of manifolds (*Mannigfaltigkeitslehre*). Husserl developed it in connection with the n-dimensional manifolds set up by H. Grassmann, W. R. Hamilton, Sophus Lie and Georg Cantor. (Milkov 2005a: 123).

[8] Of course, the direct motivation for Reichenbach’s theory of space and time was Einstein’s theory of relativity. However, his work in this realm was well prepared by the study of Hilbert’s axiomatic.

[9] We are going to discuss this problem in § 8.

for providing the foundations not only of mathematics but also of other sciences.[10] To be more specific, in his programmatic paper "Axiomatic Thought" he maintained that "anything at all that can be the object of scientific thought becomes dependent on the axiomatic method … as soon as it is ripe for the formation of a theory" (1918: 1115).

In particular, Hilbert's "general theory of forms" was closely connected with theoretical physics, so much so that, as seen by Paul Bernays, it helped Einstein to discover the general theory of relativity. Specifically, Hilbert's "mathematical formalism showed Einstein the direction. … Hilbert reduced the law of gravitation to its simplest mathematical form … and opened it for Einstein's theory to connect with it" (Bernays 1922: 98). This claim can be supported by the fact that in the last weeks of 1915 academia witnessed a raging dispute between Einstein and Hilbert over the authorship of the general theory of relativity (Bührke 2004:105).[11] In the context of this piece of history, one can view Reichenbach's project as an attempt to formalize (i.e. to axiomatize) Einstein's theory of relativity as a continuation of the joint formalist–physicist project of Hilbert and Einstein.

Another direction into which the second stage of Hilbert's axiomatics propelled him was the general philosophy of science. He realized that it could be seen as nothing less than a program of *mathesis universalis*, "i. e., a general science underlying all branches of knowledge" (Peckhaus 2003: 148). Hilbert namely maintained that

> by pushing ahead to ever deeper layers of axioms in the sense explained above we also win ever-deeper insights into *the essence of scientific thought itself*, and we become ever more conscious of the *unity of our knowledge* [12] (1918: 1115; italics added).

We can add to this claim of Hilbert's the interpretation of his axiomatics given by Robert Blanché:

> Through axiomatics, one learns much about the knowledge itself; above all, about its overall organization. … By way of discriminating different formal analogies, axiomatics reveals unexpected correspondence between various regions of the same science and also relatedness

[10] In fact, already Hilbert's sixth problem in his famous list of 23 problems in mathematics posed in the 1900 Paris lecture at the International Congress of Mathematicians, "Mathematical Treatment of the Axioms of Physics", associated axiomatics with natural science (Corry 2004). After 1917, however, Hilbert brought this connectedness to a new level.

[11] On the current stand of this dispute see https://en.wikipedia.org/wiki/General_relativity_priority_dispute.

[12] Cf. with the program for "unity of science" of the logical empiricists.

> between sciences that seem quite different. By way of explicating *invariant structures* of seemingly heterogenic theories, it makes it possible to epistemologically rule over them and to encompass them in one perspective, seeing a big landscape of ideas that we know only in its fragment more synthetically (1955: 77 f.).

This, however, was the main stance of Reichenbach's program around 1930. Among other things, he implemented it in the Berlin "Society for Empirical/Scientific Philosophy" he led between 1929 and 1933.[13] In particular, Reichenbach hoped that the "logical analysis" of different sciences, led by the axiomatic method applied to them, could bring to light important connections between the ever-changing principles of different sciences (Milkov 2011: 151, n. 14). He was especially enthusiastic about following this project in *Aims and Methods of the Modern Philosophy of Nature* (1931) which is generally considered the manifesto of the Berlin Group.

3. Reichenbach, the Berlin Group and David Hilbert

In order to bring more light on Reichenbach's debt to Hilbert, we are now going to make a short historical review of his intellectual development. In 1914–1915, he studied with Hilbert mathematics in Göttingen. To be more exact, in the summer term of 1914 Reichenbach attended Hilbert's lectures and also his seminar on statistic mechanics, and in the winter term 1914–15 he attended a Hilbert lecture on "problems and main questions [of mathematics]" and a seminar on "the structure of matter". These facts point to Hilbert's possible influence on the development of Reichenbach's philosophy of science, which, however, can be easily supported by other evidence as well.

We have just mentioned (in the last paragraph of § 2.3) that Reichenbach's philosophy of nature was substantially interdisciplinary. Above all, what Reichenbach did in his program for "logical analysis of science"—which, according to his own words, he followed since 1915—was to radically renew *epistemology* as philosophical discipline and this not via *ex cathedra* philosophical considerations but through detailed analysis of the new results achieved in science. In the 1920s, his main device to achieve this objective was the axiomatization of the new physics of Albert Einstein. In the beginning of the 1930s, Reichenbach also axiomatized the theory of probability (see § 2.2).

As previously stated (in § 1), there was considerable difference between the directions followed by the Vienna Circle and by the Berlin Group of Hans Reichenbach, Kurt Grelling

[13] On this point see also the last-but-one paragraph of § 3, and Milkov (2021b).

and Walter Dubislav. Scarcely anything better demonstrates this difference than the critical stance of the members of the so-called “First Vienna Circle” (1907–1912), Hans Hahn and Philipp Frank, towards Hilbert’s project for axiomatization as a general theory of science. Following Ernst Mach’s positivistic postulate for strict demarcation between mathematics and physics, Hahn and Frank hold the view that the axiomatic method, which strives for “*deepening of the foundations* of the individual domains of knowledge” (Hilbert 1918: 1109), is pregnant with metaphysics (Stölzner 2002). Admittedly, the fight over scientific “depths” was the motto of the Vienna Circle: “In science there are no ‘depths’: there is surface everywhere” (Neurath, Carnap, Hahn 1929: 306). It is true that Carnap, in contrast to Hahn and Frank, showed an intense interest in axiomatics. However, his *Abriss der Logistik* (1929) and *Untersuchungen zur Allgemeinen Axiomatik* (1930) were demonstrably written under Dubislav’s influence (Milkov 2015: § 9). A related attitude towards Hilbert’s axiomatics was demonstrated by the Polish logicians. They insisted that formalization of science was to be free from “philosophical assumptions”. For this reason they showed little interest in Hilbert’s program (Woleński 2017). In contrast, Reichenbach and his Berlin friends, Grelling and Dubislav, closely followed it.

Speaking about Kurt Grelling and Walter Dubislav, they also formed the basic ideas of their “natural philosophy” (*Naturphilosophie*) in Göttingen. Already in 1910, Grelling had defended his PhD thesis on the axioms of arithmetic under Hilbert’s and Zermelo’s supervision. In the summer term of 1914 Dubislav attended Hilbert’s classes but soon he had to leave Göttingen because of the Great War. In 1923, Dubislav published, together with Karl Clauberg, the *Systematisches Wörterbuch der Philosophie* which employed Hilbert’s axiomatic method in order to connect the philosophical *termini technici* to the impeccable (“gaps-free”) logical chains of definitions (*Kettendefinitionen*). Following Hilbert again, in his paper “On the Relation between Logic and Mathematics” (1925/6), Dubislav held, against Russell and Whitehead, that while the principles of mathematics could not be reduced to logic, mathematics depended upon logic in that the latter helped mathematics by formulating its proofs. Dubislav also developed what he called, a “formalist theory of science” as well as a formalist theory of definition that he advanced against Frege’s theory of definition that was part of his logic of content.

Importantly enough, the Berlin Group received a valuable feedback from Hilbert’s side. When in 1923 Reichenbach, together with Kurt Lewin, (unsuccessfully) tried to launch *Journal for Scientific Philosophy* (*Zeitschrift für wissenschatliche Philosophie*), Hilbert readily agreed to collaborate.[14] In the summer of 1926, Reichenbach was invited to deliver a lecture on the theory of space and time and on the theory of the causality at Hilbert's Colloquium in Göttingen. He had a heated discussion with Paul Bernays at it, while Hilbert showed a vivid interest.[15] Starting in 1929, Hilbert closely followed the development of the Reichenbach-led Berlin "Society for Empirical Philosophy". This could not be a surprise though since the pronounced objective of this interdisciplinary scientific community was no less than deepening the foundations of *all* sciences that was (as seen in § 2.3) of importance also to Hilbert. Furthermore, Hilbert's assistant, Paul Bernays, actively participated in the life of the Society (Hempel 1993: 4). Finally in 1931, at the insistence of Hilbert, Reichenbach renamed it "Society for Scientific Philosophy" (Jørgensen 1951: 48).

Before we further proceed with our analysis, however, we shall make some additional remarks about Reichenbach's scientific biography.

4. Reichenbach's Initial Interest in Philosophy of Science

Traditionally, Reichenbach is considered to be a philosopher of physics *par excellence*. To be sure, his main contributions as a philosopher of science are in philosophy of relativity and of quantum mechanics. His Dissertation (1915), however, was not on philosophy of physics in particular. Rather, it was dedicated to the general philosophy of science. To be more specific, its ideas were not tailored on observations and considerations supplied by physics alone. The Dissertation simply gave "a detailed account of the concept of probability as it was used in the sciences" of his time (Eberhardt 2011: 126). Besides, it defended an objectivist position, claiming that scientific knowledge reflected the laws of nature.

This point is of special interest since Reichenbach himself used to underline the importance of his Dissertation to his philosophical development. In 1932, he wrote: "I am still convinced that the basic idea of this work is very essential" (Reichenbach and Cohen 1978: 1). In particular, Reichenbach believed that his very method of "logical analysis of science", which he considered his most important contribution to philosophy of science, was already in use in his Dissertation.

As a matter of fact, in the 1910s, Reichenbach showed an avid interest in psychology. In the early teens, he studied this discipline with Carl Stumpf in Berlin and with Ernst von Aster in Munich.[16] Furthermore, when he enrolled in the University of Göttingen in 1914, he tried

[14] See Kurt Lewin's letter to Hans Reichenbach of 1.08.1923 (HR-016-36-25).

[15] See Hans Reichenbach's letter to Moritz Schlick of 2.07.1926 (HR-016-18-10).

[16] At that point in time von Aster was an assistant of the phenomenologist and philosophical psychologist

to win, as a supervisor of his PhD thesis on theory of probability in sciences, not a physicist or a philosopher but the Göttingen psychologist Georg Elias Müller. (It may be noted that at the beginning of the 20th century, the problem of probability was discussed mainly by psychologists.) More than this, in the late 1910s Reichenbach published nothing on physics. In these years he wrote, together with Kurt Lewin and Otto Lipmann, a paper on psychology (1917) and reviewed a book on the theory of probability written by a psychologist (1919). Furthermore, between 1917 and 1926, Reichenbach's closer collaborator was not Moritz Schlick or Rudolf Carnap but the just mentioned psychologist Kurt Lewin, another former student of Carl Stumpf. When his *The Theory of Relativity* was published in 1920, Reichenbach sent copies of it to Einstein and Lewin.

Reichenbach's exclusive occupation with physic started only after he, together with four other students, attended in the Winter Term of 1918/19 Einstein's legendary first seminar on the theory of relativity at the Friedrich Wilhelm University of Berlin. Reichenbach's acquaintance with Einstein's ideas had an eclecticizing effect on him. Importantly enough, this turn in his academic interests coincided with the end of his political illusions. Apparently, after the Berlin November revolution of 1918, which was ultimately put down in January 1919, he stopped believing that he personally can help to change society for good. In consequence, Reichenbach concentrated his efforts to another revolutionary task: to radically change philosophy, founding it on the new discoveries in physics. From this point on, his whole energy was concentrated in this direction. Luckily enough, Reichenbach was excellently trained for this job, studying this discipline with Max Plank, Arnold Sommerfeld and Constantin Carathéodory, among others.

Reichenbach's interest in psychology is of special importance to us because the latter is what Hilbert called, a "reality", not a "formal" science.[17] Reichenbach's exercise in this realm supported his topological understanding of physics that we are going to discuss in § 8.

5. Reichenbach's Program for Analysis of Science through Axiomatization

Hilbert's influence on Reichenbach is especially prominent the book of the latte *Axiomatization*. Some interpreters hold that this was Reichenbach's best work. According to Andreas Kamlah, it was his

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Theodor Lipps. Importantly enough, Reichenbach always underlined that the philosopher who influenced him most was Ernst von Aster.

[17] Kurt Grelling, too, strongly discriminated between "reality sciences" (*Realwissenschaften*) and "formal sciences" (Milkov 2021: § 2.4).

> biggest scientific achievement which is of interest not simply from a philosophical point of view as it presents a mathematical [axiomatic] achievement of lasting value. It is no exaggeration to say that in it he discovered a mathematical [axiomatic] theory that is in no way inferior to Euclid's geometry (1993: 251).

Glymour and Eberhardt, on their part, maintain that "the *Axiomatization* is either a work very much out of its time, or the times have not changed much" (2016).

Arguably, this is the case since the book is clearly philosophical, very ambitions, and also radically innovative. Reichenbach made it clear that the subject matter of the book is philosophical already in its "Preface": it is a discussion of the *foundations* of the exact sciences. He deplored the fact that scientists are not inclined to make philosophical analyses of their theories, while philosophers, in turn, are less disposed to discuss scientific problems in depth and in exact manner.[18] Philosophers, of course, do not strive to achieve synthetic truths (positive new results). Their objective is merely analytical: to explicate the logical structure of scientific theory.[19] This is, actually, the aim of the method of analysis of science (*wissenschaftsanalytische Methode*). In 1924, Reichenbach was categorical that this task was to be best accomplished through the axiomatic method. He clearly followed his former professor in Göttingen Hilbert on this point.

Those who are well versed in history of philosophy know that the direction Reichenbach's method of logical analysis of science points to is not new. It is well-known as the method of regressive or critical analysis that was explored in antiquity by Pappus of Alexandria and in modern times by Kant (see his 1800: § 105). In it one starts the analysis from knowledge of complex structures in order to reach its fundamental but simple principles. In contrast, the progressive analysis that is performed, for example, in the Euclidean geometry starts from most simple and evident propositions in order to build up a complex system. The task of Reichenbach's regressive analysis, in particular, was to reveal the foundations of existing science. In contrast to the a priori principles and categories of human understanding that Kant postulates to this end, however, Reichenbach's new epistemology seeks to carefully distil them from science.

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[18] In order to better understand the context of this complaint of Reichenbach's, we shall remind the reader that some 30 years before that Gottlob Frege complained that the mathematicians of his time refuse to read philosophy while philosophers refused to read mathematics (1893, "Preface").

[19] Here, Reichenbach divides academic explorations into those made in the context of discovery and those made in the context of justification, which he will develop in full form only in *Experience and Prediction* (1938: 5–6).

Hilbert’s axiomatics had similar objectives. In (1902), Hilbert maintained that in order to explicate the foundations of a specific science, one had to “set up a system of axioms which contains an exact and complete description of the relations subsisting between the elementary ideas of that science” (p. 447). Moreover, scientific theories can be logically reconstructed on different grounds with the help of different axiomatics which are, however, epistemologically equivalent. On the other hand, axiomatics can have different scientific realizations, or different interpretations.

Strikingly enough, in his program for “logical analysis of science” Reichenbach also maintained that in the case when the rational reconstruction of an established scientific theory was not to be accomplished in its full form, the philosopher could slightly change it in order to improve it. In final reckoning, Reichenbach even maintained that only his axiomatics of the space and time theory brings the theory of relativity into its ultimate form: only it clearly articulates what Einstein wanted to say. Moreover, he was convinced that “the theory of relativity stands or falls with my axiomatic” (1927: 143). Only the latter makes the rigorous philosophical discussion of the former (of the theory of relativity) possible.[20]

Moreover, pursuing his ambitious project for a new philosophy that closely followed the observations and theories of science, Reichenbach often supplemented the physical with a philosophical theory of relativity. As a matter of fact, he preferred to present the theory of relativity in a specific, philosophical way (Kamlah 1993: 261 ff.). We do not believe that this was a result of, what Kamlah called, Reichenbach’s “didactic corruption”.[21] The truth is that Reichenbach was essentially a *philosopher* who presented physical theories in a philosophical way, which he believed to be more clear and precise.

6. Reichenbach’s Constructive Axiomatic

[20] Ironically enough, on this point Reichenbach’s approach relates to that of his sworn philosophical agonist Hegel. As a matter of fact, in contrast to what Reichenbach sometimes wrote about him (1951: 67ff.), Hegel, too, was interested in science (in particular, in biology and zoology). However, similarly to Reichenbach, Hegel was convinced that the results of science alone could not reveal the ultimate truth about nature. To achieve this they are to be philosophically reprocessed and correspondingly justified and supplemented. The radical divergence between Hegel and Reichenbach was that while the former used for that purpose his “speculative logic”, the latter referred to axiomatics and mathematical logic.

[21] Kamlah meant with this term that since Reichenbach intensively worked as a scientific journalist and as a popularizer of science, he developed and adopted a technique of oversimplifying most complicated scientific problems, which he also followed when he wrote theoretically.

Following Hilbert again, Reichenbach held that there were two types of definitions that helped to clear up the physical theories: conceptual definitions in mathematics and real or coordinative definitions in physics. In other words, while mathematical axioms use implicit definitions, the axioms of physics rely on explicit definitions (see § 2.1). The objective of the latter is to describe reality—that is why the axioms of physics can be true or false. In other words, they have empirical character.[22] Accordingly, Reichenbach's *Axiomatization* "starts with *elementary facts* as *axioms*" (1924: 6). They are part of the "constructive axiomatization" which begins with the empirical facts, while in mathematics axiomatics is deductive. The ultimate objective of Reichenbach's book is to construct Einstein's theory of space and time out of them. In fact, this task was already formulated in *The Theory of Relativity* (1920: 76f.) where it was opposed to Hermann Weyl's generalization of relativity (see § 7).

In this way, Reichenbach put Einstein's theory of relativity on the firm foundations of experimental facts and definitions. His axioms fix experimental and observational content or facts, derived with the help of pre-relativistic physics so that they can be easily visualized (1928a: 84, 281). This point went well with Hilbert's program which adopted the method of "ideal elements" that connected mathematics with pieces of elementary intuition (see § 2.2, n. 4).

Reichenbach's axioms of physics are of two groups: *light axioms,* defining light signals, and *material axioms*. The first refer to "real points" that have no extension, the second to material objects. Material axioms are definitions referring to the behavior of material bodies; the light axioms refer to the behavior of light. The main tenet of *Axiomatization* is the definition of metrics achieved through light signals. This is a feasible task since the metric of material objects, which uses rigid rods and perfect clocks, clearly depends on the metric of the light signals. Importantly enough, the measurements done with the help of rods and clocks are identical with those made with light signals. This means that the whole theory of space and time can be based on light axioms alone. To put it otherwise, "a metrical determination can be made using only light signals" (Ryckman 2005: 97).

An implication of this position is that

> the topological properties turn out to be more constant than the metrical ones; and the transition from the special theory to the general one represents merely a renunciation of metrical

[22] To remind the reader, Hilbert's axiomatics, too, is a hypothetico-deductive system. The deduction starts by it with axioms understood as hypotheses. Hilbert also maintained that the epistemological value of the axioms is first to be philosophically cleared up.

characteristics, while the fundamental, topological character of space and time remains the same (Reichenbach 1924: 195).

In this way, Reichenbach adopted a *relational theory of time* that was clearly directed against Newton's conception of action at a distance.[23] Besides, he maintained the primacy of ontology over mathematics.

7. Critique of Thomas Ryckman

According to Thomas Ryckman's interpretation, Reichenbach criticized Hermann Weyl's attempt to geometrize the general theory of relativity from the position of the logical empiricism. Reichenbach's *Axiomatization*, in particular, "has immediate significance for the history of logical empiricism in that it is the first sustained attempt to give what would become known as a 'rational reconstruction' of a physical theory" (2005: 96). In this connection, Ryckman maintains that Reichenbach's "positivist metascience" was dualistic—it was built up on empirical facts and logic, while Weyl's philosophy of space and time was holistic.

In truth, however, Reichenbach inaugurated his philosophical realism and *ipso facto* his empiricism already in his 1915 Dissertation and in a more articulate form in *The Theory of Relativity* (1920) where he claimed that the metric of space and time was a property of the world. It is "an ultimate fact of nature" (p. 53). Apart from this, it is a well-known fact that Reichenbach was never a positivist. Moreover, he bitterly fought positivism, in particular, in *Experience and Prediction* (1938).

It is true that in *The Theory of Relativity* and in *Axiomatization* Reichenbach was explicitly critical of Hermann Weyl's conception as presented in (1918a). This cannot be a surprise, however, since there was a substantial difference in the approaches of both philosophers to space and time from the very beginning. Weyl investigations were based on mathematical analysis—he himself published a book on this subject (Weyl 1918b). In contrast, Reichenbach build up his axiomatics synthetically. To be more specific, while Weyl's coordinating systems are a priori available, Reichenbach, as we already have seen, *constructs* his coordinating system with the help of light signals.

Ryckman also holds that Reichenbach's project "completely departed from Hilbert's axiomatic treatment of general relativity" (p. 97). As we have seen in § 2.2, however, David Hilbert developed the second stage of his axiomatics fighting Hermann Weyl's approach. In

[23] In § 8 we are going to see that Bertrand Russell developed a related theory.

agreement with Hilbert, Reichenbach held that the value of the axiomatic method is connected with the discrimination of the definitions, i.e. the conceptual part of the theory, from its observational and experimental content (1927: 133).

8. Hans Reichenbach and Bertrand Russell

Reichenbach's reverence for Bertrand Russell can only be compared to his admiration for Einstein. Unfortunately, so far, the impact of Russell on Reichenbach has been largely neglected. Ostensibly, it is worth following the facts on this account more closely.

Already while in Germany, Reichenbach published two acclamatory papers on Russell (1928b, 1929). Apparently, they were written in the context of the increased interest of the Reichenbach-led Berlin Group into Russell's works that found expression in Grelling's translation of four books of, arguably, the father of analytic philosophy, between 1927 and 1930: *Analysis of Mind*, *Analysis of Matter*, *ABC of Relativity* and *An Outline of Philosophy* (Milkov 2005c). Years later, between September 1939 and April 1940, Reichenbach and Russell shared an office at the University of California in Los Angeles where Reichenbach was Full Professor and Russell a "Flint-visiting professor" (Reichenbach and Cohen 1978: 79). We have all the reasons to assume that the discussions the two philosophers had in these months revived Russell's interest in probability and induction that he once demonstrated in *The Problems of Philosophy* (1912). Russell's newly focused attention to these topics found expression in his last book in theoretical philosophy, *Human Knowledge: Its Scope and Limits* (1948). It is true that in it he made a lot of critical comments on Reichenbach's frequency theory of probability—Russell himself followed Keynes' conception. His extensive discussion of Reichenbach's conception in the book, however, supports the suggestion that Reichenbach directly motivated Russell to a final elaboration of his theory of probability and induction.

On Reichenbach's side, traces of this encounter are prominent in his contribution to Schilpp's Russell volume (1944) and in the paper "A Conversation between Bertrand Russell and David Hume" (1949). On March 28, 1949, Reichenbach wrote a lengthy open letter to Russell, which was published in his *Selected Writings* (1978, ii: 405–11) by way of answering Russell's critique on his theory of probability in *Human Knowledge* (1948). Furthermore, at the end of his extended visit to Europe in the summer of 1952, Reichenbach flew to UK only to have a conversation with Russell. Apparently, this meeting was very important to him. Why was this the case?

We shall begin our analysis of Reichenbach's debt to Russell referring to the fact we already mentioned (in § 2.2) that Russell's *The Principles of Mathematics* (1903) and the co-authored by him *Principia mathematica* (1910–13) stimulated Hilbert's circle in Göttingen to start working in mathematical logic. In 1917/18 Hilbert delivered his first lectures on mathematical logic and in 1928 he, together with Wilhelm Ackermann, published *Grundzüge der theoretischen Logik*, a book that "deals with mathematical logic very much after the fashion of the first volume of *Principia Mathematica*" (Langford 1930: 22).

Later Hilbert's former student Reichenbach experienced similar transition from axiomatics to logic. Actually, logic loomed large already in his *Theory of Probability* (1935); but it found its finished expression in *Elements of Symbolic Logic* (1947) and in *Nomological Statements and Admissible Operations* (1954). However, this transition was not wholly successful. This is evidenced in the fact that in his paper published in Schilpp's Russell volume, Reichenbach "voices a view of his own that was closer to Hilbert's formalism than Russell's logicism" (Salmon 1977: 70). Be this as it may, in that paper Reichenbach clearly acknowledged his indebtedness to Russell's logic (1944: viii). So, we also witness here the transition of Reichenbach's interest from Hilbert to Russell—despite the fact that, as seen in §§ 2.1 and 2.2, Hilbert's axiomatics was intrinsically connected with logic from the very beginning.

Unfortunately, as noted by Wesley Salmon, "the formal aspects of this book [Reichenbach's *Elements of Symbolic Logic*] are not very rigorous, and it is not easy pedagogically" (1977: 68). Moreover, Reichenbach also betrayed poor knowledge of the history of logic in it. For example, he did not even mention Frege's contribution in this realm and declared that Russell "has always clearly seen" the tautological character of logic (1944: 26) which is, of course, mistaken: Russell distinctly remembered his reluctance to accept this view of Wittgenstein's—it was not his initial position (1959: 157). Reichenbach also declared that Russell put logic in "close relation to conversational language" (1944: 25) referring at that to the use of quantifiers. This claim is mistaken as well. Admittedly, the technique of quantification has greater expressive power. However, it was introduced by Frege, not by Russell, who merely adopted it from Frege via Peano.

Be this as it may, Reichenbach's logic had its notable achievements. Above all, it was clearly oriented towards reality, turning its back on purely formal matters. It also showed interest in exploring ordinary language, in the tenses of verbs, and also in fictional and intentional existence. In view of these characteristics of its, it cannot be a surprise that Reichenbach criticized the belief of the logical positivists, of his friend Rudolf Carnap, in particular, that "if the mathematical logic should someday become a part of general philosophical education, the times of vague discussions and obscure philosophical systems would be over" (1944: 53). Reichenbach called this belief a "fallacy of misplaced exactness" and pleaded instead for "a true philosophical attitude" (ibid.). Russell's logic, however, was reality oriented as well. In the 1910s Russell continuously sought to renew the realistically understood by him philosophy with the help of the new symbolic logic. Later in the 1920s he tried it to apply it to the new results of psychology, in *The Analysis of Mind* (1921), and physics, in *The Analysis of Matter* (1927).

Also Reichenbach's philosophy of science was close to that of Russell. First of all, they both were philosophical realists and anti-conventionalists. Besides, both defended the topological–ontological understanding of space and time.[24] According to Russell, time is a series of autonomic events, or moments that are immediately connected one to another (Milkov 2005b). In *The Analysis of Matter* he "leave(s) open the question whether the time-order of events in one causal route can be defined in terms of causal laws" (1927: 381). This was actually the position Reichenbach presented in "The Causal Structure of the World" (1925).[25] But he was resolute that time was to be studied in topological terms.

In *Axiomatization*, Reichenbach, on his side, reduced metrical properties to those that were topological (see § 5). As we have stated in § 4, he was helped in this by his interest in "reality sciences" such as psychology. In this context, Reichenbach connected the topology of space and time with the concept of "genidentity", initially coined by Kurt Lewin in the realm of biology and physics. He referred to the concept of genidentity for the first time in *Theory of Relativity* (1920) and continued to use it in *The Philosophy of Space and Time* (1928), in connection with the theory of relativity, and in *The Direction of Time* (1956), in connection with quantum mechanics (see Milkov 2021a). This is a priori constitutive principle of human knowledge which is more fundamental than the temporal order. Importantly enough, genidentity, or the identity of the existence of individuals, is not logical but ontological identity.

The concept of "genidentity" was used also by Carnap but only in his *Aufbau* (1928, §§ 128, 159), while, as just seen, Reichenbach never lost his interest in it. Of particular interest is

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[24] Thomas Mormann correctly noticed that Russell used topological methods for analyzing space and time; but his is mistaken when we maintains that the topological analysis "never occurred to the mainstream philosophers of science" (2013: 9). It loomed large in Reichenbach's works.

[25] In *The Analysis of Matter* Russell called it "a valuable article" (1927: 381 n.). Interestingly enough, Russell did not mention in his book, which was wholly devoted to philosophy of physics as it developed after Einstein's Theory of Relativity, any other logical empiricist—neither Schlick, nor Philipp Frank, nor Carnap.

that in that book (§ 128) Carnap maintains that the term genidentity was also used by Russell in *Our Knowledge* (1914). Apparently he had in mind Russell's discussion of individuals as series of spatial-temporal relations (Milkov 2003: 64). We are not going to discuss here is Carnap's interpretation of Russell correct. It should be only noted that it indicates Russell's clear ontological commitment.

Last but not least, the topological stance of Reichenbach and Russell brought them close to the problem of modality.[26] Russell already studied modalities in *The Problems of Philosophy* (1912). In *Nomological Statements* (1954) Reichenbach, on his side, claimed that laws of nature applied to all possible worlds.[27]

By way of concluding this section, and also to the present Chapter, we can summarize that Reichenbach's philosophical development showed a transition not only from axiomatics to logic but also from epistemology to ontology.

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[26] On the connection between topology and modality see the last paragraph of § 2.2.

[27] Reichenbach's student Hilary Putnam continued to explore this idea.

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