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IMAGINATION AND IMAGING IN ECONOMIC MODEL-BUILDING

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Abstract

Modelling became one of the primary tools of economic research in the 20th century and economists understand their mathematical models as giving some kind of representation of the economic world, one adequate enough to their purpose of reasoning about that world. But when we look at examples of how non-analogical models were first built in economics, both the process of making representations and aspects of the representing relation remain opaque. Like early astronomers, economists have to imagine how the hidden parts of their world are arranged and to make images, that is, create models, to represent how they work. The case of the Edgeworth Box, a widely used model in 20th century economics, provides a good example to explore the role of imagination and images in the process of making representations of the economy.

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I Making the Economic World in Mathematical Models

Pioneers of mathematical economics in the late 19th century presented their mathematizing efforts as a matter of language change, a change that would be progressive for economics as a science, because economic ideas expressed in mathematics would be expressed more exactly, and reasoned about more rigorously, than when expressed in words. To make a mathematical economics, economists needed not only a mathematical language, terms and formulae, but also to imagine a mathematically described world within which their economic ideas could be expressed. Many of the early mathematical models came via analogies, and their

[1] I thank all those numerous individuals who have discussed these particular issues of model building with me over the last few years, including my colleagues at the University of Amsterdam and the London School of Economics and in my host departments at UC Berkeley and University of Pennsylvania during 2000-01. I thank Till Gruene and Chiara Baroni for research assistance, Tom Humphrey for permission to use his diagram, and to the British Academy for funding this research. Comments welcomed: email m.morgan@lse.ac.uk

provenance can be understood in the terms offered by Hesse (1966) (for example, see Morgan, 1999). But this was not true of all mathematical models, and for these others, it is difficult to describe them as being the result of processes of abstraction, simplification and idealization, such as might be given in some philosophical accounts of model-building in economics (see Hamminga and De Marchi, 1994). Rather, it seems, economists relied on processes of imagination and image making in modelling aspects of the world that they did not fully understand and which they could neither observe directly nor access experimentally.

Nelson Goodman's *Ways of World Making* (1978) stresses how scientists and artists are involved in making sense of the world in similar kinds of ways. Both groups make versions of the world. I interpret this here as both groups make representations of the world.[2] Economists took their earlier verbally made economic world as a matter of habit. But, their verbal economy: the nouns, verbs, descriptive phrases and relations between them all that economists still use grew up over the past centuries in such a way that their theories and descriptions of the economic world could be expressed within that domain, within that version of the world. Creating a mathematical economics was, historically, a similar process to that of creating a verbal economics. This was a process in which economists came to think about the economic world in a mathematical way and represent it to themselves in mathematical form: a process of imagining and imaging, so that over the 20th century, the elements, their meanings, how they are symbolised and what relations are assumed all came to be taken for granted. Economists had to make their mathematical version of the economic world just as their forebears had made their verbal one.

Making a mathematical economic world was a tall order. Model building, I suggest, flowered in economics during the late 19th century period and throughout the 20th century process of mathematization for two reasons. One is that exactly those kinds of qualities needed in making a new version of the world are those found in model building, namely, the abilities to be imaginative about the world and to make images of it in mathematical forms.[3] Second, model building provided a way of generating

[2] Goodman is careful not to use the term representation so broadly; R.I.G. Hughes (1997) understands Goodman's term in the context of model making as "denotation". See Weintraub 1991 for an account of world making in economics in the Goodman tradition.

[3] I want to avoid any account in which mathematization is seen either as a process of translation or as one of transcribing. Where "translation" underestimates the cognitive problem (see Latour, 1986), "transcribing" makes too much of an ontological commitment: it suggests that the laws of economics are written in mathematics and economists merely had to figure out how to decipher their own Book of Nature. (This is associated with the "perfect model" model held by some philosophers of science - see Teller (2001) for a recent attack on this view.) Recognition of the presence and importance of model building in the

the vocabulary and forms of the new way of thinking, and so of providing the "working objects" (the tag comes from Daston and Galison, 1992) on which the mathematical economic description could be refined and tested. Model building, in essence, involved new conceptual elements which could not be represented in the old forms. The nature and content of the new representations and the grammar they entailed changed the way economists picture the economy and changed the picture that economists have of the world.

II The History of the Edgeworth Box Diagram - as Told by Itself

To illustrate and make sense of these claims, I turn to the historical process by which a small mathematical diagrammatical model was formed, one which came to play a ubiquitous role in theorizing up into the mid 20th century and is still used today. In order to understand the process of imagining and imaging, I turn to the founders of the Edgeworth Box to see how they created this model.

One way to reveal this history of imagining and imaging that went on in the development of the Box is to contrast a history of the diagrams as they were actually made with the recent historical account of the development of the model by Humphrey (1996) using modernized versions of the diagram. Humphrey's account reconstructed the history of theorizing using the diagram. My question is rather different: How did economists first *make* the model which became so well known? Tom Humphrey's history and set of images could only have been written by someone who already knew the box, was familiar with what it could represent, and with how it could be used.[4] I seek to recreate the unfamiliarity erased by that history and to approach the history with the innocence of a pre-mathematical economist.

II.i Edgeworth's Image

Let us first dip into Francis Edgeworth's own account of how he made his economic world in mathematics. Edgeworth's version of the diagram was introduced in his now

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mathematization process suggests not difficulties of perception and transcription but rather ones of cognition and portrayal.

[4] His question focuses on the *usage* of the Box diagram especially its important role in the development of certain theoretical results, and its tremendous versatility to deal with theoretical questions in various domains. It is a superb history for anyone who ever thought the Box diagram was for merely for illustration, s/he need only read Humphrey's 1996 account to be enlightened. Humphrey's history is effective because he modernized along one dimension (the diagrams) in order to tell the history of the another dimension (the theoretical results). We might both be interpreted as following Lakatos' (1976) example in *Proofs and Refutations*, with actual history below the line, and reconstructed history above. We chose to reconstruct along different lines.

famous *Mathematical Psychics* (1881), a book of almost impenetrable erudition from this Irish economist. For Edgeworth, mathematics is a form of expression, a language, and because of its special qualities it is a tool or instrument both for expression of economic ideas and for reasoning about them. But in Edgeworth's mind, it is also an instrument of imagination.

Edgeworth's imagination about economic behaviour was funded by a general analogy with mathematical physics though the analogical content gets less fundamental and more illustrative as he argues about two individuals and two goods to exchange where parties are free to contract only by *mutual* consent and without competition from other traders. He defines the locus of points at which exchange might be contracted as those where, whichever direction a move is made away from that set of points, one trader gets more and the other less utility. This set of points is termed the "contract curve".

He then sets about (pp 20-28) demonstrating the qualities of his defined contract curve by a series of mixed mathematical and verbal reasoning, in terms both spatial and algebraic, partly written down and partly in the imagination, to assure himself that the characteristics of the "contract curve" are sensibly proved by several different approaches. At a certain point in his mathematical discourse, Edgeworth moves into one of his imagined worlds, and the original version of the "box" appears as his figure 1 (see Figure I) encased in the following text: (my underlinings, his italics):

> "It is not necessary for the purpose of the present study to carry the analysis further. To gather up and fix our thoughts, let us <u>imagine a simple case</u> - Robinson Crusoe contracting with Friday. The *articles* of contract: wages to be given by the white, labour to be given by the black. Let Robinson Crusoe = X. <u>Represent</u> *y*, the labour given by Friday, by a horizontal [sic] line measured northward from an assumed point, and measure x, the remuneration given by Crusoe, from the same point along an *eastward* line (See accompanying figure 1). Then any point between these lines <u>represents</u> a contract. It will very generally be in the interest of both parties to vary the articles of any contract taken as random. But there is a class of contracts to the variation of which the consent of *both* parties cannot be obtained, of *settlements*. These settlements are <u>represented</u> by an *indefinite number* of points, a locus, the *contract-curve* CC′, or rather, a certain portion of it which may be supposed to be wholly in the space between our perpendicular lines in a direction trending from south-east to north-west. This available portion of the contract-curve lies between two points, say $\eta_0 x_0$ north-west, and $y_0 \xi_0$ south-east; which are respectively the intersections with the contract-curve of the *curves of indifference* for each party drawn through the origin. Thus the utility of the contract <u>represented</u> by $\eta_0 x_0$ is for Friday zero, or rather, the same as if there was no contract. As that point he

would as soon be off with the bargain - work by himself perhaps." (pp28-9)

Thus Edgeworth imagines his Robinson Crusoe and Friday lining up at right angles to each other in the same plane, ie shoulder to shoulder, as befits those who must mutually agree before exchange can take place. Edgeworth's (*x*,*y*) space is a plane, and the indifference curves are projections from three dimensional utility surfaces; thus he imagined and made his image accordingly (and so he correctly wrote that in Figure 1 we draw the Y axis horizontally northwards). The individuals (X and Y) are not fully and separately distinguished on the diagram from those things which they have to exchange (*x* and *y*).

It seems so natural to economists nowadays to represent the two goods along these two axes, but it was not so in the late 19th century when economic diagrams were still in their infancy. Edgeworth probably began with Marshall's 1879 trade diagrams, which use this convention to show the trading relations between two countries, in which goods were represented on the two axes and the whole of the space between was open for trade. Edgeworth's diagram refers to individual traders alongside their goods, and provides an indifference curve for each individual and their contract curve. And while it seems initially that the whole space is open for trade as in Marshall, the argument defining the contract curve in conjunction with the indifference curves through the origin (ie, points at which utility is equivalent to that obtained from zero exchange) rules out some areas of the 90degree total space. Edgeworth is so impressed by his own diagram, and the way that it allowed him to work out some results which had previously failed to yield to general analysis, that he wrote (p 36) that "the figure 1, page 28, is proved to be a <u>correct representation</u>," and that the diagram provides "<u>an abstract typical representation</u>" of a process.

Because of my world-making claims, I want to delve a little further into how Edgeworth viewed this episode. Edgeworth made his model, his diagram, to represent two people in an economic situation and he used it to explain something about those things he represented. At the same time, while such diagrams represent specific cases, they seem to acquire general relevance. Although his diagram and arguments did not provide the kind of general analytical results which seem properly mathematical, they did provide convincing demonstrations of a logical kind that were more than one-off.[5]

Edgeworth understands the power of reasoning with such cases as we can see in his discussion of this form of mathematical argument. He applauds mathematics because

[5] It appears to be in the nature of geometric reasoning that particular cases are taken to provide general proofs; see Arnheim, 1969, chapter 10 for an interesting discussion of this.

its “very genius is generalisation, [which,] without dipping into particulars, soars from generality to generality” (p 86) but he also claims that mathematics can get general results from arguing particular cases:

> “Indeed the nature of the subject is such that a *single* instance - by a sort of ‘mathematical induction,’ as it has been called - a single ‘representative-particular’ authenticated instance of mathematical reasoning without numerical data is sufficient to establish the general principle.” p 83 (in 1881, Appendix I “On Unnumerical Mathematics”)

This claim about “ a sort of mathematical induction” using a “single representative-particular instance” is an apt description of his reasoning about Robinson Crusoe and Friday and this *representative-particular* label seems to be a good one for something we would now denote “a model”.

II.ii Turning Edgeworth’s diagram into a “Box”

The historical development of the Edgeworth Box enables us to explore some detailed questions about this process of model building and to open up questions about the representation. We start by asking: What does the box represent nowadays as seen in one of Tom Humphrey’s diagrams (our Figure II)? Two adjacent sides of the box denote a fixed amount of the two goods or services or resources available so that the box represents a world with given and fixed resources. The two antagonistically placed origin points mark the direction of stance of two traders, each with their own two axes (the adjacent ones), and on which their own shares of resources by endowment and by exchange can be marked. But this was not how Edgeworth imagined the economic world. The most striking thing is that Edgeworth’s 1881 diagram is not a box at all. In his diagram, the world is represented differently - there each trader measures off his resources for exchange along one axis only; one trader X is trying to make a contract by trading his own x for some y offered by Y. Note also, that with these uncapped axes imported from Marshall’s trading diagrams, there is no total amount or limit on the amount of resource that can be exchanged. What might now be taken as the irreducible shape of the box - namely a closed set of two amounts of exchangeable items represented by the sides of the box, and two traders at opposite corners, each with two axes of potential commodities to trade with - are not there from the beginning. How did Edgeworth’s diagram become a box? How did the individuals arrive in an antagonistic stance?

Vilfredo Pareto, an Italian mathematical economist of great note, without comment, places the individuals at the SW and NE corners in his 1906 book, thus lining them up opposite each other. He represents a fixed quantity of both goods, but, by extending the axes beyond the rectangle, invites the possibility of extension in some later diagrammatic treatment. Arthur Bowley, presents almost a box (in the first text book in mathematical economics in 1924) one which follows Edgeworth in orientation (ie potentially NW-SE), but it remains open, or rather, unclosed, and his axes appear flexible in length.[6] He moves what we would now call "the endowment point" (the initial amounts of both goods held by each individual) into the middle of the box, but names it as the "origin" point, the point from which trading would commence according to the indifference map, as in Edgeworth, not the zero both goods point for each trader as it later becomes. While Bowley's two axes continue but do not meet, Tibor Scitovsky (1941), like Pareto, extends his axes beyond the box, as does Abba Lerner (1933/52), who made his box represent production not exchange.

Model building is a creative, building, process: the actual historical sequence shows how very different the beginning point is from the end point seen in the later well-accepted model diagram. When economists *first* make an image of the economy, it is not that they know the world and subtract elements from it to isolate certain parts. Rather it is that they use their imagination about the workings of the economic world to make representations of those workings in equations or diagrams. Gradually over time, other economists develop the model, and further elements are added to the representation. These representations are the things that economists colloquially now call "models".

III Questions of Representation

The contents of the box show how the new conceptual elements associated with the model are developed into an analytical apparatus.[7] Edgeworth's (1881) substantial developments of Marshall's (1879) initial trade diagrams consist in mapping utility concepts into the commodity space by adding his contract curve and an indifference curve to represent the preferences of the two individual traders. These are the critical conceptual innovations that Edgeworth developed in this field, and they are discussed and diagrammed for the first time here. Pareto provides indifference maps, and shows the trading range in which welfare improvements can be negotiated in relation to (some) price rays. Bowley's innovation is to represent the possibility of initial endowments and he shows Marshall's offer curves clearly on the same map. Scitovsky

[6] Historians of economics have argued over whether Edgeworth's original diagram can properly be called a box and over the relative contributions of Edgeworth, Bowley and Pareto to its genesis and development (see particularly Creedy 1980 and 1986, and Tarascio 1980).

[7] My comments here cover only what is represented in the diagrams. The use to which all these diagrams are put is discussed by Humphrey (1996), to which the reader is referred. For a broad account of how models are used for various different functions, see Morrison and Morgan, 1999; for more focussed accounts of how we might think about their manipulation, see Morgan, 2001 and 2002b.

develops an analysis of what happens to the utility maps when the size of the commodity space changes. Finally, Wassily Leontief (1946) puts together all the conceptual elements of indifference maps, offer curves, contract curve and price rays in a representation which uses the Box to show individual preference maps, their optimum exchange points and their trading responses to different price ratios. Although some of the ideas associated with these elements have a longer history, the conceptual apparatus is not something which existed before and outside of the diagram, rather they are new with the representation and developed alongside the model.

The important thing about this conceptual apparatus is that these representations of the economic world in the Box diagram enable economists to argue in conceptual spaces, spaces beyond or behind perceptual space. The world of people and goods might be illustrated by an artist's depiction of the Box in terms of two people and their goods, but the economic concepts have to be visualized: imagined and imaged into that same space by the economist.[8]

This difference between the perceptual space of illustration and conceptual space of visualization is discussed by Michael Mahoney (1985) in relation to perspective drawing and the new mechanics of the Scientific Revolution. He sets out to destroy the Edgerton Thesis, namely that there was a direct causal link between the Renaissance improvements in drawing of machines and the development of the science of mechanics. Mahoney argues that although the engineer-artists of those days drew in new ways (they learned to provide accurate representations of physical objects in three-dimensional space) they did not draw new things. These new ways of drawing did not reveal the workings of the machines. Rather, the science of mechanics at that time treated the machine as "an abstract, general system of quantitative parameters linked by mathematical relations" so "...it is difficult to see how more accurate depiction of the basic phenomena as physical objects could have conduced to their abstraction into general systems. For the defining terms of the systems lay in conceptual realms ever farther removed from the physical space the artists had become so adept at depicting. Those terms could not be drawn; at best, they could be diagramed." (p 200). It was already the case that reasoning about mechanics was conducted in the language of mathematics; geometric reasoning and mathematical diagrams remained the main form of representing these relations and were used for reasoning through the Renaissance. But, as Mahoney remarks with

[8] In an extended version of this paper, first given at the ECHE conference in Montreal in March 2002, I also analyse an artist's depiction of the Box diagram and use this to explore both the idealization account of model building, and to demonstrate more effectively these claims about the difference between conceptual and perceptual space.

great insight, "It is the mind's eye that is looking here, and it is peering into the structural relations among quantities belonging to many different conceptual (rather than perceptual) spaces." (p 209). Mathematics marks out the difference in the source of imagination from the mind's eye compared to the body's eye.

For the economists in this case, learning to represent the economy in new ways was drawing new things. The economic elements inside the Edgeworth Box: the indifference curves, the contract curve, the points of tangency and equilibrium, etc. the mathematically expressed elements are new, mind's eye, conceptual elements - not old, body's eye, perceptual elements. Scitovsky's 1941 use of the diagram provides an excellent example of this point. The critical point of his article is the difference between allocative efficiency in which the total resources in the economy are fixed - denoted by a fixed size box, and those in which the resources change - denoted by a change in box size. The representation of the effect of this change proves to be quite difficult to understand for the modern user of boxes. It is tempting for the reader of the diagram to suppose that, by expanding the box, there are just longer axes - more cheese and wine (for example) to be exchanged for a given indifference maps (representing tastes, which have no reason to alter). But of course, these indifference contours are in conceptual space, and increasing the total resources effectively expands the box from the middle. As the axes are lengthened, perceptual space expands, but so does the conceptual space, so that the original contract curve opens out to provide a region in the middle through which the new contract curve runs.

This example provides a good illustration of the distinction between conceptual space and perceptual space, and so tells us how to distinguish when a diagram is doing any work in the argument. If the diagram is about perceptual space but the argument about conceptual space, the reasoning will take place, as Mahoney describes it, "off the diagram" and the diagram will be, at best, an illustration, rather than a tool for experimentation and demonstration.[9] Yet, as we know from Tom Humphrey's (1996) history, during the early 20th century period the diagram was a creative tool used to derive propositions and prove theorems - it was a tool for deriving and demonstrating theoretical results.

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[9] It is unwise to refer here to "mere" illustration. When every economist became familiar with the box and its results, the diagram's status in the profession dropped to that of being an "illustration", although of course it was an illustration for results derived with the box. It has remained a valid model because it links with important results derived from other forms of representation - for example game theory results can be linked with Edgeworth's box results. Further, one can never tell when a model like the Edgeworth box might not return to work: recent experimental economics by Gode et al (2001) has used the box as the basis for human and computational experiments. See Morgan 2002a for discussion of the way mathematical models are used for experimental demonstration.

IV New Things in the New Version of the World

Although economists usually (as in the examples here) fail to express the full particulars of the individuals and their goods in their model, the Box diagram enables them to place the symbolised individuals into a different form of relationship, and to say different things about their relationship, than in the verbal economics they supplant. The act of representation here involves the direct visualization of the economic world into mathematical symbols and other forms of non-verbal denotation to create a new world in the model.

As a test of this proposition of newness, imagine giving a verbal (translation) of Humphrey's box diagram with a sufficiently exact description that all the parts, and their relations to each other, are made clear. Such a description could be given, but only by using our now habitual mathematical and spatial terms expressing these economic concepts. But these same concepts and terms depended for their definition and their development on reasoning with the diagram. Thus, we can try to translate our mathematical model world into verbal terms, but it is a new world being expressed, one which we could not have expressed before we made our diagrammatic world.

I should be careful here to point out that when the Edgeworth box is described as a mathematical model, it is not only made of mathematics. We can illustrate this best by considering the allowable movements or manipulations which can be made in the model. The notion that the two traders will be at some kind of optimum when their indifference curves meet at a tangency makes use of mathematical concepts and logic. But the apparatus of offer curves, indifference curves and so, for example, the spaces in which trade is ruled out, depend on understanding the conceptual content of the elements in the model. Thus, Scitovsky's diagram showing the implications of increasing the resources requires manipulations of the diagram which are determined by the economic meaning of these curves, not by the logic of geometry. Both mathematical and subject matter conceptual knowledge constrain the details of the representation and define the allowable manipulations. This is surely not particular to models in the form of diagrams, and, indeed, it seems likely that most if not all "mathematical" models in economics depend on economic subject information to constrain or define their rules of manipulation. From this point of view, there would be as much difficulty in "translating" the Edgeworth box into "just mathematics" with no subject content as into "just words" with no mathematical content.

An even simpler test of whether the Edgeworth Box representation belongs to a new version of the world, a model version rather than a text version of the world, is suggested by an unemphasized comment by Michael Lynch (1990) in his discussion of diagrams in social theory. There he remarks, of one example, that it "is a diagram that does not obviously perform an independent representational function. If it were removed from the text, it would not be missed because it adds very little to what the surrounding text says."(p 5) This is another version, if you like, of Mahoney's observation about "reasoning off the diagram", with the additional focus on independence. Linking their points together suggests that an "independent representational function" implies that the diagram must provide some independent representational *content* not available in the text, and if we follow Mahoney, that content is conceptual .[10] The Edgeworth Box diagram does indeed carry an independent representational function: it contains conceptual apparatus which could not be represented, or manipulated, in verbal form and indeed cannot be entirely expressed in purely mathematical terms.

Thus, to go back to the original claims for mathematization, it is not just that (as economists have long argued) mathematics is more exact in expression, or a more efficient workhorse, or more rigorous in argument. The point is that mathematical models represent something *different* from verbal accounts, they involve *different* concepts, and use *different* kinds of arguments. The mathematically made version of the economic world is *different* from the verbally made one. Mathematical models represent something independently of the text, that something has conceptual content not (easily) expressible in words, and it is this quality that made models good building blocks for a mathematically made version - a newly made version - of the economic world. To arrive at that newly made version of the world requires imagination about how to represent the world and to make an image of that world in the model.

[10] This argument parallels, in a slightly different way, the claims that Morrison and I made about the autonomous functioning of models being related to a certain independence in their construction: see Morgan and Morrison, 1999, chapter 2.

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Figure 1: Francis Edgeworth

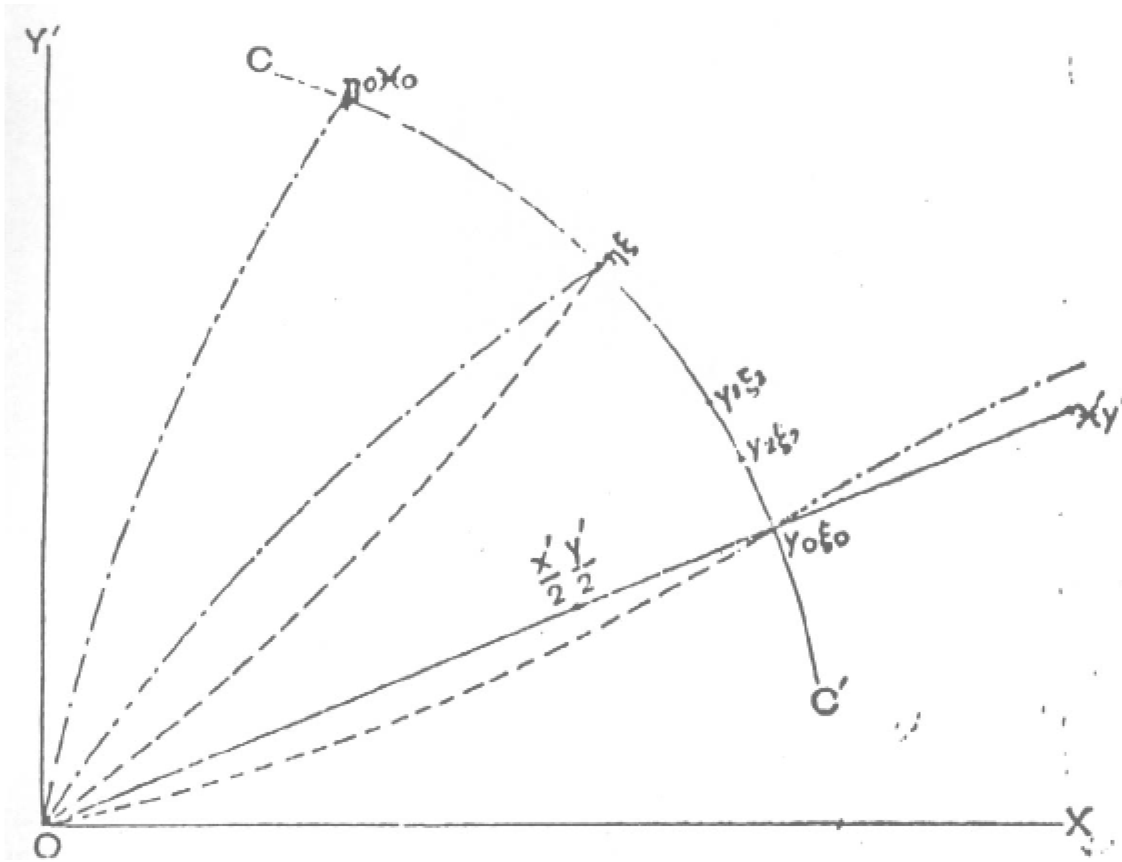

Figure 2: T. M. Humphrey