Carnap's Metrical Conventionalism versus Differential Topology

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Abstract . Geometry was a main source of inspiration for Carnap's conventionalism. Taking Poincaré as his witness Carnap asserted in his dissertation *Der Raum* (Carnap 1922) that the metrical structure of space is conventional while the underlying topological structure describes "objective" facts. With only minor modifications he stuck to this account throughout his life. The aim of this paper is to disprove Carnap's contention by invoking some classical theorems of differential topology. By this means his metrical conventionalism turns out to be indefensible for mathematical reasons. This implies that the relation between topology and geometry cannot be conceptualized as analogous to the relation between the meaning of a proposition and its expression in some language as logical empiricists used to say.

Key Words: Carnap, Conventionalism, Differential Topology, Theorems of Bonnet and Gauss-Bonnet, Euler-Poincaré characteristics.

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Carnap's Metrical Conventionalism and Differential Topology

1. Introduction . Geometry was a main source of inspiration for Carnap's general conventionalist conception of epistemology and philosophy of science (cf. Coffa 1986). If space could be endowed with different metrical structures that could be considered as different ways of codifying the same topological facts, then it was at least prima facie plausible that a similar division between conventional and factual components existed in other areas of knowledge. In this paper I'd like to show that Carnap's metrical conventionalism was doomed to fail from the outset. For purely mathematical reasons the relation between the metrical and the topological structure of space cannot be described in the conventionalist manner he proposed. Different metrical structures cannot be conceived as "alternative languages" that describe the same facts in different ways as he and many other Logical empiricists used to say.

Restricting one's attention to languages such as English, French, German, or Spanish it may be plausible to assume that any of them can be used to express the same facts of the world equally well. It is a matter of convention which of them is employed. This kind of linguistic conventionalism is fairly uncontroversial and, moreover, it is pretty trivial. If one wants to find a less trivial conventionalism one has to look elsewhere. Following Poincaré the Logical empiricists concentrated on conventionalism in geometry. One may distinguish several kinds of conventionalist doctrines dealing with geometry.[1] Perhaps the simplest (non-trivial) one is that concerned with the relation between the metrical and the topological structure of space. In any case, this was the one Carnap dealt with throughout his philosophical career from *Der Raum* (1922) until *An Introduction to Philosophy of Science* (1966).

In *Der Raum* [2] Carnap distinguished between topological matters of fact and metrical matters of convention, and attempted to elucidate their relation by the following analogy:

> "The transformation of a statement of matter of fact from one metrical space-form into another -- e.g., from the Euclidean into one of the non-

[1] For instance, one may consider the conventional distinction between the axioms of geometry and the theorems to be deduced from them (cf. Quine 1966, p. 116). This conventional aspects of geometry may be of considerable didactical interest but lacks philosophical importance.

[2] The page numbers of all quotations of *Der Raum* refer to the German original Carnap (1922). The translation is that of Michael Friedman and Peter Heath.

> Euclidean -- has been aptly compared to the translation of a proposition from one language into another. Now, just as the genuine sense of the proposition is not its presentation in one of these linguistic forms -- for then its presentation in the other languages would have to appear as derivative and less original -- but is merely that in the proposition which remains unaltered in translation; so too the sense of the statement of matter of fact is not one of its metrical presentations, but that which is common to all of them (the "invariants of topological transformations") -- and that is precisely its presentation in merely topological form." (*Der Raum*, p. 65)

In a slogan, then, Carnap's metrical conventionalism can be put as follows: The topological structure of space is to its metrical structure as the meaning of a proposition is to its specific expression in a given language (cf. Howard 1996, p. 148). Or, with a slightly different emphasis: Two metrical geometries are merely two descriptions of the same topological facts (cf Carnap 1966, p. 150).

In this paper I'd like to argue that this kind of metrical conventionalism is not tenable for mathematical reasons. The relation between the metrical and the topological is of a different kind than that between the meaning of a proposition and its expression in a contingent language. Thus the linguistic analogy quoted above is seriously misleading. To prove this contention one has to invoke some facts from differential topology philosophers may not be too familiar with. Nevertheless, the introduction of these mathematical tools is necessary, since the arguments that have been brought forward against Carnap's metrical conventionalism by Grünbaum, Nerlich, and others are less than conclusive (cf. Grünbaum 1963, Nerlich 1994). In order to keep everything on the most elementary level possible, let us restrict as far as possible our attention to 2-dimensional manifolds. Analogous arguments can be formulated for higher-dimensional spaces: the untenability of metrical conventionalism is not a matter of dimension.

The outline of this paper is as follows: in the next section 2 first we recall Carnap's early metrical conventionalism as presented in his dissertation *Der Raum* (1922). Then the theorems of Bonnet and Gauss-Bonnet are used to disprove Carnap's conventionalist theses. In section 3 some late modifications in Carnap's metrical conventionalism are discussed. It is argued that they cannot save his account from the differential topological criticism presented in section 2. In section 4 the mathematical argument against metrical conventionalism is compared with some other arguments launched forward against this doctrine, to wit, Quine's anti-conventionalist holism, and Friedman's and Ryckman's criticism based on the fact that metrical conventionalism seems to be incompatible with the general theory of relativity.

2. Metrical Conventionalism in *Der Raum* . For the following we assume the basic notions topology and the theory of differentiable manifolds to be known, see for example (James 1999, Hicks 1971). A Riemannian manifold M is a differentiable manifold endowed with a 2-tensor field g satisfying certain familiar conditions (Hicks 1971, p.20/21). A Riemannian manifold is denoted by (M, g) or simply by M if g is understood. If M is a Riemannian manifold the Riemannian curvature of M is a real-valued function R that assigns to every 2-dimensional subspace P of the tangent space M_m of M at a point $m \in M$ a real number R(P). If M is 2-dimensional the Riemannian curvature R boils down to a real-valued function K: M ------>**R** called the Gaussian curvature of M. A Riemannian manifold M is called a space form, if the Riemannian curvature R is constant, i.e., for all points m and all tangential planes P at m the Riemannian curvature R(P) is a constant K. Depending on the properties of K one may distinguish between different types of Riemannian space forms:

(2.1) Definition . Let (M, g) be a Riemannian manifold of constant curvature K.

(i) M is a hyperbolic manifold iff $K < 0$.

(ii) M is an elliptic manifold iff $K > 0$.

(iii) M is a flat manifold iff $K = 0$.

Although in *Der Raum* Carnap did not explicitly define the basic concepts of differential geometry he was to use, it transpires from his presentation that he employed concepts such as "Gaussian curvature", "Riemannian curvature" or "space of constant curvature" virtually in the same way as today. The only point Carnap was not aware of was the distinction between complete Riemannian manifolds and non-complete Riemannian manifolds. Recall that a Riemannian manifold is a metrical space in a canonical way. Hence, one may define a Riemannian manifold to be complete if it is complete as a metrical space, i.e., all its Cauchy-sequences converge. The assumption of completeness is intuitively plausible, and, moreover, it seems to be indispensible from an empiricist point of view. On non-complete manifolds many bad things happen: some geodesics cannot be extended to infinity, and some points cannot be connected by a shortest geodesic.[3] Hence, for technical and conceptual reasons all Riemannian manifolds to be considered in this paper are assumed to be complete. Perhaps the simplest Riemannian manifold that is not complete, is the punctured plane, i.e., the space $E^2 - \{b\}$, $b \in E^2$. As will become evident in the following not distinguishing between complete and non-complete manifolds lead Carnap into serious trouble.

Now let us come to the crucial problem of whether the metrical structure g of a Riemannian manifold (M,g) can be considered as a matter of conventional stipulation or not. Consider the Euclidean plane E^2 endowed with its standard Riemannian metric g. Then (E^2, g) is a flat Riemannian manifold. Poincaré (and Klein) showed that E^2 can be endowed with a different metric g- that renders (E, g-) a hyperbolian manifold of constant negative curvature. In *Der Raum* Carnap thought that he could do better than Poincaré and put forward two amazing contentions:

(C1) The 2-dimensional sphere S^2 can be endowed with a flat metric, i.e., a metric g with curvature $K = 0$.[4]

(C2) The 3-dimensional Euclidean space E^3 can be endowed with a metric g^+ of constant positive curvature $K > 0$.[5]

While (C1) was considered by Carnap only as an intuitive example for the conventional character of the metrical structure of manifolds, (C2) played an essential role for the main philosophical result of *Der Raum*, to wit, the partial justification of the Kantian thesis that Euclidean space is constitutive for spatial experience. First note that (C2) combined with Poincaré's result yields that the Euclidean space E^3 can be rendered a Riemannian space form in three essentially different ways, namely, according to (C2) one would have the Riemannian manifolds (E^3, g), (E^3, g+) and (E^3, g-), which are flat, elliptic and hyperbolian space forms, respectively.

According to Carnap, Kant erroneously considered the manifold (E^3, g) as the unique spatial structure possessing experience-constituting significance. The main result of *Der Raum* contended that the flat manifold (E^3, g) should be replaced by the topological

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[3] The most important mathematical properties that non-complete Riemannian manifolds do <u>not</u> possess, may be read of from the classical theorem of Hopf-Rinow, cf. Hicks (1971, p. 163).

[4] "In order to take an intuitive example, let it be determined that we shall consider the earth's surface S^2 as a plane. ...We could thus determine, for example, that S^2 is to have zero curvature everywhere. We could then regard the earth's surface as infinitely large with the Euclidean geometry of the plane holding everywhere upon it." (*Der Raum*, p. 47).

[5] Cf. *Der Raum*, p. 48.

space E^3, i.e., only the topological structure of E^3 was to be considered as experience-constituting while its metrical structure g was a matter of stipulation and could be chosen as flat, elliptic or hyperbolic according to one's preferences (ibidem, p. 59). Carnap took this kind of Kantian metrical conventionalism as a convincing evidence for conventionalism *überhaupt.* He considered the Riemannian metric of a manifold as a tool of the conventional arrangements of the underlying topological facts.[6] Regrettably, Carnap did not care to give rigorous proofs for (C1) and (C2). Rather, he was content to point out that these claims could easily proved by following the lines of Poincaré's proof. This cavalier attitude was to have fatal consequences: As will be explained in some detail in a moment, (C1) and (C2) cannot be proved since they are false.

As far as I am aware of, (C2) has not attracted much attention although it may be considered as the upshot of early Carnap's revisionary Kantian philosophy of geometry. On the other hand, from time to time philosophers have attacked (C1), but without mathematically convincing arguments, or so I want to argue. Particularly noteworthy is Grünbaum's criticism in *Carnap on the Foundation of Geometry* (Grünbaum 1963), and also Nerlich's arguments in Nerlich (1994). Before we come to their criticisms, let us recall the notion of a (global) topological invariant. Let T and T* topological spaces. A global topological invariant is a property P a topological space T has if and only if every topological space T* homeomorphic to T has P. In other words, global topological invariants are properties stable under topological transformations (homeomorphisms) (cf. *Der Raum*, p. 39). Typical examples of global topological invariants in this sense are properties of topological spaces such as being compact, being connected, or being Hausdorff. On the other hand, it may be noted that for a metrical space M the completeness of its metric is <u>not</u> a topological invariant.

In order to disprove (C1) Grünbaum offered the following reductio: If S^2 could be endowed with a flat metric it would be homeomorphic to the plane E^2. This is false, since S^2 is "closed" and E^2 "is "open" in both directions" (Grünbaum 1963, p. 669).[7] Hence (C1) is false. This argument is flawed. As is well-known since the beginnings of the 20th century (cf. Wolf 1979, Theorem 2.5.5) the flat Euclidean plane (E, g) is not the only complete 2-dimensional connected Riemannian manifold with K = 0. There are other 2-

[6] From this position it was only a small step to consider empirical theories as conventional arrangements of observational facts of some kind. Indeed, cashing out the results of *Der Raum* in *Über die Aufgabe der Physik und die Anwendung des Grundsatzes der Einfachstheit* (Carnap 1923) he did exactly this.

[7] In modern terminology, S^2 is compact, and E^2 is not compact. Since compactness is a global topological invariant S^2 and E^2 cannot be homeomorphic.

dimensional flat Riemannian manifolds (M, g), for instane the flat toruses. Hence, even if S^2 is proved to be non-homeomorphic to E^2 this does not prove that S^2 cannot be endowed with a flat Riemannian metric.[8]

In a different vein Nerlich argues that the "identification" of S^2 and E^2 via the well-known projection p from the "north-pole" n of S^2 onto E^2 (conceived as the tangent plane at the south-pole of S^2) leads to unacceptable "causal anomalies". For instance, moving a small area A over the north-pole of S^2, on the plane E^2 amounts to a violation the basic topological structure of that space, since the corresponding move on E^2 turns the interior of the image of A outside (cf. Nerlich 1994, p. 184f). In other words, the topological structures of S^2 and E^2 cannot be translated continuously by p from S^2 to E^2 and vice versa. This is true enough but leaves open the possibility that there exists another homeomorphism between S^2 and E^2 different from p.[9] Of course, this is excluded by Grünbaum's argument based on the global topological invariant of compactness. But, as was argued above, this is not sufficient to refute (C1).

For this task one needs a topological distinction that is a bit more sophisticated than that between compact and non-compact spaces. Differential or algebraic topology come to the rescue and offer a general framework to handle such problems. The strategy is to define appropriate global topological invariants sensitive to the possible Riemannian structures defined on the spaces to be considered; if one is lucky these invariants can be calculated and turn out to be different. This may be sufficient to prove that the topological structure of with some stipulated metrical structure. Indeed, the classical theorem of Bonnet shows that the Riemannian curvature of a manifold is related to the topological invariant of compactness in a way that can be used to refute (C2):

<u>(2.2) Bonnet's Theorem (Hicks 1971, p. 165).</u> If M is a complete connected Riemannian manifold with Riemannian curvature $K \geq a > 0$, then M is compact.

<u>(2.3) Corollary.</u> The Euclidean space E^3 cannot be endowed with a complete Riemannian metric with constant positive curvature $K > 0$.

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[8] Apparently Carnap and Grünbaum believed that a flat metric on a Riemannian manifold implies that is homeomorphic to a Euclidean space. For Carnap, see footnote 4.

[9] It should be noted, however, that the projection p_n: S^m - {n}---->E^m may be used to endow E^m with a <u>non-complete</u> Riemannian metric with constant positive curvature K, to wit, the metric induced by the standard positive metric of the <u>non-complete</u> Riemannian manifold S^m - {n}. This fact may be of some mathematical interest but lacks empirical significance since non-complete metric are empirically quite unappealing.

Proof . If E^3 could be endowed with a complete Riemann metric with constant positive curvature K, it would be compact. But E^3 is not compact. Hence, such a metric does not exist→

In order to disprove (C1) one needs another kind of topological invariants. These are defined with the aid of so-called homology theories. Informally and very roughly[10] a homology theory may be characterized as a recipe which assigns to each topological space T an algebraic object h(T) in such a way that for homeomorphic spaces T and T* one has h(T) = h(T*). That is to say, the assignment of h(T) to T is a global topological invariant of T. The one we need is the so-called Euler-Poincaré characteristic χ(T)of T which will play an important role in the following. The precise definition of χ(T) is of no interest here. Rather, the point is that for appropriate Riemannian manifolds M the Euler-Poincaré characteristic χ(M) is determined already by the curvature K of Riemannian metric g:

(2.4) Theorem of Gauss-Bonnet (Hicks 1971, p. 111). Let M be a compact connected oriented Riemannian 2-manifold with Riemannian curvature function K. Then

$$v_M K = 2\pi \chi(M)$$

(2.5) Corollary . The sphere S^2 cannot be endowed with a metric whose curvature K is 0.

Proof . S^2 is a compact connected oriented 2-dimensional manifold. Hence (2.4) applies. If there existed a Riemannian metric g with K = 0 this would imply $2\pi \chi(S^2) = 0$. But it is well-known that the Euler-Poincare characteristic $\chi(S^2) = 2$ (cf. Greenberg 1976, p.99) Hence, such a metric cannot exist.→

Summing up, Carnap's claims (C1) and (C2) have been refuted.[11] In general, the metrical structures of manifolds are not conventional with respect to the topological structures.

[10] More detailed accounts may be found in any textbook on algebraic topology. For an elementary account see Greenberg (1976).

[11] At least for complete Riemannian manifolds. But it would be a desperate move to attempt to rescue Carnap's theses by allowing him to fall back on incomplete metrics. Even if under this interpretation (C2) could be saved, (C1) remains false. Obviously Carnap considered both theses to be on an equal footing. Moreover, neither Carnap nor any other logical empiricist ever showed any sign of being conscious of the distinction between complete and non-complete metrics. Rather, they took completeness for granted.

Rather, there are non-trivial relations between the metrical structure and the underlying topological structure of M.[12] Not every kind of metric g is compatible with every kind of topological structure. Given the topological structure of a manifold M one is no longer free to assume that M can be endowed with any metric g whatsoever. It may happen that appropriate topological invariants of M are sensitive with respect to Riemannian structures to be defined on M. Hence a Riemannian metric cannot be considered as a conventional language in which the topological facts can be expressed. Some metrical languages turn out to be unsuitable for expressing some kinds of topological facts. The basic linguistic metapher of Logical empiricism's conventionalism breaks down. Of course, sometimes it may occur that a topological manifold can be endowed with metrics of different types, as Poincare showed for the plane E^2. But this is, so to speak, an exception. In general, a Riemannian metric cannot be considered as a "conventional" language that describes the topological facts in one way or other. In other words, the relation between topology and geometry cannot be conceived as the relation between an underlying base of "topological facts" conventionally described by a "geometric "language.

3. Limited Domains to the Rescue? Throughout his life, philosophy of geometry played an important role in the background of Carnap's epistemology and philosophy of science. Nevertheless he did not publish much on this topic. As far as I know, after *Der Raum* he explicitly dealt with geometry only in his *Reply to Grünbaum* (1963) and in *An Introduction to Philosophy of Science* (1966). In chapter 15 of *Introduction* we find him fiddling with problems of metrical conventionalism in virtually the same way as he did more than forty years ago in *Der Raum*. With one notable exception: in 1966 he put greater emphasis on the local geometry of space.

He cast his argumentation in Reichenbach's well-known story of the two physicists P_1 and P_2 living on a 2-dimensional sphere S^2 embedded in the standard way in Euclidean 3-dimensional space. P_1 and P_2 hold different theories about the nature of their world: according to P_1 it is the surface of a sphere, but P_2 insists that it is a plane "in which the bodies expand and contract in certain predictable ways as they move around." (ibidem, p.

[12] Often, these relations are expressed as relations between curvature and homological invariants of M. The theorem of Gauss-Bonnet is only the first and most elementary example.

147). Local constraints enter the stage in that Carnap allowed the physicists to do their investigations only in "a limited domain":

> "The sphere is gigantic in relation to their own size; they are the size of ants, and the sphere is as large as the earth. It is so large that they never travel all the way around it. In other words, their movements are confined to a limited domain on the surface of the sphere. The question is, can these creatures, by making internal measurements on their two-dimensional surface, ever discover whether they are on a plane or a sphere or some other kind of surface? (Carnap 1966, p. 146).

Obviously, the purpose of restricting the ken of P_1 and P_2 to a "limited domain" was to avoid that the ant-physicists managed to walk around their world on a great circle thereby gathering evidence that they were living on a spherical world. Since this is excluded by fiat Carnap felt justified to assume his familiar reconciliatory attitude admonishing the rival physicists to end their dispute: "There is no need to quarrel. You are simply giving different descriptions of the same totality of facts." (ibidem, p. 148). Invoking Leibniz's principle of *identitas indiscernibilium* he proposed to conceive the theories of P_1 and P_2 just as two equivalent descriptions of the same world.

Against this restriction of the domain of possible experiences of P_1 and P_2 one may object that it is not compatible with an empiricist conception of science: For an empiricist it is meaningless to be engaged in investigating the global structure of the world under the presupposition that large areas of that world are principally inaccessible to empirical investigation. Rather, if it is assumed that from a God's eye point of view the domain A accessible to P_1 and to P_2 differs from the world they are living in the question concerning the global structure of the world can be meaningfully asked only for "*their* world", i.e., the limited domain A. But then the problem Carnap wanted to eschew arises anew, no matter how limited the domain A is: one has to assume that A is a Riemannian manifold, and P_1 and to P_2 are engaged in finding out what is the global structure of A. Hence the retreat to "limited domains" does not help the metrical conventionalist.[13] Summing up we may assert that Carnap's metrical conventionalism is not tenable, neither in its global nor in its allegedly more local version for "limited domains". Even for ants it is a matter of fact and not a matter of convention what kind of world they are living in.

[13] It is known that every n-dimensional Riemannian manifold (M, g) can be embedded isometrically in a high-dimensional Euclidean space E^N endowed with the standard flat Riemannian metric (Theorem of Nash). Hence, one may always assume a Riemannian manifold to be a Riemannian submanifold of some flat Euclidean space. This is mathematically interesting but irrelevant for matters of conventionalism.

4. Conclusion . Carnap took metrical conventionalism as the paradigm for conventionalism in general. The allegedly conventional character of the metrical with respect to the topological was the driving force behind Carnap's attempts to generalize conventionalism beyond the confines of geometry culminating in his "principle of tolerance" according to which even logic is to be considered as conventional.

Carnap's general conventional stance has been attacked from various quarters. Let us mention just two: In *Two Dogmas* (Quine 1951) Quine argued against the feasibility of a neat separation of conventional and factual components of empirical knowledge but without specific reference to geometry. In Quine (1966) he addressed the case of geometry pointing out that the existence of non-Euclidean geometries per se (i.e., as uninterpreted systems) can hardly be considered as an argument in favour of a substantial conventionalism (ibidem, p. 116f). However, he did not explicitly deal with the allegedly conventional character of the metrical structure of space.

Another argument against Carnap's metrical conventionalism has been launched forward by Friedman, Ryckman and others. According to them this account is not in line with the general theory of relativity, see Ryckman (1996, p.200) and Friedman (1999, p.71ff). As was pointed out already by Einstein, the object of GTR war the space-time manifold (S, g) - without the metric g there is nothing left of it, in particular there is no residual topological space S (cf. Einstein 1961, p. 155). The ontological interdependence of space-time and matter in GTR forecloses the possibility of asserting the existence of topological structures of space-time in the absence of metrical ones dependent upon surrounding mass-energy distributions. Thus, metrical conventionalism can be blamed not to be compatible with one of the most important theories of contemporary science.

The criticism against metrical conventionalism brought forward in this paper is located on a philosophically more elementary level than the two objections just mentioned. My arguments show that metrical conventionalism is untenable for mathematical reasons. The relation between the topological and the metrical structure of space level is not conventional. Poincaré's example of the compatibility of a hyperbolic and a Euclidean metric with the same underlying topology is accidental, so to speak. In general, the metrical structure g of a Riemannian manifold (M, g) cannot be conceived as a "stipulation" of a conventionally chosen language in which the topological facts are expressed. A rather drastic evidence for this claim is the fact that there are topological manifolds that do not allow any Riemannian structure at all (Kervaire 1960). That is to say, the topological facts of these manifolds are such that they cannot be expressed in terms of a "metrical" language. This flatly refutes the idea that the metrical structure can be conceived as a kind of (neutral) language in which the topological meaning can be expressed. The topological and the metrical are entangled in a much more complicated manner.[14]

Certainly it would be highly interesting to cash out this relation in philosophical terms. In order to carry out this endeavour successfully it is necessary to elucidate as precisely as possible why and where the logical empiricists's conventionalist attempt to understand this relation failed.

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[14] How complicated this relation is may be glimpsed from the evolution of differental topology and related disciplines in the 20th century. Some of the most profound theorems of 20th century's mathematics, e.g., the Atiyah-Singer index theorem, may be conceived as attempts to elucidate the highly non-conventional and complex relation between the metrical (differential) and the topological structure of spaces.

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