Strengths and Weaknesses of Weak and Strong Supervenience

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Abstract

What is the relation between weak and strong supervenience? Kim claims that weak supervenience is weaker, that it fails to entail strong supervenience. But he mistakenly infers this in virtue of logical form. In fact, one line of reasoning suggests weak supervenience *does* entail strong. Following this line, we see that weak and strong supervenience *are* equivalent for intrinsic properties. For properties involving facts of totality, though, Kim's conclusion is correct. This is merely an epistemic point, showing the usefulness of the notion of weak supervenience. However, this raises the *metaphysical* question of whether weak supervenience relations are necessarily grounded in strong supervenience relations, whether weak supervenience relations are always mere parts of broader generalizations captured by strong supervenience claims, parts that are simply more relevant to us. I argue that they are.

Strengths and Weaknesses of Weak and Strong Supervenience

Weak and strong supervenience are standardly defined as follows, where an individual x and an individual y are A-twins just in case they have the same A-properties.[1]

> **Weak Supervenience**: A-properties weakly supervene on B-properties $=_{df}$ for any possible world w, B-twins in w are A-twins in w.
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> **Strong Supervenience**: A-properties strongly supervene on B-properties $=_{df}$ for any possible worlds w and w* and any individuals x and y, if x in w is a B-twin of y in w*, then x in w is an A-twin of y in w*.

Strong supervenience says that A properties are a function of B properties, leaving our quantifiers 'wide open', as Lewis says. That is, there is a single function f that for any possible individual maps its B properties to its A properties. In contrast, weak supervenience says that *for each world* an individual's A properties are a function of its B properties; that is, associated with each world w is some function f_w that maps B properties to A properties at that world.

Strong and Weak Supervenience are Equivalent for Intrinsic Properties

Kim points out what he calls an 'obvious' fact about the two relations: "Strong supervenience entails weak supervenience; weak supervenience does not entail strong supervenience."[2] But is the latter so obvious? True, there is no entailment in virtue of logical form, but this hardly decides the matter. One line of reasoning to the contrary goes as follows:

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[1]See, e.g., Kim, "Supervenience as a Philosophical Concept," p. 141; McLaughlin, "Varieties of Supervenience," p. 24.

[2]Kim, "Concepts of Supervenience," pp. 66-7.

Assume (reductio) that weak but not strong supervenience obtains, i.e. that there are two sets of properties, A and B, such that the A properties of the things in world w_1 are some function f_1 of their B properties, and the A properties of the things in world w_2 are some function f_2 of their B properties, yet f_2 f_1. Then, given the lack of limits on the way worlds can be, it seems there will always be a world w_3 in which some of the things are like those in w_1, i.e. whose A properties are a function f_1 of their B properties, and others are like those in w_2, i.e. whose A properties are a function f_2 of their B properties. But this contradicts our assumption that weak supervenience obtains, since at world w_3 there is no single function f_3 which maps the B properties of all things to their A properties. Or so it might seem, anyway.

In fact, given a quite plausible metaphysically necessary recombination principle, such as those Lewis, Armstrong, and others have used, we can give some substance to this intuition, showing that, for intrinsic properties anyway, weak and strong supervenience *are* equivalent.[3] I will use the following principle (where an individual x at a world w_1 and an individual y at a world w_2 are *duplicates* iff x at w_1 has exactly those intrinsic properties which y has at w_2):

> **Recombination Principle**: For any individual x in world w_1, and for any individual y in world w_2, there is a world w* containing individuals x' and y' such that x' in w* is a duplicate of x in w_1 and y' in w* is a duplicate of y in w_2.

Given the Recombination Principle, we can see that if *in one world* there is an individual whose A properties are a function f_1 of its B properties and *in another world* an individual whose A properties are a function f_2 of its B properties, then there must be *a single world* where both sorts of individuals co-exist. But this means that for *intrinsic* properties there can't be weak supervenience without strong supervenience.

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[3]Lewis tells us that "Roughly speaking, the principle is that anything can coexist with anything else, at least provided they occupy distinct spatiotemporal positions." (*On the Plurality of Worlds*, p. 88) According to Armstrong, "Any two distinct existences may be found together, or found one without the other, in a single world." (*A Combinatorial Theory of Possibility*, p. 20)

For example, one might think that there could be a property G that weakly supervenes on property F without also strongly supervening on it since there could be both a world w_1 containing only the individual a with property F and property G and a world w_2 containing only the individual b with property F but without property G. However, this sort of reasoning relies on a partial description of logical space, omitting the distribution of properties in possible worlds *other* than w_1 and w_2. And, in fact, the Recombination Principle shows that if there are worlds w_1 and w_2, then there also must be a world w_3 containing individuals a' and b' that are duplicates of a in w_1 and b in w_2, respectively. But this would mean that G does not, after all, weakly supervene on F, at least if F and G are intrinsic properties.

w_1: Fa Ga w_2: Fb Gb w_3: Fa' Ga' Fb' Gb'

In short, for all intrinsic properties weak and strong supervenience are equivalent.[4]

Both weak and strong supervenience are generalizations over *possible* worlds. Thus there are as many sorts of weak and strong supervenience as there are sorts of possibility: metaphysical, nomological, historical, epistemic, etc. Since the recombination principle is a metaphysical thesis, only the equivalence of *metaphysical* weak and strong supervenience relations, among intrinsic properties, has been shown.

What About Extrinsic Properties?

Kim's argument for the non-equivalence of weak and strong supervenience falls short. Nonetheless Kim's conclusion is correct. The property of being alone (being the only individual of a world) is such that either all or none of the members of a world will have it.[5] Therefore, the property of being alone will weakly supervene on the property

[4]Paull and Sider make a similar sort of argument in their "In Defense of Global Supervenience," pp. 835-841. They are responding to Petrie, who describes two possible worlds that he claims provide a counter-example to the equivalence of strong and global supervenience. Paull and Sider point out that two worlds do not a non-equivalence make and appeal to an isolation principle to show that the two worlds in question necessitate a third world which, in the described case, means that neither strong nor global supervenience holds.

[5]Cf. Lewis's "Extrinsic Properties."

of having negative charge — or, for that matter, on any property; throughout any world, everything that is alike with respect to charge will be alike with respect to being alone since throughout any world *everything* is alike with respect to being alone. But, clearly, being alone doesn't strongly supervene on having negative charge. Thus weak and strong supervenience cannot, in general, be equivalent.

But are they equivalent for most properties, or for those properties that most concern us? If so, then the difference between the two supervenience relations would be inconsequential; a single relation might suffice for our needs.

For some extrinsic properties, the argument for equivalence extends quite naturally. For properties such as "being near a St. Bernard" and "having a brother," e.g., the recombination principle also shows that weak and strong supervenience are equivalent. Rather than recombining the individual in question, recombining the mereological sum of the individual *together with* everything near it will guarantee that if there's a world in which x is/isn't near a St. Bernard and another world in which y is/isn't near a St. Bernard, then there will be a world in which duplicates of x and y exist and also are/aren't near a St. Bernard. Similarly, rather than recombining only an individual, we can recombine a chunk of space-time that includes the individual from the time in question back through her conception, that includes the lives of her parents from their birth forward to the time in question, and includes the lives of any sons of her parents from conception to the time in question. Recombining such chunks guarantees that two individuals can be duplicated to generate a world containing their twins with respect to having a brother.

For many extrinsic properties, however, there is no simple way to extend the idea of recombination. Take the property of "being the tallest in the world." If Bob, in world w, is the tallest in the world at 7'-11", and Sue, in w', is the tallest in the world at 8'-5", there is no way to use recombination to generate a world w* containing duplicates of Bob and Sue in which they are both the tallest in the world. There is no

local state of affairs that can be included with them to let us use recombination. Even duplicating everyone in Bob's world with him won't generate a world in which Bob's duplicate is the tallest since being the tallest rides on what *else* might be added during recombination, such as the taller individual, Sue's duplicate. What makes Bob the tallest isn't just the height of Bob, the height of his friend, the height of his neighbor, and so forth for the other 17 people of his world; it is also the fact that these twenty people are the *entirety* of the world's population. If we wanted to use recombination, we would somehow have to recombine *this* fact or property as well — what Armstrong calls a fact of totality[6] — but, of course, we can't since this fact is not independent of other facts, and this independence is just what is required for the Recombination Principle to apply.

Recombination will guarantee the equivalence of weak and strong supervenience iff the properties involve no facts of totality.[7] Notice that these facts of totality are not properties of anything *in* the world but can at most be considered properties of the world itself. Extrinsic properties can therefore be seen as consisting of two components, an intrinsic property (possibly null) of the individual — or of a chunk of space-time including the individual — and a property of the world. Bob's being the tallest person in the world consists in Bob being 7'-11" *and* of the world containing nobody taller than that. Sue's not being famous consists in nothing intrinsic to her but in the world containing few people that know of her.

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[6] *A Combinatorial Theory of Possibility*, pp. 92f.

[7] Actually, weak and strong supervenience are equivalent iff the properties *or their negations* involve no facts of totality. For supervenience is a claim about individuals that are alike with respect to the having *or* not having of the properties in question. Something may have the property of *not* being the tallest in virtue of some taller world-mate, and recombination will guarantee that we can generate an individual that is alike in this respect. But recombination will not give us everything we need in this case to show the equivalence of weak and strong, for it will not give us individuals that are alike *whether or not* they have the property in question.

The extrinsic properties we've considered show us one reason why weak supervenience is a useful relation in its own right. The property of being the tallest in the world weakly supervenes, but doesn't strongly supervene, on the height of the individual. At each world there is a function that maps an individual's height to the fact of whether or not she is the world's tallest (i.e. to the extrinsic property 'is the tallest' or the extrinsic property 'is not the tallest'). Of course, in this case it's easy to see that this function is determined by the height of the tallest individual in the world. So while the property of being the tallest in the world *weakly* supervenes on an individual's height, it *strongly* supervenes on a pair of properties, viz. the height of the individual and the height of the tallest person in the world.

The extrinsic properties we have just been considering wear their dependencies on their sleeves. Obviously, there is *one* function f applicable at all worlds such that x's being the tallest person in a world is a function f of x's height together with the height of the tallest person at x's world; equally clearly, *for each world* w, there will be a function g_w such that x's being the tallest person in world w is a function g_w solely of x's height. It may be useful to abstract away from the properties that aren't local to the individual and talk simply of the latter, world-relative function. And likely there are properties of a similar sort that don't exhibit their dependencies so clearly. In such a case the underlying non-local basis would be unknown. In both these cases, the notion of weak supervenience proves useful.

What Metaphysical Burden Does Weak Supervenience Bear?

We have seen that the notion of weak supervenience has a use beyond that of the notion of strong supervenience. This is an *epistemic* point about the *notion* of weak supervenience. But from the examples we've seen so far it looks like the usefulness of weak supervenience is simply to capture an epistemically important part of a more general relation of strong supervenience. It looks as though God's blueprint of the

world's metaphysical relations, being a complete map of logical space, can dispense with weak supervenience. Is this true? That is, are relations of weak supervenience *always* grounded in more general relations of strong supervenience? The answer is yes. In each of our two examples, we had a relation of weak supervenience only because there was an underlying relation of strong supervenience. And this, I argue, will be true in general.

One set of properties, Γ, weakly supervenes on another set, Δ, just in case at every world the subset G of properties of Γ that an individual has is a function of the subset D of properties of Δ it has — that is, iff for each world w there is a function f_w, such that for every individual i of that world, $G_i = f_w(D_i)$. But what could possibly guarantee this across all worlds? With the endless possible worlds that can be generated with the Recombination Principle, why isn't there a single world with two individuals which have the *same* subset of properties of Δ but *different* subsets of properties of Γ?

One possible answer is that Γ *strongly* supervenes on Δ. That is, if it is impossible to have two individuals *at a single world*, one with properties D and G1, and another with properties D and G2 (where D is a subset of Δ and G1 and G2 are different subsets of Γ), this could be part of a more general impossibility of there even being *an* individual with properties D and G2. Perhaps an individual's having of G1 is *necessitated* by its having of D; the conflict between the having of G2 and the having of D would, in this case, be determined solely by what G2 and D are.

So one reason that would explain why there are never two individuals at the same world, one with D and G1, one with D and G2, is that the nature of the properties precludes one of these two combinations. You can't have one person with intentions I who utters "I promise that . . ." who makes a promise and another person with the same intentions uttering the same statement who doesn't make a promise because uttering "I promise that . . ." with certain intentions *just is* to make a promise.

If, though, Γ doesn't strongly supervene on Δ, i.e. if there is nothing contradictory about an individual having D and G2, and similarly for D and G1, then why can't a world contain one of each sort of individual? Something determines that all the individuals at one world having properties D will have properties G1 while all the individuals at another world having properties D will have properties G2. Whatever it is must be something that is constant across each world, though it varies from world to world. In short, there must be a determinable totality fact F which explains our puzzle. This, though, is just another way of saying that the Γ-properties an individual has is a function of the Δ-properties it has plus the value of F at i's world. That is, this is to say that Γ strongly supervenes on Δ *and* F. A weak supervenience relation thus captures a mere part of some strong supervenience relation. More specifically, it captures that part which abstracts away from facts of totality.

And this, of course, is just what we've seen in our examples. At some worlds, all individuals who are 7'-11" are the tallest individuals of the world, whereas at other worlds, everyone who is 7'-11" is not the tallest in the world, and never do you find a world containing two individuals who are 7'-11" one who is the tallest and the other who is not. This makes sense since to know whether someone is the tallest in the world you have to know not only *their* height but also the height of the tallest person in the world. Similarly, whether a possible individual is famous depends upon how many people at that world know of her. If we wish to abstract away from the fact that being the tallest in the world depends on the height of the tallest person in the world we can instead employ the *world-relative* generalization that being the tallest in the world depends solely on one's height.

Summary

For properties intrinsic to a world, weak and strong supervenience *are* equivalent. But for properties involving facts of totality, they are *not* equivalent.

Because totality facts are common fare, though, this shows the usefulness of the notion of weak supervenience. However, useful though it is, the metaphysical basis for weak supervenience relations is the same as for strong. A more general strong supervenience relation necessarily underlies a weak supervenience relation, so a weak supervenience relation is always a restricted part of some broader strong supervenience relation, a part we find useful simply because it is known or relevant to us.[8]

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