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EFFECTS OF UNCERTAINTY AVERSION ON THE CALL OPTION MARKET

ABSTRACT. This article examines the effects of uncertainty aversion in competitive call option markets using a partial equilibrium model with the Choquet-expected utility setup. We find that the trading volume of a call option is negatively affected by uncertainty aversion, whereas the price of the call is practically independent of it.

KEY WORDS: uncertainty, ambiguity, call option

JEL CLASSIFICATIONS: D81, G12

1. INTRODUCTION

Financial asset theory is commonly presented in an expected utility framework. Some relevant results, like those on portfolio diversification and on the determination of prices for financial assets and their derivatives, are obtained using the notion of risk aversion,[1] which is equivalent to concavity of the von Neumann–Morgenstern utility function. This framework seems inadequate to capture some important aspects of the real world (the equity premium puzzle and the home-bias puzzle are only two examples). Over the last years, several more general theories have emerged, which take into account uncertainty (ambiguity) aversion, not only risk aversion. Among these theories, the Choquet-expected utility model introduced by Schmeidler (1989) and Gilboa (1987) appears particularly promising.

Uncertainty aversion seems particularly important in financial markets, where transparency matters. We can interpret transparency of an asset as the reliability of the probability distribution of its outcomes, so that the preference for more transparent assets is represented by uncertainty aversion.

Uncertainty aversion has many implications, only some of which shared by risk aversion.[2]

This article sets out to examine the effect of uncertainty aversion on asset derivatives. We use a partial equilibrium approach to analyze the effects of uncertainty aversion on the competitive equilibrium in terms of price and volume of a call option. While most current literature exclusively focuses on the value of derivatives, we analyze both the demand and the supply side to directly determine the price and the trading volume of the call option. The partial equilibrium approach limits the analysis of asset interdependencies, which are without any doubt important in financial markets in general, and for the determination of asset prices in particular. Since the scope of this work is detecting the influence of uncertainty aversion on the trading volume, this restriction is of minor importance. While the value of an asset is strongly affected by the prices of other assets and by other events affecting agents' future consumption, the relation between trading volumes and agents' information can be analyzed without these factors at a rather significant analytical simplicity. Similar approaches are fairly common in economic theory: while the theory of prices is performed in general equilibrium, a partial equilibrium approach is used for analyzing the production of a commodity.

In this article, we demonstrate that market equilibrium exchanges of call options decrease with uncertainty aversion, but that the equilibrium price is substantially independent from it. That is, we show that, even neglecting no-arbitrage conditions, the price of a call is essentially independent from the quality of information about the chances of gains and losses, i.e., from transparency (this information corresponds, with respect to Ellsberg's urn—see below in Section 2—to the knowledge on the proportion of the balls in the urn according to their colors, which is the transparency of the urn). This seemingly counterintuitive result is explained by the fact that uncertainty aversion does not only reduce Choquet-expected utility of call buyers, but also the Choquet-expected utility of call suppliers. The former effect shrinks the demand for calls, the latter decreases their supply. As a result, uncertainty aversion reduces the total trading volume, but leaves the price of the call practically unaffected.

Section 2 contains a short summary of the Choquet-expected utility model. We introduce the market for call options in Section 3 in the framework of the Choquet-expected utility model, and analyze a population of agents and their choice with respect to uncertainty aversion. Section 4 examines the partial market equilibrium. We demonstrate that the trading volume for a population of uncertainty averse agents is lower than the corresponding volume for an uncertainty neutral population, whereas no significant differences emerge with respect to the price of the call. In Section 5, we present a comparison between call market equilibria of two populations characterized by a different attitude toward uncertainty, as well as a comparison between the market equilibria for two calls with different degrees of uncertainty. Last, we provide a comparison between the expected utility model and the Choquet-expected utility model in Section 6. We point out that the expected utility model, contrarily to the Choquet-expected utility model, does not provide any relevant result in terms of risk aversion in the likely case that the size of calls relative to agents' wealth is limited. Moreover, the expected utility model does not react to differences in the information on the probability distribution of outcomes, i.e., in the transparency of assets. The main result of the article is that a lower degree of transparency reduces the trading volume of asset derivatives. This result provides a possible explanation for the empirical fact that the "history of financial markets is replete with episodes of increase in uncertainty leading to a thinning out of trade" (Mukerji and Tallon, 2001, p. 899).

2. A SHORT SUMMARY ON CHOQUET EXPECTED UTILITY

The Choquet-expected utility model differs from Savage's expected utility model in not necessarily assuming probability to be additive. Let S be the set of the states of nature and let subsets $E \subseteq S$ indicate possible events. The Choquet-expected utility model associates a capacity rather than a probability to every event. The probability function $p: 2^S \to [0,1]$ requires $p(\varnothing) = 0$, $p(S) = 1$, $p(E) \leq p(E')$ if $E \subseteq E'$, and $p(E) + p(E') = p(E \cup E') + p(E \cap E')$ for every pair $E, E' \in 2^S$ (where 2^S is the set of all the subsets of S, the empty set included). The capacity (or non-additive probability) function $v: 2^S \to [0,1]$ requires $v(\varnothing) = 0$, $v(S) = 1$ and $v(E) \leq v(E')$ if $E \subseteq E'$, but does not require the additive condition $v(E) + v(E') = v(E \cup E') + v(E \cap E')$.

A monetary act a is a function $f: S \to X$, where $X \subseteq \mathbb{R}_+$ is a set of monetary outcomes or, equally, a function (which is the inverse of the preceding function) $E: X \to 2^S$, with $E(x) = \{s \in S: f(s) = x\}$ for every $x \in X(a)$, where $X(a) = \{x \in X: x = f(s)$ for some $s \in S\}$ is the range of the function $f: S \to X$, so that $(E(x))_{x \in X}$ is a partition of S.

The preference system $\langle A, \succsim \rangle$, where A is the set of possible acts, is represented (according to the Choquet-expected utility model and assuming $X = \mathbb{R}_+$ and that the function $u: \mathbb{R}_+ \to \mathbb{R}$ has a lower bound) by the functional

$$CEU(a) = u(0) + \int_0^\infty v(t)du(t)$$

where $v(t)$ is the capacity of the event $\{s \in S: f(s) \geq t\}$.

The following definitions will be used in the analysis of the market for calls.

DEFINITION 1. (Montesano and Giovannoni, 1996; Ghirardato and Marinacci, 2002).[3] *An agent, whose preference system $\langle A, \succsim \rangle$ is represented by the Choquet-expected utility model, is averse to uncertainty if the core of the capacity is non-empty, i.e.,*

$$core(v) = \{p \in P: p(E) \geq v(E) \text{ for all } E \in 2^S\} \neq \varnothing$$

where P is the set of all probability (or additive capacity) distributions on S. In fact, the introduction of a probability belonging to $core(v)$, in place of the capacity, increases, or does not modify, the utility of every action $a \in A$. Therefore, *an uncertainty averse agent appreciates information on chances, i.e., he is willing to pay in order to know the probabilities of events (for instance, in Ellsberg's experiments, in order to know the composition of the urn).*

DEFINITION 2. (Montesano, 1999, p. 29; Ghirardato and Marinacci, 2002, Theorem 17). *Agent* 1, *with preferences* $\langle A, \succsim_1 \rangle$, *is more averse to uncertainty than agent* 2, *with preferences* $\langle A, \succsim_2 \rangle$, *if* $v_1(E) - v_2(E) \leq 0$ *for all* $E \in 2^S$, *i.e., agent* 1 *is willing to pay no less than agent* 2 *in order to know the chances.*

DEFINITION 3. *This definition introduces a notion of comparative aversion between two monetary acts with respect to agent's preferences. Act* a_1 *(function* $f_1\colon S \to X$, *with* $X \subseteq \mathbb{R}_+$*) is, for an uncertainty averse agent, more uncertain than act* a_2 *(function* $f_2\colon S \to X$*) if* $v(E_1(t)) - v(E_2(t)) \leq 0$ *and* $v(E_1^c(t)) - v(E_2^c(t)) \leq 0$ *for all* $t \in X$, *where* $E_i(t) = \{s \in S\colon f_i(s) \geq t\}$ *and* $E_i^c(t) = \{s \in S\colon f_i(s) \leq t\}$ *for* $i \in \{1, 2\}$.

An intuition is provided by Ellsberg's example that proposes an urn with 30 red balls and 60 green or black balls in an unknown proportion. Act a_1 gives a prize if a black ball is drawn (i.e., if event B occurs), act a_2 does it if a red ball is drawn (event R), i.e., act a_1 is uncertain, while act a_2 is not uncertain, only risky. For an uncertainty averse agent, we have $v(B) - v(R) \leq 0$ and $v(R \cup G) - v(B \cup G) \leq 0$, where $v(B)$ is the capacity of the event "a black ball is drawn," $v(R \cup G) = v(B^c)$ is the capacity of the event "a non-black ball is drawn," and so on. Then, $v(B)$ and $v(R)$ are the capacities of getting the prize, respectively, in case of act a_1 and act a_2, while $v(R \cup G)$ and $v(B \cup G)$ are the corresponding capacities of the complementary event, i.e., not getting the prize.

Then, either the purchase or the supply of act a_2 is preferred to the purchase or the supply of act a_1 by the uncertainty averse agent under consideration, because both the decumulative and the cumulative distribution of capacities associated to act a_1 are lower than those of act a_2.[4]

It should be noted that uncertainty significantly differs from risk in this respect. A riskier lottery can be generated from a lottery through mean-preserving spreads, which increases dispersion by modifying probabilities, so that a decrease in the probability of one outcome is compensated by an increase of the probabilities of other outcomes. A more uncertain act, on the contrary, according to the preceding definition, is generated from an act through a general decrease of capacities. For instance, if an act a_1 is riskier than act a_2, then there is a $t \in X$ for which $p(E_1(t)) > p(E_2(t))$, and, consequently, $p(E_1^c(t)) < p(E_2^c(t))$, where $E_i(t) = \{s \in S : f_i(s) \geq t\}$ and $E_i^c(t) = \{s \in S : f_i(s) \leq t\}$ for $i \in \{1, 2\}$, while, if an act a_1 is more uncertain than act a_2, then both $v(E_1(t)) \leq v(E_2(t))$ and $v(E_1^c(t)) \leq v(E_2^c(t))$ for every $t \in X$.

3. MODELING THE MARKET FOR CALLS

Consider a European call option. Analyzing the competitive partial equilibrium for the call option, we assume that there are only two dates: the date when the call can be bought or sold (date 0) and the expiry date of the call (date 1).[5] Agents have preferences represented by the Choquet-expected utility model.

The unknown variables of the model are the call price c and the trading volume Q, i.e., the quantity of exchanged calls. More precisely, c indicates the future value of the call price: i.e., if c' is the call price at date 0 and i is the interest rate for the period between date 0 and date 1, then $c = c'(1+i)$. The strike call price π at the expiry date is given, while the price π_m of the primary asset at date 1 is uncertain. Then, the set of states of the nature is the set of the possible prices π_m, which is $\mathbb{R}_+$.[6]

ASSUMPTION 1. The capacity functions $v(t)$ and $v^c(t)$, which represent, respectively, the beliefs on the possibility of the event $\pi_m \geq t$ and on the possibility of the event $\pi_m \leq t$, are continuous for all agents, with $v(t) + v^c(t) \leq 1$, so that uncertainty aversion is assumed. The utility function $u(y)$ of every agent is continuous, non-decreasing and concave, i.e., $u'(y) \geq 0$ and $u''(y) \leq 0$ for every $y \in \mathbb{R}_+$ (with $u'(w) > 0$ and $u''(w) < 0$, where

w is the agent's initial wealth), so that cardinal risk aversion is assumed.

If an agent buys q calls, then his utility at the expiry date will be $U_b = u(w - cq + q\max\{\pi_m - \pi, 0\})$. If he is uncertainty neutral and $p(t) = \Pr\{\pi_m \geq t\}$ is the decumulative probability distribution, then his expected utility will be

$$EU_b = u(w - cq) + q\int_{\pi}^{\infty} u'(w + q(t - \pi - c))p(t)dt,$$

i.e.,

$$EU_b = u(w - cq) + \pi q\int_{1}^{\infty} u'\left(w + \pi q\left(x - 1 - \frac{c}{\pi}\right)\right)p(x)dx,$$

where $x = \frac{t}{\pi}$ and $p(x) = \Pr\{\frac{\pi_m}{\pi} \geq x\}$.[7] If the agent has sold q calls, then, analogously, $U_s = u(w + cq - q\max\{\pi_m - \pi, 0\})$ and

$$EU_s = u(w + cq) - \pi q\int_{1}^{\infty} u'\left(w - \pi q\left(x - 1 - \frac{c}{\pi}\right)\right)p(x)dx.$$

If the agent is uncertainty averse and he is a buyer, then the Choquet-expected utility is

$$CEU_b = u(w - cq) + \pi q\int_{1}^{\infty} u'\left(w + \pi q\left(x - 1 - \frac{c}{\pi}\right)\right)v(x)dx,$$

where $v(x)$ is the capacity of the event $\frac{\pi_m}{\pi} \geq x$. If he is a seller, the Choquet-expected utility is

$$CEU_s = u(w + cq) - \pi q\int_{1}^{\infty} u'\left(w - \pi q\left(x - 1 - \frac{c}{\pi}\right)\right)(1 - v^c(x))dx,$$

where $v^c(x)$ is the capacity of the event $\frac{\pi_m}{\pi} \leq x$. Introducing the notation $\bar{v}(x) = 1 - v^c(x)$, the Choquet-expected utility is $CEU_s = u(w + cq) - \pi q\int_1^{\infty} u'(w - \pi q\left(x - 1 - \frac{c}{\pi}\right)\bar{v}(x)dx$.[8] Since the utility function is concave, both CEU_b and CEU_s are concave functions of q.

The agent in question is uncertainty neutral if $v(x) = \bar{v}(x)$ for every $x \in \mathbb{R}_+$, he is uncertainty averse if $v(x) \leq \bar{v}(x)$ for every $x \in \mathbb{R}_+$. The definition of capacity requires $v(0) = \bar{v}(0) = 1$, $v(\infty) = \bar{v}(\infty) = 0$, and that $v(x)$ and $\bar{v}(x)$ are non-increasing functions. Moreover, as for Assumption 1, these functions are continuous. For uncertainty averse agents, according to Definition 1, $core(v) \neq \varnothing$, i.e., there are probability distributions $p(x)$ such that $p(x) \geq v(x)$ and $1 - p(x) \geq v^c(x) = 1 - \bar{v}(x)$, so that $v(x) \leq p(x) \leq \bar{v}(x)$ for every $x \in \mathbb{R}_+$. For uncertainty neutral agents for whom $v(x) = \bar{v}(x)$ for every $x \in \mathbb{R}_+$, $core(v)$ is a set composed of a unique point and the distributions $v(x)$ and $\bar{v}(x)$ coincide with a probability distribution $p(x)$.

The agent's choice is determined by the solutions of the problems $\max_{q \geq 0} CEU_b$ and $\max_{q \geq 0} CEU_s$. The boundary solutions $q^d = 0$ and $q^s = 0$, respectively, require $\left.\frac{\partial CEU_b}{\partial q}\right|_{q=0} \leq 0$ and $\left.\frac{\partial CEU_s}{\partial q}\right|_{q=0} \leq 0$. They imply $-\frac{c}{\pi} + \int_1^\infty v(x)dx \leq 0$ if $q^d = 0$ and $\frac{c}{\pi} - \int_1^\infty \bar{v}(x)dx \leq 0$ if $q^s = 0$.

Using the notation $\alpha = \int_1^\infty v(x)dx$ and $\bar{\alpha} = \int_1^\infty \bar{v}(x)dx$, it follows that, on the one hand, uncertainty aversion implies $\alpha \leq \bar{\alpha}$ and, on the other hand, that an agent will buy calls only if $\frac{c}{\pi} < \alpha$ and will sell calls only if $\frac{c}{\pi} > \bar{\alpha}$. In the following Figure 1

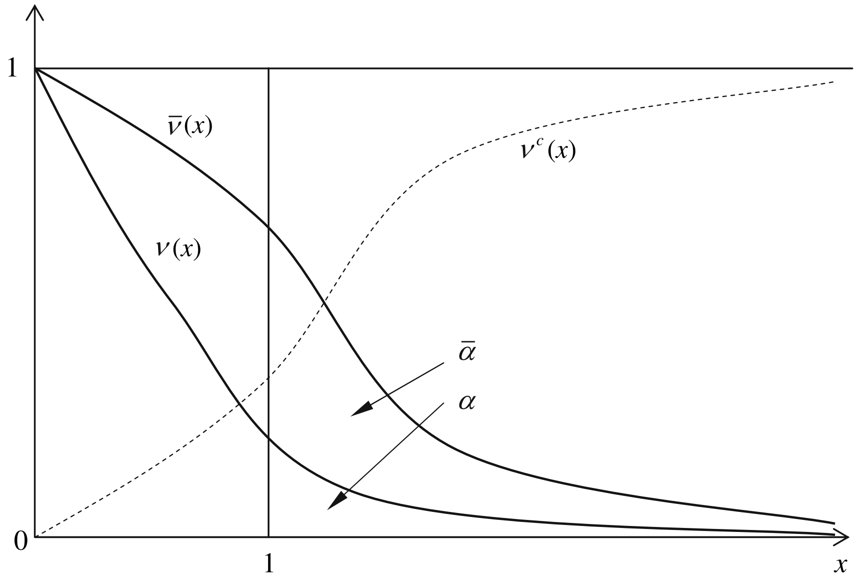

Figure 1. Functions $v(x), v^c(x)$ and $\bar{v}(x)$ and areas α and $\bar{\alpha}$ for an uncertainty averse agent.

the functions $v(x)$, $v^c(x)$ and $\bar{v}(x)$ of an uncertainty averse agent are represented and the corresponding areas α and $\bar{\alpha}$ are indicated.

Taking into account that the inequalities $\alpha > \frac{c}{\pi}$ and $\bar{\alpha} < \frac{c}{\pi}$ are incompatible for an uncertainty averse agent, for whom it is $\alpha \leq \bar{\alpha}$, the following remark emerges.

Remark 1. An uncertainty averse agent will buy calls if $\alpha > \frac{c}{\pi}$; he will sell calls if $\bar{\alpha} < \frac{c}{\pi}$; and he will neither buy nor sell calls if $\alpha \leq \frac{c}{\pi} \leq \bar{\alpha}$.

The quantity of calls that are bought or sold by an agent are determined by the first-order conditions of the problems $\max_{q\geq 0} CEU_b$ and $\max_{q\geq 0} CEU_s$, i.e.,

$$-cu'(w-cq)+\pi\int_1^\infty u'\left(w+\pi q\left(x-1-\frac{c}{\pi}\right)\right)v(x)dx$$
$$+\pi^2 q\int_1^\infty\left(x-1-\frac{c}{\pi}\right)u''\left(w+\pi q\left(x-1-\frac{c}{\pi}\right)\right)v(x)dx=0$$

and

$$cu'(w+cq)-\pi\int_1^\infty u'\left(w-\pi q\left(x-1-\frac{c}{\pi}\right)\right)\bar{v}(x)dx$$
$$+\pi^2 q\int_1^\infty\left(x-1-\frac{c}{\pi}\right)u''\left(w-\pi q\left(x-1-\frac{c}{\pi}\right)\right)\bar{v}(x)dx=0.$$

Thus, agent's demand for calls $q^d = d(c/\pi)$ is a non-increasing function,[9] with $d(\alpha)=0$ and $\lim_{\frac{c}{\pi}\to 0} d(c/\pi)=\infty$. His supply $q^s = s(c/\pi)$ is non-decreasing, with $s(\bar{\alpha})=0$.[10]

Let us assume that agents are characterized only by their attitude toward uncertainty, so that the cardinal utility function $u_i(.)$ is the same for all agents, i.e., $u_i(.) = u(.)$, and every agent i is characterized by the pair $(v_i(x), \bar{v}_i(x))_{x\in(1,\infty)}$, with $v_i(x) \leq \bar{v}_i(x)$ for every $x \in \mathbb{R}_+$ (and $v_i(x) = \bar{v}_i(x)$ in the case of uncertainty neutrality).[11] Let us assume also that the population is composed of agents whose capacity functions $(v_i(x), \bar{v}_i(x))_{x\in(1,\infty)}$ can be parameterized by $(\alpha_i, \bar{\alpha}_i)$. (For instance, this occurs if $(v_i(x), \bar{v}_i(x))_{x\in(1,\infty)}$ are stochastically ordered with $v_i(x) \geq v_j(x)$ or $v_i(x) \leq v_j(x)$ for every $x \in (1,\infty)$, and $\bar{v}_i(x) \geq \bar{v}_j(x)$ or $\bar{v}_i(x) \leq \bar{v}_j(x)$ for every $x \in (1, \infty)$, if $i \neq j$). Consequently, we can put $i = (\alpha, \bar{\alpha})$ so that every agent is indicated by the pair $(\alpha, \bar{\alpha}) \in \mathbb{R}^2_+$ with $\alpha \leq \bar{\alpha}$. Thus, the demand and supply functions for calls of the agent $(\alpha, \bar{\alpha})$ are, indicated respectively, by $q^d = d(c/\pi; \alpha)$ and $q^s = s(c/\pi; \bar{\alpha})$. (Notice that the demand function depends only on the characteristic α, and the supply function only on the characteristic $\bar{\alpha}$). Let $F(\alpha, \bar{\alpha})$ to represent the corresponding frequency distribution function (i.e., $F(\alpha, \bar{\alpha})$ is the proportion of agents whose characteristics do not exceed $(\alpha, \bar{\alpha})$). Then, $N(\alpha) = \lim_{\bar{\alpha} \to \infty} F(\alpha, \bar{\alpha})$ and $\bar{N}(\bar{\alpha}) = \lim_{\alpha \to \infty} F(\alpha, \bar{\alpha})$, where $N(z)$ and $\bar{N}(z)$ are the marginal distribution functions, which are non-decreasing, with $N(0) = \bar{N}(0) = 0$ and $N(\infty) = \bar{N}(\infty) = 1$. Since $\alpha \leq \bar{\alpha}$ for all agents, then $N(z) \geq \bar{N}(z)$ for every $z \in \mathbb{R}_+$. All this is summarized by Assumption 2.

ASSUMPTION 2. The population is composed of uncertainty averse agents characterized only by their attitude toward uncertainty. This attitude can be represented by the pair $(\alpha, \bar{\alpha}) \in \mathbb{R}^2_+$, with $\alpha \leq \bar{\alpha}$. The marginal distribution functions $N(z)$ and $\bar{N}(z)$, which respectively indicate the proportion of agents $(\alpha, \bar{\alpha})$ with $\alpha \leq z$ and $\bar{\alpha} \leq z$, thus with $N(z) \geq \bar{N}(z)$ for every $z \in \mathbb{R}_+$, are continuous.

Agents with $\alpha > \frac{c}{\pi}$ will buy calls. Since their demand depends on the function $v(x)$, but not on $\bar{v}(x)$, and function $v(x)$ can be parameterized by α, then $q^d = d(c/\pi; \alpha)$, with $\frac{\partial q^d}{\partial c/\pi} \leq 0$ and $\frac{\partial q^d}{\partial \alpha} \geq 0$. The total demand for calls can be determined taking into account the marginal distribution function $N(\alpha)$, i.e., $D(c/\pi) = \int_{c/\pi}^{\infty} d(c/\pi; \alpha) dN(\alpha)$. Agents with $\alpha \leq \frac{c}{\pi} \leq \bar{\alpha}$ neither buy nor sell calls. Agents with $\bar{\alpha} < \frac{c}{\pi}$ will sell calls and, analogously to the buyers case, we have $q^s = s(c/\pi; \bar{\alpha})$, with $\frac{\partial q^s}{\partial c/\pi} \geq 0$ and $\frac{\partial q^s}{\partial \bar{\alpha}} \leq 0$, and the total supply $S(c/\pi) = \int_0^{c/\pi} s(c/\pi; \bar{\alpha}) d\bar{N}(\bar{\alpha})$ determined by means of the marginal distribution function $\bar{N}(\bar{\alpha})$. Then, $D(c/\pi)$ is a continuous and decreasing function, with $\lim_{\frac{c}{\pi} \to 0} D(c/\pi) = \infty$ and $D(c/\pi) = 0$ for $\frac{c}{\pi} \geq \frac{c^d_{\max}}{\pi} = \min\{\alpha : N(\alpha) = 1\}$; and $S(c/\pi)$ is a

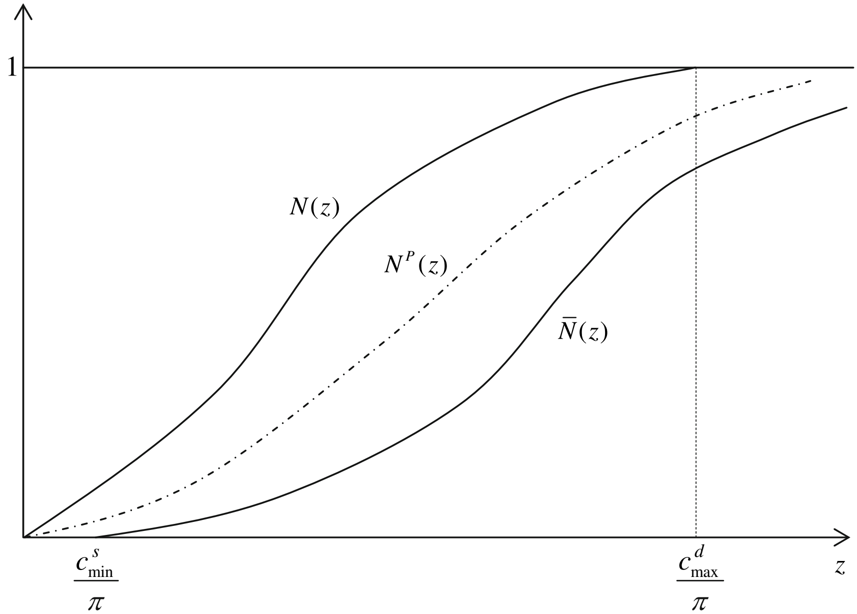

Figure 2. Marginal distribution functions of an uncertainty averse population and of an associate uncertainty neutral population.

continuous and increasing function, with $S(c/\pi)=0$ for $\frac{c}{\pi} \leq \frac{c^s_{\min}}{\pi} = \max\left\{\bar{\alpha} : \bar{N}(\bar{\alpha})=0\right\}$.

Taking into account, for every agent $i=(\alpha,\bar{\alpha})$, a probability distribution $p_i(x)$ belonging to $core(v_i)$, for which $v_i(x) \leq p_i(x) \leq \bar{v}_i(x)$ for every $x \in \mathbb{R}_+$, and defining $\alpha_i^P = \int_1^{\infty} p_i(x)dx$, we have $\alpha_i^P \in [\alpha_i, \bar{\alpha}_i]$. In this way we associate an uncertainty neutral agent to every agent i, i.e., an agent α^P with $\alpha^P \in [\alpha, \bar{\alpha}]$ is associated to agent $(\alpha,\bar{\alpha})$. Therefore, we have introduced an uncertainty neutral population of the same size of the population in examination. The frequency distribution $N^P(z)$ (where $N^P(z)$ indicates the proportion of agents with $\alpha^P \leq z$) of this uncertainty neutral population is such that $\bar{N}(z) \leq N^P(z) \leq N(z)$ for every $z \in \mathbb{R}_+$, as represented by Figure 2. This introduces the following definition.

DEFINITION 4. *Take a population* P *of uncertainty averse agents that is characterized by the marginal distribution functions* $N:\mathbb{R}_+ \to [0,1]$, $\bar{N}:\mathbb{R}_+ \to [0,1]$, *with* $\bar{N}(z) \leq N(z)$ *for every*

$z \in \mathbb{R}_+$. *Any population* P' *of the same size that is composed of uncertainty neutral agents and characterized by a distribution function* $N^P : \mathbb{R}_+ \to [0, 1]$ *such that* $\bar{N}(z) \le N^P(z) \le N(z)$ *for every* $z \in \mathbb{R}_+$, *is defined as an "associate uncertainty neutral population".*

In Sections 4 and 5, we examine the competitive partial equilibrium for call options, determining the price of the call and its trading volume and their dependence on uncertainty aversion.

4. EQUILIBRIUM OF THE MARKET FOR CALLS

Assumptions 1 and 2 imply the following total demand and supply functions:

$$D(c/\pi) = \int_{c/\pi}^{\infty} d(c/\pi; \alpha) dN(\alpha),$$

$$S(c/\pi) = \int_{0}^{c/\pi} s(c/\pi; \bar{\alpha}) d\bar{N}(\bar{\alpha})$$

Equilibrium conditions $Q = D(c/\pi) = S(c/\pi)$ determine the price c^* of the call in question and the quantity Q^* of calls. Since $D(c/\pi)$ is a continuous and decreasing function, with $\lim_{\frac{c}{\pi} \to 0} D(c/\pi) = \infty$ and $D(c/\pi) = 0$ for $\frac{c}{\pi} \ge \frac{c^d_{\max}}{\pi} = \min\{\alpha : N(\alpha) = 1\}$ and $S(c/\pi)$ is a continuous and increasing function, with $S(c/\pi) = 0$ for $\frac{c}{\pi} \le \frac{c^s_{\min}}{\pi} = \max\{\bar{\alpha} : \bar{N}(\bar{\alpha}) = 0\}$, the equilibrium exists and it is unique if $c^d_{\max} \ge c^s_{\min}$. Otherwise, i.e., if $c^d_{\max} < c^s_{\min}$, then $Q^* = 0$ and c^* is undetermined in the interval $[c^d_{\max}, c^s_{\min}]$.

Then, market equilibrium requires

$$Q^* = \int_{c^*/\pi}^{\infty} d(c^*/\pi; \alpha) dN(\alpha) = \int_{0}^{c^*/\pi} s(c^*/\pi; \bar{\alpha}) d\bar{N}(\bar{\alpha})$$

Assuming $c^d_{\max} \ge c^s_{\min}$, the following propositions concern the effects of uncertainty aversion on the equilibrium variables c^* and Q^*.

PROPOSITION 1. *If* P *is an uncertainty averse population (thus characterized by marginal distribution functions* $N : \mathbb{R}_+ \to$

$[0,1]$, $\bar{N}:\mathbb{R}_+ \to [0,1]$, *with* $\bar{N}(z) \le N(z)$ *for every* $z \in \mathbb{R}_+$) *and* P' *is any associate uncertainty neutral population* (*thus characterized by a distribution function* $N^P:\mathbb{R}_+ \to [0,1]$ *such that* $\bar{N}(z) \le N^P(z) \le N(z)$ *for every* $z \in \mathbb{R}_+$*, as introduced by Definition* 4)*, then the corresponding equilibrium conditions*

$$Q^* = \int_{c^*/\pi}^{\infty} d(c^*/\pi;\alpha)dN(\alpha) = \int_0^{c^*/\pi} s(c^*/\pi;\bar{\alpha})d\bar{N}(\bar{\alpha})$$

and

$$Q^P = \int_{c^P/\pi}^{\infty} d(c^P/\pi;\alpha^P)dN^P(\alpha^P) = \int_0^{c^P/\pi} s(c^P/\pi;\alpha^P)dN^P(\alpha^P)$$

imply $Q^P \ge Q^*$.

Proof. There are two main reasons that justify this result. On the one hand, a probability distribution belonging to the core of the capacities distributions implies a non-smaller demand or supply for calls for every agent than those generated by the capacity distributions. Formally, since $\alpha \le \alpha^P \le \bar{\alpha}$ and $\frac{\partial d(c/\pi;\alpha)}{\partial \alpha} \ge 0$, $\frac{\partial s(c/\pi;\bar{\alpha})}{\partial \bar{\alpha}} \le 0$, then $d(c/\pi;\alpha^P) \ge d(c/\pi;\alpha)$ and $s(c/\pi;\alpha^P) \ge s(c/\pi;\bar{\alpha})$. On the other hand, the proportion of agents who demand or supply calls is non-lesser. Formally, since $\bar{N}(z) \le N^P(z) \le N(z)$ for every $z \in \mathbb{R}_+$, then $\int_{c/\pi}^{\infty} dN^P(\alpha^P) \ge \int_{c/\pi}^{\infty} dN(\alpha)$ and $\int_0^{c/\pi} dN^P(\alpha^P) \ge \int_0^{c/\pi} d\bar{N}(\bar{\alpha})$. Consequently, if $c^P \le c^*$, also taking into account that $\frac{\partial d(c/\pi;\alpha)}{\partial c/\pi} \le 0$, then

$$\begin{aligned} Q^P &= \int_{c^P/\pi}^{\infty} d(c^P/\pi;\alpha^P)dN^P(\alpha^P) \ge \int_{c^*/\pi}^{\infty} d(c^P/\pi;\alpha^P)dN^P(\alpha^P) \\ &\ge \int_{c^*/\pi}^{\infty} d(c^P/\pi;\alpha)dN(\alpha) \ge \int_{c^*/\pi}^{\infty} d(c^*/\pi;\alpha)dN(\alpha) = Q^* \end{aligned}$$

and, if $c^P \ge c^*$, also taking into account that $\frac{\partial s(c/\pi;\bar{\alpha})}{\partial c/\pi} \ge 0$, then

$$\begin{aligned} Q^P &= \int_0^{c^P/\pi} s(c^P/\pi;\alpha^P)dN^P(\alpha^P) \ge \int_0^{c^*/\pi} s(c^P/\pi;\alpha^P)dN^P(\alpha^P) \\ &\ge \int_0^{c^*/\pi} s(c^P/\pi;\bar{\alpha})d\bar{N}(\bar{\alpha}) \ge \int_0^{c^*/\pi} s(c^*/\pi;\bar{\alpha})d\bar{N}(\bar{\alpha}) = Q^*. \end{aligned}$$

□

Proposition 1 says that the trading volume of calls is smaller for an uncertainty averse population than for any associate uncertainty neutral population.[12]

However, there is no such unilateral relation for the price c^* of a call; call prices appear to be mostly unaffected by uncertainty aversion. We can even obtain $c^P = c^*$, for instance taking the associate uncertainty neutral population $N^P(z) = \lambda N(z) + (1-\lambda)\bar{N}(z)$ with

$$\frac{\lambda}{1-\lambda} = \frac{\int_0^{c^*/\pi} s(c^*/\pi;\bar{\alpha})d\bar{N}(\bar{\alpha}) - \int_{c^*/\pi}^{\infty} d(c^*/\pi;\alpha)d\bar{N}(\alpha)}{\int_{c^*/\pi}^{\infty} d(c^*/\pi;\alpha)dN(\alpha) - \int_0^{c^*/\pi} s(c^*/\pi;\bar{\alpha})dN(\bar{\alpha})}.$$

In fact, the equality

$$\int_{c^P/\pi}^{\infty} d(c^P/\pi;\alpha^P)dN^P(\alpha^P) = \int_0^{c^P/\pi} s(c^P/\pi;\alpha^P)dN^P(\alpha^P)$$

requires

$$\lambda \int_{c^P/\pi}^{\infty} d(c^P/\pi;\alpha^P)dN(\alpha^P) + (1-\lambda)\int_{c^P/\pi}^{\infty} d(c^P/\pi;\alpha^P)d\bar{N}(\alpha^P)$$
$$= \lambda \int_0^{c^P/\pi} s(c^P/\pi;\alpha^P)dN(\alpha^P) + (1-\lambda)\int_0^{c^P/\pi} s(c^P/\pi;\alpha^P) \times d\bar{N}(\alpha^P)$$

and, therefore, $c^P = c^*$.

Figure 3 represents the equilibrium for an uncertainty averse population and an associate uncertainty neutral population.

5. A COMPARISON OF THE CALL MARKET EQUILIBRIUM FOR TWO POPULATIONS AND FOR TWO CALLS

The preceding analysis can be enlarged comparing equilibria for populations that differ only with regard to uncertainty aversion and equilibria for two different calls.

The following definition introduces a comparison of two populations with respect to uncertainty aversion, in order to examine the relation between their equilibria. This definition

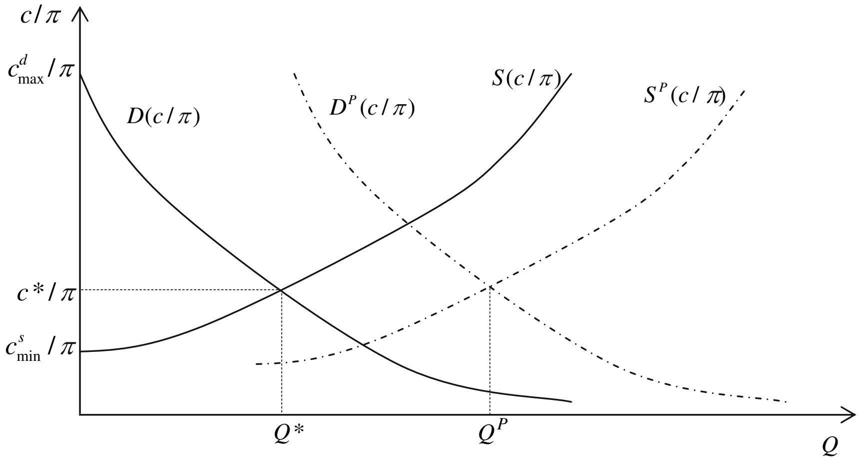

Figure 3. Market equilibria for an uncertainty averse population and an associate uncertainty neutral population.

on two populations is based on Definition 2 concerning the comparative definition of uncertainty aversion for two agents, according to which agent 1 is more averse to uncertainty than agent 2 if $v_1(E) - v_2(E) \leq 0$ for all $E \in 2^S$, i.e., if $v_1(x) - v_2(x) \leq 0$ and for all $x \in \mathbb{R}_+$, so that, since $\bar{v}(x) = 1 - v^c(x)$, also $\bar{v}_1(x) - \bar{v}_2(x) \geq 0$. This implies that $\alpha_1 \leq \alpha_2$, $\bar{\alpha}_1 \geq \bar{\alpha}_2$, $d_1(c/\pi; \alpha_1) \leq d_2(c/\pi; \alpha_2)$ and $s_1(c/\pi; \bar{\alpha}_1) \leq s_2(c/\pi; \bar{\alpha}_2)$ for every $\frac{c}{\pi} \in \mathbb{R}_+$.

DEFINITION 5. *Let two populations P and P' be respectively characterized by the marginal distribution functions* $N: \mathbb{R}_+ \to [0, 1]$*,* $\bar{N}: \mathbb{R}_+ \to [0, 1]$ *and* $N': \mathbb{R}_+ \to [0, 1]$*,* $\bar{N}': \mathbb{R}_+ \to [0, 1]$. *Population P' is defined as more uncertainty averse than population P for a given call if $N'(z) \geq N(z)$ and $\bar{N}'(z) \leq \bar{N}(z)$ for every $z \in \mathbb{R}_+$.*

Remark 2. If P is an associate uncertainty neutral population with respect to an uncertainty averse population P' (according to Definition 4), then P' is more uncertainty averse than P.

The following proposition says that the trading volume depends on the “more uncertainty averse” relation introduced by Definition 5. The trading volume is smaller for the more uncertainty averse population.

PROPOSITION 2. *If population P' is more uncertainty averse than population P according to Definition* 5, *then its trading volume is smaller, i.e., it is $Q^{*\prime} \leq Q^*$.*

Proof. The demonstration is analogous to that of Proposition 1. Equilibrium conditions require, for population P and population P' respectively,

$$Q^* = \int_{c^*/\pi}^{\infty} d(c^*/\pi;\alpha)dN(\alpha) = \int_0^{c^*/\pi} s(c^*/\pi;\bar{\alpha})d\bar{N}(\bar{\alpha})$$

and

$$Q^{*\prime} = \int_{c^{*\prime}/\pi}^{\infty} d(c^{*\prime}/\pi;\alpha')dN'(\alpha') = \int_0^{c^{*\prime}/\pi} s(c^{*\prime}/\pi;\bar{\alpha}')d\bar{N}'(\bar{\alpha}').$$

Since $\frac{\partial d(c/\pi;\alpha)}{\partial \alpha} \geq 0$, $\frac{\partial s(c/\pi;\bar{\alpha})}{\partial \bar{\alpha}} \leq 0$, $\frac{\partial d(c/\pi;\alpha)}{\partial c/\pi} \leq 0$, $\frac{\partial s(c/\pi;\bar{\alpha})}{\partial c/\pi} \geq 0$ and $\bar{N}'(z) \leq \bar{N}(z) \leq N(z) \leq N'(z)$ for every $z \in \mathbb{R}_+$, if $c^* \leq c^{*\prime}$, then

$$\begin{aligned} Q^* &= \int_{c^*/\pi}^{\infty} d(c^*/\pi;\alpha)dN(\alpha) \geq \int_{c^{*\prime}/\pi}^{\infty} d(c^*/\pi;\alpha)dN(\alpha) \\ &\geq \int_{c^{*\prime}/\pi}^{\infty} d(c^*/\pi;\alpha')dN'(\alpha') \geq \int_{c^{*\prime}/\pi}^{\infty} d(c^{*\prime}/\pi;\alpha')dN'(\alpha') \\ &= Q^{*\prime} \end{aligned}$$

and, if $c^* \geq c^{*\prime}$, then

$$\begin{aligned} Q^* &= \int_0^{c^*/\pi} s(c^*/\pi;\bar{\alpha})d\bar{N}(\bar{\alpha}) \geq \int_0^{c^{*\prime}/\pi} s(c^*/\pi;\bar{\alpha})d\bar{N}(\bar{\alpha}) \\ &\geq \int_0^{c^{*\prime}/\pi} s(c^*/\pi;\bar{\alpha}')d\bar{N}'(\bar{\alpha}') \geq \int_0^{c^{*\prime}/\pi} s(c^{*\prime}/\pi;\bar{\alpha}')d\bar{N}'(\bar{\alpha}') \\ &= Q^{*\prime}. \end{aligned}$$

□

Remark 3. No qualitative implication on the price of the call results from the comparison of the equilibria for populations P and P', i.e., $c^{*\prime} \lesseqgtr c^*$. The equilibrium conditions for the above populations P and P' (where P' is more uncertainty averse than P) may be satisfied by an identical price of the call, i.e., it can be found $c^* = c^{*\prime}$. For instance, with the distributions $N(z) = \lambda N'(z) + (1-\lambda)\bar{N}'(z)$ and $\bar{N}(z) = \mu N'(z) + (1-\mu)\bar{N}'(z)$, with $\mu \leq \lambda$ and

$$\frac{\mu}{1-\lambda} = \frac{\int_{c^{*\prime}/\pi}^{\infty} d(c^{*\prime}/\pi;\alpha')d\bar{N}'(\alpha') - \int_{c^{*\prime}/\pi}^{\infty} d(c^{*\prime}/\pi;\alpha')dN'(\alpha')}{\int_0^{c^{*\prime}/\pi} s(c^{*\prime}/\pi;\bar{\alpha}')dN'(\bar{\alpha}') - \int_0^{c^{*\prime}/\pi} s(c^{*\prime}/\pi;\bar{\alpha}')d\bar{N}'(\bar{\alpha}')},$$

the equality

$$\int_{c^*/\pi}^{\infty} d(c^*/\pi;\alpha)dN(\alpha) = \int_0^{c^*/\pi} s(c^*/\pi;\bar{\alpha})d\bar{N}(\bar{\alpha})$$

requires

$$\lambda \int_{c^*/\pi}^{\infty} d(c^*/\pi;\alpha)dN'(\alpha) + (1-\lambda)\int_{c^*/\pi}^{\infty} d(c^*/\pi;\alpha)d\bar{N}'(\alpha)$$
$$= \mu \int_0^{c^*/\pi} s(c^*/\pi;\bar{\alpha})dN'(\bar{\alpha}) + (1-\mu)\int_0^{c^*/\pi} s(c^*/\pi;\bar{\alpha})d\bar{N}'(\bar{\alpha}),$$

i.e.,

$$\int_{c^*/\pi}^{\infty} d(c^*/\pi;\alpha)dN'(\alpha) + (1-\lambda)\left(\int_{c^*/\pi}^{\infty} d(c^*/\pi;\alpha)d\bar{N}'(\alpha)\right.$$
$$\left. - \int_{c^*/\pi}^{\infty} d(c^*/\pi;\alpha)dN'(\alpha)\right) = \int_0^{c^*/\pi} s(c^*/\pi;\bar{\alpha})d\bar{N}'(\bar{\alpha})$$
$$+ \mu\left(\int_0^{c^*/\pi} s(c^*/\pi;\bar{\alpha})dN'(\bar{\alpha}) - \int_0^{c^*/\pi} s(c^*/\pi;\bar{\alpha})d\bar{N}'(\bar{\alpha})\right)$$

which is satisfied by $c^* = c^{*\prime}$.

Now, the competitive partial equilibria of two different calls C and C' presenting the same strike price π will be taken into consideration (obviously, the two calls regard two different primary assets). A given population is characterized by the marginal distribution functions $N: \mathbb{R}_+ \to [0,1]$, $\bar{N}: \mathbb{R}_+ \to [0,1]$ and $N': \mathbb{R}_+ \to [0,1]$, $\bar{N}': \mathbb{R}_+ \to [0,1]$, respectively for call C and for call C'. To further analyze this case, the following Definition 6 is introduced, based on Definition 3, according to which the call C' is, for an agent, more uncertain than call C if $v'(x) - v(x) \leq 0$ and $\bar{v}'(x) - \bar{v}(x) \geq 0$ for all $x \in (1, \infty)$. Consequently, for an uncertainty averse agent, if call C' is more uncertain than call C, then $\bar{\alpha}' \geq \bar{\alpha} \geq \alpha \geq \alpha'$.

DEFINITION 6. *For a given population of uncertainty averse agents, let two calls C and C′ be characterized by the marginal distribution functions* $N: \mathbb{R}_+ \to [0, 1]$, $\bar{N}: \mathbb{R}_+ \to [0, 1]$ *and* $N': \mathbb{R}_+ \to [0, 1]$, $\bar{N}': \mathbb{R}_+ \to [0, 1]$, *respectively. Call C′ is defined as more uncertain for this population than call C if* $N'(z) \geq N(z)$ *and* $\bar{N}'(z) \leq \bar{N}(z)$ *for every* $z \in \mathbb{R}_+$.

The following proposition says that the trading volume depends on the "more uncertain" relation introduced by Definition 6. The trading volume is smaller for the more uncertain call.

PROPOSITION 3. *If call C′ is more uncertain for a population of uncertainty averse agents than call C, according to Definition 6, then its trading volume is smaller, i.e., it is* $Q^{*\prime} \leq Q^*$.

Proof. The demonstration is formally identical to that of Proposition 2. □

It seems important to point out that the two equilibria outlined above have no implications regarding the relative prices of the calls C and C'. Similarly to Remark 3, the equilibrium conditions for the above calls C and C' (where C' is more uncertain than C for a population of uncertainty averse agents) may be satisfied by an identical price, i.e., it can be found $c^* = c^{*\prime}$.

6. A COMPARISON WITH THE EXPECTED UTILITY MODEL

Given that the results presented before are based on the Choquet-expected utility model, it seems natural to compare the results obtained through this model to those obtained within an expected utility framework.

Choquet-expected utility differs from expected utility only because probability is not additive, so that the Choquet-expected utility model coincides with the expected utility model if agents are uncertainty neutral. A first difference, which is similar to the Dow–Werlang (1992) stability effect, is indicated by Remark 1, taking into account that the expected utility model requires that all agents buy or sell calls (an agent does not exchange calls only if it is exactly $\frac{c}{\pi} = \int_1^\infty p(x)dx$). This stability effect leads to no trade and an undetermined price of the call if the intersection of the cores of individual agents is nonempty, i.e., if $c^d_{\max} < c^s_{\min}$. In this case, total demand $D(c/\pi)$ is lower than total supply $S(c/\pi)$ at every $Q > 0$, and equilibrium conditions are satisfied for $Q^* = 0$ and any $c^* \in [c^d_{\max}, c^s_{\min}]$. This result can be extended to a general equilibrium model of uncertainty averse agents (so that, according to Definition 1, every agent has a nonempty $core(v)$), stating that no trade and undetermined prices occur if the intersection of the cores of individual agents is nonempty and contains more than one probability distribution.[13]

Proposition 1 provides a second difference, stating that the market for calls is thinner for a population of uncertainty averse agents than for a corresponding population of uncertainty neutral agents.

Differences are weaker if we examine the dependence of market equilibrium on cardinal risk aversion, rather than its dependence on uncertainty aversion.[14] Although the expected utility model seems to supply similar results to those of the Choquet-expected value model, replacing uncertainty aversion with risk aversion[15] yields the following two main differences given below.

Remark 4. The attitude towards cardinal risk is irrelevant when the investment in a call is negligible with respect to agents' wealth. In this case agents behave almost as if they were risk neutral since cardinal utility is locally linear. On the contrary, even in this case, the attitude toward uncertainty matters, since it affects the decision to trade or not to trade calls, and thus also the total volume observed.

Remark 5. The expected utility model excludes that uncertainty on the probability distribution of outcomes matters. The expected utility model assumes either that the probability distribution is perfectly known or that agents' choice does not depend on the quality of information on the probability distribution (i.e., with reference to acts on an urn, information on the composition of the urn does not influence an expected utility decision maker). On the contrary, the Choquet-expected utility model is based on the possibility that this information (i.e., on the urn composition, which can be known or, totally or partially, unknown) influences choices if agents are not uncertainty neutral. Consequently, the expected utility equilibrium conditions do not depend on the quality of information on the probability distribution of outcomes, whereas this information affects the Choquet-expected utility equilibrium conditions through uncertainty aversion.[16] Therefore, taking into account two populations or two assets which are equivalent in all respects except for the quality of information on the probability distribution of outcomes, the expected utility market equilibrium conditions are equal, while market equilibrium conditions differ if the Choquet-expected utilities are taken into account. Indeed, in this case different marginal distribution functions arise for the two populations or the two calls (i.e., $N(.)$ and $\bar{N}(.)$ for population P or call C, and $N'(.)$ and $\bar{N}'(.)$ for population P' or call C', as indicated by Definitions 5 and 6). Then, the expected utility model states that the trading volume of calls is equal for the two populations and the two calls, while the Choquet-expected utility model states that such volume will generally differ.

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NOTES

1. For instance, Mas-Colell et al. (1995, pp. 188–189, 192–194, and 699–708) and Varian (1992, pp. 184–185, 187–188, and 368–385).
2. For instance, portfolio diversification is explained both by risk aversion and by uncertainty aversion. It is usually explained in the literature with reference to risk aversion only. Kelsey and Milne (1995, Proposition 4.2) make explicit reference to risk aversion in order to explain portfolio diversification, even if their analysis also includes uncertainty aversion. In case of uncertainty neutrality, i.e., with additive probability, Dekel (1989, Proposition 2) demonstrates that preference for portfolio diversification implies risk aversion (defined as aversion to mean-preserving spreads) even if the Fréchet-differentiable preference function does not satisfy the independence axiom.

   Uncertainty aversion can also produce incompleteness of financial markets (Mukerji and Tallon, 2001) as well as indeterminate equilibria (Epstein and Wang, 1994), which imply the possibility of sizable volatility in these markets. General equilibrium analysis with Choquet-expected utility is carried out by Chateauneuf et al. (2000) who indicate some interesting results, for instance if the intersection of agents' cores is not empty (i.e., there are probability distributions that dominate the capacities of all agents), then there is an equilibrium, which is indeterminate if the above intersection contains more than one probability distribution.

   The stability of the portfolio choice with respect to a variation in the asset prices (i.e., prices may vary without modifying the optimal portfolio) is incompatible with the expected utility model. As demonstrated first by Dow and Werlang (1992) and examined in a more general framework by Epstein and Wang (1994), the stability of the portfolio choice crucially depends on uncertainty aversion.
3. A stronger definition of uncertainty aversion is proposed by Schmeidler (1989). According to this definition, there is uncertainty aversion if the capacity is convex, i.e., if $v(E)+v(E')-v(E\cup E')-v(E\cap E')\leq 0$ for every pair $E, E'\in 2^S$. Note that if the capacity is convex, then its core is non-empty (Shapley, 1971).

4. If we extend to capacities the notion of the first-order stochastic dominance, this definition requires that a_2 dominates a_1 with respect to the decumulative distribution and a_2 dominates a_1 with respect to the cumulative distribution. Formally, a monetary act a_1 is first-order stochastically dominated by a_2 if $p_1(t) \le p_2(t)$ for all $t \in \mathbb{R}_+$, where $p(t)$ is the decumalitive probability distribution, i.e., $p(t) = \Pr\{s \in S: f(s) \ge t\}$, and, consequently, $p_1^c(t) \ge p_2^c(t)$, where $p^c(t)$ is the cumulative probability distribution (since $p(t) + p^c(t) = 1$ for all $t \in \mathbb{R}_+$). Definition 3 states that a_1 is more uncertain than a_2 if, with respect to the capacity distributions, both $v_1(t) \le v_2(t)$ and $v_1^c(t) \le v_2^c(t)$ for all $t \in \mathbb{R}_+$, where $v(t)$ and $v^c(t)$ are respectively the decumulative capacity distribution and the cumulative one, i.e., $v(t) = v(E(t))$ and $v^c(t) = v(E^c(t))$, where $E(t) = \{s \in S: f(s) \ge t\}$ and $E^c(t) = \{s \in S: f(s) \le t\}$.
5. This assumption rules out any trading strategy process for the period between these two dates, which is the most important part of the current analysis of the value of a call option (see, for instance, Duffie, 2001, pp. 37–39 and 88–90).
6. The following representation may be useful. Suppose an urn with various gray balls and an index of grayness from zero for white balls to infinite for black balls (for instance, if the color is produced by a mixture of a white matter and a black matter, then the proposed index of grayness may be $\frac{b}{1-b}$, where b is the proportion of the black matter in the mixture). Let us associate the "primary asset" to the act that gives a prize equal to the grayness π_m of the drawn ball. There is no uncertainty if the composition of the urn is known (i.e., the color of every ball in the urn is known), uncertainty if its composition is totally or partially unknown.

   Thus uncertainty is connected to the information on the composition of the urn, i.e., to its transparency. Uncertainty averse agents like transparency: they prefer more transparent acts and are willing to pay for transparency. In other words, an act is valued less by a more uncertainty averse agent than by a less uncertainty averse agent. And an uncertainty averse agent values a more uncertain act less than a less uncertain act (assuming that the two acts coincide in all except the degree of transparency of the urn). However, this valuation regards acts that are stochastically independent from agent's wealth. If the same state of the nature that determines the prize of the act under examination also affects agent's initial wealth and the two effects are opposite, so that the resulting total act (composed of the initial wealth and of the act under examination) is risk free (or less uncertain than initial wealth), then a more uncertainty averse agent values the act under examination more than a less uncertainty agent, because this act reduces total uncertainty. For instance, going back to Ellsberg's example, if only act a_1 (that gives a prize if a black ball is drawn) is available in the market, then it can be sold at a low price, since both risk aversion and uncertainty aversion reduces its value. On the contrary, if both act a_1 and the act that gives the same prize as act a_1 if a non black ball is drawn (i.e., if the event $R \cup G$ occurs) are available, then an agent who already holds the latter act is willing to pay a large price in order to get act a_1 and, so, hedge the risk. Consequently, if both acts are available in the market, their prices are unaffected not only by uncertainty aversion but also by risk aversion (since the sum of the prices of the two acts is equal to their prize, independently from uncertainty, and risk, aversion).

In the same way, the price of the primary asset is affected by uncertainty aversion if there is no possibility of hedging risks by means of options. Uncertainty (and risk) aversion is ineffective if options are available in the market. For instance, indicating the price of the primary asset with π_a and the price of the put option (with the same strike price π of the call) with p (more precisely, π_a and p are their future value, i.e., $\pi_a = \pi'_a(1+i)$ and $p = p'(1+i)$, where π'_a and p' are the prices at date 0 and i is the interest rate for the period between date 0 and date 1), we have the no-arbitrage condition $c + \pi - \pi_a = p$. (This condition comes out from the fact that a portfolio composed of an equal amount of the primary asset and puts and short of calls is risk free). Consequently, assuming the possibility of hedging risks, the price π_a of the primary asset is unaffected by uncertainty aversion (and risk aversion) if this happens for calls and puts. (In the present article, the price of a call is proved to be practically independent from uncertainty aversion even without considering the possibility of hedging risks).

7. Notice that $\int_{\pi}^{\infty} p(t)dt$ is a decreasing function of π, as well as, *a fortiori*, $\int_1^{\infty} p(x)dx$.
8. The possibility of default is excluded by this relationship. If it is taken into account, then $CEU_s = u(w + cq) - \int_1^{1+\frac{c}{\pi}+\frac{w}{\pi q}} u'(w - \pi q (x - 1 - \frac{c}{\pi})\bar{v}(x)dx$. The addendum $\int_{1+\frac{c}{\pi}+\frac{w}{\pi q}}^{\infty} u'(w - \pi q(x - 1 - \frac{c}{\pi})\bar{v}(x)dx$ is neglected in the following analysis.
9. The analysis of demand and supply functions, in order to demonstrate that demand is decreasing and supply is increasing with respect to the price of the call, can be performed taking into account that, integrating by parts, $CEU_b = (1 - v(1))u(w - cq) - \int_1^{\infty} u(w + \pi q(x - 1 - \frac{c}{\pi})dv(x)$. Then, $\frac{\partial CEU_b}{\partial q} = -cu'(w - cq)(1 - v(1)) - \pi \int_1^{\infty}(x - 1 - \frac{c}{\pi})u'(w + \pi q(x - 1 - \frac{c}{\pi}))dv(x)$, $\frac{\partial^2 CEU_b}{\partial q^2} = c^2 u''(w - cq)(1 - v(1)) - \pi^2 \int_1^{\infty}(x - 1 - \frac{c}{\pi})^2 u''(w + \pi q(x - 1 - \frac{c}{\pi}))dv(x)$ and $\frac{\partial^2 CEU_b}{\partial q \partial \frac{c}{\pi}} = -\pi u'(w - cq)(1 - v(1)) + \pi q c u''(w - cq)(1 - v(1)) + \pi \int_1^{\infty}(u'(w +$

$\pi q(x-1-\frac{c}{\pi}))+\pi q(x-1-\frac{c}{\pi})u''(w+\pi q(x-1-\frac{c}{\pi})))\ dv(x)$, with $\frac{\partial CEU_b}{\partial q}=0$ because of the first-order condition, $\frac{\partial^2 CEU_b}{\partial q^2}<0$ and $\frac{\partial^2 CEU_b}{\partial q \partial(c/\pi)}<0$, so that not only the second-order condition is satisfied, but also $\frac{\partial q^d}{\partial(c/\pi)}=-\frac{\frac{\partial^2 CEU_b}{\partial q\partial(c/\pi)}}{\frac{\partial^2 CEU_b}{\partial q^2}}<0$. Analogously for the supply function.

10. For instance, let $u(y)=-e^{-\sigma y}$, $v(x)=e^{-\rho x}$ and $\bar{v}(x)=e^{-\bar{\rho}x}$ with $\rho\geq\bar{\rho}>1$, so that $\alpha=\rho^{-1}e^{-\rho}$ and $\bar{\alpha}=\bar{\rho}^{-1}e^{-\bar{\rho}}$. Then $CEU_b=-(\rho+\sigma\pi q(1-e^{-\rho}))(\rho+\sigma\pi q)^{-1}e^{-\sigma w+\sigma qc}$ and the inverse demand function is, for $\frac{c}{\pi}<\alpha$, $\frac{c}{\pi}=\rho e^{-\rho}(\rho+\sigma\pi q)^{-1}(\rho+\sigma\pi q-\sigma\pi q e^{-\rho})^{-1}$. Thus, for $\frac{c}{\pi}<\alpha$, the direct demand function $q^d=d(\frac{c}{\pi};\rho)$ is $q^d=\frac{\rho}{2(e^\rho-1)\sigma\pi}\left(1-2e^\rho+\sqrt{1+4\frac{e^\rho-1}{\rho c/\pi}}\right)$, then with $\frac{\partial q^d}{\partial c/\pi}<0$, $\frac{\partial^2 q^d}{\partial(c/\pi)^2}>0$ and $\frac{\partial q^d}{\partial\rho}<0$, so that $\frac{\partial q^d}{\partial\alpha}>0$ since $\frac{d\alpha}{d\rho}<0$. Analogously, $CEU_s=-(\bar{\rho}-\sigma\pi q(1-e^{-\bar{\rho}}))(\bar{\rho}-\sigma\pi q)^{-1}e^{-\sigma w-\sigma qc}$ and, for $\frac{c}{\pi}>\bar{\alpha}$, $\frac{c}{\pi}=\bar{\rho}e^{-\bar{\rho}}(\bar{\rho}-\sigma\pi q)^{-1}(\bar{\rho}-\sigma\pi q+\sigma\pi q e^{-\bar{\rho}})^{-1}$, $q^s=\frac{\bar{\rho}}{2(e^{\bar{\rho}}-1)\sigma\pi}\left(2e^{\bar{\rho}}-1-\sqrt{1+4\frac{e^{\bar{\rho}}-1}{\bar{\rho}c/\pi}}\right)$, thus, for $q^s=s(\frac{c}{\pi};\bar{\rho})$, with $\frac{\partial q^d}{\partial c/\pi}>0$, $\frac{\partial^2 q^s}{\partial(c/\pi)^2}<0$, $\lim_{\frac{c}{p}\to\infty}q^s=\frac{\bar{\rho}}{\sigma\pi}$, $\frac{\partial q^s}{\partial\bar{\rho}}>0$ and $\frac{\partial q^s}{\partial\bar{\alpha}}<0$. Moreover, with respect to cardinal risk aversion, which is measured by the parameter σ in this example, we find that $\frac{\partial q^d}{\partial\sigma}<0$ and $\frac{\partial q^s}{\partial\sigma}<0$.

11. The assumption that the cardinal utility function is the same for all agents is introduced for the sake of simplicity. If cardinal utility functions are heterogeneous, then the demand and supply functions will depend in a complicated way on differences in attitudes toward risk as well as on differences in attitudes toward uncertainty. Then this assumption allows determining the effects of the differences in uncertainty aversion without the necessity of separating them from the effects of the differences in risk aversion.

12. If the demand and supply functions for calls are specified, then specific results can be obtained. For instance, if the demand and supply functions are constant, i.e., $d(c/\pi;\alpha)=s(c/\pi;\bar{\alpha})=k$, then the equilibrium conditions $Q^*=k\int_{c^*/\pi}^{\infty}dN(\alpha)=k\int_0^{c^*/\pi}d\bar{N}(\bar{\alpha})$ imply $\frac{1}{k}Q^*=1-N(c^*/\pi)=\bar{N}(c^*/\pi)$. Thus, if all agents are uncertainty neutral, so that $N(c^*/\pi)=\bar{N}(c^*/\pi)$, then $Q^*=\frac{1}{2}k$ and $\frac{c^*}{\pi}$ is equal to the median of the distribution $N(.)$. Consequently, if all agents are uncertainty averse, so that $N(c^*/\pi)\geq\bar{N}(c^*/\pi)$, then $\frac{1}{k}Q^*=1-N(c^*/\pi)=\bar{N}(c^*/\pi)\leq\frac{1}{2}$ (since $\bar{N}(c^*/\pi)>\frac{1}{2}$ would imply $N(c^*/\pi)>\frac{1}{2}$ and $1-N(c^*/\pi)<\frac{1}{2}$, so that $1-N(c^*/\pi)\neq\bar{N}(c^*/\pi)$). If the demand and supply functions are linear, with $d(c/\pi;\alpha)=(\alpha-c/\pi)k$ and $s(c/\pi;\bar{\alpha})=(c/\pi-\bar{\alpha})k$, then the equilibrium conditions $Q^*=k\int_{c^*/\pi}^{\infty}(\alpha-c^*/\pi)dN(\alpha)=k\int_0^{c^*/\pi}(c^*/\pi-\bar{\alpha})d\bar{N}(\bar{\alpha})$ imply $Q^*=$

$k\int_{c^*/\pi}^{\infty}\alpha dN(\alpha)-k(1-N(c^*/\pi))c^*/\pi = k\bar{N}(c^*/\pi)c^*/\pi - k\int_0^{c^*/\pi}\bar{\alpha}d\bar{N}(\bar{\alpha})$. Thus, if all agents are uncertainty neutral, so that $\alpha=\bar{\alpha}$ and $N(c^*/\pi)=\bar{N}(c^*/\pi)$, then $\frac{c^*}{\pi}$ is equal to the average of the distribution $N(.)$, i.e., $\frac{c^*}{\pi}=\int_0^{\infty}\alpha\, dN(\alpha)$, and $Q^*=k\int_0^{c^*/\pi}N(\alpha)d\alpha$. Consequently, if all agents are uncertainty averse, so that $\alpha\leq\bar{\alpha}$ and $N(c^*/\pi)\geq\bar{N}(c^*/\pi)$, then $\frac{1}{k}Q^*=\frac{c^*}{\pi}\bar{N}(c^*/\pi)-\int_0^{c^*/\pi}\bar{\alpha}d\bar{N}(\bar{\alpha})=\int_0^{c^*/\pi}\bar{N}(\bar{\alpha})d\bar{\alpha}\leq\int_0^{c^*/\pi}N(\alpha)d\alpha$.

13. These indeterminate equilibria are analyzed by Chateauneuf et al. (2000), as indicated in footnote 2.
14. See Carr and Madan (2001) for an analysis where only risk aversion matters.
15. See, for instance, the end of footnote 10.
16. This aspect can be clarified by means of the following example. The outcome (which is the price of the asset at the expiry date) can take two possible values, B or W, according to the action chosen in the set $\{a,b,c\}$ by the management of a firm and to the quantity of rain after the action is chosen. Let us take under consideration two situations, according to the information on the action of the management. In the former situation, the action is unknown. In the latter it is known: it is action b. The probability of the best outcome, which is B, is equal to 60% (thus, that of the worst outcome W is equal to 40%) in case of action a, 50% in case b, and 40% in case c, if it rains sufficiently; and it is equal to 40% in case of action a, 50% in case b, and 60% in case c, if the weather is dry. The expected utility model implies that choice is characterized in both situations by probabilities: if they are 50% for the best outcome, i.e., $p(B)=0.5$, and 50% for the worst outcome, i.e., $p(W)=0.5$ in the former situation, then the choice in the former situation (no information on the action chosen by the management) is equal to the choice in the latter situation (where the action is known). The presence of information (on the action of management) does not matter. On the contrary, the Choquet-expected utility model introduces capacities when the former situation is analyzed: this can be rationalized assuming that an uncertainty averse agent attributes a capacity smaller than 50% to the best outcome if he is a buyer of a call (for instance, $v(B)=0.4$, more or less as he is focused on the pessimistic hypothesis that the management has chosen the wrong action), whereas he attributes a capacity smaller than 50% to the complementary event if he is a seller of a call (for instance, $v(W)=0.4$, more or less as he is focused on the pessimistic hypothesis, for the seller, that the management has chosen the right action). There are capacities in place of probabilities, for the Choquet-expected utility model in the former situation, because the information on the action of the management

is lacking. A deeper information makes weaker, *ceteris paribus*, the effect of uncertainty aversion. If the agent is fully informed, as in the latter situation, then uncertainty aversion is ineffective, because there is no uncertainty.

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