This is a preprint of an article published by Taylor & Francis in *Social Epistemology*, available online: https://doi.org/10.1080/02691728.2020.1789776

Increasing specialization: Why we need to make mathematics more accessible

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Abstract Mathematics is becoming increasingly specialized, divided into a vast and growing number of subfields. While this division of cognitive labor has important benefits, it also has a significant drawback: it can sometimes impede mathematical progress by making it difficult for mathematicians to make connections to subfields other than their own. Mathematicians can address this by making their *own* subfield more accessible to researchers working in *other* areas. One way they can do this is by engaging in exposition, as I illustrate with the User's Guide Project in algebraic topology. However the current reward structure of mathematics does not appropriately credit mathematicians who make their subfields more accessible via exposition. I thus conclude that the reward structure of mathematics should be changed to more highly value such work, with changes being adopted at the level of departments, professional societies and funding agencies.

Key Words: Mathematics, reward structure, division of labor

1. Introduction

Mathematicians, like scientists, divide their cognitive labor. For example, they organize into different groups, such as algebraists, topologists, and geometers, according to the skills they learn and the types of problems they pursue.[2] Philosophers of science and social epistemologists have investigated questions about the division of cognitive labor in science, asking questions such as (see e.g. Kitcher (1990), Strevens (2003) and Weisberg and Muldoon (2009)): (i) How should researchers distribute their labor?; and (ii) How should they be incentivized to do so effectively?

Here I focus on a consequence of the continued division of cognitive labor in mathematics: hyperspecialization of the discipline. The mathematician Piergiorgi Odifreddi describes the situation as follows:

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[2] Nonetheless, it is important to point out that some mathematicians span these divisions. For example, algebraic topologists use algebraic tools to study topological spaces and analytic number theorists use the tools of analysis to study the integers.

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> The problems that are simple and easy to solve are few, and once they have been solved a discipline can only grow by tackling complex and difficult problems, requiring the development of specific techniques, and hence of specialization. This is indeed what happened in the twentieth century, which has witnessed a hyperspecialization of mathematics that resulted in a division of the field into subfields of ever narrower and strictly delimited borders (Odifreddi, 2006, p. 2)

To further illustrate how mathematics is split up into a vast number of subfields, consider the 2010 Mathematics Subject Classification. This has 63 top level areas which get further subdivided to yield over 5,000 mathematical specialities (Dunne & Hulek, n.d.).

There are benefits to specialization in mathematics. For example, Casadevall and Fang (2014) identify the following benefits of specialization in science: 'efficiency, reduced time to production, improved quality, and the partitioning of vast quantities of knowledge into more-manageable units' (Casadevall and Fang, 2014, p. 1356). Presumably these benefits apply to mathematics as well. Nonetheless, the drawbacks of increasing specialization can be significant and include 'monotony, lack of mobility, monopoly, isolation, and the costs of coordination' (Casadevall & Fang, 2014, p. 1356). My focus here will be on a related drawback of increasing specialization in mathematics: it can sometimes impede mathematical progress by making it difficult for mathematicians to make connections to subfields other than their own.

More precisely, let the research community[3] of a mathematical subfield *S* be the collection of mathematicians who conduct research in *S*, publish their work on *S* in journals, attend conferences on *S* and otherwise interact with other researchers working in *S*. For brevity I will drop the “of a subfield *S*” and simply use “a research community” to mean the research community of an arbitrary subfield *S*. I will argue that making connections to other subfields often helps, and is sometimes necessary for, members of a research community to solve problems in their own subfield. I will also argue that increasing specialization can make it difficult for members of a research community to make connections to other subfields. By making it hard for members of a research community to make connections to other subfields, increasing specialization can thus sometimes slow the progress of that community. As that research community was arbitrary, however, this means that increasing specialization can sometimes impede the progress of mathematical research communities in general.

This significant drawback of increasing specialization in mathematics can be addressed if mathematicians make their *own* subfields more accessible to researchers working in *other* areas. I illustrate how mathematicians can make their own subfields more accessible by examining the User’s Guide Project in algebraic topology. Participants in this project wrote an expository paper to serve as a guide to one of their research papers.

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[3] I am assuming that there is a unique research community associated with a subfield. This is an idealizing assumption, since there may be multiple distinct research communities associated with a given subfield.

The reward structure of mathematics, however, gives mathematicians credit primarily for proving new and important theorems. When writing expository papers, mathematicians are not proving new theorems and thus their efforts to make their work more accessible are not directly rewarded. As mathematicians need to accumulate credit to get hired and to be promoted, they may thus choose to pursue more rewarding activities, like proving new theorems, instead. Under such circumstances, it is unlikely that mathematical subfields will be made widely accessible and so it will remain difficult for mathematicians to make connections between them.

The current reward structure of mathematics is thus not always beneficial, just like the reward structure of science is not always beneficial either (see e.g. Heesen (2018) and Zollman (2019)). Consequently, the reward structure of mathematics should be changed in order to give mathematicians more credit for making their work accessible.

The plan of this paper is as follows. In section (2) I show that making connections to other subfields is often important to a research community's progress. I then argue that increasing specialization can make it difficult for mathematicians to find these connections. In section (3) I suggest that these problems can be ameliorated if mathematicians work to make their own subfield more accessible to others and illustrate how this can be achieved using exposition with the User's Guide Project. In section (4) I argue that the reward structure of mathematics does not give mathematicians sufficient credit for making their own subfields more accessible via exposition and warn that mathematicians may thus decide not to undertake this important work. I then suggest that the reward structure of mathematics be changed to encourage mathematicians to make their subfields accessible. Otherwise continued specialization may sometimes stymie the progress of mathematical research communities.

2. Specialization, connections and progress[4]

2.1 Making connections between subfields promotes and is sometimes required for mathematical progress

Many mathematicians have explicitly acknowledged that making connections to other subfields stimulates and is sometimes necessary for a research community to make continued progress. Below are some illustrative quotations:

> Making unexpected connections between apparently unrelated areas of mathematics often lies at the heart of solving a great problem (Stewart, 2013, p. 44).
>
> [M]any of the results I proved [in abelian group theory] that most impressed other mathematicians were obtained very cheaply, by using results that I knew from fields that

[4] A modified and condensed version of the argument in this section also appears in Morris (2020b).

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> seemed far removed from abelian group theory and which most abelian group theorists either weren't aware of, or at least had never paid much attention to (Lady, n.d.)

> One characteristic that most of the failed attempts [to prove the Poincaré conjecture, a now proven theorem in topology] share is a reliance on topological arguments. But, noted John Morgan of Columbia University, “It seems like this problem does not succumb to that type of argument.” Rather, he said, one needs tools from outside topology [...] (Jackson, 2006, p. 897).

> The mystery as well as the glory of mathematics lie not so much in the fact that abstract theories do turn out to be useful in solving problems, but in that wonder of wonders, in the fact that a theory meant for one type of problem is often the only way of solving problems of entirely different kinds, problems for which the theory was not intended. These coincidences occur so frequently that they must belong to the essence of mathematics (Rota, 1991, p. 488).

Such quotations are admittedly anecdotal. However, the general point they raise, that making connections between subfields stimulates and is sometimes necessary for making progress on a problem, is further supported by a study carried out by the Committee on the Mathematical Sciences in 2025. The study aimed to assess the status of research in the mathematical sciences, to evaluate their effects on a variety of other important fields, and to make recommendations to ensure that they continue to flourish (National Research Council, 2013, pp. 16–17).

One of the themes that emerged from the study was the 'increasing importance of connections for mathematical sciences research' (National Research Council, 2013, p. 93 section heading). The authors summarize their findings as follows:

> Based on testimony received at its meetings, conference calls with leading researchers [...], and the experiences of its members, the committee concludes that the importance of connections among areas of research has been growing over the past two decades or more. This trend has been accelerating over the past 10-15 years, and all indications are that connections will continue to be very important in the coming years (National Research Council, 2013, p. 93).

More specifically, the authors highlight that connections to different subfields are necessary for a research community to make mathematical progress:

> The discipline itself—research that is internally motivated—is growing more strongly interconnected, with an increasing need for research to tap into two or more fields of the mathematical sciences (National Research Council, 2013, p. 93).

The authors also present a wide variety of examples within core mathematics, physics and kinetic theory to illustrate how connections to other subfields help a research community to make progress. They mention, for example, connections between random matrix theory, combinatorics and number theory, commutative algebra and statistics, as well as geometry and quantum field theory. The authors then observe:

> The sorts of connections exemplified here are powerful, because they establish alternative modes by which mathematical concepts may be explored. They often inspire further work because surprising connections hint at deeper relationships (National Research Council, 2013, p. 96).

As one concrete example of connections between fields leading to new discoveries, I will mention Jean Bourgain's (1999) work on the Kakeya problem. This is a geometric problem but Bourgain improved upon existing work by using arguments from additive number theory (Łaba, 2008). His work was important not just for its impact on the Kakeya problem, but for introducing mathematicians working on this problem to tools from different subfields. In fact, Łaba suggests that without Bourgain's work connecting different subfields, 'The Green-Tao theorem and many other developments might never have happened' (Łaba, 2008, p. 91). Making connections to other subfields is thus important to the progress of mathematical research communities.[5]

2.2 Increasing specialization makes it hard to find connections between subfields

We have seen, above, that making connections to other subfields promotes, and is sometimes required, for a mathematical research community to make progress. But what is required to make such connections? Bateman and Hess (2015) explain that in the context of science:

> [C]onducting research that bridges two or more knowledge domains requires acquiring and applying different content and methodological expertise, or at least considering multiple styles of thought, research traditions, techniques, and discipline-specific terminologies that are difficult to translate across domains (Bateman & Hess, 2015, p. 3654).

This applies to mathematics as well. In other words, making connections to other mathematical subfields requires, at the very least, that mathematicians are familiar with the general approaches taken by those subfields, as well as with their specific resources, including, for example, their concepts, methods, theorems and proofs. Increasing specialization makes this a difficult task, however. In particular, different subfields diverge significantly as a result of specialization, each

[5] As a reviewer notes, however, the idea that discoveries can be made by finding connections between subfields is a modern one. For example, Aristotle maintained that arithmetic theorems could not be used to prove geometric results (see e.g. (Mendell, 2019, sec. 5)).

with their own increasingly technical set of resources. This plausibly makes it difficult for mathematicians to become familiar with subfields other than their own.

For example, mathematicians Don Larson, Kristen Mazur, David White and Carolyn Yarnall remark:

> Over the past 100 years mathematics has become increasingly specialized, and currently it can be difficult to understand research in fields other than your own (Larson et al., 2020, p. 412)

They note that it is difficult to become familiar with algebraic topology in particular:

> [Algebraic topology's use of] overly-technical mathematical machinery [...] makes it hard for fledgling algebraic topologists to get situated and hinders those outside the field from leveraging its tools and accurately judging topological work (Larson et al. 2020, page 418).

The difficulty of learning different subfields has also been noted by other mathematicians. For example, the mathematician William Thurston comments:

> Basic concepts used every day within one subfield are often foreign to another subfield. Mathematicians give up on trying to understand the basic concepts even from neighboring subfields, unless they were clued in as graduate students (Thurston, 1994, p. 166).

The mathematician Felix Browder reportedly echoed similar complaints:

> [He] also warned that mathematics was becoming too specialized and siloed, such that experts in different subfields, not to mention in different sciences, no longer shared a common language or mutual understanding (Lin, 2016).

Quotes such as these are admittedly anecdotal, but they do offer some support to the intuitive idea that specialization, and the corresponding development of increasingly technical tools in each subfield, can make it difficult for researchers in one subfield to become familiar with other subfields.[6] And if it is hard for researchers to become familiar with different subfields, it will be difficult for them to find connections between them.

[6] A potential avenue for future work would be to conduct a survey of a large number of mathematicians to see if the attitudes reported in these quotes are indeed widely held, either within specific subfields or more generally across all of mathematics. If so, this would lend more support to the claim that increasing specialization makes it difficult to become familiar with new subfields.

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3. What can be done?

3.1 Making it easier to find connections between subfields

So far, we have seen that (i) making connections to other subfields promotes and is sometimes necessary for a mathematical research community to make progress; (ii) increasing specialization can make it difficult for members of a research community to find connections to other subfields. Increasing specialization can thus sometimes hamper the progress of many mathematical research communities. Nonetheless specialization has important benefits, so preventing further specialization is not desirable. What can be done to improve this situation?[7]

As we have seen, specialization can make it hard for members of a research community to find connections between subfields by making it difficult for mathematicians to become familiar with subfields other than their own. One way for members of a given research community to avoid this difficulty is thus for them to avoid becoming familiar with other subfields themselves and to collaborate instead. Indeed the study carried out by the Committee on the Mathematical Sciences in 2025 emphasized the increasing importance of collaboration:

> [T]he various subfields in mathematics depend on one another in ways that are unpredictable but almost inevitable, and so more individuals need to collaborate in order to bring all the necessary expertise to bear on today's problems (National Research Council, 2013, p. 94).

However, collaboration still requires mathematicians to become familiar with other subfields, at least to some extent. For example, suppose a mathematician is working on a problem in their own subfield and starts to think they may need to make connections to a different subfield to help them solve it.[8] If they want to collaborate to find such connections, they'll need to have an idea of where to look to find a suitable collaborator. That is to say, they'll need to have an idea of which other subfields it might be helpful to make a connection to. And this means they'll need to have some basic familiarity with other subfields, knowing the general approach and some of the important resources used in other areas. Indeed, if a mathematician doesn't have basic familiarity with other subfields, their situation seems parallel to that of the scientists, engineers and medical researchers described by the Committee on the Mathematical Sciences in 2025:

> Often, scientists, engineers and medical researchers do not know what mathematics and statistics are available that might be relevant to their problem, and they do not know

[7] I also discuss this question in Morris (2020b).

[8] This happens in practice. The Committee on the Mathematical Sciences in 2025 reports: 'Fields in the mathematical sciences are mature enough so that researchers know the capabilities and limitations of the tools provided by their field, and they are seeking other tools from other areas' (National Research Council, 2013, p. 97).

> whom to turn to. Likewise, mathematical scientists are often sitting on expertise that would be just what is needed to solve an outside problem, but they are unaware of the existence of these problems or of who might possess the relevant data (National Research Council, 2013, p. 100).

Because of this, I will not focus on collaboration as a strategy for avoiding the problem caused by hyperspecialization. Instead I will focus on making it easier for mathematicians to become familiar with other subfields, which will help mathematicians to make connections between them, either on their own or collaboratively. In the next subsection, I will examine the User's Guide Project, which serves as an example of how this can be achieved via exposition. It is not the only example of mathematical exposition, however. In fact, online sources,[9] especially MathOverflow[10] and mathematical blogs, such as Terence Tao's,[11] provide much needed exposition, as do graduate textbooks.

3.2 The User's Guide Project

The User's Guide Project, created by Luke Wolcott, was explicitly designed to make research in algebraic topology more accessible. This project was active for 3 years, from 2014 to 2017, and generated 13 peer-reviewed "user's guides" to accompany 13 research papers in algebraic topology (Larson et al., 2020, pp. 412–413). A "user's guide" is a guide to its accompanying research paper, written by the same author, with the aim of 'providing further exposition and context for the results' (Malkiewich et al., 2015, p. 187). Although they are expository, user's guides are aimed at mathematicians with sufficient training to read the corresponding research papers (Enchiridion, 2015). Indeed, the User's Guide Project website states 'We envision the source paper and the user's guide being read side-by-side' (Enchiridion, 2015).

Each user's guide is structured in the same way and includes the following four sections: (1) Key insights and organizing principles; (2) Conceptual metaphors and mental imagery; (3) Story of the development; (4) Colloquial summary. Section (1) answers the question of 'What's really going on?' (Enchiridion, 2015) in the paper. In particular, it makes the background of the paper explicit, by identifying key ideas and important principles, numbering them like definitions and theorems are numbered in research papers, and in some cases includes a discussion of the main proofs in the corresponding research paper.

As an example of the type of content found in section (1), consider David White's user's guide written to accompany his (2014) paper. In section (1) of the user's guide, he explains the importance of the notion of model category: 'Having the structure of a model category allows for the tools of homotopy theory to be applied, and in this way parts of homological algebra,

[9] I am grateful to an anonymous reviewer for reminding me of these sources of exposition.
[10] https://mathoverflow.net/
[11] https://terrytao.wordpress.com/

algebraic geometry, representation theory, logic, graph theory, and even computer science can be viewed as special cases of homotopy theory' (White, 2015, p. 2). This is then incorporated into White's Organizing Principle 1.2 'In settings where one has a notion of weak equivalence or something like a homology theory to compress complicated information into simple information, one should try to build a model structure so that the tools of abstract homotopy theory can be applied' (White, 2015, p. 3).

Section (2) of a user's guide documents the author's ways of thinking about the subject matter, including any analogies, metaphors or images that are particularly helpful for grasping the results of the paper. This sort of information is often provided in conference talks, but is omitted from research papers (Enchiridion, 2015). For example, in the user's guide accompanying her (2015b) research paper, Carolyn Yarnall describes how she thinks about certain mathematical objects called Mackey functors as lattices and notes that thinking of them like this 'is especially useful for performing computations' (Yarnall, 2015a, p. 6). She also notes that it can sometimes be helpful to look at mathematics differently: 'While we might often describe building the tower from the bottom up, I find it useful to imagine beginning at the top and working down …' (Yarnall, 2015a, p. 8).

Section (3) of a user's guide answers questions such as 'Where and when did these ideas and results arise? What is the logistic and psychological context of this paper? What was the process of arriving at these particular statements and proofs?' (Enchiridion, 2015). White's user's guide focuses on the human aspect of the development of his paper, describing what the process of researching a PhD thesis in mathematics looked like in his case. Other user's guides include mathematical information that was crucial to obtaining the results. For example, Yarnall describes the content of section (3) of her user's guide as follows: 'While much of the early work on this project was essential to determining the final result, these computations play no part in the actual proof and thus all discussions concerning this part of the process were omitted from the paper. Here, I will lay out the early ideas and useful notions that ultimately lead to the main result' (Yarnall, 2015a, p. 12).

Finally section (4) of a user's guide provides a summary of the paper aimed at non-mathematicians. For example, White uses simple examples to illustrate the core concepts from his paper and Yarnall draws analogies between the tools used in her work and medical equipment.

In their paper reflecting on the User's Guide Project, Larson et al illustrate how user's guides can help make algebraic topology more accessible with a hypothetical example. They describe a case in which an important research paper is posted on arXiv, the mathematical preprint server, with an abstract whose very first sentence contains technical terminology, for example 'Let *E* be an *E*-infinity ring spectrum' (Larson et al., 2020, p. 419). They continue:

> Suppose [...] that a graduate student in algebraic topology and a researcher from another field both see the paper on the arXiv and suspect the paper might be helpful for their own work. Ideally, the paper would be accepted to the journal, the grad student would find the paper readable and be able to leverage it for her dissertation, and the outside researcher would quickly surmise the connections with her field and use them to build a bridge to algebraic topology. And yet, it is quite conceivable that none of these things would occur, because the concept of an *E*-infinity ring spectrum is a prime example of technical mathematical machinery (Larson et al., 2020, p. 419).

In other words, this is a particular instance of specialized terminology and tools making a paper inaccessible both to new researchers in algebraic topology and researchers in other mathematical or scientific fields. However, as Larson et al note, the situation could be improved if the author also wrote a user's guide. They especially emphasize the importance of sections (2) and (3) in such a document:

> But, what if the author also wrote a user's guide to accompany the paper that provided intuition for how to think about an *E*-infinity ring spectrum in Topic 2: Metaphors and imagery, like the way [13] does for Greek letter elements in the stable homotopy groups of spheres? What if this user's guide chronicled how these intuitions were obtained in Topic 3: Story of development, like the way [20] does for equivariant bundles? Then the guide could come to the rescue in a couple of ways. For one, the grad student and outside researcher could leverage the user's guide to more easily trudge beyond the author's daunting opening sentence. For another, the author could revamp their abstract using the intuition provided in the user's guide, thereby making the abstract more inviting and favorable with the wide-audience journal editor (Larson et al., 2020, p. 419).[12]

I'll add that sections (1) and (4) of a user's guide can also help make the paper more accessible. First, if the graduate student wanted to work through the details of the paper, section (1) would make this task easier by giving her a roadmap so she could see where the author was going and how they would get there. If the outside researcher was from a very different mathematical subfield or was a scientist rather than a mathematician then section (4) may help her to get a quick idea of what the paper is about and how relevant it is to her own research. This in turn can help her to decide if it would be worthwhile to spend more time studying it in detail.

Unfortunately the User's Guide Project is no longer active, since its founder is no longer in academia (Larson et al., 2020, p. 413). However, the expository papers it generated are still available and continue to help mathematicians become familiar with work in algebraic topology.

[12] References [13] and [20] in the quotation are (Larson, 2017) and (Merling, 2015), respectively.

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4. Accessibility and the reward structure of mathematics

We have seen in the previous section that one way to make a subfield more accessible, and thus easier for others to become familiar with, is by engaging in expository writing. As I will argue, however, the current reward structure of mathematics does not incentivize mathematicians to undertake expository work.[13] Thus I suggest that the reward structure be changed so as to encourage mathematicians to make their subfields more accessible in this way.

4.1 Credit for theorems

The reward structure in mathematics primarily gives researchers credit for proving important theorems. For example, Thurston refers to 'a common currency that many mathematicians believe in: credit for theorems' (Thurston, 1994, p. 172). However, expository work, such as the User's Guide Project for algebraic topology, does not involve proving new theorems. Mathematicians thus receive little reward for this kind of work. In fact, Larson et al highlight this was a problem encountered by participants in the User's Guide Project: 'When asked if their current institution values the fact that they wrote the user's guide (in the sense of promotion and tenure) only one author said yes while six authors said no and four were unsure of how the project would be perceived' (Larson et al., 2020, p. 424).

Other mathematicians and even professional societies have noted the lack of appropriate reward for expository writing. The Society for Industrial and Applied Mathematics (SIAM) have a journal dedicated to publishing review articles, which are a kind of expository writing, but their executive director Jim Crowley notes that 'such articles seem to be more highly regarded (and hence rewarded) in other disciplines' (Crowley, 2014).[14] Further the mathematician Steven Krantz notes 'American mathematics does not prize expository work as highly as do other American sciences, nor as highly as does European mathematics' (Krantz, 2005, p. 18). While Krantz does temper this somewhat by citing Rota who titles a section of his 1997 paper 'You are more likely to be remembered by your expository work' (Rota, 1997a, p. 1999), being remembered does not necessarily entail being rewarded. Indeed, Rota also remarks that 'high-caliber expository work is more exploited than rewarded by the mathematical community' (Rota, 1997b, p. 175). While these quotes are anecdotal, they do agree with the small survey of 11 of the User's Guide Project authors reported above.[15]

[13] I make a condensed version of this argument in Morris (2020b).
[14] In his blog post, Crowley (2014) also announces a new prize awarded by SIAM to reward exposition.
[15] Future work could collect data about the tenure and promotion requirements of different institutions to obtain more concrete information about how exposition is or is not valued.

4.2 Making changes

If members of a research community make their subfield more accessible by engaging in expository writing, it will be easier for members of other research communities to become familiar with it. This, in turn, will make it easier for mathematicians working in other subfields to make connections to it. It is thus important that mathematicians are appropriately rewarded for making their subfields accessible to other researchers in this way. Otherwise, they may be less likely to undertake such work. After all, they need to obtain credit to get jobs and be promoted, so if they are not rewarded for making their subfields more accessible to others, it will be less attractive to them than other work. And if members of a research community do not undertake this work, it will be difficult for mathematicians in other research communities to make connections to their subfield, potentially slowing progress.

The reward structure of mathematics should thus be changed to directly reward members of a research community who work to make their subfield more accessible via exposition. This will involve changes at the level of academic institutions, professional societies and funding agencies. Below I will sketch some suggestions for potential changes. These suggestions are meant to start discussion rather than serve as concrete proposals, so they are not fully developed.

At the institutional level, a first step would be for exposition to be more highly valued in hiring and tenure and promotion decisions. I thus strongly agree with Larson et al:

> We call upon the mathematics community and institutions of higher learning to value expository projects like the User's Guide Project in the contexts of hiring, promotion and tenure. It is frustrating that efforts to illuminate mathematics often do not receive official support or recognition, while research articles that only a handful of mathematicians comprehend are valued (Larson et al., 2020, pp. 424–425).

Professional societies like the American Mathematical Society and the Mathematical Association of America can help mathematicians to see the value in exposition by highlighting how and why it is important and training them to write it well. This will hopefully help the mathematical community become more willing to reward it. Further, as publication venues for certain kinds of expository pieces are somewhat slim (see e.g. (*Which journals publish expository work?*, n.d.)), professional societies could create new journals to fill this gap. Having publication venues backed by professional societies could help ensure expository work is taken seriously and is more likely to receive credit in hiring, promotion and tenure decisions. Finally, professional societies could advocate for further funding to help support expository work.

Funding agencies like the National Science Foundation could help to make mathematics more accessible by requiring funded research projects to produce accompanying expository documents as well as research articles. Since many research active mathematicians would then have to engage in expository writing, perhaps this would increase its prestige and help make mathematicians more willing to reward it. If funding agencies are made aware of the potential importance of exposition to mathematical research, then they may also be willing to explicitly solicit and fund some projects that have a significant expository component.

5. Conclusion

While specialization has important benefits, it also has a significant side-effect: it can make it difficult for mathematicians to find connections between subfields. However, making these connections can sometimes be crucial to a mathematical research community's continued progress. One way to address this is to have mathematicians make their subfields more accessible to mathematicians working in other fields, such as by engaging in exposition. However, the current reward structure of mathematics does not appropriately credit mathematicians who engage in such work. It should thus be modified so that work to make subfields accessible is more highly valued, with changes being implemented at the level of departments, professional societies and funding agencies.

My purpose in this paper has been to identify a drawback of increasing specialization and to point to a potential solution at a high level. I have thus provided very little in the way of concrete advice about changing the reward structure of mathematics. Further work is thus required and should be interdisciplinary. Philosophers of mathematics have already emphasized the importance of exposition (see e.g. (Corfield, 2012)) and may provide further insights about other ways of making mathematics more accessible (see e.g. (Avigad, 2020; Morris, 2020a)). Work done in mathematics education about how to disseminate knowledge from teachers to students may also transfer to the situation of making a subfield more accessible to other professional mathematicians. Finally, public policy may be able to identify and evaluate specific changes to the reward structure in mathematics.

Acknowledgment
I would like to thank the anonymous referees for their valuable feedback and suggestions. I would also like to thank Erich Kummerfeld for helpful conversations about earlier drafts of this work.

Disclosure statement
The author reports no potential conflict of interest.

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