The Technology of Analogical Models: Irving Fisher's Monetary Worlds

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Mary Hesse's well-known work on models and analogies gives models a creative role to play in science, which rests on developing certain analogical properties considered neutral between the two fields. Case study material from Irving Fisher's work (*The Purchasing Power of Money*, 1911), in which he used analogies to construct models of monetary relations and the monetary system, highlights certain omissions in Hesse's account. The analysis points to the importance of taking account of the negative properties in the analogies and to certain differences between "ready-made" analogies (models of systems based on existing analogical structures) and "designed" analogies (models built up from separate analogical features).

1. Introduction. Mary Hesse's well-known work on models and analogies (*Models and Analogies in Science*, 1966) suggests that models play a creative role in science. Her account hinges on the use of analogies between a preexisting structure in a particular field and another structure in the new field to suggest possible theory developments in the new field. In using analogies from one field as a model for another field, we make a comparison between the set of properties of the analogy in the two fields: (a) positive analogies (exactly the same or like properties); (b) neutral analogies (likeness not known); and (c) negative analogies

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(unlike properties). Recall, from Hesse, that it is the *neutral analogies* in the model which form the "growing points" for the theory, for they suggest new ideas to the scientist in the receiving field. It is by suggesting specific ways to extend the theory that models based on analogies are potentially creative for theory building. Hesse says little about the role of any *negative analogies* involved, or their relationship to the positive and neutral analogies.

Case study materials from Irving Fisher's work, in which he used analogies to construct models of monetary relations and the monetary system, conform in certain respects to Hesse's account, but the cases also highlight certain omissions, particularly in the role played by negative analogies. The first case I want to consider concerns a "ready-made" analogy, one in which properties are linked together within a preexisting structure that is carried over completely. The neutral properties allowed Fisher to develop a new theoretical account. However, there was also a negative element in the analogy between the two structures, one which was not independent of the positive and neutral characteristics and so threatened to undermine earlier theoretical results supported by the positive elements of the analogy.

The second case concerns not a ready-made analogy but a "designed" analogical model built up from analogical properties. The workings of this model, and the range of demonstrations it achieved, depended on certain adjustable features of his design. On careful analysis, these features turn out to have been resting on a negative analogy. As we shall see, in both cases the negative analogies were used constructively.

2. The Ready-Made Analogy Of Monetary Relations. Irving Fisher (1867–1947) was the foremost American monetary economist of his day. He was obsessed with the problem of stabilizing the purchasing power of the dollar, and in pursuing this aim (in his *The Purchasing Power of Money*, 1911) developed both the theoretical and applied policy analysis of monetary economics as well as innovating new measurement systems.

The first case discussed here concerns Fisher's adoption of the mechanical balance as an analogy for the equation of the aggregate exchanges in an economy. Fisher's equation of exchange (which forms the building block for much subsequent monetary economics) is an accounting relation describing the exchange between the total goods and the money available in an economy in a given period. Fisher took these to be equal by definition, for any individual exchange represents equal value of money for goods, and the general equation is the aggregation of individual exchanges (with adjustment on the money side for the cir-

culation of money, known as "velocity"). Fisher presented this accounting relation in three forms. First, an "arithmetic illustration":

$5m money × 20 times a year (circulation)
= (200m loaves × $.10 a loaf)
+ (10m tons coal × $5 a ton)
+ (30m yards cloth × $1 a yard);

then in the form of an illustration of a mechanical balance shown in my Figure 1 (where I have added labels to his Figure 3); and finally in the form of an algebraic relation: MV = PT.

All three "illustrations" were used to demonstrate the quantity theory of money: a proportional cause-effect relation from changes in the amount of money in the system (M) to the average level of prices (P), given that the circulation of money, "velocity" (V), and the quantities of goods traded, "transactions" (T), are held constant. In order to support his use of the mechanical balance as an analogical model of the equation of exchange in this way, Fisher relied on two positive analogies[1] between the mechanical balance and the arithmetic version of the accounting relation: (1) the four elements in the arithmetic illustration of the accounting identity can easily be transposed onto the arms and weights of the balance; and (2) the equality in the accounting relation can be mapped onto the balance in equity of the mechanical model.

The theory development in this case depended on an important neutral analogy: namely that the mechanical balance has the physical prop-

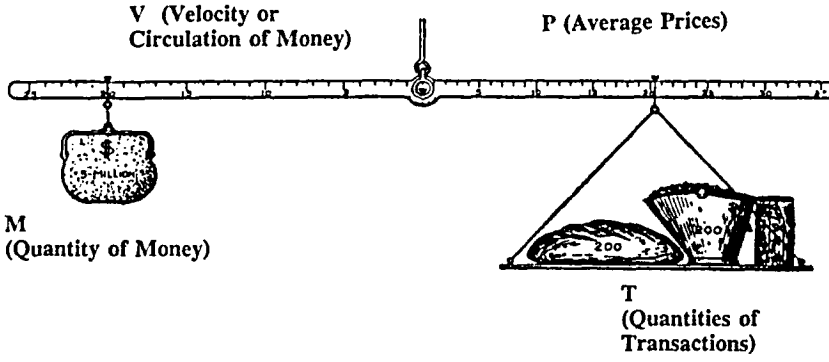

Figure 1: Fisher's Mechanical Balance Illustration

1. There were at least two more positive analogies supporting the usage of the mechanical model. The model, its extensions and its various functions (such as developing measurement concepts) are discussed fully in Morgan forthcoming.

erty that it oscillates in the process of coming back to rest. Fisher used this property of the balance to motivate his discussion of the transition effects of a change in any of the four elements in the equation of exchange. Instead of returning immediately to balance, the transition oscillations could be portrayed as an economic cycle based on changes in credit (money). Thus, the neutral aspect of the analogy allowed Fisher to develop a theoretical account of an economic world which behaved like a mechanical balance, in which both the quantity theory of money and the credit cycle are integrated within the same mechanism. This extended the monetary theory of the time for, although neither the quantity theory nor the credit cycle theory were new, their integration within the same structure was new, and this followed directly from his use of the analogical (mechanical) model.

However, there were also negative properties lurking in the analogy between the mechanical balance and the arithmetic or algebraic versions of the equation of exchange. First, as soon the oscillation process begins, the strict accounting identity of the equation of exchange is destroyed. Second, although the balance comes to rest after the oscillation, it can come to a point of rest out of equity; i.e., at a point at which the accounting identity does not hold. For example, if some event increases the M weight on the balance, the mechanism will come to rest at some point, but not at equity: it obeys the laws of mechanics. Although we can see how to bring it back to balance, there is no requirement that this be done. In the accounting case, the two sides of the exchange must always balance. There has to be some matching change in another item to retain the identity: it is governed by the rules of accounting.

This difference is critical, not only because these two negative properties threaten to undermine the basic relationship in the equation of exchange—namely its continued equality—but because they also threaten Fisher's earlier results on the quantity theory of money. This is because the quantity theory (that P must change when M changes, and by exactly the same proportional amount, provided V and T are constant) depends upon the accounting identity constraint holding.

How did Fisher solve this? Effectively he reintroduced the accounting identity to hold over the mechanical balance as a kind of higher order constraint; and redefined the nature of that identity. He assumed that the identity holds not exactly all the time, but in the nature of a tendency of the elements in the system towards an equilibrium at the equity point of balance. So the accounting identity is reestablished, but is reinterpreted as an equilibrium tendency, and it no longer constrains so tightly as before: the balance never is exactly in equity. This substantially changed the nature of the equation of exchange.

The problem arose here because the properties which go to make up the analogy are not independent of each other. By moving to the mechanical balance, Fisher gained the advantage of the oscillation process, which at first seemed a safe neutral point upon which interesting theoretical developments could rest. But this neutral property was associated with two negative properties, which created problems for Fisher, for they undercut the quantity theory claims and the basis of the equation of exchange which both relied on the initial positive property of the analogy (equity in balance). Because the negative, neutral, and positive analogies were not independent, Fisher could not just take the positive and neutral analogies and brush aside the negative ones.

This case, in which negative analogies are correlated with neutral and positive ones, clarifies a difference in the role played by analogical reasoning and analogical models. The former, analogical reasoning, relates to the positive properties providing a basis for inferring that we are justified in adopting the analogy as a model for the new field and exploring its neutral properties. But the power of analogical models lies in their ability to help us investigate and extend our knowledge of the economic system, and here it is the not just the neutral but also the negative properties which are important. The neutral analogies, as Hesse suggests, have the potential for creative theory building. The negative properties, can also be turned to good account. As this first case shows, one strategy is to adapt such negative properties to the model in the new field and to reconfigure the model in such a way as to neutralize the negative qualities while still defining an economic system. The process of making the neutral and negative properties work, learning what will fit and what has to be adjusted in the model for the new field, is one of the ways in which the modeling process extends knowledge.

3. The Designed Analogical Model Of The Monetary System. The second case concerns Fisher's analogical model of the monetary system. This is no "ready-made" analogy with preexisting structure to be carried over (as with the mechanical balance which was taken over in its entirety). Rather, as is common with analogical models, Fisher built up his second model by choosing a set of parts (properties, elements, relations) which had analogical qualities and putting them together.

In the previous case, Fisher had explored the effects of changes in M, V, P, and T within the equation of exchange. Here he assumes that the quantity theory relation (between the amount of money, M, and the price level, P) holds throughout by carefully embedding that theoretical relationship in the design of his new model. His problem now is to explore and demonstrate how the amount of money is determined by the "natural" elements of supply and demand for gold and silver

and by the "institutional" arrangements and controls set by the government. He did this by designing a model, consisting of a laboratory equipment set-up, and assuming that the economic principles which determine the value of money and its movements around the equipment are analogous to hydraulic principles[2] at work on money conceived as a liquid. One illustration of the model is shown here as my Figure 2 (using Fisher's Figure 7 with my labels and inventory).

Only a full analysis of the various different diagrams he used reveals the complexity of Fisher's model. Briefly, the design incorporates at

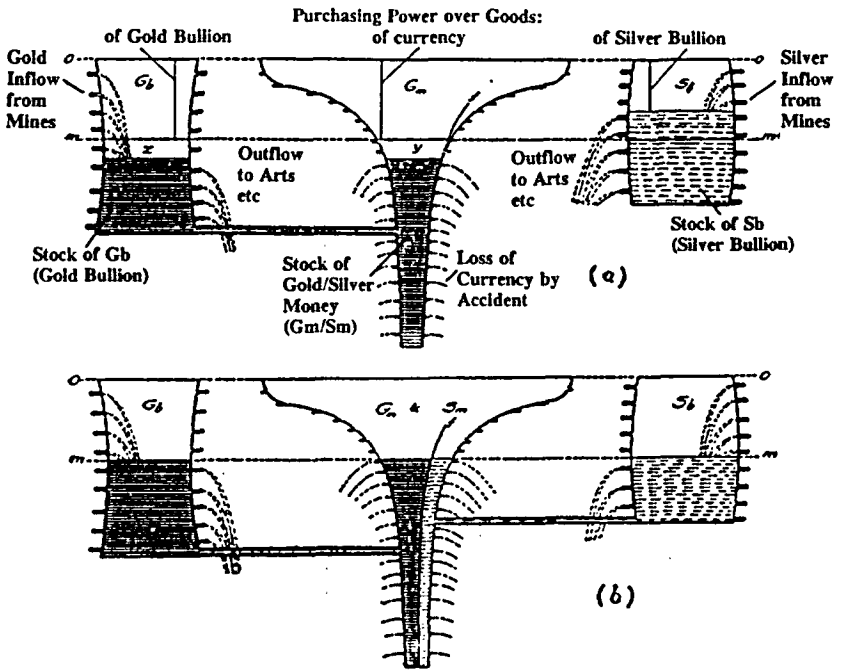

Figure 2: Fisher's Hydraulic Model of the Monetary System
(1) Purchasing Power increases as Stocks decrease in Gm and Gb
(2) If Gb Stock rises, Purchasing Power falls (prices have risen)
(3) Connected and open pipes allow control of bullion flows into Gm/Sm vessel ensuring equal purchasing power of bullion and money
(4) Shape of Gm/Sm: if Money Stock doubles, Purchasing Power halves by Quantity Theory
(5) If volume above mean level (m) in Sb vessel, and pipe connected and open, silver bullion flows into currency vessel and pushes gold (partly) out of circulation (as in above case of bimetallism)

2. Fisher's thesis (written for Willard Gibbs and William Sumner at Yale) involved building a real hydraulic model of a general equilibrium economic system.

least three sorts of positive analogies: (1) between the hydraulic principles governing the behavior of liquids and the laws of equal value governing the behavior of money; (2) between laboratory equipment determining the inflow and outflow of liquids (metals) into bullion, coinage, and other uses and the marginalist economic principles determining production and consumption; and (3) between the controlling actions of the laboratory scientist over the design, arrangement and working of the equipment and of the government monetary authorities through legal and direct controls over currency.

This combination of positive analogical characteristics between the hydraulics and the economics of money are used to demonstrate well-accepted economic theories (e.g., law of equal value between currency and bullion), empirical laws (e.g., Gresham's Law), and the institutional and economic conditions under which the legal arrangement known as bimetallism (the acceptance of both gold and silver currency as legal tender) could be made to work in practice. All of these demonstrations are achieved within the world defined by the model: the monetary economy depicted as a laboratory system.

In achieving these various demonstrations, the workings of his model depended not only on all three of the analogical properties outlined above, but also on certain adjustable features of his model design. For example, Fisher shows how the position of the silver vessel is critical: if the level of liquid in the silver bullion vessel (Sb) is below the level of gold in the currency vessel (Gm), no silver liquid will flow into the currency vessel, and bimetallism will not occur regardless of institutional arrangements (whether the pipe is open or closed). If the liquid level is above the currency (as in Figure 2a), and the pipe is open, silver will flow through. But then, whether the gold is all pushed out by Gresham's law (the empirical law that bad money always drives out good) or bimetallism occurs (dual circulation of both metals, as shown in my Figure 2b above) depends on the relative volumes above and below the *mm* level of silver in the Sb vessel compared to both gold in Gb and currency in the (Gm & Sm) vessel.

We have been alerted to the importance of where the vessels are drawn in relation to each other, and to the relevance of volumes as an alternative adjustable (determining) characteristic. But there is also the issue of the shape of the bullion vessels. This is important because the volume above/below the mean level can be altered not only by the position of the vessels but also by their shapes. For the present, I will label these three adjustable characteristics of the bullion flasks (shape, position and volume) as neutral properties and observe that it is by altering these characteristics that Fisher can "adjust" his model equipment to "demonstrate" both the various outcomes of alternative mon-

etary arrangements and some standard empirical observations about the "natural" behavior of money within the same designed structure.

How does such adjustability arise? In the process of designing or building analogical models out of a set of parts, there is always a certain degree of open choice about the bits chosen and how to put them together. It is one of the advantages of such designed analogical models that the model builder can choose those features with positive (or at least neutral) properties, to make the demonstrations work, and try to avoid the negative properties. Fisher could choose to have certain volumes in the vessels, and could choose to draw them in certain positions and with particular shapes. But this same openness of choice means that these elements are potentially unconstrained. To a greater extent than in ready-made analogies, the model-builder of a designed model can set his own constraints. This may create problems, for while a model which is unconstrained might be able to address lots of questions, it might not provide very concrete answers.[3] Indeed, the freedom for adjustability built in to Fisher's model makes his choices appear almost arbitrary, for he can alter shape, position and volume at will to provide the answers he seeks. Are there any constraints in the analogy which limit his choices and thus give meaning to them?

Given the very mixed analogical character of Fisher's model, the constraints might come in from either the economic or the physical side. The physical principles of hydraulics, and the economic principles of equal values (mentioned above) give little in the way of constraints, for there are wide possibilities in volumes, shapes, and positions which still allow these operative principles at work in the model to demonstrate Fisher's various results. Both the physical laws and the economic laws allow these elements to be genuinely adjustable.

But, we must also ask: what are the analogical economic counterparts of the vessels and of the volumes and how are these constrained? The currency flask (Gm & Sm) contains the specially partitioned element of metal that has been pressed into currency: the flask's shape has been carefully chosen to represent the quantity theory of money; and its volume can be controlled by the policy of the monetary au-

3. In the case of a ready-made analogy, a certain level of constraints is built in by virtue of the fact that the structure and relations of the system are already decided in the other field. It is important to note, however, that the problem of unconstrained flexibility *need not* arise with designed analogical models. In choosing the parts, the elements may be as well tied together as in the ready-made analogy. For example, compare with Morrison's (1992) analysis of Maxwell's vortex aether model, where the design of the idle wheels are chosen to have certain specific properties (by analogy with certain real elements) and their role in the structure is also well defined—they do not produce different results in different runs of the model.

thorities via the connecting pipes. It is constrained on both accounts to share positive properties with economic theories and government control of money. By contrast, the shape and position of the bullion flasks (Gb and Sb), and their volumes, appear to be completely unconstrained by the economic side of the model. Fisher mentions no justification for the slight differences in shape between the gold and silver bullion vessels and it is difficult to see what such differences in the monetary world these might refer to. The volumes refer by analogy, in both theory and in the real world, to stocks of solid gold and silver bullion. These amounts are not necessarily inherently immeasurable. Indeed there were heroic attempts in the late 19th century to make such measures in terms of weights of gold and silver mined. But these measured weights could not be mapped into anything like volumes inside flasks, because one would have to make arbitrary translations of the weights into arbitrarily defined volume spaces. Fisher tried briefly, but unsuccessfully, to outline such a mapping of weights into volumes, and so failed to provide any empirical constraints on the way volumes were represented in his model.[4]

My analysis suggests that the flexible (and even arbitrary) character of these two bullion flasks and their volumes stems, in fact, from a broader negative analogy. In introducing this case, I noted a general positive analogy (the third in my list) between the laboratory scientists designing, arranging, and controlling the experiment and the government designing institutions and controlling money. Both laboratory scientists and economic authorities can control the flow between the vessels, but under a metallic monetary standard (such as in the 19th century) neither economic theorists, nor the government, has any real design control over the bullion flasks, their shape, position, and volumes. In these respects, the level of design control used by the model builder (Fisher) had no counterpart at that time by the monetary authorities either in theory or in practice. The apparent arbitrariness in Fisher's design really points to a hidden negative analogy between the scientist's general level of design control compared with the more restricted level available to the monetary authorities. Because of the hidden negative analogy, Fisher's model does not provide a technology of intervention to control the value of money in the real world economy as it initially seemed to promise, though this does not detract from its powers to demonstrate results within the model world.[5]

4. This contrasts with Fisher's first case, where he successfully mapped the statistics of real data on M, V, P, and T onto his mechanical balance model.

5. In fact, this was a fundamental design fault in his model, for although there was an alternative choice open to Fisher (of keeping the vessels fixed and altering the ratio of exchange), it would have suffered from similar problems.

Fisher's analogical model can be usefully contrasted with an earlier metaphor which was popular in this field, and which Fisher himself used when introducing his hydraulic model. In the metaphor, money is conceived as water flowing not between laboratory flasks, but between a lagoon and the sea, both of uncharted depth and shape. The outcome level from a flow between them depends on the vagaries of nature, and this matches the government's dependency on the unknown depths, distribution, and flows of gold bullion in the world. The positive analogy in the metaphor hooks onto the real world more accurately than does Fisher's model. But it was precisely this lack of information (evident in the metaphorical account) which both hampered economists' ability to turn their knowledge about bullion stocks into useful tools for policy intervention, and fed the academic and public debates of the late 19th century about monetary institutions.

It was exactly the absence of such information that required Fisher in his model building to imagine what kind of institutional arrangements would be necessary to obtain interventionist tools for monetary policy in the economy and then to simulate their various effects under the different kinds of natural conditions which might occur. In effect, the negative analogy in the model allowed Fisher to explain what kind of control would be needed in order for specific institutions to work, and it provided the flexibility in the model which enabled him to integrate both economic institutions and natural laws of economics into one structure and to explore their interaction.

4. Conclusions. I have made a distinction between two types of analogical model: ready-made analogies and designed (built-up) analogies. Ready-made analogies are constrained only to pose and answer questions within the originating structure. Designed analogies, because their structure is open to choice to some degree, may provide greater flexibility and range, but it may be more difficult to provide answers using such models if they suffer from an associated lack of constraints. In the examples of such models analyzed here, Fisher used the negative analogical features of his models to generate answers beyond those obtained from analogical reasoning from the positive to the neutral analogies. In the first case, the negative property was neutralized by reconfiguring the model, thus altering the status of the basic equation of exchange relating money to goods in the economy. In the second case, Fisher used the negative property as a counterfactual base within the model from which to explore the possibilities of what would be necessary to provide both a full economic account of monetary relations and give full control over the existing (or proposed) monetary system. In both cases the negative analogies played a critical role in

generating knowledge using the model. These examples argue for taking seriously, rather than ignoring, the negative properties in analogical model building.

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