# Thomistic Foundations for Moderate Realism about Mathematical Objects

## Contents



## 1. Extended Abstract

Contemporary philosophers of mathematics are deadlocked between two alternative ontologies for numbers: Platonism and nominalism. According to contemporary mathematical Platonism, numbers are *real abstract objects*, i.e. particulars which are nonetheless "wholly nonphysical, nonmental, nonspatial, nontemporal, and noncausal."[1] While this view does justice to intuitions about numbers and mathematical semantics, it leaves unclear how we could ever learn anything by mathematical inquiry. Mathematical nominalism, by contrast, holds that numbers do not exist extra-mentally, which raises difficulties about how mathematical statements could be true or false. Both theories, moreover, leave

[1] Mark Balaguer, "Fictionalism in the Philosophy of Mathematics," in *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta, 2013, http://plato.stanford.edu/archives/fall2013/entries/fictionalism-mathematics/.

inexplicable how mathematics could have such a close relationship with natural science, since neither abstract nor mental objects can influence concrete physical objects.[2]

The Thomist understanding of quantity as an accident inhering in concrete substances breaks this deadlock by granting numbers a foundation in extra-mental reality[3] which explains why numeric expressions are relevant in natural science and how we come to know the truth or falsity of mathematical statements. Further, this kind of moderate realism captures the semantic advantages of the Platonists[4] and the ontological parsimony of the nominalists (since for Thomists quantity is not a separate, free-standing ontological addition). Despite these advantages, the Thomistic understanding of number has been out of favor since the work of Cantor, who argued that actual infinities (regarded by Aristotelians as impossible) are required for modern developments in mathematics.[5] This paper frees philosophers of mathematics to embrace the advantages of Thomistic realism by showing that Cantor's understanding of actual and potential infinity is not the same as Aquinas's, and that potential infinities (in Aquinas and Aristotle's sense)[6] can do all the work Cantor claims for actual infinities in modern mathematics.

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[2] Eugene P. Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences. Richard Courant Lecture in Mathematical Sciences Delivered at New York University, May 11, 1959," *Communications on Pure and Applied Mathematics* 13, no. 1 (1960): 1–14; R. W. Hamming, "The Unreasonable Effectiveness of Mathematics," *The American Mathematical Monthly* 87, no. 2 (1980): 81–90; Arthur M. Lesk, "The Unreasonable Effectiveness of Mathematics in Molecular Biology," *The Mathematical Intelligencer* 22, no. 2 (March 1, 2000): 28–37, https://doi.org/10.1007/BF03025372; Sundar Sarukkai, "Revisiting the 'unreasonable Effectiveness' of Mathematics," *Current Science* 88, no. 30 (2005): 415–23; Arkady Plotnitsky, "On the Reasonable and Unreasonable Effectiveness of Mathematics in Classical and Quantum Physics," *Foundations of Physics* 41, no. 3 (March 1, 2011): 466–91, https://doi.org/10.1007/s10701-010-9442-2; Roland Omnès, "Wigner's 'Unreasonable Effectiveness of Mathematics', Revisited," *Foundations of Physics* 41, no. 11 (November 1, 2011): 1729–39, https://doi.org/10.1007/s10701-011-9587-7.

[3] Thomas Aquinas, *Super Boetium De Trinitate, Expositio Libri Boetii De Ebdomadibus*, Leonine, Sancti Thomae de Aquino opera omnia 50 (Paris: Le Cerf, 1992), http://archive.org/details/operaomniaiussui02thom, 5.3.

[4] Richard Pettigrew, "Platonism and Aristotelianism in Mathematics," *Philosophia Mathematica* 16, no. 3 (October 1, 2008): 15–16, https://doi.org/10.1093/philmat/nkm035.

[5] Georg Cantor, *Gesammelte Abhandlungen*, ed. Ernst Zermelo (Heidelberg: Springer, 1932), http://www.springer.com/gp/book/9783662002544.

[6] Thomas Aquinas, *In Duodecim Libros Metaphysicorum Aristotelis Expositio.*, ed. M. R. Cathala and Raymundi M. Spiazzi (Taurini: Marietti, 1950), https//catalog.hathitrust.org/Record/001913444, Bk. XI, lect. 10.

## 2. Introduction

In the second half of the twentieth century there has been an ongoing fascination with the "unreasonable effectiveness" of mathematics in natural science. In the predominant contemporary metaphysical accounts of mathematical beings, that effectiveness is indeed condemned to remain a mystery. While the Thomist account, in which mathematical objects are abstractions from the real category of quantity, does a great deal to explain the effectiveness of mathematics in describing the physical world, it was broadly rejected due to seeming incompatibilities with nineteenth and twentieth century developments in the logical foundations of mathematics. The most historically important claims of incompatibility, however, are apparent rather than real, and reflect limitations in the claimants understanding of both Aquinas's views and the implicit metaphysics of mathematical formulae. I show this by first situating the Thomist position within contemporary debates in philosophy of mathematics, second illustrating the need for quantity in motivating accounts of number rich enough for analysis, and third showing that Thomism is not incompatible with the mathematical analyst's need for infinite sets.

## 3. Formal and Material Mathematics

### 3.1 Why should math have anything to do with matter?

Wigner, who popularized the widely repeated notion of mathematics' "unreasonable effectiveness,"[7] describes it as "the miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics" which "is a wonderful gift which we neither understand nor deserve."[8] When Sarukkai revisited Wigner's thesis, he observed that "The mysteriousness that is enshrined in the phrase 'unreasonable effectiveness' of mathematics reflects as much a confusion about

[7] "The Unreasonable Effectiveness of Mathematics in the Natural Sciences." with almost 2,000 citations in Google Scholar; Hamming, "The Unreasonable Effectiveness of Mathematics"; Lesk, "The Unreasonable Effectiveness of Mathematics in Molecular Biology"; Sarukkai, "Revisiting the 'unreasonable Effectiveness' of Mathematics"; Plotnitsky, "On the Reasonable and Unreasonable Effectiveness of Mathematics in Classical and Quantum Physics"; Omnès, "Wigner's 'Unreasonable Effectiveness of Mathematics', Revisited."

[8] "The Unreasonable Effectiveness of Mathematics in the Natural Sciences.," 14.

the mysteriousness of what mathematics is…as its applicability."[9] For Wigner, "mathematics is the science of skillful operations with concepts and rules invented just for this purpose…the principal emphasis is on the invention of concepts."[10] It is natural that pragmatic applicability to physics would seem wondrous on such a nominalist view of what mathematics is. Even if mathematics is understood as more closely related to logic, it is unclear why it should be such a rich "tool for exploring the universe as we perceive it at present" given the fairly minimal constraint and assistance logic normally provides for theorizing.[11]

One might expect less difficulty in understanding the applicability of mathematics for those with a realist understanding of mathematics in which "mathematical entities exist independently of humans just as trees and tables do" but "Platonism about mathematical entities is the dominant realist tradition" and "Platonic entities…do not have spatial or temporal characteristics."[12] More technically, Platonists take numbers to be abstract objects, "and abstract objects, platonists tell us, are wholly nonphysical, nonmental, nonspatial, nontemporal, and noncausal."[13] Unsurprisingly then, "Platonism, although popular among mathematicians and scientists, runs into serious problems when confronted with the applicability of mathematics"—human epistemic access and applicability to physics are both deeply mysterious for such entities.[14] In response to this difficulty, Sarukkai proposes that "mathematics is a product of human imagination" but "grounded in our experience with the world" such as "the activity of counting and aggregating."[15] Since Sarukkai's proposal grants a role for both objective ontology and human intellection, it is a kind of middle way between nominalist and Platonist accounts of

[9] Sarukkai, "Revisiting the 'unreasonable Effectiveness' of Mathematics," 417.
[10] Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences.," 2.
[11] Hamming, "The Unreasonable Effectiveness of Mathematics," 90.
[12] Sarukkai, "Revisiting the 'unreasonable Effectiveness' of Mathematics," 416.
[13] Balaguer, "Fictionalism in the Philosophy of Mathematics."
[14] Sarukkai, "Revisiting the 'unreasonable Effectiveness' of Mathematics," 416.
[15] Sarukkai, 417.

mathematics which he suggests “functions like a language.”[16] If “mathematics actually ‘arises’ from the world then the unnatural connection is no longer there, thereby diluting the puzzle as far as this relation is concerned.”[17] Sarukkai’s proposal is reasonable given the epistemic and scientific mysteries created by nominalist and Platonist accounts of mathematics, but he fails to advert to the history or constraints of the middle way often vaguely categorized as “anti-nominalism” or “lightweight Platonism.”[18]

### 3.2 Not just any matter will do

One of the principal philosophical arguments for mathematical Platonism is Frege’s contention that numbers function as proper names in mathematical discourse.[19] After all, it seems that no concrete object could provide a referent for many important mathematical symbols:

> no Euclidean straight line, regular polygon, etc. can be instantiated in concrete objects, or can be used as description of concrete objects. In fact, in the age of the theory of relativity and of quantum mechanics, when it becomes justifiable to represent the universe as having a finite diameter and containing a finite number of elementary particles, and to conceive of space-time as having a curvature different from 0, which concrete object could be considered as a straight line without breadth and infinite in length having 0 curvature, or as an instantiation of a transfinite cardinal number greater than $2^{\aleph_0}$?[20]

If mathematical symbols are proper names for objects ,then, they must be the proper names of abstract objects. Further, since it seems that mathematicians make truth claims about the referents of their symbols, this argument has been persuasive enough that many nominalists have become fictionalists.[21] Mathematics may be a language, as Sarukkai suggested, but considerations from philosophy of language

[16] Sarukkai, 417.
[17] Sarukkai, 417.
[18] Øystein Linnebo, “Platonism in the Philosophy of Mathematics,” in *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta, 2013, http://plato.stanford.edu/archives/win2013/entries/platonism-mathematics/.
[19] Bob Hale and Crispin Wright, *The Reason’s Proper Study: Essays Towards a Neo-Fregean Philosophy of Mathematics* (Clarendon Press, 2003), 7–8.
[20] Gianluigi Oliveri, *A Realist Philosophy of Mathematics*, Texts in Philosophy 6 (London: College Publications, 2007), 82.
[21] Balaguer, “Fictionalism in the Philosophy of Mathematics.”

seem to worsen rather than ease the difficulties philosophy of science faces in understanding the application of mathematics.

A potential escape route is charted by Pettigrew, who shows that neither formal nor semantic tests can discriminate between Platonist and free variable theories for interpreting mathematical language.[22] Pettigrew's free variable semantics is compatible with a Thomist understanding of the relationship between matter and mathematics since it ""posits as separate what is not separate, the very thing that the arithmetician and the geometer do,"[23] with the axioms as the posits of that separation. I will not give an account in this paper of the material grounds of that separating abstraction in rational psychology, focusing instead on the formal, metaphysical question of whether quantity has necessary and sufficient content for the objects of contemporary mathematics. Sarukkai does hint, however, at Aquinas's claim that mathematical understanding resolves to the imagination, averring that:

> There are innumerable examples of the importance of symbolic manipulation based on formal similarity. In fact, I would go to the extent of saying that the effectiveness of mathematization significantly depends on the power of symbols to act like pictures of ideas, concepts and events. The role of mathematics in the sciences seems to be essentially dependent on the possibility of using mathematical symbols as 'pictures'. For example, we could look upon 1/2ab2 as the 'picture' of kinetic energy. So even in contexts that are very different we can still recognize that picture and identify it with the kinetic energy of that object or system.[24]

Plotnitsky, too, recognizes that any account of the development and effectiveness of mathematics must involve idealization, since we "disregard those features of [objects] that are not thus mathematically idealized"[25]—Aquinas would suggest that these are matter and motion. This role for the imagination, however, must rely on real abstraction for its content, however, rather than mere representation, since

[22] "Platonism and Aristotelianism in Mathematics," 15–16.
[23] Aristotle, *Metaphysics*, trans. Joe Sachs, 2nd edition (Santa Fe, N.M: Green Lion Press, 2002), bk. XIII.4 (1078a22).
[24] Sarukkai, "Revisiting the 'unreasonable Effectiveness' of Mathematics," 422.
[25] Plotnitsky, "On the Reasonable and Unreasonable Effectiveness of Mathematics in Classical and Quantum Physics," 467.

the mathematical ‘picture’ of kinetic energy also holds in quantum physics, where we cannot represent the underlying reality in our imaginations at all.[26] Matter must be involved in the grounding of mathematics for it to work in physics, but the semantics of that work demand that it must be abstracted from matter itself. This is exactly the account Aquinas gives of abstracting mathematicals from quantity, a real mode of being.

### 3.3 The Goldilocks Problem

The difficulty for the Thomist, then, is not making sense of mathematics in the first act of the intellect but in the second. Are the mathematicals abstracted from quantity really necessary for mathematical reasoning itself, or only for its application to physics? If the latter, again a strange coincidence arises which suggests an unreasonable effectiveness. Can the objects needed for mathematical analysis, like infinite sets, be abstracted from quantity or are they precluded by its being a real mode of being? If the latter, it again seems that mathematics is working in two modes with their compatibility as an awkward and unexplained coincidence. Both of these objections must be met for the Thomist position to regain currency in the philosophy of mathematics, despite the difficulties in which the others find themselves.

## 4. Quantity is Necessary for Mathematics: A Ratio Measure

One concern repeatedly raised for the Thomist is whether mathematics really requires an extra-mental foundation. The arguments of the Fregean platonists against the nominalists, with which the Thomist can agree in part, are one possible avenue of response. The difficulty can be put in a more acute form, however, internal to the Thomist project. The Thomist says that there are three persons in God, ten categories which divide being, and a multitude of the heavenly host. Aquinas may aver that these numbers are not the same as those of quantity,[27] but what can he offer to ground the distinction?

[26] Plotnitsky, 468.
[27] Aquinas, *In Meta.*, 476 Bk. X, lect. 4, n. 1997.

Not only do these values seem suspiciously like numerical quantities, but logical non-identity, which they surely presuppose, can be used to give definitions for the ordinal numbers which found mathematics.[28] The unique claim Aquinas makes for magnitude, or real quantity, is that it "is said to be divisible into infinity" by the mathematical operation of division.[29] This matters because mere sets are not enough for mathematical analysis: the sets have serve as the basis for rings which define the operations on those sets. As Stevens illustrated, the origin of the sets matters if the operations are to be meaningful.[30] If numbers merely indicate nominal differences or ordering, multiplication and division will give meaningless results: the set might be defined, but the ring is not defined on the set. Division is undefined for transcendental multitudes: there is no meaningful claim that can be made about half a divine person, half an angel (since they are simple), or half of a mode of being. If mathematics is to include the operations of division and multiplication, it must define them on a ring, and that ring must be grounded in a set on which those operations make sense. As Aquinas tells us, that set is continuous quantity, which is grounded in the fundamental divisibility of matter. As such, quantity is necessary for mathematical objects—mere logical difference will not do.

## 5. Quantity is Sufficient for Mathematics: Actual Potential Infinity

Cantor's theory of sets, which most accept as the basis of contemporary mathematics, "requires a commitment to the existence of actual, completed infinities."[31] Perhaps unsurprisingly, both he and the subsequent traditions of mathematics and philosophy of mathematics understood this commitment

[28] Rchard E. Hennessey, "Thomas Aquinas: Beyond Aristotle's Aristotelian Conception of the Numerical," *After Aristotle* (blog), April 1, 2014, https://afteraristotle.net/2014/04/01/thomas-aquinas-beyond-aristotles-aristotelian-conception-of-the-numerical/.
[29] "Alius modus secundum quod infinitum dicitur ablatione et divisione, secundum quod magnitudo dicitur divisibilis in infinitum." Aquinas, *In Meta.*, 550 Bk. XI, lect. 10.
[30] Stanley Smith Stevens, "On the Theory of Scales of Measurement," *Science* 103, no. 2684 (June 7, 1946): 677.
[31] Anne Newstead, "Cantor on Infinity in Nature, Number, and the Divine Mind," *American Catholic Philosophical Quarterly* 83, no. 4 (2009): 534.

as "overthrow[ing] a long-standing scholastic and Aristotelian tradition."[32] If Aristotle's arguments against the existence of actual infinities in nature were sound, then so much the worse for Aristotle's entire philosophy: completed infinities must be platonic objects. I suggest, however, that the sense in which Cantor demanded that infinities be complete is not at odds with Aquinas's scholasticism, and thus need not entail any rejection of the Aristotelian project.

Let us begin our analysis by examining the formal statements of set theory in its later, more precise formalization by Zermelo and Fraenkel. They begin by borrowing from J.S. Mill the definition of a set as "the indefinite multitude of individuals denoted by a general name."[33] Mill in turn defines a general name as "capable of being truly affirmed, in the same sense, of each of an indefinite number of things."[34] They then define the successor relation: for any set a, its successor $a^+ = a \cup \{a\}$ and at this point the Axiom of Infinity can be precisely stated: $(\exists A)[\emptyset \in A \,\&\, (\forall a \in A) a^+ \in A]$.[35] Cantor's motivation for moving to this formulation, and his readiness to call it actual or completed infinity, is due to the contrast with what he considered informal constructions like [0,1,2,3…]. These open formulae, which suggest some process external to that explicitly denoted on the page, seemed insufficiently rigorous to Cantor.

The older formulation was a more natural fit for Cantor's late-scholastic interlocutors, since it seems like a fair symbolic representation of Aquinas's first sense of infinity, which he explicitly applies to number, since "any number is always given to succession by one, and thus number is by augmentation into infinity."[36] Thus it is a possible infinity, since the successor can be given, but is not already extant.

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[32] Newstead, "Cantor on Infinity in Nature, Number, and the Divine Mind."
[33] John Stuart Mill, *A System of Logic, Ratiocinative and Inductive, Being a Connected View of the Principles of Evidence, and the Methods of Scientific Investigation*, 3rd Ed, vol. 1 (London: J. W. Parker, 1851), 28.
[34] Mill, 1:24.
[35] Herbert B. Enderton, *Elements of Set Theory* (Academic Press, 1977), 68.
[36] "Semper enim cuilibet numero dato est apponere unitatem, et sic numerus est augmentabilis in infinitum." Aquinas, *In Meta.*, 550 Bk. XI, lect. 10.

Each ordinal number enumerated within the set and the cardinality of the set as a whole are numerable, so they are subject to Aquinas's argument that they must each be finite, since "nothing numerable is infinite, because the numerable must be passed through by number: therefore no number is infinite."[37] Cantor's new formulation, however, asserted the existence of a complete extant set, presupposing rather than using the successor relation. Is this not, then, the kind of infinite Aquinas found "repugnant to cognition" and unable to be understood?[38] Where the ellipsis signified "potential infinite always consist[ing] in succession" is not the Axim of Infinity the posit of a number line lacking termini, and thus an actual infinite?[39] Aquinas's concern is that

> it is possible that our intellect should perfectly know in a certain manner the infinite continuum; but in no way discrete infinities, for the reason that it is not possible to understand many things through one species; and thence it is that if many things must be considered, it is necessary that one be known after another, and thus discrete quantity is known through the means of infinity. Whence if the infinite multitude should be known in act, it should follow that he would know the infinite through the way of infinity; that is impossible.[40]

Since the Axim of Infinity asserts the existence of $\aleph_o$ rather than $\mathbb{R}$, or in contemporary parlance discrete rather than continuous infinity, this might seem like an insuperable difficulty.

Cantor's set theory and its further developments are, however, genuine breakthroughs in mathematics unknown to Aquinas. Aquinas, living before the advent of analysis, was unaware of any

[37] "nullum numerale est infinitum, quia numerale est pertransibile numerando: ergo nullus numerus est infinitus." Aquinas, 551.

[38] "Infinitum autem, sicut repugnat cognitioni, ita et repugnat transitioni: infinitum enim nec cognosci nec transiri potest" Thomas Aquinas, *Quaestiones disputatae de veritate, qq. 1-7.*, ed. P. Antoine Dondaine, Leonine, vol. 1.2, Sancti Thomae de Aquino opera omnia 22 (Rome: Ed. di san Tommaso, 1970), 73, http://gallica.bnf.fr/ark:/12148/bpt6k9482m Q. 2, a. 9, co.

[39] "Dicendum, quod duplex invenitur infiniti distinctio. Uno modo distinguitur per potentiam et actum; et dicitur infinitum potentia quod semper in successione consistit, ut in generatione et tempore et divisione continui, in quibus omnibus est potentia ad infinitum, semper uno post aliud accepto; actu autem infinitum, sicut si poneremus lineam terminis carentem." Aquinas, 1.2:75–76 Q.2, a. 10, co.

[40] "sic possibile est ut intellectus noster quodammodo infinitum continuum perfecte cognoscat; sed infinita discrete nullo modo, eo quod non potest per unam speciem multa cognoscere; et inde est quod si multa debet considerare, oportet quod unum post alterum cognoscat, et ita quantitatem discretam per viam infiniti cognoscit. Unde si cognosceret infinitam multitudinem in actu, sequeretur quod cognosceret infinitum per viam infiniti; quod est impossibile." Aquinas, 1.2:73 Q. 2, a. 9, co.

non-geometric means of bringing many numbers under a single act of understanding. Note the way he assumes that discrete quantity is known by succession, one after another. This is exactly what the formal statement of the Axiom of Infinity avoids, however: the successor relation is understood formally, and so the set's act of existence and its intelligibility are also singular. This was Cantor's achievement. The idea of passing through the infinite set by actually using the successor relation is what Cantor wished to avoid. The infinity is completed neither by an infinite actual enumeration nor, as in Peano Arithmetic, by an infinity of axioms.[41] Instead, the mathematical object of discrete infinity can be abstracted, because Cantor has provided a method for doing so which does "not lead to cognition of a singular as insomuch as those things by which singulars are mutually distinguished, but only insomuch as their common nature."[42] The definition of the infinite set that Cantor provides is not done by enumeration from the elements, nor does it give the distinguishing marks of those elements (their actual successions): it only asserts that the set does actually contain all of the possible successors. As such, for Aquinas it counts as a potential infinity, since the species before the mind is that of the set, not that of the elements.[43] The set is not repugnant to cognition because there is no necessity to traverse it. The elements of the set are of course untraversable, and thus properly infinite, but it is not the elements which must be passed through to give a measure to the set: Cantor's infinite sets are measured by transfinite cardinals without enumeration, and therefore they are not actual infinities in Aquinas's

[41] Usually written as a single second-order axiom of induction, but which is equivalent to an infinite number of first order succession axioms.

[42] "sed similitudo rei recepta in intellectu, est absoluta a particularibus conditionibus, unde cum sic elevatior, est ductiva in plura. Et quia una forma universalis nata est ab infinitis singularibus participari, inde est quod intellectus quodammodo infinita cognoscit. Sed quia illa similitudo quae est in intellectu, non ducit in cognitionem singularis quantum ad ea quibus singularia ad invicem distinguuntur, sed solum quantum ad naturam communem;" Aquinas, *De Veritate (v1)*, 1.2:73 Q. 2, a. 9, co.

[43] "inde est quod intellectus noster per speciem quam habet apud se non est cognoscitivus infinitorum nisi in potentia;" Aquinas, 1.2:72 Q. 2, a. 9, co.

metaphysical sense.[44] This is precisely the possibility Aquinas allows for, though his intended model is the continuum rather than discrete infinity:

> If some infinite were known through the way in which it is infinite, in no way could it be perfectly known; if however it should be known not through the way [in which] it is infinite, then he will be able to know it perfectly: because the ratio [notion] of infinity befits quantity, according to Aristotle in the first book of the Physics, and every quantity has from its own ratio an order of parts; it follows that now when it is apprehended part after part the infinite would be known through the way [in which it is] infinite.[45]

Infinity actually befits quantity, because quantity gives a way to bring the infinite under a single species, the line segment or surface.

Beyond an analogy between Cantor's infinite discrete set and Aquinas's understanding of the continuum, there are other reasons to suppose that Aquinas would be pleased rather than displeased with Cantor's advance. He recognizes, for instance, that mathematical relationships (namely equality) can exist among infinities: "Just as a certain finite is equal to a certain finite, thus an infinite is equal to another infinite."[46] Outside of Cantor-inspired set theory, however, there is no known way to formalize and prove this fact. Further, Aquinas recognizes inductive results in number theory:

> If odd numbers are added to unity in order, the same form of number is always produced. Suppose, if one is added to three, which is the first odd they make four, which is a square number: to which again if the second odd, namely five, is added it makes nine, which again is a square number, and thus always into infinity. But in even numbers it always makes a different form. For if unity is added to binary, it makes ternary, which is a triangular number; to which again if quaternary, which is the second even, is added, it makes septenary, which is of septangular form, and thus into infinity.[47]

[44] "infinitum proprie est, quod mensurando pertransiri non potest. Tot ergo modis dicitur infinitum, quot modis dicitur intransibile." Aquinas, *In Meta.*, 550 Bk. XI, lect. 10.

[45] "si aliquod infinitum cognoscatur per viam per quam est infinitum, nullo modo perfecte cognosci potest; si autem cognoscatur non per viam infiniti, sic perfecte cognosci poterit: quia enim infiniti ratio congruit quantitati, secundum philosophum in I Phys., omnis autem quantitas de sui ratione habet ordinem partium; sequitur quod tunc infinitum per viam infiniti cognoscatur, quando apprehenditur pars post partem." Aquinas, *De Veritate (v1)*, 1.2:73 Q. 2, a. 9, co.

[46] "sicut quoddam finitum est aequale cuidam finito, ita infinitum est aequale alteri infinito." Aquinas, 1.2:52–53 Q. 2, a. 3, ad 4.

[47] "si supra unitatem impares numeri per ordinem adduntur, semper producitur eadem figura numeralis. Puta, si supra unum addantur tria, qui est primus impar consurgunt ipsum quatuor, qui est numerus quadratus: quibus

In Robinson Arithmetic, without Peano's infinite axioms or Cantor's so-called actually-infinite sets, all of number theory's claims about particular numbers can be proven, but not the inductive results which go on into infinity.[48] If Aquinas is committed to these claims about number theory being real mathematical results, then, he is implicitly committed to the Axiom of Infinity.

## 6. Conclusion

Not only is the Thomist best-equipped to explain the origin and application of mathematical concepts in quantity, he can explain why quantity is both necessary and sufficient for the mathematical beings of analysis. In abstraction from matter and motion that resolves the imagination, mathematicals are well-suited for physics. In providing a unique ground for ratio numbers, they ground the rings which define the range of mathematical operations. With Cantor's axiomatic treatment they ground the infinite sets necessary for analytic proofs, contrary to Cantor's own misunderstanding of the scholastic tradition. Thomism should be a live and even leading option in the philosophy of mathematics.

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rursus si addatur secundus impar, scilicet quinarius, consurgit novenarius, qui item est numerus quadratus, et sic semper in infinitum. Sed in numeris paribus semper surgit alia et alia figura. Si enim unitati addatur binarius, qui est primus par, consurgit ternarius, qui est numerus triangularis; quibus si rursus addatur quaternarius, qui est secundus par, consurgit septenarius, qui est septangulae figurae, et sic in infinitum." Thomas Aquinas, *Sentencia libri de Anima*, Sancti Thomae de Aquino opera omnia. 45 (Paris: J. Vrin, 1985), 33, http://gallica.bnf.fr/ark:/12148/bpt6k9497f Bk. I, lect. 7, n. 93 at Bekker 406b30.

[48] "Robinson Arithmetic," in *Wikipedia*, February 1, 2017, https://en.wikipedia.org/w/index.php?title=Robinson_arithmetic&oldid=763214648.

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