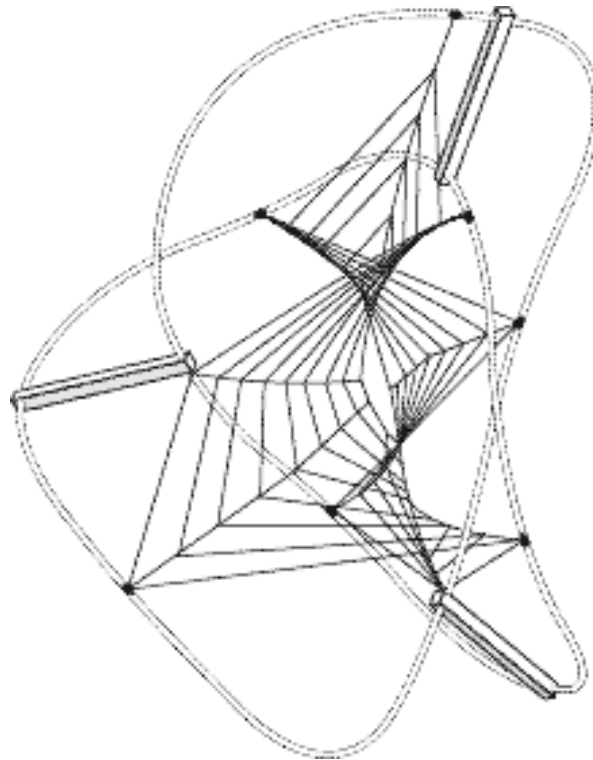


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*Epistemic Virtues and Theory Choice in Economics*Ivan Moscati  
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# Epistemic Virtues and Theory Choice in Economics

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## **Abstract**

When economists have to choose between competing theories, they evaluate not only the theories' empirical relevance, but also qualities like their simplicity, tractability, parsimony and unifying power. These are called the epistemic virtues of a theory. The present paper proposes a formal definition for some epistemic virtues and investigates their role for theory choice in economics.

## **Keywords**

Epistemic virtues, Syntactic properties, Unifying power, Exactness of implications, Systematicity, Semantic Virtuosity

## **JEL Codes**

B41, B13, D00

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## 1. INTRODUCTION

In defending an economic theory from allegations of lack of realism and factual relevance, mainstream economists often appeal to some non-empirical quality of the theory: among others, to its simplicity, tractability, parsimony of assumptions, exactness of implications or unifying power. In epistemological discourse, these qualities are called the *theoretical* or *epistemic virtues* of a theory. Epistemic virtues are relevant not only in theory defence but also in theory choice. When economists have to choose between a theory  $T^0$ , which is at odds with robust empirical evidence but is parsimonious and has numerous clear-cut implications, and an alternative theory  $T^1$ , which captures better the empirical data but is more cumbersome, has many *ad hoc* assumptions, or less exact implications, economists typically choose  $T^0$ .

As an instance of the relevance of epistemic virtues in economic theory choice, we can think of the case of Expected Utility Theory (EUT). Since the 1950s a large amount of experimental evidence has been collected showing that individuals in many relevant circumstances do not behave as expected-utility maximizers. In the last thirty years decision theorists have proposed numerous alternative theories that capture the individual behavior under uncertainty better than EUT<sup>1</sup>. However, it is difficult to find these non-expected utility theories used in branches of theoretical economics different from decision theory or in applied economics (for instance, in game theory strategic uncertainty is modeled almost exclusively through EUT). As Larry Samuelson pointed out in a recent survey on the relationship between economic theory and experiments, the informal explanation for this state of affairs “typically is that for most applications, expected utility theory’s lack of realism is a reasonable price to pay for its simplicity” (Samuelson 2005: 76).

So the question is: Why are epistemic virtues so relevant, in science in general and in mainstream economics in particular? How much weight should the epistemic virtues of a theory receive, compared with its empirical relevance? And further: How many epistemic virtues are there? What exactly does each of them amount to?

There is no shared opinion about these issues among philosophers of science, and this seems due to the qualitative and vague nature of epistemic virtues. As Thomas Kuhn observed with respect to the role of epistemic virtues in theory choice,

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<sup>1</sup> For a review of the alternatives to EUT, see Starmer 2000.

Two sorts of difficulties are regularly encountered by men who must use these criteria in choosing [among theories]. [...] Individually, the criteria are imprecise: individuals may legitimately differ about their application to concrete cases. In addition, when deployed together, they repeatedly prove to conflict with one another (Kuhn 1977: 322).

In my opinion, the difficulties encountered in defining and handling epistemic virtues have caused an underestimation of their role in scientific theory choice, especially in economics. Traditional methodological appraisals of economic science focus on the verification, falsification, testing, or the predictive power of a theory, but rarely discuss the role of its epistemic virtues in determining its acceptance and success among economists. In the present paper I propose a formal definition for some epistemic virtues and attempt to assess their role for theory choice in economics.

The work is organized as follows. Section 2 introduces the distinction between the syntactic and semantic view of theories. From a syntactic point of view, a theory is only a logical-mathematical structure. On the other hand, when we consider the relationship between the theory and the world, the viewpoint is semantic. Section 3 looks at epistemic virtues as syntactic properties of a theory. Section 4 proposes an analytical treatment of a number of epistemic virtues. In particular, the epistemic virtue of “systematicity” appears to summarize most of the other epistemic virtues of a theory. Section 5 defines the notion of “semantic virtuosity” of a theory, and points out that economists typically face a trade-off between the pursuit of systematicity and that of semantic virtuosity. Section 6 suggests that, facing that trade-off, economists attempt to maximize the semantic virtuosity of their theories, yet without lowering the level of systematicity already attained. In other words, theory choice in economics appears to be driven by the principle “more semantic virtuosity, but without loss of systematicity”. The Appendix presents a case-study showing how this rule allows us to rationalize the early developments of demand theory. Section 7 summarizes the paper and discussed the issues it leaves open.

Even if the paper addresses epistemological topics, its style should nevertheless be familiar to economists. In fact, not only are the examples illustrating the notions introduced in the work taken from microeconomics, but the notions themselves are often defined through concepts commonly used in economics. In particular, the paper exploits the concepts of ordering, completeness, trade-off, constrained maximization, as well as the “as if” methodology typical of economics.

## 2. A SYNTACTIC-SEMANTIC VIEW OF THEORIES

Following the so-called Received View of scientific theories, I distinguish the theory as a logical-mathematical structure from the interpretations of the theory, which relate it to the world<sup>2</sup>. When we look at a theory as a logical-mathematical structure, we consider it from a *syntactic* viewpoint. When we are interested in the interpretations of the theory and its empirical relevance, our viewpoint is *semantic*.

The theory as a logical-mathematical structure can be thought of as a hypothetico-deductive machinery. A theory  $T$  starts with a set of definitions  $D = \{d_1, d_2, \dots, d_w\}$ , a set of assumptions  $A = \{a_1, a_2, \dots, a_n\}$  on the objects defined in  $D$ , and uses various deductive procedures to generate a set of implications  $I = \{i_1, i_2, \dots, i_m\}$  concerning the objects in  $D$ . In other words, a theory  $T$  can be looked at as a function mapping a set of definitions  $D$  and assumptions  $A$  into a set of implications  $I$  over  $D$ :  $T : D \times A \rightarrow D \times I$ <sup>3</sup>.

For instance, EUT defines what a decision maker and his preference relation are, as well as certain properties of the preference relation and the notion of “lottery”. Then EUT assumes that the decision maker’s preference relation over lotteries is complete, transitive, continuous, and that it satisfies the independence axiom. Finally, EUT proves that under these assumptions the decision maker’s preferences over lotteries can be represented by an expected utility function, in the sense that the decision maker chooses among the lotteries as if he maximized his expected utility.

The interpretations assign an empirical meaning to the definitions, assumptions and implications of a theory, so connecting it to the world. If the elements of a theory can be interpreted in different ways, the theory can be applied to different portions of the world. For instance, a function  $f(x, y)$  can be interpreted as the utility function of a consumer as well as the production function of a firm. In the first case the function is a brick of a theory of consumer demand, in the second one it belongs to a theory of production.

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<sup>2</sup> On the Received View of scientific theories see Suppe 1977. In economics, T.C. Koopmans (1957, especially the second essay) and G. Debreu (1959, especially the preface) stressed the distinction between the logical structure of the theory and its interpretations.

<sup>3</sup> This notion of theory is sufficiently abstract to cover what in the epistemological literature are called “models”, as distinct from theory. On the role of models in economics, see Morrison-Morgan (1999). What I call “implications of the theory” are often called in the economic discourse “predictions of the theory”. However, the term prediction has a semantic connotation that is better avoided in considering the theory from the syntactic viewpoint.

To be sure, “the world” is not a ready-made thing that is out there. The world, and in particular the world of a scientist like the economist, is an entity constituted through perception, introspection, commonsense considerations, anecdotal empiricism, statistical data, experimental findings, as well as through theoretical spectacles. Even conjectures and thought experiments contribute to constitute the world: if something might exist, in some sense it is part of the world. For instance, Giffen goods are more conjectural than real, but they certainly belong to the world of the demand theorist<sup>4</sup>.

This is not the place to address metaphysical questions about what constitutes “the world”. Here I simply propose to identify the world with the set of evidences  $E = \{e_1, e_2, \dots, e_s\}$  available to the theorist. These evidences can be produced by perception, introspection, real and thought experiments, commonsense considerations, statistical data etc. The interpretation of a theory connects its definitions, assumptions and implications to the set  $E$  of available evidences.

In general, the more the definitions, assumptions and implications of a theory are in accord with the available evidences, the more the theory is descriptively adequate and valuable. The lack of realism and empirical relevance often reproached in mainstream economic theories can be seen as a failure of some of their parts to correspond to the evidences in  $E$ . If we think of a theory as a function mapping a set of definitions  $D$  and assumptions  $A$  into a set of implications  $I$  over  $D$ , a very rough measure of the degree of correspondence between theory and evidence could be given by the number of elements of  $D$ ,  $A$ , or  $I$  that match some element in  $E$ <sup>5</sup>.

Evidently, the problem is how to assess this “matching”, “correspondence” or “accord” between the constitutive elements of a theory and the evidences in  $E$ . The verificationism of Neopositivists, Popper’s falsificationism, probabilistic criteria for theory confirmation *à la* Bayes or Neyman-Pearson are possible answers to the problem, which however has found no unanimous solution. Moreover, the so called Duhem-Quine thesis made clear that the assessment of the correspondence between theory and evidence always depends on a series of auxiliary hypotheses<sup>6</sup>. This cir-

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<sup>4</sup> For Giffen goods an increase in the price leads to an increase in the quantity demanded. Such a phenomenon is rare, but economically possible. Some economists, notably George Stigler (1947), argued that the Giffen goods do not exist in reality. More on Giffen goods and their importance for the development of consumer demand analysis in the Appendix.

<sup>5</sup> This idea is formalized in Section 5 below.

<sup>6</sup> On these issues, with particular reference to economics, see Hands 2001 and Guala 2005.

cumstance renders such assessment tricky, and this independently of the preferred epistemological approach.

In the present work I do not address the semantic problem concerning the assessment of the theory-evidence correspondence. I assume that in some way this problem can be solved, and that we can state whether a certain definition  $d$ , assumption  $a$  or implication  $i$  of the theory matches some evidence in  $E$ . The focus here is on the syntactic properties of a theory, given its assessed degree of correspondence with the available evidences.

### **3. EPISTEMIC VIRTUES AS SYNTACTIC VIRTUES**

In epistemological discourse there is no consensus about what the relevant epistemic virtues are. A partial list of characteristics that have been labeled as epistemic virtues of a theory includes: simplicity, testability, fruitfulness, explanatory power, systematicity, neatness, generality, conservativeness, tractability, parsimony, exactness, unifying power, predictive power, coherence, breadth of scope, elegance, consistency, and accuracy. Moreover, even if a certain feature of a theory is commonly recognized as an epistemic virtue (as in the case of simplicity), such a feature is defined in different ways.

To develop a more constructive treatment of epistemic virtues, I propose to employ the label “epistemic virtues” only for the syntactic properties of a theory. In this acceptance, epistemic virtues concern exclusively the formal features of a theory as a logical-mathematical structure. The qualities related to the semantic adequacy of the theory with the available evidences in  $E$  should be referred to in a different way, namely as the *semantic virtues* of a theory. Therefore, if the accuracy of a theory is defined as the circumstance that it is “in demonstrated agreement with the results of existing experiments and observations” (Kuhn 1977: 321), then accuracy is a semantic, not an epistemic virtue of the theory. Similarly, if predictive power is viewed as the aptitude of a theory to yield empirical predictions not contradicted by factual evidence, then predictive power is a semantic, not an epistemic virtue.

### **4. FORMALIZING AND ORDERING EPISTEMIC VIRTUOSITY**

When epistemic virtues are considered as syntactic virtues of a theory, a formal treatment for some of them can be put forward. In particular, I discuss the virtues of parsimony, generality, explanatory power, unifying power, exactness of implica-

tions, systematicity, tractability and simplicity.

When possible, I suggest a numerical index that represents each of these virtues. The virtuosity index of  $T^0$  with respect to, say, parsimony, has a higher value than the index of  $T^1$  if and only if  $T^0$  is more parsimonious than  $T^1$ . This is exactly the way the utility index represents the preferences of an individual.

However it is not always possible to compare any two theories with respect to a certain epistemic virtue. In more technical terms, for some epistemic virtues the “virtuosity order” among theories is not complete. In these cases, no general index of epistemic virtuosity can be constructed, and we can only compare the virtuosity of particular pairs of theories.

#### 4.1. Parsimony

It appears natural to relate the parsimony of a theory to the number of its assumptions: the more assumptions a theory has, the less parsimonious it is. More formally, if  $|A|_T$  is the number of assumptions of theory  $T$ , and  $P(T)$  is the parsimony of  $T$ , we can assign to  $P(T)$  a number that is decreasing in  $|A|_T$ :  $P(T) = p(|A|_T)$ , where  $p(\cdot)$  is a real-valued function with  $p' < 0$ . According to the parsimony index  $P(T)$ , if a theory  $T^0$  has fewer assumptions than a theory  $T^1$ , that is, if  $|A|_{T^0} < |A|_{T^1}$ , then  $T^0$  is more parsimonious than  $T^1$ , that is,  $P(T^0) > P(T^1)$ .

A possible specification for the function  $p(\cdot)$  is the inverse function. In this case  $P(T) = 1/|A|_T$ , and the parsimony index ranges between 1 (maximal parsimony) and 0 (minimal parsimony). If in a theory a certain set of implications  $I$  can be drawn from different sets of assumptions, it seems reasonable to choose the smallest  $A$  to compute the parsimony index of the theory.

#### 4.2. Generality

Generality is a further property that is often mentioned as an epistemic virtue of a theory. The appreciation of this virtue is typically made in comparative judgments: we say that one theory is more general *than another* rather than that a theory is general in some absolute sense.. Moreover, we compare the generality of theories dealing with the same objects rather than theories concerning different fields. In particular, given two theories  $T^0$  and  $T^1$  dealing with the same set of objects  $D$ ,  $T^0$  is said to be more general than  $T^1$  if the assumptions of  $T^0$  on these objects are less restric-



tive than the assumptions of  $T^1$ .

For instance, in game theory psychological games generalize standard games. Whereas in standard games the players' utilities depend only on the actions they choose, in psychological games utilities may also hang on the belief the players harbor about some other element of the game. An example of psychological game is the tipping-game, where a client tips as much as he expects that the waiter expects to get, and suffers from guilt if he tips less<sup>7</sup>.

More formally, let us consider an object  $d_s$  belonging to  $D$ , and label as  $[d_s]$  the set of all theoretically possible forms, behaviors or realizations of  $d_s$ . For instance, if  $d_s$  is the preference relation over lotteries of a decision maker,  $[d_s]$  is the set containing complete and incomplete, transitive and intransitive preferences. If  $d_s$  is the utility representation of a player's preferences,  $[d_s]$  contains all possible functional forms for the utility representation. Then let us label as  $a_s^T$  the assumption of theory  $T$  on object  $d_s$ , and as  $[d_s]_{a_s^T}$  the possible forms, behaviors or realizations of  $d_s$  as restricted by  $a_s^T$ . For instance, EUT restricts the decision maker's preference to be complete and transitive. Standard game theory restricts the domain of the utility function to the set of actions chosen by the players. By definition  $[d_s]_{a_s^T} \subseteq [d_s]$ .

If two theories  $T^0$  and  $T^1$  concern the same objects  $d_1, d_2, \dots, d_n$ , and make on these objects the assumptions  $a_1^{T_0}, a_2^{T_0}, \dots, a_n^{T_0}$  and  $a_1^{T_1}, a_2^{T_1}, \dots, a_n^{T_1}$  respectively, then we can say that  $T^0$  is more general than  $T^1$  if  $[d]_{a_s^{T_1}} \subseteq [d]_{a_s^{T_0}}$  for every  $s = 1, 2, \dots, n$ .

Clearly, this generality ordering is incomplete because it can be easily the case that neither  $[d]_{a_s^{T_1}} \subseteq [d]_{a_s^{T_0}}$  nor  $[d]_{a_s^{T_0}} \subseteq [d]_{a_s^{T_1}}$  for every  $s$ . Therefore, it makes little sense to propose an index for generality as we did in the case of parsimony.

However, there exists an important relationship between generality and parsimony that allows us to focus on the latter: if  $T^0$  is more general than  $T^1$ , then  $T^0$  is also more parsimonious than  $T^1$ , that is, generality is a special case of parsimony. In fact, if  $T^0$  is more general than  $T^1$  we can think of  $T^1$ 's assumptions as obtained from  $T^0$ 's assumptions by adding to them some further restrictive hypotheses. Hence  $T^1$

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<sup>7</sup> On psychological games see Geanakoplos-Pearce-Stacchetti 1989 and Battigalli-Dufwenberg 2005.

has all the assumptions of  $T^0$  plus such additional restrictive hypotheses, so that  $T^1$  is less parsimonious than  $T^0$ . The contrary doesn't hold: if  $T^0$  has less assumptions than  $T^1$ , it is not always the case that every assumption of  $T^1$  can be thought of as an assumption of  $T^0$  plus some additional restriction.

#### 4.3. Explanatory power

A theory as a logical-mathematical structure produces implications about some of the objects it deals with, that is, about some of the elements of  $D$ . For instance, EUT produces implications about the preferences of the decision maker over lotteries, game theory yields implications about the actions chosen by the players in a game, and consumer theory carries implications about the demand for commodities. It seems reasonable to say that the more implications a theory yields, the more powerful it is in explaining the behavior of the objects it deals with.

If a  $|I|_T$  is the number of implications of theory  $T$ , and  $EP(T)$  the explanatory-power of  $T$ , we can then assign to  $EP(T)$  a number that is increasing in  $|I|_T$ :  $EP(T) = e(|I|_T)$ , where  $e(\cdot)$  is a real-valued function with  $e' > 0$ . If two theories  $T^0$  and  $T^1$  concern the same objects in  $D$ , the explanatory-power index assigns a higher value to the theory that yields more implications over these objects: if  $|I|_{T^0} > |I|_{T^1}$ , then  $EP(T^0) > EP(T^1)$ .

A possible specification for the function  $e(\cdot)$  is the identity function. In this case,  $EP(T)$  is simply equal to  $|I|_T$ .

#### 4.4. Unifying power

In general, there exists a trade-off between the parsimony and generality of a theory on the one side, and its explanatory power on the other side. In fact, we typically need more restrictive assumptions to obtain more implications. As far as generality can be considered as a special case of parsimony, the above trade-off can be reduced to the trade-off between parsimony and explanatory power: the number of implications of a theory is inversely correlated to the number of its assumptions.

The unifying power of a theory can be seen as the epistemic virtue that combines its parsimony and explanatory power. More exactly, the unifying power of a theory  $T$  can be defined as its power to derive the largest possible set of implications from the smallest possible set of assumptions. The unifying-power index of  $T$  – labeled as

$UP(T)$  – can then be thought of as an increasing function of the ratio between the number of implications and the number of assumptions of  $T$ :  $UP(T) = u(|I|_T / |A|_T)$ , with  $u' > 0$ . A possible specification for the function  $u(\cdot)$  is the identity function. In this case,  $UP(T)$  is simply equal to  $|I|_T / |A|_T$ .

#### 4.5. Exactness of implications

The implications of a theory do not simply concern some of the objects it deals with. Indeed, the implications of a theory restrict the possible behaviors, forms or realizations of these objects. For instance, the EUT decision maker prefers lotteries with a higher expected utility; the rational players of game theory do not play dominated strategies; consumer demand theory states that the substitution effect is non positive. It seems sensible to say that the more a theoretical implication  $i$  restricts the possible behaviors of a certain object  $d$ , the more this implication is exact. It also seems reasonable to affirm that the aptitude to deliver implications as exact as possible is a further epistemic virtue of a theory.

In order to formalize the notion of exactness of implications, let us employ the notation already introduced in discussing generality.  $[d_s]$  is the set of all theoretically possible forms, behaviors or realizations of an object  $d_s$ ,  $i_s^T$  is an implication of theory  $T$  on object  $d_s$ , and  $[d_s]_{i_s^T}$  are the possible forms, behaviors or realizations of  $d_s$  as restricted by  $i_s^T$ . By definition  $[d_s]_{i_s^T} \subseteq [d_s]$ . When no comparison among different implications and theories is at issue, we can omit the indexes  $T$  and  $s$ , and simply talk of an object  $d$ , the set of its possible realizations  $[d]$ , and the set  $[d]_i$  of the possible realizations of  $d$  as restricted by implication  $i$ .

Let us consider first the simpler case when  $d$  displays only a finite number of possible behaviors or realizations. For instance, let us imagine that  $[d]$  is the set of all strategy profiles that could be played in a simultaneous, one-shot game among two players.  $|[d]|$  is the number of elements in  $[d]$ , and  $|[d]_i|$  is the number of elements of  $[d]$  whose realization is not excluded by implication  $i$ . In our game-theoretic example, if each of the two players has 3 strategies available, then  $|[d]|$  is equal to 9. If the players are rational and this fact is commonly known, an implication of the theory of games is that the players will not play strategies dominated, even iteratively,

by other strategies<sup>8</sup>. If the strategy profiles surviving to the iterated elimination of dominated strategies (IEDS) are 4, then  $|[d]_i| = 4$ .

When  $[d]$  is a finite set it is possible to calculate the ratio  $|[d]_i|/|[d]|$ . If  $i$  is a sharp implication of the theory, the ratio  $|[d]_i|/|[d]|$  is small. If instead  $i$  is a loose implication, the ratio  $|[d]_i|/|[d]|$  tends to 1. In our game-theoretic example, if  $|[d]_i| = 4$  then  $|[d]_i|/|[d]| = 4/9$ . Instead, if only a strategy profile survives to the IEDS, then  $|[d]_i|/|[d]| = 1/9$ .

In the finite case, it seems then quite natural to introduce the following index for the Exactness of an Implication  $i$ , labeled as  $EI(i)$ :

$$EI(i) = 1 - \frac{|[d]_i|}{|[d]|}$$

For clear-cut implications  $EI$  is close to 1, whereas for loose implications it is close to 0. As an extreme case, if a theory does not generate restrictive implications on  $d$ , that is, if according to the theory “anything goes” with respect to  $d$ ,  $[d]_i$  is equal to  $[d]$  and  $EI$  is equal to zero. In our game, if no strategy profile is eliminated by the IEDS, then  $EI = 1 - (9/9) = 0$ .

If all implications of a theory  $T$  concern objects that have only a finite number of behaviors or realizations, it is possible to define an index capturing the global exactness of the implications of  $T$ . For each implication  $i_s$  concerning a specific object  $d_s$  we can compute the  $EI$  relative to  $i_s$ :

$$EI(i_s) = 1 - \frac{|[d_s]_{i_s}|}{|[d_s]|}$$

If theory  $T$  has  $m$  implications  $i_1, i_2, \dots, i_m$ , a possible index for the Global Exactness of its Implications – let us label it  $GEI(T)$  – is given by the sum of the indexes  $EI(i_s)$ :

$$GEI(T) = \sum_{s=1}^m EI(i_s) = m - \sum_{s=1}^m \frac{|[d_s]_{i_s}|}{|[d_s]|}$$

The more implications a theory has, and the more exact these implications are, the higher is  $GEI$ . If two theories  $T^0$  and  $T^1$  concern the same objects, and  $GEI(T^0) > GEI(T^1)$ , on the whole the implications of  $T^0$  are more exact than those of

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<sup>8</sup> On this, see for instance Brandenburger 1992.

$T^1$ .

Things get much more complex when the range of variation of an object  $d$  is not finite. For instance, if  $d$  is a price then  $[d]=[0,+\infty)$ , or if  $d$  is the substitution effect on the demand for a good,  $[d]$  might theoretically range in an interval  $[-a,b]$ , where  $a$  and  $b$  are positive numbers.

If  $[d]$  is an infinite set,  $[d]_i$  can be finite as well as infinite. For instance, demand theory generally implies that there is only one equilibrium price; in this case  $|[d]_i|=1$ . Yet,  $[d]_i$  can also be an infinite set: one implication of standard consumer theory is that the substitution effect is non-positive, so that  $[d]_i=[-a,0]$ . In both cases, it makes little sense to calculate the ratio  $|[d]_i|/|[d]|$ . If  $[d]_i$  is finite,  $|[d]_i|/|[d]|$  is equal to zero and this independently from the value of  $|[d]_i|$ . This means that two implications, e.g., one stating that there is only one equilibrium price and another allowing for 100 possible equilibrium prices, would be equally exact. This is counterintuitive. On the other hand, if  $[d]_i$  is an infinite set, it is not clear how to evaluate the ratio between the two infinite sets  $[d]$  and  $[d]_i$ . Therefore, if  $[d]$  is an infinite set we have to set aside the ratio  $|[d]_i|/|[d]|$  and renounce to the indexes of exactness  $EI$  and  $GEI$ .

In the infinite- $[d]$  case, a surrogate and much coarser indicator for the exactness of an implication  $i$  is  $[d]-[d]_i$ , that is, the set containing all realization of  $d$  excluded by  $i$ . It seems reasonable to assume that the larger  $[d]-[d]_i$  is, the more exact  $i$  is.

This indicator can be employed to produce a partial ordering among specific implications of different theories. If two theories  $T^0$  and  $T^1$  display two different implications  $i^{T_0}$  and  $i^{T_1}$  on the same object  $d$ , and  $[d]-[d]_{i^{T_1}} \subseteq [d]-[d]_{i^{T_0}}$ , then implication  $i^{T_0}$  is more exact than implication  $i^{T_1}$ . Notice that  $[d]-[d]_{i^{T_1}} \subseteq [d]-[d]_{i^{T_0}}$  if and only if  $[d]_{i^{T_0}} \subseteq [d]_{i^{T_1}}$ . The latter expression is a more transparent way to state the condition assuring that  $i^{T_0}$  is more exact than  $i^{T_1}$ . The exactness order among specific implications of different theories is only partial because it can be easily the case that neither  $[d]_{i^{T_0}} \subseteq [d]_{i^{T_1}}$ , nor  $[d]_{i^{T_1}} \subseteq [d]_{i^{T_0}}$ .

A possible ordering of theories according to the global exactness of their implications may be obtained in the following way. Consider two theories  $T^0$  and  $T^1$  concerning the same objects in  $D$  and recall that  $[d_s]_{i_s^T}$  are the possible forms, behaviors

or realizations of object  $d_s$  as restricted by implication  $i_s$  of theory  $T$ . We can imagine assigning one point to  $T^0$  and zero point to  $T^1$  for every  $s$  such that  $[d_s]_{i_s^{T_0}} \subseteq [d_s]_{i_s^{T_1}}$ , that is, for every implication of  $T^0$  that is more exact than the corresponding implication of  $T^1$  on the same object  $d_s$ . We can think of these points as “exactness points”. Similarly, for every  $s$  such that  $[d_s]_{i_s^{T_1}} \subseteq [d_s]_{i_s^{T_0}}$ , that is, for every implication of  $T^1$  that is more exact than the corresponding implication of  $T^0$  on  $d_s$ , we can assign to  $T^1$  one exactness point and no point to  $T^0$ . If for some  $s$   $[d_s]_{i_s^{T_1}} = [d_s]_{i_s^{T_0}}$ , then both theories get one point. Finally, if a theory has a restrictive implication on a certain object  $d_s$  whereas the other carries no implication about the same object, the former is more exact and gets the exactness point.

The theory that in the end gets more points can be considered the one whose implications are globally more exact. As far as it can easily be the case that neither  $[d_s]_{i_s^{T_0}} \subseteq [d_s]_{i_s^{T_1}}$  nor  $[d_s]_{i_s^{T_1}} \subseteq [d_s]_{i_s^{T_0}}$ , also this order concerning the implications of a theory is incomplete.

#### 4.6. Systematicity

Probably since Plato and Aristotle, in the Western tradition of thought, systematicity has been judged as a major virtue of scientific theories<sup>9</sup>. In particular, in the *Critique of Pure Reason* Immanuel Kant presents systematicity as the fundamental cognitive ideal that drives reason in the construction of scientific knowledge:

Systematic unity is what first raises ordinary knowledge to the rank of science. [...] Under the government of reason, our diverse cognitions [...] must constitute a system. Only so they can support and advance the essential ends of reason [...].

What reason quite uniquely prescribes and seeks to achieve concerning [the cognitions of our understanding] is their *systematization*, that is, to exhibit their connection with a single principle (Kant 1929 [1787]: B860 and B673, translation of N. Kemp Smith, slightly modified)<sup>10</sup>.

Also for the Marburg School, a Neo-Kantian philosophical movement which attained a leading position in German philosophy during the late 19<sup>th</sup> and early 20<sup>th</sup> century, systematicity is the essential feature of scientific knowledge. Ernst Cassirer,

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<sup>9</sup> See on this Rescher 1974.

<sup>10</sup> On the importance of systematicity in Kant’s theory of knowledge, see Rescher 2000, especially pp. 64-98.

the main exponent of the Marburg School, writes in his *Substance and Function* that:

Science does not have, and cannot have, a criterion of truth higher than unity and coherence in the systematic construction of the whole experience (Cassirer 2000 [1910]: 203, my translation)<sup>11</sup>.

More recently, the role of systematicity in scientific knowledge has been highlighted by C. Hempel (1965: 278-288), N. Rescher (1979, 2003) and P. Hoyningen-Huene (2001).

The notion of systematicity used by these authors is not just syntactic. For them the systematicity of a scientific theory is also related semantically to the theory's capacity of explaining the empirical phenomena it investigates. Here I follow a different path. Even if I maintain the central role Kant or Cassirer assign to systematicity, I define it in a purely syntactic way. More exactly, by systematicity of a theory I mean its power to derive the largest possible set of exact implications from the smallest possible set of assumptions. In this acceptation, systematicity summarizes the five epistemic virtues described before – parsimony, generality, explanatory power, unifying power, and exactness of implications – and can then be thought of as the main epistemic virtue of a theory.

As far as the unifying power of a theory  $T$  already synthesizes its generality, parsimony and explanatory power, we can consider the systematicity of  $T$  – to be labelled as  $S(T)$  – as depending only on the unifying power and exactness of implications of  $T$ . In particular, this dependence is positive: the more unifying a theory is, and the more exact its implications are, the more it is systematic. In the finite- $[d]$  case, when it is possible to construct an index  $GEI$  for the global exactness of  $T$ 's implications, we can also assign a numerical value to the systematicity of a theory. In this case, the systematicity index has the form:

$$S(T) = s(UP(T), GEI(T))$$

where  $s(\cdot)$  is a real-valued function increasing both in  $UP(T)$  and  $GEI(T)$ . Even in the case when no index  $GEI$  is available, we can try to compare two theories with respect to their systematicity. If  $T^0$  has more unifying power than  $T^1$ , and  $T^0$ 's implications are on the whole more exact than  $T^1$ 's, then  $T^0$  is more systematic than  $T^1$ .

The systematicity order is clearly incomplete, and its incompleteness originates from

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<sup>11</sup> On Cassirer and his philosophy of science see Friedman 2004 and 2005.

two independent sources. First, it can happen that two theories are not comparable with respect to the global exactness of their implications. Second, even if they are comparable and, say, the implications of  $T^0$  are more exact than the implications of  $T^1$ ,  $T^1$  can have more unifying power than  $T^0$ . In this case it is not clear which theory is more systematic.

#### *4.7. Tractability*

Tractability is a further property that is often mentioned as an epistemic virtue of a theory. It appears related to the specific mathematical tools a theory employs and to the complexity of its demonstrations. If we look at a theory as a hypothetico-deductive machinery with assumptions as inputs and implications as outputs, tractability is a virtue of the theory's technology. Or, if we consider a theory as a function mapping  $D \times A$  into  $D \times I$ , the tractability of the theory depends on the functional form itself of the theory.

For real-valued functions, some tractability indexes based on the number of parameters have been suggested<sup>12</sup>. However, a theory is not a real-valued function, and actually is a function only in a very abstract way. As far as I know, for complex functional objects like theories no tractability index has been proposed, and I am not able to suggest a convincing one here.

With respect to theory choice in economics, such flaw appears less severe if we consider that economic theories generally employ the same mathematical tools: basic logic, set theory, topology, function analysis, optimization techniques, and matrix algebra. Moreover, theories concerning the same topic typically use the identical analytical apparatus and can therefore be considered equally tractable. When this is the case, tractability plays no role with respect to the choice among competing theories.

#### *4.8. Simplicity*

Simplicity is by far the most cited epistemic virtue. The problem with it is that simplicity is too rich a notion, that in fact has been understood in a number of disparate ways: as paucity of parameters, parsimony, tractability, but also as plausibility, strong falsifiability, informativeness, and even as elegance and communicability<sup>13</sup>.

I submit that all the elements that characterize simplicity as a syntactic virtue are ex-

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<sup>12</sup> For an introduction on these issues see Forster-Sober 1994.

<sup>13</sup> On simplicity see Sober 1975, Zellner-Keuzenkamp-McAleer 2001, and Baker 2004.



pressed analytically by the epistemic virtues discussed in Sections 4.1-4.7. Therefore in what follows I will avoid referring to the all-embracing and hence confusing notion of simplicity.

## 5. SYSTEMATICITY VS. SEMANTIC VIRTUOSITY

Based on the previous analysis of epistemic virtues, let us go back to our original questions: What is the role of epistemic virtues in economic theory choice? In particular: How much weight do the epistemic virtues of an economic theory receive compared with its descriptive accuracy, predictive power or other semantic virtues? As observed in the Introduction, epistemic virtues seem often to have more weight than semantic virtues in economic theory choice. When mainstream economists have to choose between a theory  $T^0$ , which is at odds with robust empirical evidence but is parsimonious, has large unifying power and numerous exact implications, and a theory  $T^1$ , which captures better empirical data, but has many *ad hoc* assumptions and vague implications, economists typically choose  $T^0$ . This does not mean that economists are not interested in the descriptive accuracy, the empirical relevance, or the predictive power of a theory, as many critics of mainstream economics have argued. Rather, such a choice simply indicates that for economists the epistemic virtues of a theory are often more important than its semantic virtues. The present and the following sections attempt to spell out more exactly the relationship between epistemic and semantic virtues in economic theory choice.

With respect to the epistemic virtues of a theory  $T$ , I focus on its systematicity, labeled as  $S(T)$ . As argued in the previous section, systematicity summarizes most of the other epistemic virtues. Therefore, if two competing theories are equally tractable, it is their systematicity that informs the choice between them.

As observed in Section 2, the semantic virtues of a theory, like its descriptive accuracy, empirical relevance, realism, or predictive power, can be eventually traced back to the correspondence between the theory's definitions, assumptions, and implications and the available evidences belonging to the set  $E$ . If we assume that there is no problem in assessing such correspondence, we can construct an index measuring how much a given theory is semantically virtuous.

More exactly, we can think of three functions  $f_d$ ,  $f_a$ , and  $f_i$  defined (respectively) from  $D \times E$ ,  $A \times E$  and  $I \times E$  into  $\{-1,0,+1\}$ :  $f_d: D \times E \rightarrow \{-1,0,+1\}$ ,  $f_a: A \times E \rightarrow \{-1,0,+1\}$ , and  $f_i: I \times E \rightarrow \{-1,0,+1\}$ . We can call these functions *interpre-*

*tation functions*. Each interpretation function is equal to +1 in the case of a positive match between its argument and some element in  $E$ , whereas it is equal to -1 in the case of a negative match, that is, when its argument is at odds with some element in  $E$ . The interpretation function is equal to zero when its argument is neither in accord nor at odds with the elements of  $E$ . This may happen because the argument is an abstract mathematical condition with no immediate empirical meaning. Based on the interpretation functions, an index  $SV(T)$  measuring the Semantic Virtuosity of a theory can be defined as follows:

$$SV(T) = \sum_{d_s \in D} f_d(d_s) + \sum_{a_s \in A} f_a(a_s) + \sum_{i_s \in I} f_i(i_s)$$

$SV(T)$  measures the excess of positive over negative matches of a theory. Clearly, the larger  $SV(T)$ , the more semantically virtuous  $T$  is.

In general, there is a trade-off between the systematicity and the semantic virtuosity of a theory. This is typically due to the fact that assumptions that have a high degree of correspondence with the available evidences tend to produce few exact implications, so lowering the theory's systematicity. For instance, the usual assumption that the economic agent is rational in the sense that he maximizes his expected utility and updates his beliefs according to Bayes' law is at odds with much empirical evidence. From a syntactic viewpoint, it would be more general and hence more parsimonious to make less restrictive assumptions on the agent's psychology. For instance, the agent might also have passions, non-selfish motivations, cognitive limits, as well as cultural or sociological biases that let him deviate from expected utility maximization and Bayesian updating. From a semantic viewpoint, providing the traditional *homo oeconomicus* with richer psychological traits would increase the accord between the assumptions of decision analysis and the available evidences, thereby rendering the former semantically more virtuous. However, a theory of decision making based on such loose assumptions about the agent's psychology would produce few exact implications for his behavior. What asset an investor purchases, what strategy a player chooses, how many commodity units a consumer buys would depend on the way the agent perceives the prices and frames the situation, on how much emotions and social values are important for his choice, or on the specific cognitive limits he displays. The agent's possible choices would be barely restricted by the implications of the theory, which would be in accord with the available evidences, but only in the trivial sense of stating that anything can occur. In effect, the theorist's task is typically that of weakening or relaxing the theory's assumptions, so increasing their ac-

cord with the available evidences, yet without ending up with an anything-goes-theory that has no clear-cut implications.

To recapitulate. When economists have to choose between two theories  $T^0$  and  $T^1$ , they take into account both the epistemic and the semantic virtuosity of them. If the two theories are equally tractable, the epistemic virtues relevant for theory choice boil down to the theories' systematicity. The semantic virtuosity of a theory can be traced back to the degree of correspondence between its definitions, assumptions and implications, and the available evidences. If  $T^0$  is both more systematic and more semantically virtuous than  $T^1$ , then in all probability  $T^0$  will be the chosen theory. However, in general there is an inverse correlation between systematicity and semantic virtuosity, so that it will often be the case that one theory is more systematic but less semantically virtuous than the other. In such circumstances, which theory will economists tend to choose?

## **6. MORE SEMANTIC VIRTUOSITY, BUT WITHOUT LOSS OF SYSTEMATICITY**

Like other empirical scientists, economists attempt to construct theories that capture the empirical evidences available to them as completely as possible. Using the terms introduced in the present work, we could say that economists try to maximize the semantic virtuosity of their theories. However, for economists also the systematicity of a theory is important. In particular, it seems that systematicity acts as a kind of constraint in the economists' pursuit of semantic virtuosity: if a new theory  $T^1$  is semantically more virtuous than the currently accepted theory  $T^0$ , but  $T^1$  is not as systematic  $T^0$ , it is often the case that economists stick to  $T^0$ .

Using the constrained-maximization language of basic microeconomics, we could say that economists attempt to maximize the semantic virtuosity of economic theory under the constraint of preserving its systematicity. In symbols, if  $SV(T)$  indicates the semantic virtuosity of a theory  $T$ ,  $S(T)$  its systematicity, and  $S^0$  the systematicity of the currently accepted theory  $T^0$ , then the economic theorist's problem can be stated as follows:

$$\textit{Maximize } SV(T), \textit{ under the constraint } S(T) \geq S^0$$

My thesis is that the epistemological principle "more semantic virtuosity, but without loss of systematicity" regulates theory choice in economics. This means that, if

$T^1$  is semantically more virtuous than  $T^0$ , i.e.  $SV(T^1) \geq SV(T^0)$ , but  $T^1$  is less systematic than  $T^0$ , i.e.  $S(T^1) \leq S(T^0)$ , then economists will choose  $T^0$ .

What is put forward here is an epistemological metatheory of economic theory choice. In particular, this is descriptive, not normative metatheory: it attempts to capture the principles that actually regulate theory choice in economics, not those that, according to some alleged right methodological criterion, should regulate it.

The set of objects  $D$  this metatheory deals with are economic theories and their virtues, both epistemic and semantic. The metatheory makes assumptions about the systematicity and the semantic virtuosity of economic theories (for instance, that  $T^1$  is semantically more virtuous but less systematic than  $T^0$ ), and delivers implications about the economists' theory choice (that the chosen theory is  $T^0$ ) and hence about the dynamics of economic theory.

From a syntactic viewpoint, this epistemological metatheory presents a serious flaw since it is not always able to deliver exact implications. In fact, as noticed in Section 4.6, the systematicity order may result incomplete. And if  $T^0$  and  $T^1$  are not comparable with respect to their systematicity, then the principle "more semantic virtuosity, but without loss of systematicity" has no clear implications about the choice between  $T^0$  and  $T^1$ . In such circumstances, theory choice will probably depend on different factors (e.g. of sociological, academic, or political nature) that are not captured by the present metatheory.

The set of evidences this metatheory refers to is constituted by the history and present status of economic science. The metatheory is semantically virtuous if large parts of the developments and current state of economics, can be rationalized as the result of the theory-choice principle "more semantic virtuosity, but without loss of systematicity".

A correspondence between the proposed metatheory and the available evidences would not mean that economists calculate explicitly the systematicity and semantic-virtuosity indexes of economic theories, and then choose the theory that maximizes semantic virtuosity without lowering systematicity. It would only mean that economists choose among theories *as if* they were following that principle. In the same sense, EUT does not state that the decision maker calculates mentally the expected utility of different lotteries, and then chooses the one with the highest expected utility. EUT just states that if the decision maker's preferences satisfy certain assumptions, then he chooses among lotteries as if he maximized his expected utility. In

other words, what matters for the semantic virtuosity of an economic theory and an epistemological metatheory is the same: that the set of available evidences can be described and rationalized *as if* it were the result of the principle assumed by the theory or the metatheory.

The evidences relevant to an epistemological metatheory are probably even more constructed than the evidences relevant to an economic theory. What has been labeled as “the history and current status of a science” is in fact inevitably the product of historical reconstructions, epistemological interpretations, and theoretical beliefs. However, this does not mean that epistemological metatheories are arbitrary. The difficulties of assessing the correspondence between a metatheory and the evidences relevant to it are probably more severe, but in essence not different from those concerning the assessment of the semantic virtuosity of an economic theory.

Indeed, in a companion paper (Moscati 2007) I attempt to demonstrate that the historical development of consumer demand theory can be convincingly rationalized as driven by the theory-choice rule “more semantic virtuosity, but without loss of systematicity”. That companion paper is historical in nature and discursive in style, and it makes little use of the formal notions illustrated in the current work. In order to show how to employ these notions, the Appendix of the current work reconstructs the history of demand analysis between 1871 and the mid 1930s, applying to it the formal framework introduced in the previous Sections.

## **7. FINAL REMARKS AND OPEN ISSUES**

In the present paper epistemic virtues were interpreted as syntactic properties of a scientific theory, and a formal theory of some of them was proposed. In particular, the epistemic virtue “systematicity” turned out to summarize most of the other epistemic virtues of a theory. Subsequently, the paper introduced a measure for the semantic virtuosity of a theory and pointed out that in the construction of theories scientists typically face a trade-off between the pursuit of systematicity and the pursuit of semantic virtuosity. Theory choice in science, and in particular in economic science, crucially depends on this trade-off. The main thesis of the work is that economists’ theory choice is driven by a specific rule, namely “more semantic virtuosity, but without loss of systematicity”. The Appendix shows how the early development of demand theory can be rationalized as driven by that rule. In what follows, some of the issues the present work leaves open and may be the topic for future research

are discussed.

First, the formal treatment of epistemic virtues proposed in the paper is very elementary and some refinements may be appropriate. In particular, an analytical definition of tractability (see above Section 4.7) is still missing. However, these limitations seem typical of a new approach. In fact, as far as I know, the idea of considering the epistemic virtues as syntactic properties of a theory, and modeling them in a formal way is a novelty in economic methodology. As said in the Introduction, in my opinion the difficulties encountered in defining exactly what epistemic virtues amount to led traditional economic methodologies to underestimate their role in theory choice. Therefore I think that, despite its limitations, the formalistic approach introduced in the current paper opens a promising perspective for understanding and appreciating the importance of epistemic virtues.

Second, even if the history of consumer theory can be rationalized as shaped by the rule “more semantic virtuosity, but without loss of systematicity”, it is not obvious that the same holds for other branches of economics. As regards decision theory, game theory, general equilibrium theory as well as other parts of microeconomics, much anecdotal evidence suggests that also in these areas theory choice can be thought of as driven by the rule at issue. However, a detailed study investigating the history of these branches of microeconomics in the light of the trade-off between systematicity and semantic virtuosity is still missing. As for macroeconomic theories, not only are historical studies lacking, but even the anecdotal evidence about the hierarchy between systematicity and semantic virtuosity in theory choice (in the sense that when the pursuit of semantic virtuosity conflicts with the commitment to systematicity, the latter prevails) is weaker.

Third, even if the development and current state of all parts of mainstream micro and macroeconomics could be rationalized as the result of the rule “more semantic virtuosity, but without loss of systematicity”, the question concerning the epistemological legitimacy of this rule would remain undecided. Is this rule only a methodological idiosyncrasy of that specific research program known as mainstream or neoclassical economics? Or, as the knowledge theory of Kant and the Marburg School suggests, does the pursuit of systematicity in science constitute the main principle that drives thought in the construction of scientific knowledge? Other economic approaches – as e.g. classical political economy, institutional economics, Austrian economics or Post-Keynesian economics – assign much less importance to the pursuit of sys-

tematicity. Does this not mean that the rule “more semantic virtuosity, but without loss of systematicity”, far from being connected to some cognitive ideal of scientific knowledge, has only a contingent nature, and hence should be traced back to historical and sociological circumstances rather than to cognitive factors?

The current paper does not take a stance on this point. Based on the present discussion, the rule “more semantic virtuosity, but without loss of systematicity” could be interpreted as a methodological idiosyncrasy of mainstream economics as well as a regulative principle of scientific cognition.

I am convinced that the latter interpretation is the right one, and that mainstream theory superseded the other economic approaches and became the prevailing one precisely because the way it manages the trade-off between systematicity and semantic virtuosity is in accord with the fundamental principles of scientific cognition. However, I can offer here no substantial argument to sustain this conviction. As far I can tell, not even Kant or the Marburg Neo-Kantians were able to demonstrate in a compelling way that the pursuit of systematicity constitutes the main regulative principle of scientific thought.

Nowadays the issues concerning cognition and thought processes are investigated by cognitive sciences rather than philosophy. Indeed, cognitive scientists have investigated systematicity as a fundamental property of thought. Jerry Fodor and others (1988, 1990) pointed out that thought is systematic in the sense that anyone who can think a thought of a certain form  $F$  can also think thoughts whose form is just a variant of  $F$ . For instance, anyone who can think “John loves Mary”, can also think “Mary loves John”; or anyone who can infer  $Q$  from  $P$ , can also infer  $Q$  from  $P \& Q$ <sup>14</sup>. Unfortunately, this notion of systematicity of thought concerns very basic cognitive processes, and has little relation to the systematicity pursued in higher cognitive functions like those operating in the elaboration of a scientific theory. Therefore, for the time being the question concerning the cognitive status and legitimacy of the rule “more semantic virtuosity, but without loss of systematicity” remains undecided.

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<sup>14</sup> On systematicity in this sense see also Cummins 1996 and Aizawa 2003.

## APPENDIX

### FROM ADDITIVE TO GENERAL UTILITY IN DEMAND THEORY

The present Appendix reconstructs the story of the acceptance in consumer demand theory of a general utility function in place of the additive utility function originally employed by the founding fathers of neoclassical theory. The Appendix is based on the first part of Moscati (2007), where the reader can find further details and bibliographical references.

#### *A1. The Jevons-Walras-Marshall theory ( $T^1$ )*

Consumer demand theory originated in the marginalist value theories of Carl Menger, W. Stanley Jevons and Leon Walras, and became an autonomous body of doctrine with Alfred Marshall's *Principles*. According to the marginalists, the economic value of a commodity depends on the evaluation the subjects give to its marginal units. This evaluation was called "final degree of utility" by Jevons (1871), who first wrote down total utility as a cardinal function  $u(x)$  of the quantity of the good, and marginal utility as the function obtained by differentiating total utility. Jevons also assumed that marginal utility is a positive and decreasing function, and that total utility is an additively separable function. In mathematical terms, Jevons postulated that  $u(x_1, x_2, \dots, x_n) = u_1(x_1) + u_2(x_2) + \dots + u_n(x_n)$ , where  $x_i$  is the quantity consumed of commodity  $i$ , and  $u_i(x_i)$  is the total utility function relative to commodity  $i$ , with  $i = 1, 2, \dots, n$ . The marginal utility of commodity  $i$  is given by  $\partial u / \partial x_i$ , which is assumed to be positive ( $\partial u / \partial x_i > 0$ ) and decreasing ( $\partial^2 u / \partial^2 x_i < 0$ ) for each  $i$ . Additive separability implies that the cross-partial derivatives of the utility function are equal to zero, that is,  $\partial^2 u / \partial x_i \partial x_j = 0$  for  $i \neq j$ .

Based on these assumptions about total utility and marginal utility, Jevons obtained his famous "equations of exchange". The equations state that, for the maximization of the traders' utility under the budget constraints, when in equilibrium each trader demands all commodities and there are no trades at disequilibrium exchange ratios, for each trader the ratio between the marginal utilities of the commodities must be equal to the exchange ratio between the commodities (Jevons 1871: 95 ff.). That is, in equilibrium for each trader and any commodity  $i$  and  $j$ , it must be that

$\frac{\partial u / \partial x_i}{\partial u / \partial x_j} = \frac{p_i}{p_j}$ , where  $p_i$  and  $p_j$  are the prices of commodity  $i$  and  $j$ , respectively.



These are the first order conditions for utility maximization, which in Jevons' setting also ensures that the second order conditions are satisfied. Therefore, if all commodities are demanded, first order conditions are sufficient for maximizing utility. Besides, in Jevons' setting the commodity bundle that maximizes utility is unique.

Based on the same assumptions as Jevons, in the *Éléments* Walras systematized the subjective value theory in a coherent and general price theory, and first fixed the exact relationship between the marginal utility of a good and its demand (Walras 1874: 77 ff.). In particular, when total utility is additively separable and marginal utility is positive and decreasing, the demand for a good definitely decreases (increases) as its price rises (falls) (*i.e.*  $\partial x_i / \partial p_i < 0$ ). In other words, under these assumptions the so-called "law of demand" holds, and demand curves are always downward sloping.

Jevons and Walras analyzed demand with respect to an exchange situation, where agents have an initial endowment of commodities and the relative prices at which they trade these commodities are the result of the exchange process. In his *Principles* (first edition 1890) Marshall considered instead the demand of an isolated agent, who initially has a monetary endowment to purchase commodities and for whom the commodity prices are fixed by the market. In this way, Marshall limited consumption analysis to the current boundaries and separated exchange theory from consumption theory. Apart from these innovations, Marshall's theory of demand was substantially equivalent to that elaborated by Jevons and Walras, and in particular it crucially depended on the assumptions that total utility is additively separable and marginal utility is positive and decreasing.

If we put together the elements of the theories of Jevons, Walras and Marshall, we obtain the theory of demand which was the dominant one during the period 1871-1910. Let us label this theory  $T^1$  and express it as a logical-mathematical object, with its definitions  $D_{T^1}$ , assumptions  $A_{T^1}$ , and implications  $I_{T^1}$ .

$T^1$ : The Jevons-Walras-Marshall theory		
$D_{T^1}$	$A_{T^1}$	$I_{T^1}$
1. Consumer 2. Commodity 3. Quantity of commodity $i$ : $x_i$ 4. Price of commodity $i$ : $p_i$ 5. Commodity bundle: $\underline{x} = (x_1, \dots, x_n)$ 6. Consumer's wealth: $w$ 7. Budget constraint: $\sum_{i=1}^n x_i p_i = w$ 8. Consumer's utility function: $u(\underline{x})$ 9. Optimal bundle: the one that maximizes $u(\underline{x})$ under $\sum_{i=1}^n x_i p_i = w$ 10. Demand function: $\partial x_i / \partial p_i$	1. The consumer maximizes $u(\underline{x})$ under $\sum_{i=1}^n x_i p_i = w$ 2. $u(\underline{x})$ is a cardinal function 3. $u(\underline{x})$ is additively separable: $u(\underline{x}) = u_1(x_1) + \dots + u_n(x_n)$ 4. The marginal utility of any commodity is positive: $u_i = \partial u / \partial x_i > 0$ for every $i$ 5. The marginal utility of any commodity is decreasing: $u_{ii} = \partial^2 u / \partial^2 x_i < 0$ for every $i$ 6. $u(\underline{x})$ is continuously differentiable	1. First-order (necessary) conditions for utility maximization (internal solutions): $\frac{\partial u / \partial x_i}{\partial u / \partial x_j} = \frac{p_i}{p_j}$ for any $i$ and $j$ 2. The cross-partial derivatives of the utility function are null: $\partial^2 u / \partial x_i \partial x_j = 0$ for any $i$ and $j$ 3. Second order conditions: since $\partial^2 u / \partial x_i \partial x_j = 0$ , $u_{ii} = \partial^2 u / \partial^2 x_i < 0$ for every $i$ is sufficient for utility maximization 4. If the first and second order conditions are satisfied, the optimal bundle is unique 5. For any commodity, the demand function is decreasing: $\partial x_i / \partial p_i < 0$

### A2. The Edgeworth theory ( $T^2$ )

One of the elements of the Jevons-Walras-Marshall theory that appeared more at odds with the available evidences was the assumption that utility is additively separable. In his *Mathematical Psychics* (1881) Edgeworth pointed out that in many cases the utilities of goods are interdependent. In particular, it can be the case that the marginal utility of one good diminishes when the quantity of another increases (that is,  $\partial^2 u / \partial x_i \partial x_j < 0$ ), as for tea and coffee. These goods were called rival or competitive goods by Marshall and the other economists of the period. On the other hand, it can be the case that the marginal utility of one good increases when the quantity of another increases (that is,  $\partial^2 u / \partial x_i \partial x_j > 0$ ), as for coffee and sugar. These were labeled as complementary goods. Yet, to capture the phenomenon of competitive and complementary goods, additive utility must be abandoned, since it implies that  $\partial^2 u / \partial x_i \partial x_j = 0$ .

However, to make room for a utility function with a general form was not an easy task, since by weakening the additive assumption most of the exact implications of  $T^1$  faded away. First, in a general-utility setting the first order conditions for utility maximization no more assure that also the second order conditions are satisfied. And until V. Pareto's *Manual of Political Economy* (Italian edition 1906; French edition

1909), it was not clear what were the appropriate second order conditions with any number of commodities. Second, with a general utility functions there can be a multiplicity of optimal bundles. Third, with a general utility function the demand for a good might decrease as well as increase when its price rises. The problem with this was that until the works of Pareto, W.E. Johnson and E. Slutsky (see below) it was not clear under which theoretical circumstances each case occurs, so that the theory vacuously stated that “anything goes”.

Let us express this state of affairs in more formal terms, labeling as  $T^2$  the demand theory resulting from Edgeworth’s suggestion of relaxing the additive utility assumption.

$T^2$ : The Edgeworth theory		
$D_{T^2}$	$A_{T^2}$	$I_{T^2}$
1. Consumer 2. Commodity 3. Quantity of commodity $i$ : $x_i$ 4. Price of commodity $i$ : $p_i$ 5. Commodity bundle: $\underline{x} = (x_1, \dots, x_n)$ 6. Consumer’s wealth: $w$ 7. Budget constraint: $\sum_{i=1}^n x_i p_i = w$ 8. Consumer’s utility function: $u(\underline{x})$ 9. Optimal bundle: the one that maximizes $u(\underline{x})$ under $\sum_{i=1}^n x_i p_i = w$ 10. Demand function: $\partial x_i / \partial p_i$	1. The consumer maximizes $u(\underline{x})$ under $\sum_{i=1}^n x_i p_i = w$ 2. $u(\underline{x})$ is a cardinal function 3. The marginal utility of any commodity is positive: $u_i = \partial u / \partial x_i > 0$ for every $i$ 4. The marginal utility of any commodity is decreasing: $u_{ii} = \partial^2 u / \partial^2 x_i < 0$ for every $i$ 5. $u(\underline{x})$ is continuously differentiable	1. First-order (necessary) conditions for utility maximization (internal solutions): $\frac{\partial u / \partial x_i}{\partial u / \partial x_j} = \frac{p_i}{p_j}$ for any $i$ and $j$ 2. For rival goods $\partial^2 u / \partial x_i \partial x_j < 0$ ; for complementary goods $\partial^2 u / \partial x_i \partial x_j > 0$ ; for independent goods $\partial^2 u / \partial x_i \partial x_j = 0$ 3. There can be a multiplicity of optimal bundles 4. For any commodity, the demand function is typically decreasing but can also be increasing

### A3. Theory choice between $T^1$ and $T^2$

Let us compare the semantic virtuosity and systematicity of the Jevons-Walras-Marshall theory ( $T^1$ ) and the Edgeworth theory ( $T^2$ ). Notice that the two theories are equally tractable, since they employ the identical analytical apparatus. Therefore, tractability is not relevant for theory choice here.

To evaluate the semantic virtuosity of the two theories we use the index  $SV(T)$  introduced in Section 5. The objects defined in  $T^1$  and  $T^2$  – that is,  $D_{T^1}$  and  $D_{T^2}$  – coincide. Therefore also the index measuring the correspondence between these sets of objects and the available evidences  $E$  takes the same value:  $\sum_{d_s \in D_{T^2}} f_d(d_s) = \sum_{d_s \in D_{T^1}} f_d(d_s)$ .

The assumptions of the two theories – i.e.  $A_{T^1}$  and  $A_{T^2}$  – coincide with the exception

of the additive utility assumption, which is absent in  $T^2$ . Since this assumption appears at odds with the available evidences, the interpretation function  $f_a$  assigns to it the value -1. Therefore, by removing such assumption the correspondence level between assumptions and evidences increases:  $\sum_{a_s \in A_{T^2}} f_a(a_s) > \sum_{a_s \in A_{T^1}} f_a(a_s)$ .

As regards the implications of the two theories, – i.e.  $I_{T^1}$  and  $I_{T^2}$  – the implications of  $T^2$  seem to correspond to the available evidences better than those of  $T^1$ . The implication concerning the first-order conditions for utility maximization is identical among the two theories. As already observed,  $T^1$ 's implication stating that the cross-partial derivatives of the utility function are equal to zero is at odds with the apparent interdependence of the commodity utilities. Therefore for this implication,  $f_i = -1$ . On the contrary,  $T^2$  allows for the existence of rival and complementary goods. Hence, for  $T^2$ 's second implication  $f_i = +1$ . As regards  $T^2$ 's implication stating that there can be a multiplicity of optimal bundles, it is simply a loosening of  $T^1$ 's implication according to which the optimal bundle is unique. Therefore, if the evidences in  $E$  support the latter they also support the former. With reference to the implication of  $T^1$  that rules out an increasing demand function, it is at odds with the possible existence of the Giffen goods ( $f_i = -1$ ).  $T^2$  does not rule out an increasing demand function ( $f_i = +1$ ). Finally,  $T^1$ 's second order conditions for utility maximization – that the marginal utility of every commodity is decreasing – does not seem at odds with the available evidence, so that we can imagine that  $f_i$  assigns to it the value +1. Summing up, it turns out that  $\sum_{i_s \in I_{T^2}} f_i(i_s) > \sum_{i_s \in I_{T^1}} f_i(i_s)$ . This means that the implications of  $T^2$  match the available evidences better than  $T^1$ 's implications.

If we put together the results concerning definitions, assumptions and implications we obtain that  $SV(T^2) > SV(T^1)$ . This means that, as claimed above,  $T^2$  appears to be semantically more virtuous than  $T^1$ .

What about the systematicity of the theories? First, we notice that  $T^1$  has more unifying power than  $T^2$ . In fact, the ratio between the number of implications and the number of assumptions for  $T^1$  is equal to 5/6, whereas for  $T^2$  the same ratio displays a lower value, 4/5.

As regards the exactness of implications of the two theories, we are in the lucky case when the implications of the two theories can be always be compared. We do this using the exactness-points index introduced in Section 4.5. The first implication is

identical among the two theories.  $T^2$ 's second implication – that about the sign of  $\partial^2 u / \partial x_i \partial x_j$  – is more exact than the analogous implication of  $T^1$ , since the latter is only a special case of the former. The third and fourth implications of  $T^2$  are instead a loosening of the corresponding implications of  $T^1$ , so that with respect to them  $T^1$  is more exact than  $T^2$ . Finally,  $T^1$ 's implication on the second order conditions for utility maximization has no correspondent in  $T^2$ , so that also in this case  $T^1$  is more exact than  $T^2$ . Summing up,  $T^1$  gets four exactness points whereas  $T^2$  gets only two, so that the implications of  $T^1$  are globally more exact than those of  $T^2$ .

Since  $T^1$  has more unifying power than  $T^2$ , and  $T^1$ 's implications are on the whole more exact than  $T^2$ 's, then we can say that  $T^1$  is more systematic than  $T^2$ :  $S(T^1) > SV(T^2)$ . Hence, in choosing between the Jevons-Walras-Marshall theory ( $T^1$ ) and the Edgeworth theory ( $T^2$ ), the economists of the late 19<sup>th</sup> - early 20<sup>th</sup> century faced the typical trade-off between systematicity and semantic virtuosity: the Edgeworth theory is semantically more virtuous, but less systematic than the Jevons-Walras-Marshall theory. In this state of affairs, the rule “more semantic virtuosity, but without loss of systematicity” predicts that economists hold to the Jevons-Walras-Marshall theory until the difficulties raised by the introduction of general utility are resolved. And this is what in fact happened.

Between the publication of Edgeworth's *Mathematical Psychics* and that of Pareto's *Manual*, neoclassical economists generally recognized that the additive utility assumption did not match the perceived evidences, but stuck to it in all their works. The subsequent editions of Walras' *Éléments* (1889-1900) and Marshall's *Principles* (1890-1920) maintained the additive utility groundwork of the first editions. The theories of P.H. Wicksteed (1888), K. Wicksell (1893), E. Barone (1894) and, although in a non-mathematically explicit form, M. Pantaleoni (1889 and 1898), F.F. von Wieser (1889), and J.B. Clark (1899) were also based on the additive utility assumption. Also at the very beginning of the 20<sup>th</sup> century, the contributions in demand theory were still developed within the additive framework, as the works of A. Aupetit (1901), C. Colson (1901), U. Ricci (1904), P. Boninsegni (1904), A.W. Flux (1904) and H. Cunyngame (1904) show. Things changed only when Pareto, Johnson and Slutsky showed how to construct a theory of demand based on general utility not less systematic than the Jevons-Walras-Marshall theory.

#### *A4. The Pareto-Johnson theory ( $T^3$ )*

Pareto is not usually remembered for having introduced a general utility function in demand theory, but for the attempt he made after 1900 to restate consumer equilibrium analysis without reference to cardinally measurable utility. However, he should also be credited for some important results that paved the way for the acceptance of general utility in consumer demand theory.

In his first cardinalist phase Pareto built up general utility analysis, yet without using his own results. In fact, in the five-part article, “Considerazioni” he recognized the soundness of Edgeworth’s generalization, stated the exchange analysis in general utility terms, but then developed it within the additive framework (Pareto 1892-93, parts I-III). He subsequently resumed the general utility analysis and first provided the exact expression of  $\partial x_i / \partial p_i$  in the general utility case and for any number of goods, but then continued by using the additive utility which he declared “approximately true” (Pareto 1892-93, part V: 306-7). Similarly, in the *Cours*, Pareto developed the entire analysis with additive utility functions and discussed the general case only in a footnote (Pareto 1896-97: 332-4).

After 1900 Pareto changed his approach to the topic. In the *Manual* he recognized that the additive assumption was an approximation made “in order to simplify the problems” but affirmed that “it is time now to take a step forward and also consider the case in which the ophelimity of a good depends on the consumption of all other goods” (cf. Pareto 1906: 241, and 1909: 253; ophelimity is the Paretian term for utility). Accordingly, in the Appendix to the *Manual* Pareto provided a sufficient second order condition for utility maximization with any number of goods, namely that the Hessian matrix of the utility function is negative definite (Pareto 1906: 550, and 1909: 577). Moreover, in the Appendix to the French edition of his work, Pareto re-introduced the general expression of  $\partial x_i / \partial p_i$  that he had already obtained in his 1892 “Considerazioni” (cf. Pareto 1909: 580-1). Although Pareto’s second order conditions are too strong and the economic meaning of the Paretian formula for  $\partial x_i / \partial p_i$  is not clear, these results offered the first satisfactory solution to the problems induced by a general utility function.

In 1913, the Cambridge logician W.E. Johnson published an important article on demand theory in the *Economic Journal*. Johnson’s treatment of the subject strongly resembles that set forth by Pareto in the *Manual*, even if the Cambridge logician did not cite the Italian economist. Independently of the question concerning Johnson’s

knowledge of Pareto's work, the 1913 paper contains some original elements. In particular, Johnson analyzed the effects on demand of a variation in individual income as well as in commodity prices in the two-commodity case. At a geometrical level, he constructed the income-consumption curve as well as the price-consumption one. At a mathematical level, he provided an analytical treatment of "the case in which an increased price leads to an *increase* of the amount of the commodity bought (*i.e.* [the] Giffen's paradox)" (Johnson 1913: 484) and demonstrated that the Giffen goods are a subset of the inferior goods, that is, of the goods whose demand decreases when income increases.

The definitive systematization of consumer demand theory in a general utility framework was accomplished by Slutsky in his celebrated 1915 paper (Slutsky 1915). Slutsky, who referred admiringly to Pareto but did not cite Johnson, provided most of the results of consumer demand theory we still use today. However, Slutsky's paper remained basically ignored until the mid 1930s, so it did not play any significant role for the developments of demand analysis between 1915 and 1935. Therefore, with respect to the acceptance of a general utility function in demand analysis after 1910, here only the theory emerging from the contributions of Pareto and Johnson will be taken into account. We can label the Pareto-Johnson theory as  $T^3$  and express it in the usual logical-mathematical terms.

$T^3$ : The Pareto-Johnson theory		
$D_{T^3}$	$A_{T^3}$	$I_{T^3}$
1. Consumer 2. Commodity 3. Quantity of commodity $i$ : $x_i$ 4. Price of commodity $i$ : $p_i$ 5. Commodity bundle: $\underline{x} = (x_1, \dots, x_n)$ 6. Consumer's wealth: $w$ 7. Budget constraint: $\sum_{i=1}^n x_i p_i = w$ 8. Consumer's utility function: $u(\underline{x})$ 9. Optimal bundle: the one that maximizes $u(\underline{x})$ under $\sum_{i=1}^n x_i p_i = w$ 10. Demand function: $\partial x_i / \partial p_i$	1. The consumer maximizes $u(\underline{x})$ under $\sum_{i=1}^n x_i p_i = w$ 2. $u(\underline{x})$ is a cardinal function 3. The marginal utility of any commodity is positive: $u_i = \partial u / \partial x_i > 0$ for every $i$ 4. The marginal utility of any commodity is decreasing: $u_{ii} = \partial^2 u / \partial^2 x_i < 0$ for every $i$ 5. $u(\underline{x})$ is continuously differentiable	1. First-order (necessary) conditions for utility maximization (internal solutions): $\frac{\partial u / \partial x_i}{\partial u / \partial x_j} = \frac{p_i}{p_j}$ for any $i$ and $j$ 2. For rival goods $\partial^2 u / \partial x_i \partial x_j < 0$ ; for complementary goods $\partial^2 u / \partial x_i \partial x_j > 0$ ; for independent goods $\partial^2 u / \partial x_i \partial x_j = 0$ 3. Second order (sufficient) conditions for utility maximization: the Hessian matrix of the utility function is negative definite 4. If the first and second order conditions are satisfied, the optimal bundle is unique 5. Theoretical conditions under which $\partial x_i / \partial p_i < 0$ or $\partial x_i / \partial p_i > 0$ (Giffen goods)

#### A5. Theory choice between $T^1$ and $T^3$

Let us now compare the semantic virtuosity and systematicity of the Jevons-Walras-Marshall theory ( $T^1$ ) and the Pareto-Johnson theory ( $T^3$ ). Notice that also these two theories employ the identical analytical apparatus and are therefore equally tractable. Hence, not even in this case is tractability relevant for theory choice.

As regards semantic virtuosity, notice that the definitions and assumptions of the Pareto-Johnson theory ( $T^3$ ) coincide with those of the Edgeworth theory ( $T^2$ ). Therefore, based on what was argued in Section A3, we can affirm that

$$\sum_{d_s \in D_{T^3}} f_a(d_s) = \sum_{d_s \in D_{T^1}} f_a(d_s) \text{ and } \sum_{a_s \in A_{T^3}} f_a(a_s) > \sum_{a_s \in A_{T^1}} f_a(a_s).$$

To evaluate the correspondence between the available evidences and the implications of the two theories, a table displaying these implications may be useful:

Implications of $T^1 : I_{T^1}$	Implications of $T^3 : I_{T^3}$
1. First-order (necessary) conditions for utility maximization (internal solutions): $\frac{\partial u / \partial x_i}{\partial u / \partial x_j} = \frac{p_i}{p_j}$ for any $i$ and $j$	1. First-order (necessary) conditions for utility maximization (internal solutions): $\frac{\partial u / \partial x_i}{\partial u / \partial x_j} = \frac{p_i}{p_j}$ for any $i$ and $j$
2. The cross-partial derivatives of the utility function are null: $\partial^2 u / \partial x_i \partial x_j = 0$ for any $i$ and $j$	2. For rival goods $\partial^2 u / \partial x_i \partial x_j < 0$ ; for complementary goods $\partial^2 u / \partial x_i \partial x_j > 0$ ; for independent goods $\partial^2 u / \partial x_i \partial x_j = 0$
3. Second order conditions: since $\partial^2 u / \partial x_i \partial x_j = 0$ , $u_{ii} = \partial^2 u / \partial^2 x_i < 0$ for every $i$ is sufficient for utility maximization	3. Second order (sufficient) conditions for utility maximization: the Hessian matrix of the utility function is negative definite
4. If the first and second order conditions are satisfied, the optimal bundle is unique	4. If the first and second order conditions are satisfied, the optimal bundle is unique
5. For any commodity, the demand function is decreasing: $\partial x_i / \partial p_i < 0$	5. Theoretical conditions under which $\partial x_i / \partial p_i < 0$ or $\partial x_i / \partial p_i > 0$ (Giffen goods)

The first and fourth implications are identical for the two theories. As observed in Section A3, the second and fifth implications of  $T^1$  appear to be at odds with the perceived evidences in  $E$  (hence  $f_i = -1$  for both of them), whereas the third one does not conflict with  $E$  ( $f_i = +1$ ). As regards  $T^3$ , its second and fifth implications seem in accord with the perceived existence of rival, complementary and Giffen goods (therefore,  $f_i = +1$  for both of them). With respect to the third implication of  $T^3$ , it appears quite difficult to assess whether the available evidence supports the negative definiteness of Hessian matrix of the utility function. Therefore, I submit that the value the interpretation function should assign to this implication is zero ( $f_i = 0$ ). Summing up, we have that  $T^3$ 's implications match the available evidences



better than the implications of  $T^1$ :  $\sum_{i_s \in I_{T^3}} f_i(i_s) > \sum_{i_s \in I_{T^1}} f_i(i_s)$ . If we put together the results concerning definitions, assumptions and implications we obtain that  $SV(T^3) > SV(T^1)$ , which means that the Pareto-Johnson theory is semantically more virtuous than the Jevons-Walras-Marshall theory.

Let us now compare the systematicity of the two theories. As observed in Section A3, the ratio between the number of assumptions and the number of implications for the Jevons-Walras-Marshall is equal to  $5/6$ . For the Pareto-Johnson theory this ratio is  $5/5$ , which is greater than  $5/6$ . Therefore  $T^3$  has more unifying power than  $T^1$ .

As regards the exactness of implications, it turns out that those of  $T^3$  are more exact than those of  $T^1$ . In fact, implications one and four are identical for the two theories, whereas the second, third and fifth implications of  $T^1$  are only a special case of the analogous implications of  $T^3$ . Therefore,  $T^1$  gets two exactness-of-implications points whereas  $T^3$  gets five.

Summing up,  $T^3$  has more unifying power than  $T^1$ , and  $T^3$ 's implications are also more exact than  $T^1$ 's, so that  $T^3$  is more systematic than  $T^1$ :  $S(T^3) > SV(T^1)$ . In conclusion,  $T^3$  is both more systematic and semantically virtuous than  $T^1$ . In this state of affairs, the rule “more semantic virtuosity, but without loss of systematicity” predicts that economists embrace the Pareto-Johnson theory, which is what actually happened.

The works of Pareto and Johnson demonstrated that the introduction of general utility did not imply lowering the systematicity of the consumer demand theory. Indeed, from the 1910s on general utility became the standard whereas additive utility became a special case, as the works of Wicksteed (1910), G. Borgatta (1911-12a and 1911-12b), A. Osorio (1913), W. Zawadzky (1914), L. Amoroso (1921 and 1928), A.L. Bowley (1924), P. Boninsegni (1925), M. Fanno (1925-26), R. Frisch (1926), A. de Pietri-Tonelli (1927), F. Divisia (1928), G.C. Evans (1930), O. Weinberger (1930), A. Bordin (1932), V. Dominedò (1933) and H. Schultz (1933) show.

In effect, between the publication of Pareto's *Manual* and the mid 1930s, the main challenge for consumer analysis was related to the introduction of ordinal utility rather than general utility into the theory's assumptions. It turns out that the rule “more semantic virtuosity, but without loss of systematicity” makes sense also of this phase of the development of demand analysis. This part of the story can be found in Moscati 2007.

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