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Updating, undermining, and perceptual learning

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Abstract As I head home from work, I'm not sure whether my daughter's new bike is green, and I'm also not sure whether I'm on drugs that distort my color perception. One thing that I am sure about is that my attitudes towards those possibilities are evidentially independent of one another, in the sense that changing my confidence in one shouldn't affect my confidence in the other. When I get home and see the bike it looks green, so I increase my confidence that it is green. But something else has changed: now an increase in my confidence that I'm on color-drugs would undermine my confidence that the bike is green. Jonathan Weisberg and Jim Pryor argue that the preceding story is problematic for standard Bayesian accounts of perceptual learning. Due to the 'rigidity' of Conditionalization, a negative probabilistic correlation between two propositions cannot be introduced by updating on one of them. Hence if my beliefs about my own color-sobriety start out independent of my beliefs about the color of the bike, then they must remain independent after I have my perceptual experience and update accordingly. Weisberg takes this to be a reason to reject Conditionalization. I argue that this conclusion is too pessimistic: Conditionalization is only part of the Bayesian story of perceptual learning, and the other part needn't preserve independence. Hence Bayesian accounts of perceptual learning are perfectly consistent with potential underminers for perceptual beliefs.

Keywords Bayesianism · Credences · Undermining defeat · Epistemology · Rigidity

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1 Introduction

On the way home from work I find myself wondering what color my daughter's new bike is. I think it might be blue, or red, or maybe green—I'm not sure. I'm also not sure whether my colleague was joking when he claimed to have slipped a (slow acting) color-hallucination-inducing drug in my afternoon coffee. One thing I am sure about at this point is that facts about my perceptual sobriety and facts about the color of my daughter's bike are evidentially unrelated: since I haven't yet seen the bike, changing my confidence in the one shouldn't affect my confidence in the other. Later I see the bike, and since appears it to be green I become confident that it is green. But something else has changed as well: now if I were to increase my confidence that I'm on color-drugs, I would begin to doubt the veridicality of my perceptual experience as of the greenness of the bike, and as a result I would reduce my confidence that the bike is green. In other words, my belief about whether I'm on color-drugs is no longer evidentially unrelated to my belief about the color of the bike; the former now serves as a potential defeater for the latter. In particular, it's an undermining defeater: instead of telling directly against the truth of *the bike is green*, it tells against the evidential support that I have for believing that proposition.

Weisberg (2009, 2014) and Pryor (2013) have argued that the case as described is in tension with the Bayesian's account of perceptual learning. That's because any two propositions that start out probabilistically independent cannot lose that independence as a result of conditionalizing on one of them. Conditionalization[1] being the primary means of rationally permissible credence revision in any Bayesian account of perceptual learning, and the loss of probabilisitic independence being essential to at least some cases of undermining defeat, they conclude that undermining defeat and Jeffrey Bayesianism are in tension or even inconsistent.[2] In this essay I argue that Weisberg's and Pryor's conclusion is overly pessimistic, and that the Bayesian account of perceptual learning is perfectly consistent with undermining defeat.

2 The puzzle

I'll begin with a quick sketch of the Bayesian account of perceptual learning that I'll be discussing. Agents assign subjective probabilities or credences to propositions (e.g. $P(A)$), with those assignments subject to norms of probabilistic coherence (call

[1] Although I am primarily interested in Jeffrey Conditionalization, I will also discuss Classical Conditionalization. Any unqualified references to 'Conditionalization' should be understood as applying to both versions of that rule.

[2] Thought both Pryor's and Weisberg's written work supports this reading, in conversation they both take the lesson of the puzzle to be somewhat weaker: Weisberg takes it as a reason to abandon *subjective* Bayesianism for an *objective* version that permits updates directly upon perceptual states, while Pryor takes the lesson to be very similar to what I argue below. In this essay I'll be responding to their written work.

that thesis 'Probabilism'). Probabilities are also assigned to propositions conditional on other propositions (e.g. $P(A \mid B)$), which for our purposes I'll understand as being defined in terms of unconditional probabilities according to the formula $P(A|B) =_{df} \frac{P(A\&B)}{P(B)}$. Perceptual experience leads agents to revise some subset of their credences, which by a process of conditionalizing on this new evidence leads to revisions in other credences.

Bayesians understand the process of conditionalizing on new evidence in slightly different ways. According to Classical Bayesians, upon changing credence in B to 1 (due to a perceptual experience, or whatever) the agent updates by setting her new credence in A to her old credence in: A conditional on B. In other words, where $P_{old}(\cdot)$ is the probability function accepted by the agent before having the relevant perceptual experience and $P_{new}(\cdot)$ is the function accepted by the agent after having the experience and updating on B, Classical Bayesians claim that for any A and B,

Classical Conditionalization: $P_{new}(A) = P_{old}(A|B)$

Jeffrey Bayesians[3] generalize the Classical program by relaxing the requirement that all conditionalization be on propositions assigned a credence of 1. I'll go into more detail about how Jeffrey Conditionalization works below, but here's a rough sketch: the process begins with an assignment of credences to some subset of the propositions to which the agent assigns credences. These credence assignments are laundered (see below) into a partition of the agent's state space, in which that space is divided into an exhaustive and exclusive set of ways that the world might be (the 'elements' of the partition), with each way assigned a credence. Finally, the agent conditionalizes on this partition with its weighted elements—call them the B_i—using the following rule:

Jeffrey Conditionalization: $P_{new}(A) = \sum_i P_{old}(A|B_i)P_{new}(B_i)$

With these preliminaries in place, let's return to the puzzle of the color-drugs and the bike. Before seeing the bike I regarded the veridicality of my own color perception as irrelevant to the greenness of the bike, and hence I regarded the propositions *I'm on color-drugs* and *the bike is green* as being probabilistically independent. Taking $P_{old}(\cdot)$ as the probability function that I accepted before having a perceptual experience as of the greenness of the bike, that means that:

(1) $P_{old}(green \mid color - drugs) = P_{old}(green)$

After I've had an experience as of the bike being green and shifted my partition accordingly, I adopt the credence function $P_{new}(\cdot)$ that results from the relevant conditionalization procedure. At this point I no longer regard the two propositions as being independent, but instead regard *I'm on color-drugs* as a defeater for *the bike is green*. I'll interpret this as saying that:

[3] For the purposes of this essay a Jeffrey Bayesian is any Bayesian who accepts Richard Jeffrey's generalization of the Classical Conditionalization, what I'm calling 'Jeffrey Conditionalization'. Our 'Jeffrey Bayesians' needn't share Richard Jeffrey's particular views about the motivations for accepting that rule (Jeffrey 1992), Radical Probabilism (Jeffrey 2004) or anything else.

(2) $P_{new}(green \mid color - drugs) < P_{new}(green)$

Weisberg and Pryor observe that the introduction of a negative probabilistic correlation between two propositions through a process of updating on one of them is problematic within the Bayesian framework. That's because Jeffrey Conditionalization is rigid:[4]

> Jeffrey Rigidity: If $P_{new}(\cdot)$ is the credence function resulting from accepting $P_{old}(\cdot)$ and then updating on a shift in partition $\{B_i\}$, then for any proposition A and any $B_i \in \{B_i\}$, $P_{new}(A|B_i) = P_{old}(A|B_i)$

and rigid updating rules preserve independence:[5,6]

> Rigidity is Independence Preserving (RIP): If the transition from $P_{old}(\cdot)$ to $P_{new}(\cdot)$ is rigid on partition $\{B_i\}$ and $P_{old}(B_i|A) = P_{old}(B_i)$ for every $B_i \in \{B_i\}$, then $P_{new}(B_i|A) = P_{new}(B_i)$ for every $B_i \in \{B_i\}$

Hence Weisberg's puzzle, as I'll call it, is this: our intuitions about undermining defeaters commit us to both (1) and (2), but if learning from an experience as of a green bike involves updating on *the bike is green* using a rigid updating rule such as Jeffrey Conditionalization, then that combination is impossible.[7,8] As Weisberg puts

[4] See Jeffrey (1992, p. 80) and Weisberg (2014, p. 125).

[5] See Weisberg (2014, p. 126). For the Classical versions of the Rigidity and RIP principles take the partition to consist of a single cell weighted to 1.

[6] The independence-preserving nature of rigid update rules also creates problems for what we might call 'promoters'. My confidence that the bike is green might be very high after an experience as of its greenness, and then become higher still when I learn that I'm on drugs that make my color-perception especially reliable. This would require that my new credence function include a *positive* correlation between those propositions, which cannot be introduced via a rigid updating rule (assuming that were independent before the experience). Thanks to an anonymous referee for pointing this out.

[7] One could model the undermining effect of *I'm on color-drugs* on *the bike is green* by updating instead on something like *it appears as if the bike is green*, which in turn raises my credence in *the bike is green* only if I already have a low credence in *I'm on color-drugs*. This allows us to regard anything that raises that last credence as an undermining defeater for the greenness of the bike; in the language of Pryor (2013) this could be modeled as a case of 'quotidian' undermining. But this is beside the point. Weisberg's puzzle presents a problem for anyone who thinks that the propositions conditionalized upon—whatever those happen to be — can themselves be undermined, and so updating upon beliefs about how things seem offers a solution only if we agree that (i) *those* beliefs cannot be undermined, and (ii) all cases of undermining defeat are quotidian. An evaluation of that approach is outside of the scope of this essay—I'll be arguing that the Bayesian has a solution to Weisberg's puzzle requiring neither indefeasible updates nor pan-quotidianism about undermining defeat.

[8] Note the conditional structure of the preceding sentence. An alternative possibility is that episodes of perceptual learning that seem to require failures of Rigidity are simply inapt to be modeled using Jeffrey Conditionalization. Jeffrey thought of Rigidity not as feature of Conditionalization, but as a precondition for that rule's applicability (see Jeffrey 1970, pp. 172–179). Since what I learn from my experience as of the greenness of the bike is vulnerable to undermining defeat, this case seems to require just such a failure of Rigidity, and so in this case the precondition is not satisfied and Jeffrey Conditionalization does not apply. Weisberg implicitly rejects this picture, proceeding as if Jeffrey Conditionalization either must apply in every case of perceptual learning or it must be rejected. Since Jeffrey Conditionalization is rigid, he reasons, it doesn't apply to cases of perceptual learning that are vulnerable to undermining defeat, so it doesn't apply to every case, so it must be rejected. (For more on this dispute, see Sect. 5.) I argue that neither side has it quite right: pace Jeffrey, Conditionalization applies in all cases of perceptual learning,

it, "...[perceptual underminers are] irrelevant to the supported proposition at first, but negatively relevant after the perceptual state has lent its support. And this is precisely what 'Rigidity is Independence Preserving' rules out. If the underminer is irrelevant before the perceptual state supports the proposition, it is irrelevant after as well. So Rigidity prevents perceptual undermining when it obviously shouldn't." (Weisberg 2014, p. 126)

3 Bayesian learning more carefully

Below I'll be arguing that Weisberg's conclusion is too strong, but in order to do so we must first take a closer look at Bayesian perceptual learning and the ways that it's constrained by Rigidity.

3.1 Bayesianism is incomplete

Bayesianism is at best an incomplete theory of epistemology, in the sense that there are at least two very important varieties of constraints upon rational credence assignments that it is unable to explain. The first variety of incompleteness concerns rationally permissible *starting credence functions*, functions held by agents who possess no evidence at all (so-called 'super-babies'). There are many intuitively impermissible starting credence functions that are nonetheless perfectly consistent with Probabilism, and hence whose impermissibility cannot be explained by anything within the Bayesian formalism. An agent's choice of starting credence function will of course determine how perceptually acquired information is to affect other credences via Classical or Jeffrey Conditionalization, as it will determine their conditional probabilities.

I'll return to the significance of the Bayesian formalism's underdetermination of rationally permissible starting credence functions in Sect. 5, but right now I want to focus on another type of incompleteness in the Bayesian account of perceptual learning. Just as Probabilism alone is too weak to rule out all of the intuitively impermissible starting credence functions, both Classical and Jeffrey Conditionalization are too weak to rule out all intuitively impermissible credence revisions. That's because not all permissible credence revisions proceed via Conditionalization, and those that don't are only minimally constrained by the Bayesian formalism.

The most important credence revisions that don't proceed via Conditionalization come as a result of a perceptual experience. Why am I rational in believing that the stove is warm? Because it feels warm. Why am I rational in believing that the cat is on the mat? Because I had a perceptual experience as of a cat on the mat. Those

Footnote 8 continued
and pace Weisberg, this needn't lead to Rigidity failures, so this creates no significant problem for Jeffrey Conditionalization.

experiences make it rationally permissible to form those beliefs.[9] On the sort of subjective Bayesian picture that we're considering, probabilities are understood as partial belief states, and the only sorts of things that can be partially believed are *propositions*. Experiences as of warm stoves or cats on mats might have propositional content (I think that they do), but they are not themselves propositions and so they cannot be assigned credences. Hence they are not the sorts of things that can be conditionalized upon. Hence Conditionalization cannot be the whole story when it comes to rationally permissible credence revisions.[10]

Bayesians construct formal models of rationally permissible credence revisions, and all of the revisions that they model proceed via Conditionalization. As we've seen, many rationally permissible credence revisions do not proceed via Conditionalization and hence not all rationally permissible credence revisions are modeled. This distinction will be important to what follows, so I'll introduce some terminology: call the revisions modeled by the Bayesian *endogenous* credence revisions (as in *endogenous to the model*), and call the rest *exogenous* revisions.[11]

3.2 Rigidity and independence, carefully this time

With the distinction between endogenous and exogenous revisions in mind, let's take a closer look at the rigidity of both Classical and Jeffrey Conditionalization. To that end (and I swear this is relevant) note that it's common for a single perceptual experience to affect one's rational confidence in many different propositions. For example, if I have a perceptual experience as of a red, spherical ball, I might shift my confidence in *the ball is red* and in *the ball is spherical*, along with lots of other propositions (experience is pretty rich, after all). Though the details of how to model this phenomenon will differ slightly on the Jeffrey and the Classical Bayesian accounts, they share some important similarities, and in both cases those details have important implications for the rigidity of Bayesian perceptual learning.

Consider first how the Classical Bayesian will model a case in which an agent exogenously revises her credence in more than one proposition at a time. At t_1 Clara accepts a credence function such that $P_{t_1}(A) = P_{t_1}(B) = P_{t_1}(A|B) = .5$, and then at t_2 she exogenously shifts her credences in A to 1 and in B to 1. As discussed above, this exogenous shift alone will fix some subset of her credences at t_2—in this case that set will include her credences in A and in B—with others being determined by conditionalizing upon that subset. But what exactly does it mean to update not on a single proposition, but on a set of propositions? For the Classical Bayesian, the

[9] For those who prefer a picture on which agents update on propositions about how things seem rather than how things are, the question becomes: why am I rational in believing *I've had an experience as of a cat on the mat*? The answer is the same: because of my experience.

[10] Note how minimal I've been in describing the role of experience in fostering rationally permissible credence revision. The point applies not only to those (such as myself) who think that a perceptual experience can be *evidence* that *justifies* belief, but also to those who think that it can play only a non-evidential, non-justificatory role in making certain beliefs or credence revisions rationally permissible (e.g. Davidson, Jeffrey, and Williamson).

[11] See Miller (2016, p. 773); the terminology originates with Howson and Urbach (1993, p. 82).

answer is very simple: update on all of the new evidence acquired by updating on the conjunction of all of the propositions whose probabilities have just been exogenously revised to 1, which in this case means updating on $A\&B$.

Classical Conditionalization is rigid, meaning that updating on $A\&B$ never changes the probability of any other proposition conditional on $A\&B$. Importantly, though, conditionalizing on that conjunction will not in every case preserve the probability of some proposition C conditional on one of the conjuncts of that conjunction, i.e. $P(C|A)$ or $P(C|B)$. Suppose for reductio that that's false, and so for any A, B and C, $P(C|A) = P_{A\&B}(C|A)$. No matter what values are assigned to $P(C|A)$ and $P(C|A\&B)$, it must be the case that $P_{A\&B}(C|A) = P_{A\&B}(C|A\&B)$; after all, at that point I've assigned a credence of 1 both to A and to $A\&B$. The rigidity of Classical Conditionalization ensures that $P_{A\&B}(C|A\&B) = P(C|A\&B)$, and so given our supposition it follows that $P(C|A) = P(C|A\&B)$ for any A, B and C. But this last equality is often false—my credence that the table is delicate given that it's made out of glass is much higher than my credence that it's delicate given that it's made out of glass and it's incredibly sturdy—and so our supposition is false.

The lesson so far is not that Classical Conditionalization isn't rigid; it is. The two-part lesson is that (i) the proposition that's conditionalized upon might be just one of the many propositions that are exogenously revised, and (ii) though Classical Conditionalization is rigid with respect to the one proposition that's conditionalized upon, it's not rigid with respect to those other exogenously revised propositions.

Having appreciated both (i) and (ii) we're now in a position to sketch a possible response to Weisberg's puzzle. As far as that puzzle goes, Rigidity is only interesting because rigid updates preserve independence between the proposition updated upon and other propositions. This is puzzling only if we assume that the propositions losing their independence with potential underminers are the ones that we update upon directly, rather than conjuncts in a larger conjunctive proposition that we update upon. If we drop that assumption, then we are free to concede the rigidity of our preferred version of Conditionalization without thereby conceding that Conditionalization preserves the independence of exogenously revised, perceptually justified propositions with their potential underminers.

Below I'll develop this response on behalf of the Jeffrey conditionalizer, but first let's note that it's hopeless for the Classical conditionalizer. The case here is overdetermined, but I'll mention just one reason that's particularly salient to our discussion. Classically conditionalizing upon $A\&B$ requires assigning assigning it a credence of 1, which requires assigning A a credence of 1. But any proposition assigned a credence of 1 is probabilistically independent of any other proposition,[12] and so any proposition that's been updated upon, or any of their logical implications, will be independent of all other propositions. It follows that if A and C are independent before I Classically conditionalize on $A\&B$, then they'll be independent afterward. Hence while Classically conditionalizing on $A\&B$ can *change* credences conditional on A or on B, it can't *destroy the independence* of A or B with some other proposition.

[12] Assuming that the propositions in question have credences greater than zero.

Jeffrey Conditionalization avoids this particular problem by allowing updates on propositions with credences less than 1. For example, Jeffrey Bayesianism describes how to conditionalize when an experience makes it rationally permissible to exogenously revise my credence in *the ball is red* to .7 and my credence in *the ball is spherical* to .9. But this creates a new problem: while the Classical Bayesian can handle cases of multiple propositions whose credences have been revised exogenously by conditionalizing on their conjunction, in most cases this move is unavailable to the Jeffrey Bayesian. A probability assignment of 1 to each conjunct ensures that the probability of the conjunction will be 1 as well, but assignments of probabilities strictly between 0 and 1 to both conjuncts is consistent with a range of probability assignments to their conjunction. For example, if I think that $P(red) =$ .7 and $P(spherical) = .9$, the value of $P(red\&spherical)$ can be anywhere between .6 and .7, and where in that interval the probability of that conjunction lies is undetermined by the probabilities of the conjuncts themselves.

Jeffrey conditionalizers face a second complication in selecting what to conditionalize upon. While Classical Bayesians update on a weighted *proposition*,[13] Jeffrey Bayesians update on a weighted *partition* of the state space, where a partition is simply a division of that space into mutually exclusive and jointly exhaustive parts, each weighted according to its probability. Propositions such as *the ball is red* and *the ball is spherical* are neither exclusive nor exhaustive, and so they typically won't partition the relevant state space (though they might: see fn. 14).

Jeffrey (1983, p. 173) resolves the issue in a very simple way. He begins by identifying an initial set of propositions—he calls them 'originating propositions'—whose probabilities shift exogenously, but which typically are not elements of the partition. Those elements are instead conjunctions constructed by taking each originating proposition or its negation as a conjunct. For example, taking A and B as our originating propositions we wind up with four conjunctions as our partition elements: $A\&B$, $A\&\neg B$, $\neg A\&B$ and $\neg A\&\neg B$. These four conjunctions (Jeffrey calls them 'atoms'; I'll follow more recent authors and call them 'elements') are mutually exclusive and jointly exhaust any probability space, so taking each conjunction as one of our B_i's, the set $\{B_i\}$ will form a partition, allowing us to Jeffrey conditionalize upon it.[14]

[13] This is slightly misleading—see fn. 14.

[14] Many authors—Weisberg and Pryor included—omit this aspect of Jeffrey's theory in their summaries. I speculate that this is because in certain circumstances the effect of updating on the originating propositions and updating on the elements is the same, and because Jeffrey's most widely discussed example of how his system works just happens to be one of those circumstances. In the example we are asked to imagine seeing a cloth in poor lighting, which results results in exogenous revisions to the probabilities of *the cloth is green (=G), the cloth is blue (=B)* and *the cloth is violet (=V)*. Strictly speaking, this should lead to an update on a partition whose elements include the eight conjunctions that we can construct from those three originating propositions and their negations, yet Jeffrey (together with many later authors discussing this example) omits discussion of the conjunctions and simply talks of updating on these three propositions. The reason that this isn't disastrous in Jeffrey's example is because we're asked to also suppose that the agent seeing the cloth is already certain that nothing is more than one color (all over, at the same time, etc) and is also certain that the cloth is either green or blue or violet. Given those suppositions the probability of five of our eight conjunctions is zero, and so they can safely be ignored as elements of the partition. The three remaining conjunctions will each be closely identified

With all of this in mind, let's revisit Rigidity with an eye to clarifying precisely what's rigid with respect to what on the Jeffrey picture. Recall that Rigidity says:

Jeffrey Rigidity If $P_{new}(\cdot)$ is the credence function resulting from accepting $P_{old}(\cdot)$ and then updating on a shift in partition $\{B_i\}$, then for any proposition A and any $B_i \in \{B_i\}$, $P_{new}(A|B_i) = P_{old}(A|B_i)$

We've just seen that each $B_i \in \{B_i\}$ is a conjunction of originating propositions, not an originating proposition itself. Hence what Jeffrey Rigidity rules out is a change in conditional probabilities on conjunctions of originating propositions, not changes in conditional probabilities on the originating propositions themselves. As with Classical Conditionalization, the probabilities of the individual conjuncts—the originating propositions—conditional on other propositions will not be so-constrained.

Consider again my perceptual experience as of the red, spherical ball. Before that experience I assigned a probability of .5 to each proposition, and I assign $P_{old}(red \mid spherical) = .5$. Upon having that experience I set $P_{new}(red)$ to .7 and $P_{new}(spherical)$ to .9. Since those originating propositions don't form a partition, I now need to assign credences to the four relevant conjunctions. Suppose that I do so as follows:

$P_{new}(red \ \& \ spherical) = .6$
$P_{new}(red \ \& \ \neg(spherical)) = .1$
$P_{new}(\neg(red) \ \& \ spherical) = .3$
$P_{new}(\neg(red) \ \& \ \neg(spherical)) = 0$

Now my credence in $P_{new}(red \mid spherical) = 2/3$. We therefore have a case analogous to the one observed above: we have an episode of perceptual learning in which the probability of an originating proposition on something else has changed, all while respecting the rigidity of Jeffrey Conditionalization.

That's the first lesson of this example. The second lesson is actually a bit more interesting. The exogenously revised values that I assigned to my two originating propositions constrain the values that I assign to the elements of my partition—to my four conjunctions—but do not determine them completely. Since (i) the probability of *the ball is red* conditional on *the ball is spherical* is by definition (we are supposing) the ratio of their conjunction to the unconditional probability of *the ball is red*, and (ii) the probabilities of two propositions (sometimes) underdetermines the probability of their conjunction, it follows that (iii) assigning probabilities to two originating propositions (sometimes) underdetermines the probability of one of them conditional on the other. For example, I might just as easily have assigned the following credences after my observation of the red, spherical ball:

Footnote 14 continued
with one of our originating propositions: *the cloth is green* with $G\&\neg B\&\neg V$, *the cloth is blue* with $\neg G\&B\&\neg V$, and *the cloth is violet* with $\neg G\&\neg B\&V$. Given the particulars of the case it's harmless to speak of updating on a partition with elements G, B, and V, but since those particulars will not generally obtain this harmlessness does not generalize.

$P_{new} * (red \; \& \; spherical) = .7$
$P_{new} * (red \; \& \; \neg(spherical)) = 0$
$P_{new} * (\neg(red) \; \& \; spherical) = .2$
$P_{new} * (\neg(red) \; \& \; \neg(spherical)) = .1$

In that case my credence in $P_{new} * (red \mid spherical) = 7/9$, yet just as before my credences in *the ball is red* and *the ball is spherical* are .7 and .9, respectively.

The probabilities assigned exogenously to originating propositions are only minimally constrained by the Bayesian formalism. As we now see, even once those probabilities are selected the probability of their conjunction is sometimes underdetermined, in which case the probability of one originating proposition conditional upon another is also underdetermined. It's frequently the case that experience makes it rationally permissible to revise the probabilities of several originating propositions at once, and as a result it's frequently necessary for agents to go further and determine the values of their conjunctions in order to form the partition required for updating. The upshot, then, is that perceptual learning as understood by the Jeffrey Bayesian effectively involves changes to conditional probabilities that are unmediated by Jeffrey Conditionalization and hence are unconstrained by Rigidity.

We're now in a position to draw a broader lesson regarding the significance of the rigidity of Conditionalization for Bayesian perceptual learning. Episodes of perceptual learning involve an exogenous assignment of credences to some propositions (as a result of an experience or something else) and also an endogenous assignment of credences (via Conditionalization) to others. The endogenous assignments reflect the bearing of the exogenously set credences upon the rest.

Weisberg correctly points out that Rigidity prevents the introduction of probabilistic entanglement between *the bike is green* and *I'm on color-drugs* via Conditionalization. But why is that problematic? What's clear is that this entanglement must be introduced into my credence function as one of the effects of the experience. What's not clear is that this introduction must be among the *endogenous* effects of the experience—those that proceed via Conditionalization—rather than among the *exogenous* effects that do not proceed via Conditionalization. Put another way, the intuition driving Weisberg's Puzzle is not that the probabilistic entanglement is introduced via Conditionalization, but merely that it's among the results of my perceptual experience.

As we've seen, the Bayesian account of perceptual learning involves more than just Conditionalization: it also involves exogenous credence revisions that don't proceed via Conditionalization. Moreover, those exogenous revisions commonly result in changes to the probability of one originating proposition conditional upon another, as we saw in the case of the red, spherical ball. Finally, even once the exogenously set unconditional probabilities of our originating propositions are determined, there's considerable flexibility in setting their new conditional probabilities.

4 Formal proposal

My proposed response to Weisberg's puzzle is fairly simple. Intuitively, having a perceptual experience as of the greenness of my daughter's bike should result in (i) an increase in my confidence in *the bike is green*, (ii) no change to my confidence in *I'm on color-drugs*, and (iii) the introduction of a negative correlation between *I'm on color-drugs* and *the bike is green*, i.e. it should now be the case that $P_{new}(green \mid color - drugs) < P_{new}(green)$. Rigidity prevents the introduction of this negative correlation endogenously via Jeffrey Conditionalization on a partition that includes *the bike is green* as an element, and so it must not be an element of the partition. Assuming that my confidence in that proposition is to be increased exogenously, the introduction of the negative correlation in (iii) requires that my new credence in both *the bike is green* and *I'm on color-drugs* must be among the conjuncts of the elements of the input partition. Hence what must happen is that my credence in both of those propositions must be set exogenously.

I'll defend this proposal in Sect. 5, but for now let's just get a sense for how it works out formally. We take as our originating propositions *the bike is green* (=*G*) and *I'm on color-drugs* (=*D*), and hence we partition our state space into four elements, correlating with the four possible combinations of those propositions and their negations. For simplicity assume that each of the four elements starts out with a probability of 1/4. The introduction of the negative correlations looks like this (Fig. 1):

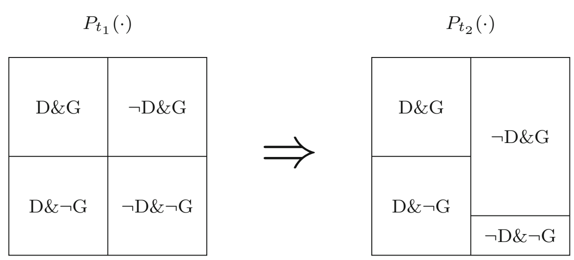

Fig. 1 Introduction of negative correlation between *D* (=I'm on color drugs) and *G* (=the bike is green)

Here my confidence that I was on color drugs when I had my perceptual experience as of the green bike hasn't changed, and my confidence that the bike is green has increased. If we suppose that *I'm on color-drugs* is a complete undermining defeater—a defeater that deprives the perceptual experience of all of its evidential force—then if I were to become certain that I was on color drugs, then my epistemic situation vis-à-vis *the bike is green* before I had the perceptual experience should be the same as my situation after having the experience and becoming certain of the underminer. In pictures (Fig. 2):

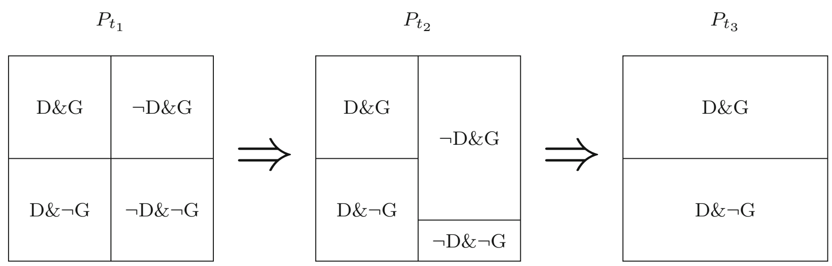

Fig. 2 Becoming certain of a full undermining defeater

If at t_3 I become more confident that I was on color drugs without becoming certain of it the result is a net decrease in the *G* space (Fig. 3):

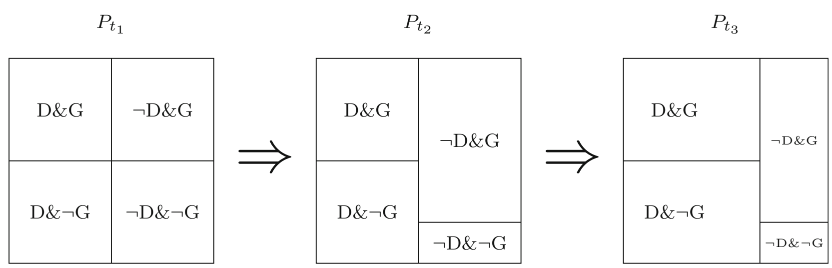

Fig. 3 Increased confidence in a full undermining defeater

What if instead of varying my degree of confidence in a full undermining defeater, we vary the degree to which *G* is undermined? For example, suppose that the color-drugs only somewhat decrease the reliability of my color perception, so that *D* is a *partial* undermining defeater. This will be set at that initial exogenous revision in response to the experience. The particular mechanism will be that it will increase the size of the *D&G* space at the expense of the ¬*D&G* space, where a greater increase means a weaker undermining effect. Assuming that this doesn't reduce my new (at t_2) credence of *G*, that means that the ratio of ¬*D&G* to ¬*D&*¬*G* will decrease sightly. If my confidence in *D* increases at t_3 (but not quite to 1) the picture is that of Fig. 4.

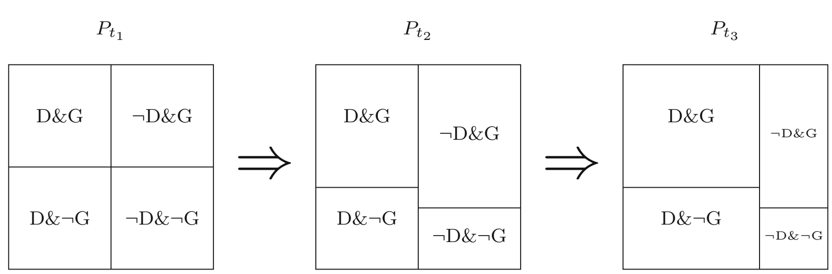

Fig. 4 Uncertainty in a partial undermining defeater

5 Defending the proposal

Weisberg's puzzle illustrates that the introduction of a negative probabilistic correlation between an originating proposition and its potential undermining defeater cannot be modeled within the Bayesian formalism. Weisberg takes this to be a reason to reject Bayesianism. I have proposed instead that it is a reason to move the introduction of that correlation outside of the model, so that it is already achieved once Jeffrey Conditionalization is applied to the partition. I've shown that this is consistent with Jeffrey Bayesianism, which already assumed the existence of credence revisions taking place off-model (exogenous revisions) that can change conditional probabilities on propositions involved in those off-model revisions, and so includes a formal mechanism for incorporating those revisions into the model.

Weisberg (2014, pp. 142–145) anticipates this type of response, calling it the 'appeal to richer inputs'. Though he concedes that it 'produce[s] the desired results', he raises two further objections to the way that those results are produced. (142) His first objection is that my proposal requires input partitions that are far more complex than the simple four or eight cell partitions that I've diagramed in Sect. 4. After all, there are lots and lots of potential underminers for instances of perceptual learning, and each of them will need to become negatively correlated with the proposition that they have the potential to defeat. On my proposal each of those propositions will need to be treated as an originating proposition, and as a result the input partitions will be fairly fine-grained. Moreover, because the determination of which fine-grained partition to adopt given a particular experience will take place outside of the formal model, my proposal involves a loss of explanatory power for Bayesianism. I'll return to this objection below.

More troubling to Weisberg than a mere loss of explanatory power is exactly what is being left unexplained:

> An update rule is supposed to determine our new credences as a function of our old beliefs and the new evidence. But on the current proposal, "the new evidence" is not really the new evidence. The complex distribution we would be plugging into Jeffrey Conditionalization would be produced by considering how an experience as of a red-looking sock and our background beliefs about optics combine to warrant new beliefs about the quality of the air and the colour of the sock. And this is precisely the kind of work our update rule was supposed to do. (*ibid.* 144)

This second objection to my proposal can be interpreted in two ways, each of which amounts to (i) a proposed criterion of adequacy for any update rule, and (ii) the claim that Jeffrey Conditionalization together with my proposal does not satisfy this proposed criterion. On both interpretations the idea seems to be that if the objection is correct, then the Jeffrey conditionalizer is left with a dilemma. If they reject my proposal, then they are left with a rule that satisfies Weisberg's criterion of adequacy only to founder on rigidity puzzle. But if they embrace my proposal, then they avoid the rigidity puzzle only to run afoul of Weisberg's criterion. In what follows I argue that the first horn of the dilemma is illusory, as no plausible version of Jeffrey Conditionalization satisfies (either interpretation of) Weisberg's criterion, regardless of whether my proposal is accepted. Hence the choice before the Jeffrey Conditionalizer is a much simpler one: either reject my proposal and be saddled with the problematic consequences highlighted by the Rigidity Puzzle, or embrace my proposal and avoid those consequences (recall that the capacity of my proposal to 'produce the right results' is not disputed).[15]

The two interpretations of Weisberg's criterion are distinguished by what we take the 'new evidence' to be. Like Weisberg, I think that perceptual experience is one type of evidence. This suggests an interpretation of Weisberg's criterion on which any adequate update rule must take experiences and old beliefs as inputs and determine new beliefs as outputs. The problem with Weisberg's criterion so-interpreted is that the 'kind of work' that's being demanded is one that no version of Conditionalization is capable of doing, independent of whether my proposal is accepted.

The problem is that perceptual experience is the wrong sort of thing to be conditionalizing upon. As discussed in Sect. 3.1, only *endogenous* credence revisions are governed by a Bayesian update rule, and the only thing that can spark an endogenous revision is an exogenously revised credence. Having a perceptual experience and exogenously revising a credence are two very different things,[16] and hence we never conditionalize upon perceptual evidence. It follows that if the adequacy of an update rule demands that it take us from 'old beliefs and... new evidence' (in the form of an experience) to a new credence function, then Jeffrey Conditionalization is not an adequate update rule.[17]

Some authors deny that experience can serve as evidence. Taking inspiration from Davidson (1986, p. 311), Richard Jeffrey (1983, pp. 184–185, 211) holds that only a belief can justify a belief (i.e. can be evidence), and since experiences aren't beliefs it follows that experiences can't be evidence. On his (and Davidson's) view, experience may *cause* credences to shift, but those shifts are inapt for rational evaluation and hence not within the purview of epistemology. Williamson (2000, pp. 197–200) thinks something very similar: though only known propositions count as evidence, some propositions are known *because* (yes, that's his term) of the agent's experiences, which are not themselves evidence.

If all evidence is propositional, then my objection to Weisberg's criterion (first interpretation) of adequacy for an update rule is moot, as Jeffrey Conditionalization is now capable of taking agents from old beliefs and new (propositional) evidence to new credences. But this response is inconsistent with the spirit of the criterion, which seems to be that an update rule should model the epistemic significance of

[15] A third option is to reject Jeffrey Conditionalization. Evaluating this option would take us well outside the scope of the present essay. My aim is to defend to defend Jeffrey Conditionalization from Weisberg's objections rooted in the rigidity puzzle, i.e. to defend the consistency of Jeffrey Conditionalization with the phenomenon of underminable perceptual evidence. If Jeffrey Conditionalization's failure to satisfy Weisberg's criterion is disqualifying, then the rigidity-based argument to that effect is irrelevant, a mere exercise in dead-horse beating. If that failure is not disqualifying, then my proposal offers a low-cost rebuttal to any such argument.

[16] See Plantinga (1993, pp. 82–83).

[17] Since Classical Conditionalization also requires updates on propositions rather than on experiences, it too fails to satisfy Weisberg's criterion (first interpretation).

experiences, whether or not we label those experiences ‘evidence’; this is something that Jeffrey Conditionalization cannot do. For Davidson and Jeffrey, note that two agents with identical beliefs/ credence functions might not be rationally alike, as one might have some beliefs caused by a perceptual experience, and hence capable of justifying other beliefs, while the other has beliefs with some other causal origin; one agent possesses propositional evidence that the other agent lacks. On this view the epistemic difference between the two agents can’t be explained without accounting for the etiology their beliefs, which will require an account of the relationship between propositions and experiences—between propositions and non-propositions—something Bayesians cannot do within their formal model. Hence even for someone with Jeffrey-like views on perceptual justification, Jeffrey Conditionalization cannot satisfy the spirit of Weisberg’s criterion.[18]

Williamson’s views are a bit more complicated. Whereas for Jeffrey *beliefs* caused by experience can be evidence for other propositions, Williamson thinks that only *knowledge* plays that role. If Jeffrey is right, then we can hold the initial beliefs fixed while changing the epistemic status of inferred beliefs by changing the etiology of those initial beliefs. But if it’s *knowledge* that serves this evidential role, then that same trick won’t work, at least not given the rest of Williamson’s view. Williamson thinks that evidential relations are objective relations between propositions: input a set of evidence propositions (the ones that the agent knows) into their credence function and out comes the probability that ought to be assigned to every other proposition (op. cit. Sect. 10.2). Against subjective Bayesians he claims that this probability function itself (in contrast to its inputs) is eternal/ insensitive to the beliefs of the agent. On this view it really doesn’t matter *why* or *on what grounds* the agent knows evidence proposition *A*, only that it is known.

Nonetheless, Williamson’s update rule is also inconsistent with the spirit of Weisberg’s criterion. After all, the epistemologist will still want to know why, in virtue of what, particular agents have the evidence that they do in fact have/ know the things that they know non-inferentially. In some cases the agent will know that *A* in virtue of their experiences. The epistemological significance of experience does not disappear simply because we stop calling it ‘evidence’. (As above, I don’t mean to suggest that Williamson thinks otherwise.)

On the first interpretation of Weisberg’s criterion, the objection is that if my proposal is accepted, then the inputs to Jeffrey Conditionalization cannot include new experiential evidence, and hence that that rule cannot ‘determine our new credences as a function of our old beliefs and the new [perceptual] evidence’. I’ve argued that that’s a feature of every version of Conditionalization, and hence that it’s not a special problem for my proposal.

On the second interpretation of Weisberg’s criterion, the complaint is not that my proposal requires updates on partitions rather than experiences, but that the partitions that my proposal requires are defective, and that this defect is not shared by versions of Jeffrey Conditionalization that do not adopt my proposal. This

[18] I don’t mean to suggest that Jeffrey himself ever thought that it could do something like that; he didn’t. I want simply to dispense with the notion that adopting Jeffrey’s views on evidence renders Jeffrey Conditionalization consistent with Weisberg’s criterion.

putative defect is not formal; formally speaking Jeffrey Conditionalization can take any weighted partition of the agent's old beliefs as an input. Instead the objection seems to be couched in a specific idea about the role that an update rule should play in a complete theory of perceptual learning.

A complete theory of perceptual learning would be one that satisfies the first interpretation of Weisberg's criterion: it would determine new beliefs from old beliefs and experiences; it would be a theory of the form (Fig. 5):

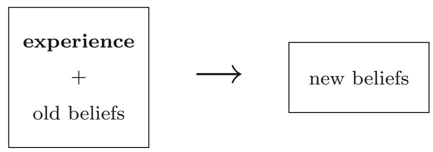

Fig. 5 The form of a complete theory of perceptual learning

In contrast, Jeffrey Conditionalization is of the form (Fig. 6):

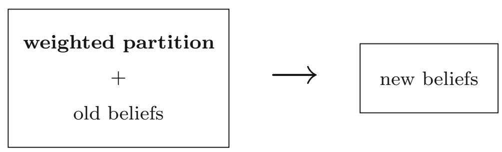

Fig. 6 The form Jeffrey Conditionalization

If Jeffrey Conditionalization is to have any role to play in a complete theory of perceptual learning, then there must be a part of that theory that spells out how experiences determine the inputs to that rule: weighted partitions. Hence any complete theory of perceptual justification that is broadly Bayesian in nature will consist in two distinct update rules: a heretofore unknown rule that determines a weighted partition from the experience (possibly together with old beliefs—more on this below), and Conditionalization, which determines new beliefs as a function of old beliefs plus that partition. The form of this broadly Bayesian theory of perceptual justification is (Fig. 7):

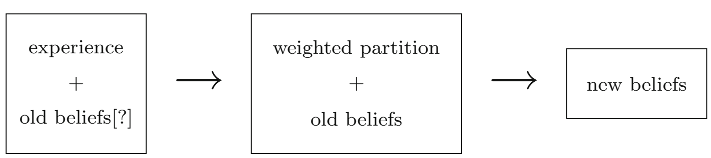

Fig. 7 The form of a broadly Bayesian theory of perceptual learning

We're now in a position to begin fleshing out the second interpretation of Weisberg's criterion. The question turns on the role of old beliefs in determining the weighted partitions that agents update upon. When Weisberg objects to partitions determined by 'considering how an experience as of a red-looking sock and our background beliefs about optics combine to warrant new beliefs about the quality of the air and the color of the sock' because 'this is precisely the sort of work that our update rule was supposed to do', he's suggesting that background beliefs should not play a role in partition determination. Instead the partition should be identified with the direct epistemic effects of the experience, i.e. those credence revisions that are unmediated by background beliefs.

My proposal does not satisfy this version of Weisberg's criterion because it requires that some propositions that are not directly affected by experience appear in the partition as originating propositions: the undermining defeaters for the other originating propositions in that partition. On my proposal, if my experience as of the red, spherical ball leads me to increase my confidence in *the ball is red* and also to come to regard *I'm on color-drugs* as an undermining defeater for that proposition, then both of those propositions must appear in the partition as originating propositions. But it will not generally be the case that such an experience will directly affect my beliefs about my own color-sobriety, so the partition is underdetermined by those direct effects.

As before, however, this version of Weisberg's criterion amounts to a general indictment of Jeffrey Conditionalization rather than of my proposal in particular. The general problem is that in very many cases, at least some of the agent's posterior credences will be determined neither by the experience alone—they will not be among the direct effects of the experience—nor by conditionalizing upon those direct effects. As a result, some indirect effects of experience will be determined exogenously, so Jeffrey Conditionalization will fail to do the 'kind of work' that Weisberg's criterion (second interpretation) demands of any adequate update rule.

To illustrate, suppose that we reject my proposal and retain Jeffrey Conditionalization. Plausibly, among the direct effects of my experience as of the red, spherical ball are an increased credence in *the ball is red* and an increased credence in *the ball is spherical*, and so presumably those propositions will be among the originating propositions in my partition. Supposing further that these are the *only* relevant originating propositions, the partition will contain four conjunctions as elements: *red* & *spherical, red* & ¬*spherical*, ¬*red* & *spherical*, ¬*red* & ¬*spherical.* That means that my credence in each of those conjunctions will be revised exogenously, i.e. not via Jeffrey Conditionalization.

Is it plausible to claim that my credence in each of those conjunctions is determined solely by experience, with no input from background beliefs? I'm inclined to say no, and my inclination only strengthens once we drop the simplifying supposition that our conjunctive partition elements are composed of only two conjuncts each. For in addition to appearing red and spherical, the ball might appear to be dirty, punctured, and three feet to the left of the tree,[19] in which case our partition consists of thirty-two elements with five conjuncts each. I feel no inclination to say that my credence in the following is a direct effect of my

[19] There's no reason to stop at a mere *five* ways that the ball might appear; experience is pretty rich, after all.

experience: *red* & ¬*spherical* & ¬*dirty* & *punctured* & ¬*three feet to the left of the tree*.[20]

One more example to drive the point home. By the definition of conditional probability, once $P(A\&B)$ and $P(\neg A\&B)$ are determined, so is $P(A|B)$. That means that if both A and B are originating propositions in an exogenously determined partition, then once the weighted elements of that partition are determined, the probability of each originating proposition conditional on every other originating proposition is determined as well. For example, the partition described in the previous paragraph would determine my credence that the ball is not red given that it's punctured, and also my credence that it's dirty and not three feet to the left of the tree given that it's not spherical. As before, I feel no inclination to say that these credences are among the direct effects of my experiences, and yet they aren't determined via Jeffrey Conditionalization either. Hence even without my proposal, it's not plausible that the input partitions required by Jeffrey Conditionalization are determined entirely by experience.

I have proposed that the best way for Bayesians to accommodate the phenomenon of perceptual learning that is itself vulnerable to undermining defeat is to include potential undermining defeaters among the originating propositions of the input partition, and hence to determine the negative correlation between defeater and new belief exogenously. The identity of and the posterior credence in those underminers are not plausibly among the direct effects of experience, and hence Jeffrey Conditionalization together with my proposal does not satisfy the second interpretation of Weisberg's criterion: some indirect effects of experience are determined independently of Jeffrey Conditionalization. But as I've argued, this is not a radical departure from Jeffrey Conditionalization *without* my proposal, which also fails to satisfy that criterion.

That does not mean that my proposal is without cost. Any complete theory of perceptual learning that employs Jeffrey Conditionalization to determine new beliefs from old beliefs and a weighted partition will require a second rule for determining partitions from experience (possibly together with old beliefs). The explanatory work to be done by the complete theory of perceptual learning will be divided between these two rules, and the more of this work that Jeffrey Conditionalization can do the better supported it will be.

We're left, then, with Weisberg's first objection: that my proposal involves a loss of explanatory power. In this he is completely correct. It is unwelcome news that the Bayesian is unable to model the introduction of a negative correlation between an

[20] To be clear: the issue is whether my credence in the *conjunction* is determined by experience alone, not whether my credence in each *conjunct* is so-determined. Importantly, it is not generally the case that the probability of a conjunction is determined by the probabilities of its conjuncts. Note, however, that in the special case in which the elements of the input partition all have credences of either 0 or 1—as will be the case with any partition taken as an input to Classical Conditionalization—the new credences of the originating propositions do in fact determine the new credences of those partition elements. If $P(\phi) = P(\psi) = 1$ then $P(\phi\&\psi) = 1$, in which case $P(\neg\phi\&\psi) = P(\phi\&\neg\psi) = P(\neg\phi\&\neg\psi) = 0$. Hence if the shifting of $P(\phi)$ and $P(\psi)$ to 1 is a direct effect of an experience, then so is the determination of the new credences of all partition elements. What this suggests, of course, is that Classical Conditionalization satisfies Weisberg's criterion (second interpretation).

exogenously revised proposition and its underminers. He's also correct that the input partitions will need to be more complicated than those in my examples from Sect. 4, and so the auxiliary theory bridging the gap between experience and input partition will be more complex than the Bayesian might have initially supposed.

These are real objections to my proposal, and the best that can be done in response is to mitigate their badness. Two considerations to that effect. First, though the input partitions required by my proposal will involve a significant number of originating propositions, that number is dwarfed by the number of propositions that are not involved in it. Though the formal model will be unable to explain the introduced negative correlation between perceptually justified beliefs and their potential underminers, it will be able to explain how *those* changes ought to affect the agent's credences in all other propositions and hence to determine a posterior credence function. Even in its reduced state the explanatory power of the Bayesian formalism is quite robust. Second, as I argue below, Bayesians are already committed to a limitation upon starting credence functions that's closely analogous to this limitation upon input partitions, and it's unclear why the one constraint should be considered more problematic than the other. Hence it's unclear why Weisberg's objection to my proposal doesn't generalize into a broader indictment of Bayesianism.

As noted, Probabilism ensures that certain evidential relations will be encoded in any permissible credence function. For example, it ensures that any evidence that makes it rationally permissible to set $P_{new}(A)$ to .7 also makes it rationally permissible to set $P_{new}(\neg A)$ to .3, and prohibits setting $P_{new}(\neg A \& B)$ any higher than that. But not all intuitively mandatory evidential relations—those to which all rational agents are obliged to conform—follow from Probabilism, and hence many probabilistically coherent credence functions are intuitively impermissible. Famously, Probabilism fails to ensure that the observation of lots of green emeralds and no non-green ones supports (H_1) *all emeralds are green* more than it supports (H_2) *all emeralds are grue*.[21] Because both H_1 and H_2 entail E = *all observed emeralds are green*, conditionalizing on E will increase (or leave the same) my confidence in both of them, and yet intuitively my posterior credence in H_1 should be much higher than that of H_2. According to the Bayesian, that means that before acquiring evidence E it must be the case that $P_{old}(H_1 \& E) > P_{old}(H_2 \& E)$. In other words, if we wish to ensure that conditionalizing on E determines a rationally permissible credence function, then we must constrain our prior credence functions in ways that go well beyond Probabilism.

In many cases this phenomenon appears innocuous, as when an agent starts out thinking that $P_{start}(H_1 \& E) \leq P_{start}(H_2 \& E)$ and then adopts the desired inequality after acquiring new evidence and updating in the normal way. But this merely pushes the bump in the rug. Take E^* to be the conjunction of E and all of the other evidence that the agent has acquired to t. In that case a necessary and sufficient

[21] See Goodman (1946) and (1983, pp. 72–83).

condition for the agent holding that $P_t(H_1) > P_t(H_2)$ is that their starting credence function $P_{start}(\cdot)$ be such that $P_{start}(H_1 \& E^*) > P_{start}(H_2 \& E^*)$.[22]

The narrow point is that if the Bayesian is to regard inductive inference as more epistemically respectable than counter-induction or non-induction, they'll need to go beyond mere Probabilism and impose further constraints upon starting credence functions. The broader point is this: for any E and H such that $E \nvDash H$ and $H \nvDash E$, ensuring that E supports H more than some competing hypothesis depends crucially on the choice of starting credence function. We have lots of intuitions about evidential relations that go beyond deductive entailment (e.g. the intuition that induction is preferable to counter-induction), and in order to require of agents that they satisfy those intuitions we have to constrain their starting credence functions. The Bayesian formalism (= Probabilism + (some version of) Conditionalization) does not impose those restrictions, and hence additional constraints on starting credence functions are needed in order to ensure that their prior credence functions are rationally permissible, which are themselves required in order for Conditionalization to determine a rationally permissible posterior credence function given some permissible exogenous revision.

For the Bayesian, there are obvious parallels between what's required by Goodman's New Riddle and what I'm proposing in response to Weisberg's puzzle: just as the former requires a constraint upon rationally permissible starting credence functions, the latter requires a constraint upon exogenous revisions. Ideally, both of those constraints would be imposed by the formalism itself, but in both cases that's proven not to be the case. If we assume that Weisberg is objecting to proposals like mine, rather than to Bayesianism in general, then the problem can't simply be with the existence of intuitively compelling constraints upon the formalism for which we have no widely accepted formal theory; we have no such theory for distinguishing 'projectable' predicates like *green* from 'unprojectable' ones like *grue*, either. Presumably, then, the objection must be either (i) that such constraints are more objectionable at the exogenous revision side of the model than at the starting credence function side, or (ii) to some other feature of undermining defeat that distinguishes it from broader inductive practice, and in virtue of which my proposal is the more problematic. (i) seems arbitrary, and (ii) is not forthcoming. Seen in this light it's unclear how my proposal presents any special problem for the Bayesian that isn't closely analogous to a problem that they already have, and hence it's unclear how Weisberg is objecting to my proposal in particular rather than to Bayesianism in general.

There's one last objection that I'd like to consider. Weisberg (2014, 129) considers a response a bit like mine, which he characterizes as the claim that Jeffrey Conditionalization doesn't 'apply' in cases of perpetual undermining.[23] The thought here seems to be rooted in the late career Richard Jeffrey's somewhat unorthodox views about the motivations for Jeffrey Conditionalization. One prominent view

[22] For Classical Bayesianism at least—the possible non-commutativity of Jeffrey Bayesianism makes that case less straightforward. See Domotor (1980).

[23] See Wagner (2013) for a defense of this view.

among Bayesians is that agents ought to conditionalize because failure to do so leads to the sort of pragmatic defeat illustrated by the Lewis/ Teller dynamic Dutch book argument. (Teller (1973)) Jeffrey thinks that such considerations are beside the point,[24] and that Jeffrey Conditionalization is motivated—when it is motivated—by considerations of coherence alone. The Total Probability theorem follows from the probability axioms plus the definition of conditional probability:

Total Probability $P_{new}(A) = \sum_i P_{new}(A \mid B_i) P_{new}(B_i)$

When the transition from an agent's old credences to her new ones is rigid on some B_i, $P_{new}(A \mid B_i) = P_{old}(A \mid B_i)$. Hence in such cases, by simple substitution on Total Probability we get:

Jeffrey Conditionalization $P_{new}(A) = \sum_i P_{old}(A \mid B_i) P_{new}(B_i)$

The upshot is that concerns of synchronic coherence alone require that we Jeffrey conditionalize upon our new evidence any time Rigidity holds. On this way of seeing things, Rigidity is a precondition that must be satisfied in order for Jeffrey Conditionalization to be applicable at all, rather than a feature of every case of perceptual learning that must be accommodated by all Bayesians (see footnote 8). That just leaves us with the following question: when is this precondition satisfied? Not always, says Jeffrey. And therein lies a possible answer to the puzzle: perhaps cases involving undermining defeaters are cases in which Rigidity does not hold, and hence they are cases in which Jeffrey Conditionalization is unmotivated and inappropriate.

To this Weisberg very reasonably objects that perceptual justification is nearly always vulnerable to undermining defeat, and hence if Jeffrey Conditionalization is inapplicable in cases involving the possibility of underminers, then it's inapplicable in nearly every case of perceptual learning.

Weisberg is no doubt correct about the near ubiquity of potential underminers for perceptual experience, and so if Jeffrey Conditionalization is to be rejected in all such cases, then rational agents won't be doing much conditionalizing. But while this might be a serious problem for some other proposals, it's no objection to mine (to be clear: Weisberg never says that it is). On my proposal, Jeffrey Conditionalization applies in every case of perceptual learning. Underminable perceptual learning requires changes to conditional probabilities, changes that cannot be achieved endogenously through a rigid updating rule like Jeffrey Conditionalization. I've proposed that any time the probability of some proposition conditional on an originating proposition needs to change, that this change occur exogenously rather than via Jeffrey Conditionalization. This is important because exogenous credence revisions are not constrained by Rigidity[25] and hence need not preserve

[24] Interestingly, Jeffrey (2004) motivates Total Probability with a Dutch Book argument (Sect. 1.4) and then goes on to motivate Jeffrey Conditionalization by appeal to Total Probability (Sect. 3.2). Hence there's a sense in which he *does* rely on considerations of pragmatic defeat to motivate Jeffrey Conditionalization, but only because those pragmatic considerations motivate Probabilism.

[25] It's not that exogenous revisions are anti-rigid, in the sense that they provide counterexamples to Rigidity, i.e. cases involving updates on a partition $\{B_i\}$ with element B_i such that $P_{new}(A|B_i) \neq P_{old}(A|B_i)$; that's just confused. The inputs to an exogenous revision include experiences,

independence. But once all such changes are encoded into the partition, the rigidity of Jeffrey Conditionalization is completely unproblematic. Jeffrey Conditionalization then 'applies' to the partitions that are determined by these (non-Rigid) exogenous revisions, and in this regard it's just like other versions of Bayesianism.[26]

6 Conclusion

The introduction of a negative correlation is an essential aspect of acquiring new information that is itself vulnerable to undermining defeat. Weisberg's puzzle is important because it illustrates that Bayesians can't model this effect in any straightforward way. Weisberg himself concludes that this is a reason to reject subjective Bayesianism. I have argued that this conclusion is too strong—the lesson instead is that Bayesians should reduce their explanatory ambitions, moving problematic aspects of undermining defeat off-model. This move is appealing for several reasons. First, it restores the consistency of the Bayesian formalism with our intuitions about undermining defeat. Second, the Jeffrey Bayesian's account of perceptual learning has always presupposed that some credences will be revised exogenously, revisions that do not proceed via Jeffrey Conditionalization, and so my proposal represents only an incremental increase to an already existing aspect of the theory rather than a new, dramatic departure. Third, while Jeffrey Conditionalization is rigid, Bayesian perceptual learning is not: since both Classical and Jeffrey Bayesians are committed to exogenous revisions that change the ratio of the probability of conjunctions to the probability of their conjuncts, it's inevitable that conditional probabilities themselves will change exogenously. So again, what I'm proposing isn't a great departure from the pre-Weisberg status quo. Fourth, my proposal doesn't involve commitment to any cases in which Jeffrey Conditionalization doesn't 'apply'. Fifth, and finally, there's a long tradition of Bayesians imposing extra-formal constraints upon their theory in order to deal with counter-intuitive consequences of the minimal Probabilism + Jeffrey Conditionalization account, as they do in response to Goodman's New Riddle.

Acknowledgments Many thanks to Josh Dever, Sinan Dogramaci, Jim Pryor, Miriam Schoenfield, David Sosa, Jonathan Weisberg, audiences at the University of Texas at Austin and the Arché Centre at the University of St Andrews, and an anonymous referee.

Footnote 25 continued
so they're not just partitions, and hence the antecedent of the Rigidity conditional is always false in cases of exogenous revision. For that reason it's more precise to say that exogenous revisions *are* rigid, but only trivially so. The essential point is simply that this 'trivial rigidity' does not preserve independence: there is no analogue of the RIP principle for exogenous credence revisions.

[26] Thanks to an anonymous referee for call to my attention this aspect of my proposal.

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