ıantum Physics, Marcel

ln Interactive Interpre-

:al proposition systems,

in P. Lahti and P. Mit-
ıdern Physics 1985: 50
ment, World Scientific,

:luwer Academic, Dor-

:lean noninvariance of
'3-278.

ɛe particle, Progress of

of reference, I, II and
}-479, 723-742, and 86

pilot-wave dynamics,

of quantum mechanics
of Physics, 25 (1995),

BRADLEY MONTON

QUANTUM-MECHANICAL SELF-MEASUREMENT*

ABSTRACT

The idea of self-measurement by a quantum-mechancal automaton is presented, and the conclusions that are typically reached about what we can come to know from doing self-measurements are shown to be mistaken. Specifically, it is shown that, while we are capable of *predicting* and *measuring* the values of two incompatible observables, we are incapable of *knowing* both these values simultaneously. This is an example of the interesting limitations quantum mechanics places on knowledge.

1 INTRODUCTION

There's a story about quantum-mechancal self-measurement and knowledge that David Albert (1983, 1987, 1992) has told, and it's a very interesting story, and while I think the story is wrong, I also think we can learn a bit about quantum mechanics by figuring out exactly how the story is wrong, and that's what this paper is going to be about.

To get the story of the ground, we need two presuppositions. First, you and I and Cathy and everything else that we consider to be good observers are presumed to be purely physical systems that obey the laws of quantum mechanics (just like stones and trees and cats). If you're unhappy with this presupposition, then consider the story that's about to be told to concern not people but *quantum-mechancal automata* (which are physical systems constructed out of microscopic particles that *do* obey the laws of quantum mechanics, and systems to which it would be natural to attribute mental states).

The second presupposition of the self-measurement story is that the correct interpretation of quantum mechanics is a *no-collapse* one; specifically, that the correct interpretation is a modal interpretation. (A self-measurement story can be told for other sorts of no-collapse interpretations, but I will focus on modal interpretations in this paper.)

---

*For helpful comments on previous versions of this paper, I would like to thank Jeff Barrett, Jeff Bub, Rob Clifton, Dennis Dieks, Tim Maudlin, Chad Mohler, Jitendra Subramanyam, Bas van Fraassen, Pieter Vermaas, and Bob Weingard. I would also like to thank David Albert for inspiring me to think about self-measurement. This material is based upon work supported under a National Science Foundation Graduate Research Fellowship.

D. Dieks & P.E. Vermaas (eds.), The Modal Interpretation of Quantum Mechanics, 307-318.

We all know what happens when an electron is prepared in the quantum state $|z\text{-spin up}\rangle_{\mathrm{e}}$ and the z-spin is measured using a Stern-Gerlach apparatus by a competent observer like Cathy: the electron/Cathy system evolves from the quantum state

$$|z\text{-spin up}\rangle_{\mathrm{e}}\ |\text{ready to measure } z\text{-spin}\rangle_{\mathrm{Cathy\#}}\ |\&\rangle_{\mathrm{Cathy*}} \tag{1}$$

to the quantum state

$$|z\text{-spin up}\rangle_{\mathrm{e}}\ |\text{records } z\text{-spin up}\rangle_{\mathrm{Cathy\#}}\ |\&\rangle_{\mathrm{Cathy*}}\ . \tag{2}$$

Here *Cathy#* is that part of Cathy that is involved in the measurement of the z-spin. Thus, Cathy# would include that part of her brain where she records the value of the z-spin, and parts of her eye and optic nerve, which record the location of the photons that are emitted from the phosphorescent screen. The state $|\&\rangle_{\mathrm{Cathy*}}$ is the state of the rest of Cathy, where Cathy* in this case is all of Cathy other than Cathy#.

Consider a case where an electron is prepared in the quantum state $|z\text{-spin up}\rangle_{\mathrm{e}}$ and the *x-spin* is measured by Cathy. According to modal interpretations, the system evolves from the quantum state

$$|z\text{-spin up}\rangle_{\mathrm{e}}\ |\text{ready to measure } x\text{-spin}\rangle_{\mathrm{Cathy\#}}\ |\&\rangle_{\mathrm{Cathy*}} \tag{3}$$

to the quantum state

$$\begin{aligned}&\tfrac{1}{\sqrt{2}}\big\{|x\text{-spin up}\rangle_{\mathrm{e}}\ |\text{records } x\text{-spin up}\rangle_{\mathrm{Cathy\#}}\\ &\qquad + |x\text{-spin down}\rangle_{\mathrm{e}}\ |\text{records } x\text{-spin down}\rangle_{\mathrm{Cathy\#}}\big\}\ |\&\rangle_{\mathrm{Cathy*}}\ .\end{aligned} \tag{4}$$

Here the system is in a *superposition* of states, where in one branch of the superposition Cathy records x-spin up, and in the other branch Cathy records x-spin down. We can *measure* whether or not the electron/Cathy# system is in the state (4); that is, there's some quantum-mechanical *observable*, call it Q, such that when Q is measured on the electron/Cathy# system the value 1 is always obtained if the system is in the state (4), and the value 0 could be obtained otherwise. When the electron/Cathy system starts in the state (3) and Cathy does her measurement, and one takes a Q measurement on the resulting electron/Cathy# system, one always finds that Q is 1.

Since we know that at the end of the measurement Cathy either does record up or does record down, we know that *something* must be *added* to the quantum state to get the correct empirical predictions. In the modal interpretations, what is added is the *value state*. The viable modal interpretations differ in their specifications of what this value state is, but all the viable interpretations agree that the value state ensures that at the end of the measurement, either Cathy records that x-spin is up, or she records that x-spin is down.

Some modal interpr
the property that Q act
every time a Q measure
as if Q can actually ha
where this is not the cas
always returns 1 as the
someone believes that Q
the person believes that
Q has value 1 believes t
with certainty be obtain

2 THE STORY

Albert has an interesti
know when they make
selves (assuming that t
assuming that some mo
story goes like this.

Suppose that Cathy
electron. The quantum
value state determines
Now suppose that a frie
system is in the state (
Q is 1.

More specifically, th

$\frac{1}{\sqrt{2}}\{|x$-spin
$+\ |x$-s
$\otimes\ |\&\rangle_{\mathrm{Cat}}$

to the quantum state

$\frac{1}{\sqrt{2}}\{|x$-spin
$+\ |x$-s
$\otimes\ |\&\rangle_{\mathrm{Ca}}$

and here the quantum
friend will with certaint

Now, suppose that t
is, and suppose that C
of the friend's measure

prepared in the quantum
ı Stern-Gerlach apparatus
:athy system evolves from

$$\text{thy\#}\ |\&\rangle_{\text{Cathy}*} \qquad (1)$$

$$|\&\rangle_{\text{Cathy}*}\ . \qquad (2)$$

d in the measurement of
t of her brain where she
e and optic nerve, which
from the phosphorescent
f Cathy, where Cathy* in

d in the quantum state
ıccording to modal inter-
ate

$$\text{hy\#}\ |\&\rangle_{\text{Cathy}*} \qquad (3)$$

$$\text{athy\#}\}\ |\&\rangle_{\text{Cathy}*}\ . \qquad (4)$$

ere in one branch of the
ıer branch Cathy records
ctron/Cathy# system is
ıancal *observable*, call it
athy# system the value
l, and the value 0 could
stem starts in the state
ı Q measurement on the
hat Q is 1.
nent Cathy either does
thing must be *added* to
dictions. In the modal
viable modal interpre-
ue state is, but all the
ures that at the end of
up, or she records that

Some modal interpretations guarantee that the system in state (4) has the property that Q actually has value 1, while others simply guarantee that every time a Q measurement is made, 1 is obtained as the result. I will talk as if Q can actually have value 1. To accommodate modal interpretations where this is not the case, one can say that in a case where a Q measurement always returns 1 as the result, Q *effectively* has value 1. When I say that someone believes that Q has value 1, this can be interpreted as meaning that the person believes that Q effectively has value 1. Thus, a person who believes Q has value 1 believes that if a measurement of Q is taken, the result 1 will with certainty be obtained.

2 THE STORY

Albert has an interesting story to tell about what observers can come to know when they make certain quantum-mechancal measurements on themselves (assuming that these observers are quantum-mechancal systems, and assuming that some modal interpretation of quantum mechanics is true). The story goes like this.

Suppose that Cathy starts in state (3), ready to measure the x-spin of the electron. The quantum state of the system evolves into state (4), and the value state determines that Cathy records that the value of X is, say, 'up'. Now suppose that a friend of Cathy's measures Q; since the electron/Cathy# system is in the state (4), that friend will with certainty get the result that Q is 1.

More specifically, the quantum state of the system will evolve from

$$\begin{aligned}&\tfrac{1}{\sqrt{2}}\{|x\text{-spin up}\rangle_e\,|\text{records } x\text{-spin up}\rangle_{\text{Cathy\#}}\\ &\qquad + |x\text{-spin down}\rangle_e\,|\text{records } x\text{-spin down}\rangle_{\text{Cathy\#}}\}\\ &\qquad \otimes |\&\rangle_{\text{Cathy}*}\,|\text{ready to measure } Q\rangle_{\text{friend}}\end{aligned} \qquad (5)$$

to the quantum state

$$\begin{aligned}&\tfrac{1}{\sqrt{2}}\{|x\text{-spin up}\rangle_e\,|\text{records } x\text{-spin up}\rangle_{\text{Cathy\#}}\\ &\qquad + |x\text{-spin down}\rangle_e\,|\text{records } x\text{-spin down}\rangle_{\text{Cathy\#}}\}\\ &\qquad \otimes |\&\rangle_{\text{Cathy}*}\,|\text{records } Q{=}1\rangle_{\text{friend}}\end{aligned} \qquad (6)$$

and here the quantum state of the electron/Cathy# system entails that the friend will with certainty measure Q to be 1.

Now, suppose that the friend tells Cathy what his Q measurement result is, and suppose that Cathy is in a ready state for hearing what the result of the friend's measurement is. The system evolves from state (6) to the quantum state

$$\begin{aligned}\tfrac{1}{\sqrt{2}}\{&|x\text{-spin up}\rangle_{\text{e}}\,|\text{records } x\text{-spin up}\rangle_{\text{Cathy}\#}\\ &+ |x\text{-spin down}\rangle_{\text{e}}\,|\text{records } x\text{-spin down}\rangle_{\text{Cathy}\#}\}\\ &\otimes |\text{records } Q{=}1\rangle_{\text{Cathy}\$}\,|\&\rangle_{\text{Cathy}*}\,|\text{records } Q{=}1\rangle_{\text{friend}}\,.\end{aligned} \tag{7}$$

Here, Cathy\$ is that part of Cathy that is involved in the recording of the outcome of the friend's Q measurement. The state $|\&\rangle_{\text{Cathy}*}$ is the state of the rest of Cathy, where Cathy$*$ in this case is all of Cathy other than Cathy# and Cathy\$.

Now, we have reached the central claim of the story that is told about self-measurement and knowledge. The central claim is that Cathy knows that X is 'up' (because of the value state), and Cathy simultaneously knows that Q is 1 (because of the quantum state), and yet X and Q are two *incompatible* observables, and it is standardly understood by those who talk about quantum mechanics that one cannot have simultaneous knowledge of two incompatible observables. The conclusion of Albert's story is that the standard understanding is wrong.

It's worth spelling out why X and Q are incompatible observables.[1] X and Q are incompatible because if one were to take a measurement of Q, then X, on a system, one would sometimes get different outcomes than if one were to take a measurement of X, then Q. Consider the system in state (4). When one measures Q on that, one with certainty gets the value 1, and when one subsequently measures X on that, one gets the value, say, 'up'. But if, after this X measurement, we try to measure Q, we do *not* necessarily get the value 1. After the X measurement, the system has evolved into the quantum state

$$\begin{aligned}\tfrac{1}{\sqrt{2}}\{&|x\text{-spin up}\rangle_{\text{e}}\,|\text{records } x\text{-spin up}\rangle_{\text{Cathy}\#}\\ &\qquad\otimes |\text{records } x\text{-spin up}\rangle_{\text{someone}}\\ &+ |x\text{-spin down}\rangle_{\text{e}}\,|\text{records } x\text{-spin down}\rangle_{\text{Cathy}\#}\\ &\qquad\otimes |\text{records } x\text{-spin down}\rangle_{\text{someone}}\}\,|\&\rangle_{\text{Cathy}*}\,,\end{aligned} \tag{8}$$

and (if the reader refers back to the definition of a Q measurement, she can confirm that) a Q measurement made on *this* state might give the result that Q is 0. And so X and Q are incompatible.

3 SELF-MEASUREMENT WITHOUT MEASUREMENT?

This self-measurement story might to some seem too complicated.

---

[1] Officially, it's $X \otimes I$ and Q that are incompatible, where I is the identity observable for Cathy#, but I will leave off the $\otimes I$ for convenience.

"Look," these people w
values of two incompatible
to do is take an x-spin mea
and then think about the
the dynamics, Cathy can
such that if one were to
system, one would with c
incompatible with X. Th
dynamics, Cathy can kno
knows what the value of X
incompatible observables."

The first problem with
measurement, since Cathy
this is a story about is sel
surement she has taken, a
figure out the value of th
obtain simultaneous knowl

I believe that the answ
is not sufficient for knowl
check whether some propo
It is certainly the case th
mechancal dynamics to g
patible observables. But
say that Cathy simultane
ables, unless Cathy does
has predicted.

Perhaps this is just a m
the reader does not share
is a paper about self-mea
position, the question I w
taneous knowledge of the
'no.' From this answer, I
neous knowledge of the va
who believes theoretical p
to take this extra step.

4 PROBLEMS WITH

A small problem with th
foreshadowing for the bi
for Cathy to *know* the va
have implicitly specified t
only if there is a part of

#

wn$\rangle_{\text{Cathy\#}}\}$ (7)

rds $Q{=}1\rangle_{\text{friend}}$.

d in the recording of the
$|\&\rangle_{\text{Cathy}*}$ is the state of
Cathy other than Cathy#

story that is told about
is that Cathy knows that
multaneously knows that
and Q are two *incompat-*
by those who talk about
ous knowledge of two in-
story is that the standard

patible observables.[1] X
ke a measurement of Q,
ent outcomes than if one
r the system in state (4).
ts the value 1, and when
value, say, 'up'. But if,
lo *not* necessarily get the
volved into the quantum

(8)

n$\rangle_{\text{Cathy\#}}$

one$\}\ |\&\rangle_{\text{Cathy}*}$,

measurement, she can
ight give the result that

REMENT?

complicated.

I is the identity observable

"Look," these people would say. "If the goal is to simultaneously know the values of two incompatible observables, this is very easy to do. All Cathy has to do is take an x-spin measurement on an electron that is prepared z-spin up, and then think about the quantum-mechancal dynamics. By thinking about the dynamics, Cathy can figure out that there's some observable, call it Q^*, such that if one were to take a Q^* measurement on the electron/Cathy# system, one would with certainty obtain the value 1, and such that Q^* is incompatible with X. Thus, just by thinking about the quantum-mechancal dynamics, Cathy can know that the value of Q^* is 1. Since Cathy already knows what the value of X is, Cathy simultaneously knows the values of two incompatible observables."

The first problem with this idea is that this is no longer a story about self-*measurement*, since Cathy is not taking any measurement on herself. What this is a story about is self-*reflection*: Cathy is reflecting on the x-spin measurement she has taken, and is using the quantum-mechancal dynamics to figure out the value of the Q^* observable. Can Cathy use self-reflection to obtain simultaneous knowledge of the values of two incompatible observables?

I believe that the answer is 'no.' I believe that theoretical argumentation is not sufficient for knowledge; I believe that one must go out and actually check whether some proposition is true before one can know that proposition. It is certainly the case that Cathy can use self-reflection and the quantum-mechancal dynamics to get a good *prediction* for the values of two incompatible observables. But that is just a theoretical prediction. I would not say that Cathy simultaneously *knows* the values of two incompatible observables, unless Cathy does the appropriate experiments to confirm what she has predicted.

Perhaps this is just a metaphysical bias I have against theory, and perhaps the reader does not share this bias. If so, I will retrench, and say that this is a paper about self-measurement, not self-reflection. From the retrenched position, the question I want to ask is: can one achieve *experimental* simultaneous knowledge of the values of two incompatible observables? I answer, 'no.' From this answer, I will also conclude that one cannot achieve simultaneous knowledge of the values of two incompatible observables, but the reader who believes theoretical prediction is sufficient for knowledge is not obligated to take this extra step.

4 PROBLEMS WITH THE STORY

A small problem with the self-measurement story (which will provide some foreshadowing for the big problem) is that nowhere is one told what it is for Cathy to *know* the value of an observable. In the state vectors above, I have implicitly specified that Cathy *records* the value of an observable if and only if there is a part of her which is storing, in the appropriate fashion, the information that the observable has that value. But this does not capture what we mean by knowledge: knowledge requires, among other things, *consciousness.* We do not say, for example, that the phosphorescent screen of a Stern-Gerlach apparatus *knows* what the spin of the electron is, though we do say that the phosphorescent screen *records* what the spin of the electron is.

It seems to follow from this discussion that Cathy cannot know something unless she is *consciously aware* of that something. (A better account would say that Cathy cannot know something unless she is *capable of* becoming consciously aware of that something. This would allow us to say that Cathy knows some fact, even during those times when she is not consciously aware of that fact. However, this also makes the self-measurement discussion more confusing, and since in the end it doesn't affect my conclusion about what we can know from self-measurement, I will stick with the simpler account.)

As a definition of knowledge, I will specify that Cathy *knows* a proposition A if and only if she is consciously aware of A, A is true, and her belief that A is true is warranted. Since conscious awareness is ill-defined, I will specify that Cathy is consciously aware of a proposition if and only if she *subvocalizes* it; that is, if and only if she says it under her breath, without actually speaking. While this specification probably doesn't work for a full-fledged theory of consciousness, it will be sufficient for our purposes of knowing propositions about values of observables.

It follows from this definition of knowledge that when Cathy# is in for example the state $|\text{records } x\text{-spin up}\rangle_{\text{Cathy\#}}$, Cathy does not necessarily *know* anything about what the spin of the electron is. This follows because, when Cathy is in that state, Cathy is not necessarily *conscious* of *anything.* There could simply be a part of Cathy's brain which is recording, in the appropriate fashion, the fact that the x-spin is up, and that part of Cathy's brain could have nothing to do with the part of Cathy's brain that is involved in consciousness. Moreover, it could be the case that Cathy is incapable of becoming conscious of the x-spin: she could be in a coma, for example.

The central claim of the self-measurement story is that one can achieve *simultaneous knowledge* of the values of two incompatible observables. I will define simultaneous knowledge as follows. Cathy *simultaneously knows* two propositions if and only if she is simultaneously consciously aware of them, they are both true, and her belief that they are true is warranted.[2]

But how does one gain such *simultaneous conscious awareness*? It would not be sufficient to say that Cathy is simultaneously consciously aware of the values of two observables if and only if she subvocalizes one and then

---

[2] If we were to go with the more complicated account of knowledge discussed above, where a requirement for knowing a proposition is that one is capable of becoming consciously aware of that proposition, then analogously a requirement for simultaneous knowledge of two propositions is that one is capable of becoming simultaneously consciously aware of the two propositions.

---

subvocalizes the other, bec
calization. During the time
the first observable, she mig
other time period in which
servable, she might know t
be any period in which Cat
at the same time.

The definition of *simult*
is simultaneously consciou
combines the two propositi
can one subvocalize two pr
propositions into one using
our example, where there
'up', and X is 'down'. Si
consider four propositions,
walk," B "I will hop," C "I
following decision table to

$Q=$
$Q=$

Thus, given that one
hop," one is subvocalizin
'down'.[3] One can have sim
and this is one way to do

I hope that these defini
intuitive understanding of
background in mind, let's
measurement story.

Consider the state (7),
and after she has been t
ambiguous, since it did n
X and *in what way* she r

Let's call all of Cathy
subvocalize anything, *Ca*
value of Q, but which do
that part of Cathy which

Let's specify that, du
never subvocalizes anythi
and see if we can add su
allow Cathy to simultane
ables.

---

[3] Albert would endorse thi

subvocalizes the other, because that would ignore the time factor in subvocalization. During the time period in which Cathy subvocalized the value of the first observable, she might know that observable's value, and during some other time period in which Cathy subvocalized the value of the second observable, she might know that second observable's value, but there wouldn't be any period in which Cathy had simultaneous knowledge of *both* the values, at the same time.

The definition of *simultaneous conscious awareness* I will give is that one is simultaneously consciously aware of two propositions if and only if one combines the two propositions and subvocalizes them both at once. But how can one subvocalize two propositions at once? One way is to combine the two propositions into one using an appropriate translation mechanism. Consider our example, where there are four possible propositions: Q is 1, Q is 0, X is 'up', and X is 'down'. Since these can be combined in four different pairs, consider four propositions, A, B, C, and D. (For definiteness, let A be "I will walk," B "I will hop," C "I will skip," and D "I will jump.") One can use the following decision table to combine the two propositions into one:

	X='up'	X= 'down'
Q=1	A	B
Q=0	C	D

Thus, given that one has used this table, when one subvocalizes "I will hop," one is subvocalizing two propositions: that Q is 1, and that X is 'down'.[3] One can have simultaneous conscious awareness of two propositions, and this is one way to do it.

I hope that these definitions sound reasonable, and cohere with the reader's intuitive understanding of knowledge and simultaneous knowledge. With this background in mind, let's figure out what more can be said about the self-measurement story.

Consider the state (7), after Cathy has measured the x-spin of the electron, and after she has been told that Q is 1. The story told above was a little ambiguous, since it did not specify *in what way* Cathy records the value of X and *in what way* she records that Q is 1. Let's go ahead and specify this.

Let's call all of Cathy which records the value of X, but which does not subvocalize anything, *Cathy-box 1*. Let's call all of Cathy which records the value of Q, but which does not subvocalize anything, *Cathy-box 2*. Let's call that part of Cathy which subvocalizes things *Cathy-box 3*.

Let's specify that, during the self-measurement story told above, Cathy never subvocalizes anything. We can consider that story as the framework, and see if we can add subvocalization to the framework in such a way as to allow Cathy to simultaneously know the values of two incompatible observables.

---

[3] Albert would endorse this use of decision tables: see his (1992, p. 188).

Thus, when we consider state (4), after Cathy has measured X, we see that state (4) can be rewritten as

$$\begin{aligned}&\tfrac{1}{\sqrt{2}}\{|x\text{-spin up}\rangle_{\mathrm{e}}\,|\text{records } x\text{-spin up}\rangle_{\text{Cathy-box 1}}\\&\quad+|x\text{-spin down}\rangle_{\mathrm{e}}\,|\text{records } x\text{-spin down}\rangle_{\text{Cathy-box 1}}\}\,|\&\rangle_{\text{Cathy}*}\,.\end{aligned}\tag{9}$$

Now, Q is the observable such that a Q measurement would obtain value 1 with certainty when the electron/Cathy-box 1 system is in the state given by (9), and value 0 could be obtained otherwise. After Cathy has been told the value of Q, the system evolves into the quantum state

$$\begin{aligned}&\tfrac{1}{\sqrt{2}}\{|x\text{-spin up}\rangle_{\mathrm{e}}\,|\text{records } x\text{-spin up}\rangle_{\text{Cathy-box 1}}\\&\quad+|x\text{-spin down}\rangle_{\mathrm{e}}\,|\text{records } x\text{-spin down}\rangle_{\text{Cathy-box 1}}\}\\&\quad\otimes\,|\text{records } Q{=}1\rangle_{\text{Cathy-box 2}}\,|\&\rangle_{\text{Cathy}*}\,,\end{aligned}\tag{10}$$

where from now on we'll leave out the state of the friend since he's not important to our story (and since Cathy could in principle have made the Q measurement herself).

Now, we can say that Cathy records the value of X, and we can say that Cathy records the value of Q, but this is only true in the weak sense that both values are stored somewhere in Cathy. It could be that Cathy is storing the value of X in the left hemisphere of her brain, and it could be that Cathy is storing the value of Q in the right hemisphere, and it could be that Cathy has had her corpus callosum, which connects her two hemispheres, severed. Thus state (10) does not imply that Cathy is recording the values of both observables together; the recording could be going on simultaneously but separately, in unconnected parts of Cathy's brain.

Let's go on. Suppose Cathy wants to obtain simultaneous knowledge of the values of X and Q; thus, she wants to combine the two facts about the values of the observables and subvocalize them together. Suppose that Cathy does this subvocalization; the system evolves from state (10) into the quantum state

$$\begin{aligned}&\tfrac{1}{\sqrt{2}}\{|x\text{-spin up}\rangle_{\mathrm{e}}\,|\text{records } x\text{-spin up}\rangle_{\text{Cathy-box 1}}\\&\qquad\otimes\,|\text{subvocalizes } X{=}\text{`up' and } Q{=}1\rangle_{\text{Cathy-box 3}}\\&\quad+|x\text{-spin down}\rangle_{\mathrm{e}}\,|\text{records } x\text{-spin down}\rangle_{\text{Cathy-box 1}}\\&\qquad\otimes\,|\text{subvocalizes } X{=}\text{`down' and } Q{=}1\rangle_{\text{Cathy-box 3}}\}\\&\otimes\,|\text{records } Q{=}1\rangle_{\text{Cathy-box 2}}\,|\&\rangle_{\text{Cathy}*}\,.\end{aligned}\tag{11}$$

Alternatively, we can imagine that Cathy uses the decision table above for her subvocalization, so that the system evolves from state (10) into the quantum

---

state

$\frac{1}{\sqrt{2}}\{|x$-spin u

+ $|x$-spi

⊗ |records

Now suppose we do
recalls the definition of a
might give the result th
box 1 system is no long
electron/Cathy-box 1/C
electron/Cathy-box 1 re

Thus, when Cathy is
Q is 1, but she does no
belief is false because, if
(11), one might get the

Note that the evolut
Cathy to be conscious c
her brain where the res
two results are put toge

Thus, Cathy cannot
and Q; once she becom
Q is false. Thus, self-m
knowledge of the values

5 A COUNTERAR

I believe that the conc
mechancal observers lil
values of two incompat
might mount against th
ments now.

---

[4] If we interpret Cathy's
then we can say that Catl
observables *were*. Note th
use my suggestion at the
that Q has value 1 as the
give 1 as the result. Settin
simultaneous knowledge of
sort of knowledge that Alb
use her knowledge to make

state

$$\begin{aligned}\tfrac{1}{\sqrt{2}}\{&|x\text{-spin up}\rangle_{\mathrm{e}}\ |\text{records } x\text{-spin up}\rangle_{\text{Cathy-box 1}}\\ &\qquad\otimes\ |\text{subvocalizes `A'}\rangle_{\text{Cathy-box 3}}\\ &+\ |x\text{-spin down}\rangle_{\mathrm{e}}\ |\text{records } x\text{-spin down}\rangle_{\text{Cathy-box 1}}\\ &\qquad\otimes\ |\text{subvocalizes `B'}\rangle_{\text{Cathy-box 3}}\}\\ &\otimes\ |\text{records } Q{=}1\rangle_{\text{Cathy-box 2}}\ |\&\rangle_{\text{Cathy}*}\ .\end{aligned} \tag{12}$$

Now suppose we do a Q measurement on state (11). One can see, if one recalls the definition of a Q measurement, that a Q measurement on this state might give the result that Q is 0. This follows because the electron/Cathy-box 1 system is no longer in a superposition, as it was in state (10). The electron/Cathy-box 1/Cathy-box 3 system is in a superposition, but the electron/Cathy-box 1 reduced state is a mixture.

Thus, when Cathy is in state given by (11), she consciously *believes* that Q is 1, but she does not *know* that Q is 1, because her belief is false. Her belief is false because, if a Q measurement were taken on the system in state (11), one might get the result that Q is 0, while Cathy believes that Q is 1.

Note that the evolution into a state like (11) or (12) is quite general; for Cathy to be conscious of both results together, there has to be some part of her brain where the results of box 1 and box 2 are combined. But once the two results are put together, it is no longer the case that Q is 1.

Thus, Cathy cannot simultaneously know the values of the observables X and Q; once she becomes consciously aware of them both, her belief about Q is false. Thus, self-measurement does not allow one to gain simultaneous knowledge of the values of two incompatible observables.[4]

5 A COUNTERARGUMENT CONSIDERED

I believe that the conclusion I reached above is the correct one: quantum-mechancal observers like Cathy cannot have simultaneous knowledge of the values of two incompatible observables. But there are counterarguments one might mount against this conclusion; let's consider one of those counterarguments now.

---

[4] If we interpret Cathy's subvocalization as being about the *past* values of X and Q, then we can say that Cathy has simultaneous knowledge of what the values of the two observables *were*. Note that this assumes that Q actually does have a value; we cannot use my suggestion at the end of the introduction, which was to interpret Cathy's belief that Q has value 1 as the belief that an upcoming measurement of Q will with certainty give 1 as the result. Setting that issue aside, I am willing to grant that Cathy can have simultaneous knowledge of the past values of two incompatible observables. This isn't the sort of knowledge that Albert is interested in, however; Albert wants Cathy to be able to use her knowledge to make *predictions*: see his (1983, p. 251), (1992, p. 188).

Here's the counterargument.[5] The observable that we really ought to be concerned with is not Q, but Q^*, where Q^* has value 1 with certainty when the electron/Cathy-box 1/Cathy-box 3 system is in the quantum state given by

$$\begin{aligned} \tfrac{1}{\sqrt{2}}\{&|x\text{-spin up}\rangle_{\mathrm{e}}\,|\text{records } x\text{-spin up}\rangle_{\text{Cathy-box 1}} \\ &+|x\text{-spin down}\rangle_{\mathrm{e}}\,|\text{records } x\text{-spin down}\rangle_{\text{Cathy-box 1}}\} \\ &\otimes|\text{ready}\rangle_{\text{Cathy-box 3}}\,|\text{ready}\rangle_{\text{Cathy-box 2}}\,|\&\rangle_{\text{Cathy}*} \end{aligned} \tag{13}$$

or in the quantum state given by

$$\begin{aligned} \tfrac{1}{\sqrt{2}}\{&|x\text{-spin up}\rangle_{\mathrm{e}}\,|\text{records } x\text{-spin up}\rangle_{\text{Cathy-box 1}} \\ &\quad\otimes|\text{subvocalizes } X\text{=`up' and } Q^*\text{=}1\rangle_{\text{Cathy-box 3}} \\ &+|x\text{-spin down}\rangle_{\mathrm{e}}\,|\text{records } x\text{-spin down}\rangle_{\text{Cathy-box 1}} \\ &\quad\otimes|\text{subvocalizes } X\text{=`down' and } Q^*\text{=}1\rangle_{\text{Cathy-box 3}}\} \\ &\otimes|\text{records } Q^*\text{=}1\rangle_{\text{Cathy-box 2}}\,|\&\rangle_{\text{Cathy}*}\,, \end{aligned} \tag{14}$$

and Q^* can have value 0 otherwise. Thus, Q^* has value 1 with certainty when the state vector of the system is in the subspace of Hilbert space spanned by state (13) and state (14). State (13) is the state where Cathy-box 1 has just measured the value of the x-spin of the electron, and state (14) is the state where Cathy-box 3 has subvocalized the values of X and Q.[6]

After Cathy measures X, the electron/Cathy system will be in state (13), so when the friend measures Q^*, he will with certainty obtain the value 1. After the friend tells Cathy the value of Q^*, and after she subvocalizes the values of X and Q^* together, the electron/Cathy system will be in state (14). Hence, Q^* will still be 1. X and Q^* are still incompatible observables though, because were someone else to take a measurement of X, the electron/Cathy system would no longer be in a superposition, so Q^* could give 0 as the result. Hence, according to the counterargument, Cathy *is* able to simultaneously know the values of two incompatible observables.

I believe that this counterargument does not work. One worry is that Cathy does not appear to have sufficient *warrant* for believing that Q^* is 1 after she has subvocalized. Her warrant for recording, in Cathy-box 2, that Q^* is 1 is that the friend has measured Q^* to be 1 and the friend is a good measuring device. But after Cathy has subvocalized, the state of the

---

[5] This counterargument is similar to the way the self-measurement story is told by Albert and Putnam (1995, pp. 18-19). It may be that Albert and Putnam tell the story this way because they've realized that the self-measurement story as told by Albert (1983, 1987, 1992) doesn't work. I believe that the Albert and Putnam story has problems as well; the problems are the same as those presented below for the counterargument.

[6] State (13) is like state (9), except that in state (13), I have specified that the Cathy-box 2 and box 3 systems are in ready states. State (14) is like state (11), except that in state (14), Q^* is the relevant observable, not Q.

---

system that the friend
measurement no longer
the case because the ca
based on the state of th
that Q^* is 1 *after* subvo
the system ends up. Si
Q^* is 1, Cathy's belief t
have knowledge of the v

One may argue in re
believe that Q^* is 1, be
the evolution of the syst
problems with this resp

The first problem is
fication of Cathy's belie
based on what her friend
about how the system w

But let us grant tha
based on reasoning abo
Even so, there is a seco
not self-measurement.
is based on reflection a
cal dynamics. Thus, Ca
she is not experimental
theoretical prediction d
but even if the reader di
Cathy has for believing
version of the counterar
not self-measurement.

6 WHAT CAN WE

The self-measurement st
knowledge of the values
are interesting things to
erations about self-meas
some of them.

It's interesting enou
there *are* two incompati
ues. (As discussed at th
tations we can say that
definite effective values.
and X are incompatible

Here's something mo

nat we really ought to be
ue 1 with certainty when
the quantum state given

box 1

$\text{wn}\rangle_{\text{Cathy-box 1}}\}$ (13)

$\text{k}\rangle_{\text{Cathy*}}$

$1\rangle_{\text{Cathy-box 3}}$

ny-box 1 (14)

$*{=}1\rangle_{\text{Cathy-box 3}}\}$

ne 1 with certainty when
ilbert space spanned by
re Cathy-box 1 has just
l state (14) is the state
and Q.[6]
em will be in state (13),
nty obtain the value 1.
er she subvocalizes the
em will be in state (14).
ble observables though,
X, the electron/Cathy
uld give 0 as the result.
able to simultaneously

k. One worry is that
r believing that Q^* is
ding, in Cathy-box 2,
e 1 and the friend is a
lized, the state of the

nent story is told by Albert
am tell the story this way
ld by Albert (1983, 1987,
has problems as well; the
rgument.
ecified that the Cathy-box
(11), except that in state

---

system that the friend measured has *changed.* Q^* is still 1, but the friend's measurement no longer provides a *good reason* to believe that Q^* is 1. This is the case because the causal chain that leads Cathy to believe that Q^* is 1 is based on the state of the system *before* Cathy subvocalizes, while the reason that Q^* is 1 *after* subvocalization has to do with the particular state in which the system ends up. Since Cathy no longer has good reason to believe that Q^* is 1, Cathy's belief that Q^* is 1 is no longer warranted, so Cathy does not have knowledge of the value of Q^*.

One may argue in response that Cathy actually *does* have good reason to believe that Q^* is 1, because she knows that her subvocalization is part of the evolution of the system from state (13) to state (14). But there are two problems with this response.

The first problem is that the response is unfairly giving a God's-eye justification of Cathy's belief. Cathy's belief at the time of her subvocalization is based on what her friend has told her; her belief is *not* based on her reasoning about how the system will evolve once she does the subvocalization.

But let us grant that there is some way for Cathy to get her belief to be based on reasoning about how the system will evolve when she subvocalizes. Even so, there is a second problem, for here Cathy is doing *self-reflection*, not self-measurement. After she has subvocalized, her belief that Q^* is 1 is based on reflection about her own state and about the quantum-mechancal dynamics. Thus, Cathy is making a *theoretical prediction* that Q^* is 1; she is not experimentally determining that Q^* is 1. I believe that Cathy's theoretical prediction does not have enough warrant to be called knowledge, but even if the reader disagrees with that, it is still the case that the warrant Cathy has for believing that Q^* is 1 comes from self-reflection. Hence, this version of the counterargument renders Albert's story one of self-reflection, not self-measurement.

6 WHAT CAN WE LEARN ABOUT QUANTUM MECHANICS?

The self-measurement story, in the end, doesn't allow us to have simultaneous knowledge of the values of two incompatible observables. Nevertheless, there are interesting things to be learned about quantum mechanics from considerations about self-measurement, and by way of finishing up I'll talk about some of them.

It's interesting enough that, according to some modal interpretations, there *are* two incompatible observables that simultaneously have definite values. (As discussed at the end of the introduction, for other modal interpretations we can say that two incompatible observables simultaneously have definite effective values.) We can see this in the self-measurement story: Q and X are incompatible observables which both have definite values.

Here's something more interesting. We can make a *theoretical prediction* of what the values of the two incompatible observables are, by doing a combination of an experimental measurement and a theoretical application of the quantum-mechancal dynamics. Cathy can measure the value of X, and can think about the quantum-mechancal dynamics to figure out that there must be some Q^*, incompatible with X, which has value 1.

Here's something even more interesting. We can *measure* the values of two incompatible observables, and we can simultaneously *record* what these values are. Cathy can measure the value of X, and Cathy can measure the value of Q. There is a part of Cathy, Cathy-box 1, which records the value of X, and there is another part of Cathy, Cathy-box 2, which simultaneously records the value of Q.

But we cannot have simultaneous *knowledge* of the values of two incompatible observables. And that really is the most interesting lesson to be learned from this discussion of self-measurement: quantum mechanics lets us get so close to achieving simultaneous knowledge, but always maintains a barrier. What we can come to know is constrained by the dynamics of quantum mechanics; the epistemic lives of automata who could transcend that barrier will forever remain (for us) unimaginably rich.

Princeton University

REFERENCES

Albert, D. Z (1983), "On Quantum-Mechanical Automata", *Physics Letters* **98A**: 249-252.

Albert, D. Z (1987), "A Quantum-Mechanical Automaton", *Philosophy of Science* **54**: 577-585.

Albert, D. Z (1992), *Quantum Mechanics and Experience*, Chapter 8. Cambridge: Harvard University Press.

Albert, D. Z and H. Putnam (1995), "Further Adventures of Wigner's Friend", *Topoi* **14**: 17-22.

THE BARE

In this paper I res
quantum mechani
the theory. The m
ical predictions ar
measurement resu
would typically e
results, measurem
nomenal content.
thought of as alte
they only succeed
account of the act
discuss some of th

1 THE BARE THE

The bare theory is the st
mechanics without the c
link. Albert (1992, 116-1
standing Everett's relati
if the bare theory canno
the end of a measureme
having recorded mutuall
pretation of states (the i
link) is a state where the
record whatsoever.

Because of its simplic
to anyone interested in n
further, it is surprising h
experience; or, more pre
why we *falsely* believe th
than explain our determi
seeks to explain it away
telling us what a good o
experience in various me
context of the bare theor
one's experiences and be

D. Dieks & P.E. Vermaas (eds
© 1998, *Kluwer Academic pu*