ever it is, in this issue one side is probably correctly represented by the insistence on the proposition; but I suspect—my hunch is—that the other side is the right one, but is *not* correctly represented by objecting to the presentation in a proposition.

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EXTENSIONAL AND NON-TRUTH-FUNCTIONAL CONTEXTS

THERE is a large class of sentences, each sentence s of which contains all the terms of a sentence s'. I shall denote s as $C[s']$, as a "context containing s'," but one must bear well in mind that this notion of containment is not one under which s' has to appear as a unit in $C[s']$, although all the terms of s' must occur as terms of s. I shall let what follows explain the relevant sense of 'contain', although the way the first sentence puts it, it is more dependent than I would want on the accidental features of a language. It is the sense in which 'Milly has a big nose' is contained in 'Milly has a very big nose' or in 'Milly probably has a big nose' (a distinguishing mark, informally, is that you must understand s' to understand s). My intention is to suggest how semantics may be given for a certain subclass of these sentences which satisfy the following three conditions:

(a) Extensionality: if t is obtained from s by substituting predicate B for predicate A, then

$$(x)(A(x) \equiv B(x)) \supset (C[s] \equiv C[t])$$

(b) Referential transparency: If t is obtained from s by substituting a name of b for a name of a, then

$$(a = b) \supset (C[s] \equiv C[t])$$

(c) Non-truth-functionality: we do *not* have

$$(s \equiv t) \supset (C[s] \equiv C[t])$$

nor do we have

$$(x)[(A(x) \equiv B(x)) \supset (C[A(x)] \equiv C[B(x)])]$$

(which may look sufficiently like (a) to cause confusion). I am not assuming that any $C[s]$ can make sense with any s. Most $C[s]$ have evident restrictions, though often (I don't know why) they have intensional variants with weaker restrictions. 'a saw ···', for example, can contain far fewer sentences than its intensional variant 'a saw that ···'.

The semantics given will be Fregean in tone: predicates will be represented as functions from individuals to truth values, and truth-functional connectives as functions from truth values to truth values. The main point is to show that (c) does not prevent (a) and (b) from allowing an extensional analysis. To begin let us restrict our attention to cases where the contained sentence is of the form $A(a)$. Let the predicate A be represented by the function f. Now $C[A(\cdots)]$ corresponds to some function from the domain D of individuals under consideration to $\{T,F\}$; the problem is to express this function in terms of f. The most obvious method would be to take it as the result of composing f with some function c, so that, if we write 'δs' for 'the truth value of s', $(x)(\delta C[A(x)] \equiv c(f(x)))$. But of course (c) rules this out, since $f(x)$ must for each x be either T or F.

Yet (a) and (b) afford a simple and workable procedure. They amount to the assertion that there is a function $C(g;x)$ of functions from D to $\{\mathrm{T,F}\}$ (the g's) and individuals in D (the x's) such that $(x)(\delta C[A(x)] = C(f;x))$. Nothing startling yet, but even this much suffices to block an argument of Frege's (refined by Gödel, Church, and Quine) to the effect that (a) and (b) imply not-(c).

The argument runs: take two objects Tom and Frank and a sentence-operator δ such that '$\delta(s) = \mathrm{T}$' is logically equivalent to s and '$\delta(s) = \mathrm{F}$' to not-s.[1] (Tom and Frank need not be The True and The False. Many objects will do: if they are $\{\phi\}$ and ϕ, then $\delta(s)$ is $\{x{:}x = \phi \ \&\ s\}$.) Then if r and s are both two true sentences and (i) $C[r]$ is true, and if $C[\cdots]$ preserves its truth under substitution of logical equivalents, then (ii) $C[\delta(r) = \mathrm{T}]$, and therefore (iii) $C[\delta(s) = \mathrm{T}]$, by (b), since $\delta(r) = \mathrm{T} = \delta(s)$. Therefore (iv) $C[s]$.

Now try to make the argument go through in our $C(g;x)$ notation. r and s are then true sentences $A(a)$ and $B(b)$, where A and B are represented by functions f and g from D to $\{\mathrm{T,F}\}$. Then, writing '$\lambda x(\cdots\ x\ \cdots)$' for 'the function whose value for each x is $\cdots\ x\ \cdots$' and talking in terms of equality of functions rather than equivalence of sentences,[2] we translate (i) as

$$\text{(i)}'\ C(f;a)$$

(ii) can be *either*

$$\text{(ii)}'\ C(\lambda y(\delta(f(y) = \mathrm{T}));a)$$

or

$$\text{(ii)}''\ C(\lambda y(\delta(f(a) = y));\mathrm{T})$$

And so we have two versions of (iii):

$$\text{(iii)}'\ C(\lambda y(\delta(g(y) = \mathrm{T}));b)$$

[1] There's a certain amount of use-mention sloppiness here and elsewhere. Enough circumlocution could dispose of it.

[2] To copy the argument very carefully we should, of course, duplicate an inference with an inclusion. (So 'p therefore q' is translated as '$\{x{:}f(x) = \mathrm{T}\} \subseteq \{x{:}g(x) = \mathrm{T}\}$'.)

and

$$\text{(iii)}'' \quad C(\lambda y(\delta(g(b) = y));\mathrm{T})$$

(iv) becomes

$$\text{(iv)}' \quad C(g;b)$$

Trying to reproduce the argument, we find that (i)′ is in general equal to (ii)′, but that (ii)′ is not in general equal to (iii)′ (because $f(a) = g(b)$ doesn't imply $f = g$; not even when $a = b$). The double-prime way looks more as if it gets there: (ii)″ is in general equal to (iii)″. The trouble is that (ii)″ is not in general equal to (i)′ (nor (iii)″ to (ii)′ or to (iv)′).

What this amounts to is that, since our construal of $C[A(a)]$ gives it a relational form and separates the A and the a in the representing function $G(f;a)$, it makes $C[\delta(A(a)) = \mathrm{T}]$ ambiguous. It can be a relation between a and $\delta(A(\cdots)) = \mathrm{T}$ or between T and $\delta(A(a)) = \cdots$, and these are different. This shows how careful one must be with the assumption, which seems so clear, that these contexts preserve logical equivalence. For if s is logically equivalent to t and $C[s]$ is unambiguous, then there will be *some* reading of $C[t]$ on which it is logically equivalent to $C[s]$ (got by subjecting the representing form to transmutations corresponding to those which give t from s), but not all readings of $C[t]$ will be so.

We have represented the logical form of contexts $C[s]$, of the right sort, as functions of those functions and names which represent the logical form of s. In this respect the truth value of 'I shall be home before sundown', say, is like the value of the derivative of $\sin(x)$ at $x = 0$. And wanting to make 's before sundown' depend just on the truth of s is on this account like the mistake made by beginning students of calculus of evaluating "$d/dx \sin(x)$ at $x = 0$" as $d/dx \sin(0), = 0$ rather than as $\cos(x)$ at $x = 0, = 1$. A more ordinary such function-and-argument-containing context is given by 'The present mayor of Port Arthur is, *ex officio,* a member of the Association of Ontario Mayors and Reeves'. The mayor of Port Arthur is Saul Laskin, but it would be a similar mistake to infer "Saul Laskin is, *ex officio,* a member of the Association of Ontario Mayors and Reeves", which seems false, if even well-formed. This is, of course, because "The ϕ of ψ is, *ex officio,* $\cdots$" depends on the *function* 'the ϕ of $\cdots$' and not just on its value at ψ.

Our method so far has constituted an abandonment of the composition-of-functions method. That method was a lot less extravagant, however, and we can in a way go back to it by the following method.

Suppose that P were a set of objects ("P-values") such that the context $C[s]$ was "P-functional"; i.e., there was a function c from P

to $\{T,F\}$ such that $(x)(\delta C[s] = c(p(s)))$ for any s, where $p(s)$ gives the P-value of s. Then we could take predicates as functions from individuals to P-values, and so on. P-values could be taken in one way as events and facts, and in another as the values of many-valued logics, and both these lines can be followed. It is interesting that for every $C[s]$ there is a trivial P of four members defined as follows:

$p(s) = p_1$ if s and $C[s]$ are true
$p(s) = p_2$ if $C[s]$ is true and s false
$p(s) = p_3$ if $C[s]$ is false and s true
$p(s) = p_4$ if $C[s]$ is false and s false

Then $c(p_i)$ is T if $i = 1$ or 2, and F if $i = 3$ or 4.

P-domains are intermediate between individuals and truth values in that every function f from individuals to $\{T,F\}$ can be expressed as the composition of a function f_1 from individuals to P and one f_2 from P to $\{T,F\}$. Without this characterization P-domains are not worth having. For a construction like the above can equally well be made for intensional contexts, though the p_i will not then constitute an intermediate domain. (And if we were not wanting that characteristic, we might as well have used '$p(s) = v_1$ if $C[s]$ and $P(s) = v_2$ if not-$C[s]$'.)

The easy intermediate domains will each do for only a few contexts; so we must stick them together or make new ones to deal with numbers of contexts. And this, showing the dependence on C of the particular way that f is decomposed into f_1 and f_2, indicates that the method of intermediate domains must be a development of the function-of-functions method rather than a method that can proceed all by itself.

It is important that we have ways to deal with quantified sentences and others besides $A(a)$ sentences. This has a lot to do with being able to make the distinction between contexts that contain a sentence and those which just contain its parts. The tenor of the discussion so far has been that in one sense there is no such distinction. For we have represented the contexts as dissecting their contained sentences, and I do not see how we could have done otherwise. But still one feels that there is a type of context distinguished by the fact that it can contain sentences of any form (though not necessarily all sentences of any form). 'Possibly ⋯' and 'John sees ⋯' are like this, or seem to be, while '⋯ very ⋯' and 'on Tuesday ⋯' are not.

We could take quantified sentence-forms as functions from sets of individuals to truth values, and extensional contexts as functions of these functions and their arguments. Since a context should be represented by the same function whether it contains a quantified or an unquantified sentence, we must represent contexts that are also to contain quantified sentences as rather hybrid $F(a;b)$, where either a is a function of functions and b a function of individuals or a a function of individuals and b an individual. Functions like this of suitable intricacies can be used to make a large variety of sentences embeddable in a context. But none will deal with all types of sentences, and the exceptions can always be easily found.

Perhaps this is the best we should expect, for while the upshot of the Frege-Gödel-Church-Quine argument is not that all extensional contexts are truth-functional, it *is* that any analysis of them must make them depend on the particular structure of the sentences they contain. The point of this paper is to show that this can be done without harm to their extensionality, and it would in some measure be achieved even if the semantics of every context had to include a list of what types of sentence could be put into it. (The importance of Frege's argument should not be underestimated. Largely for awe of it in its various forms, treatments of non-truth-functional contexts have assimilated them to intensional contexts, either to shade them with the same dark incorrigibility or to honor them with all the mathematical and philosophical sophistication that the intensional requires.)

A little more greed might not be unreasonable, for one might well want extensional semantics for a number of non-truth-functional contexts in which sentences of any form seem to fit, e.g., the connectives of some logic. What follows is intended to provide this for a certain class of contexts that seems to include most of the real sentence-containing contexts of any interest.

'John is very happy' could be rewritten from a God's-eye view as 'In the world in which the only ordinarily happy are not happy (but which is otherwise like this world), John is happy'; and 'We are almost in Princeton' as 'In the world in which Princeton is a little bigger, we are in Princeton'. The new sentences have nothing to commend them, and are besides automatically false on many readings of them. But they are useful because they can hold any sort of sentence; so we can say "In the world in which the only gulls are those in the sight of Francis, all the gulls flew to the tower" for "Francis saw all the gulls fly to the tower."

The formal version has no need of the notion of the identity of individuals in different worlds, and all the talk of worlds is definitely just heuristic. By *the world* I shall mean the structure consisting of the set of individuals, the set of functions from individuals to truth values, the set of functions from these to truth values, and so on, including functions of mixed orders. Some variety of type theory must be assumed to govern this description. Luckily the rather doubtful complete world will not play a large role. Call any substructure of it *a world* as long as its domain of individuals is a subset of that of the world, and the domains of all its functions are contained in the hierarchy starting from its domain of individuals. Now with every sentence containing context $C[s]$ we associate a representing mapping c from worlds to worlds; more exactly, from the smallest world containing the functions used in representing the form of s, to some other. $C[s]$ is represented as having the same form as s, with each term t replaced by ct—what c maps it into. So if s has the form $F(a,b)$, for example, we take $C[s]$ as $cF(ca,cb)$.

There are several features of this method worth discussing, but let me just mention an issue that arises both with regard to it and in the discussion of intermediate domains. It may well be that some $C[s]$ may require that s be taken as having a certain form and another context require that it have another not incompatible but more involved form. For example, if s is 'all things are A', then for most contexts it will suffice to represent s as '$(x)A(x) = \mathrm{T}$' for some function A, and then take $C[s]$ as '$(x)(cA(x) = \mathrm{T})$'; but some imaginable context could require that s be taken as having the form '$F(A)$' where F is a function of functions of individuals, and then that $C[s]$ be taken as '$cF(cA)$'. So the question might arise: 'What is the form of s?', just as we might ask whether predicates represent functions to $\{\mathrm{T,F}\}$ or to P-values. I think that it is pretty evident that in this context the question is wrong-headed. We do not want semantics to tell us what sentences really are, but to provide us with a method of discussing certain notions (truth, consequence) about them.

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COMMENTS AND CRITICISM

REASONS AND INTENSIONALITY

THERE is something appealing about a causal account of reasons. That is, there is something appealing about asserting an equivalence to hold between the following:

(1) Jones's desire to smoke and his belief that the object in his hand was a smokable thing, were his reasons for lighting the cigarette