Axiomathes (2014) 24:549–558
DOI 10.1007/s10516-014-9237-8

ORIGINAL PAPER

Benedykt Bornstein's Philosophy of Logic and Mathematics

Roman Murawski

Received: 3 December 2013 / Accepted: 6 May 2014 / Published online: 13 May 2014
© The Author(s) 2014. This article is published with open access at Springerlink.com

Abstract The aim of this paper is to present and discuss main philosophical ideas concerning logic and mathematics of a significant but forgotten Polish philosopher Benedykt Bornstein. He received his doctoral degree with Kazimierz Twardowski but is not included into the Lvov–Warsaw School of Philosophy founded by the latter. His philosophical views were unique and quite different from the views of main representatives of Lvov–Warsaw School. We shall discuss Bornstein's considerations on the philosophy of geometry, on the infinity, on the foundations of set theory and his polemics with Stanisław Leśniewski as well as his conception of a geometrization of logic, of the categorial logic and of the mathematics of quality.

Keywords Bornstein · Philosophy of logic · Philosophy of mathematics

Benedykt Bornstein was a significant Polish philosopher who now is almost completely forgotten. Although he wrote his doctoral dissertation under the supervision of Kazimierz Twardowski, the founder of the famous Lvov–Warsaw School of Philosophy,[1] he was not a member of this school—mainly because of his metaphysical views. In some way he was an individualist; his research did not follow the main trend.

Bornstein was born in Warsaw on 31 January 1880. He studied in Warsaw and Berlin. In 1907, he received his doctoral degree at the University of Lvov under the supervision of Kazimierz Twardowski. From 1915 he lectured on logic, epistemology and ontology within the framework of the Warsaw Society of Science Courses

[1] For the Lvov-Warsaw School of Philosophy see the monograph by Woleński (1989).

R. Murawski (✉)
Faculty of Mathematics and Computer Science, Adam Mickiewicz University, ul. Umultowska 87, 61-614 Poznan, Poland
e-mail: rmur@amu.edu.pl

and from 1918 in the Free Polish University (Polish: Wolna Wszechnica Polska). From 1928 he also worked in the Łódź branch of the Free Polish University. After World War II he held the Chair of Logic and Ontology at the University of Łódź. He died suddenly after a surgery in Łódź on 11 November 1948.

Bornstein's scientific interests were on the border of philosophy and mathematics. His conceptions did not win recognition and greater interest of his contemporaries. He worked in relative isolation although he participated in philosophical congresses and published his works in the major periodicals both in Poland (such as *Przegląd Filozoficzny*, *Wiedza i Życie*, *Przegląd Klasyczny*) and abroad. His scientific activities can be divided into three periods: in the first one he translated Kant's works and developed his ideas in a critical way; the second period was dedicated to investigations concerning the philosophy of mathematics, and the third period—to problems of metaphysics cultivated in the spirit of the classical trend. His works written in the second period raised some interest of Polish philosophers. His investigations concerning the philosophy of mathematics led to the formulation of a new philosophical method in the form of categorial geometrical logic. The theme of this paper makes us focus on the latter investigations.

Let us begin by discussing Bornstein's reflections on the philosophy of geometry. Here Bornstein referred to Kant's transcendental aesthetics and Twardowski's theory of images and concepts (cf. his 1894). At the same time, he criticised the idea of constructing geometry on the basis of set theory or topology; he also distanced himself from Poincaré's conventionalism. In his opinion, constructing a geometry should be begun by constructing proper geometrical concepts, which have their objective references. In his book *Prolegomena filozoficzne do geometryi* [Philosophical Prolegomena to Geometry] (1912) he distinguished between the image of physical space and the concept of geometrical space, and he followed the idea that the so-called background image must be an image the object of which exists and is truly perceived, which is to guarantee that the common features of the object of the concept of geometrical space and the object of the background image will not only concern the world of objective images but also be grounded in the experiential reality.[2] According to Bornstein, one of the common features of both objects is three-dimensionality. He wrote in *Prolegomena filozoficzne do geometryi*:

[2] It is worth mentioning here Leśniewski's ideas concerning the distinction between objects and concepts. The reason is that both Leśniewski and Bornstein were students of Twardowski, so they came from the same philosophical school. Below we shall present the discussion between Leśniewski and Bornstein dealing with the foundations of set theory. It is the other reason to consider Lesniewski's ideas.

Leśniewski spoke not about concepts but abort names (in particular abort general names) and about individual objects that could be of arbitrary nature. Further one should distinguish object in his ontology (it was in fact a calculus of names) and in his mereology (that was the theory of sets in the collective sense). According to Leśniewski a name is any expression which can play a role of *B* in sentences of the form "*a* is *B*". Hence Leśniewski proposed in his ontology a theory of names of one category only and liquidated the dualism of nominal expressions (individual names vs. general names). He did not say anything on the nature of objects that exist except that they are individual objects. His ontology is "metaphysically" neutral—it cannot be deduced from its theses whether anything does exist and what does exist. He says only that *A* exists if and only if for some *x*, *x* is *A*, and that *A* is an object if and only if for some *x*, *A* is *x*. Add that on Leśniewski's ontology were founded ontological considerations of Tadeusz Kotarbiński. His conception is called reism—it is claimed in it that individual objects are things and that only they do exist.

> If we analyse this image with respect to spatiality we will be always convinced that its object is three-dimensional, i.e. it has length, width and height (or depth); that from each of its points we can draw three perpendicular lines, belonging to the given object in some space. This objective spatiality, characterising three-dimensionality, is a common feature of our background image and the object of the concept of geometrical space, based on that image (1912, p. 8).[3]

Three-dimensionality is determined by experience and is not—as Poincaré claimed—a separate mental construction.[4]

As far as the question of the choice between Euclidean and non-Euclidean geometries is concerned, Bornstein thought that:

> From the purely logical or analytical point of view the theorems or formulas of non-Euclidean geometry contain no contradictions, and it is logically possible that they are equally eligible as the theorems and formulas of Euclidean geometry (1912, p. 89).[5]

At the same time, experience cannot help us choose one, true and correct geometry. Bornstein wrote in *Prolegomena*:

> If now the followers of the purely logical or analytical concept of geometry turn to experience with the question which of the three logically possible systems of theorems is important to experience and is confirmed by it, they must be prepared not to receive any answer to their question. […] In other words, when we turn to experience to show us which of the possible logical systems is confirmed by it, which is true, then experience will never give us any answer since its data will present a constant in the equation with two unknowns (one geometrical and the other physical), and so they will be insufficient to solve precisely this geometrical unknown in the equation (1912, pp. 89–90).[6]

[3] 'Jeżeli zanalizujemy takie wyobrażenie pod względem przestrzenności przekonamy się zawsze, że przedmiot jego jest trójwymiarowy, t.j. że posiada długość, szerokość i wysokość (względnie głębokość), że w każdym jego punkcie można poprowadzić trzy prostopadłe linie, należące na pewnej przestrzeni do danego przedmiotu. Ta przestrzenność przedmiotowa, którą charakteryzuje trójwymiarowość, jest cechą wspólną przedmiotu naszego wyobrażenia podkładowego i przedmiotu pojęcia przestrzeni geometrycznej, opartego na tem wyobrażeniu.'

[4] For the particular remarks on Bernstein's views concerning the problem of essence and structure of geometrical space see Śleziński (2009).

[5] 'Z punktu widzenia czysto logicznego lub czysto analitycznego twierdzenia lub formuły geometryi nieeuklidesowej nie zawierają sprzeczności, a logicznie możliwe, są równie uprawnione, jak twierdzenia i formuły geometryi euklidesowej.'

[6] 'Jeżeli teraz zwolennicy czysto logicznego lub czysto analitycznego pojmowania geometryi zwrócą się do doświadczenia z pytaniem, który z trzech logicznie możliwych systemów twierdzeń jest ważny dla doświadczenia i znajduje w niem potwierdzenie, to muszą być przygotowani na to, że odpowiedzi na to pytanie nie otrzymają. […] Słowem, gdy zwracamy się do doświadczenia, by nam wskazało, który z możliwych logicznie systemów znajduje w niem potwierdzenie, który jest prawdziwy, to doświadczenie na to pytanie nigdy nie będzie mogło dać nam odpowiedzi, gdyż jego dane będą przedstawiały wielkość stałą w równaniu z dwiema niewiadomymi (jedną geometryczną, drugą fizyczną), a więc będą niedostateczne do ścisłego rozwiązania tego równania co do niewiadomej geometrycznej.'

Bornstein claimed that real spatial extensiveness could not be identified with the extensiveness defined by the continuum of real numbers. The latter has no space character. Therefore, the attempts to transfer theorems from one domain to the other are not justified. In particular, one cannot assume a priori that a geometrical line does not correspond to any continuous function. In his article ‘Problemat istnienia linji geometrycznych’ [The Problem of the Existence of Geometrical Lines] (1913) he showed that such lines corresponded to some solid functions and did not correspond to other ones. Assuming that all geometrical curves have tangents we have the result that only functions with derivatives correspond to them. Consequently, if every movement must have speed, and speed is the derivative of distance with respect to time, movement cannot occur along curves without tangents. Thus not all functions are of geometrical character, in particular it concerns those functions that have no derivatives.

Bornstein also dealt with the problem of infinity. In his opinion an infinite set can be given only as a certain whole embracing infinitely many elements. At the same time, the actual infinity is never given as the infinity of its particular elements—only a finite number of them can actually be given.[7] Thus, a question arises whether all elements of an infinite set (in the sense of actual infinity) exist physically or whether they exist in themselves independently from their actualisation. Bornstein examined these questions in his book *Elementy filozofii jako nauki ścisłej* [Elements of Philosophy as an Exact Science] (1916) asking whether an actual segment is a set of potential or actual points. He concluded that an infinite set of points situated between two points of a geometrical line existed physically in nature but not all of its elements necessarily did.

Thus we come to Bornstein’s considerations on the foundations of set theory. We must above all mention his work ‘Podstawy filozoficzne teorji mnogości’ [The Philosophical Foundations of Set Theory] (1914). This work was criticised by Stanisław Leśniewski in his article ‘Teorja mnogości na “podstawach filozoficznych” Benedykta Bornsteina’ [Set Theory on the ‘Philosophical Foundations’ of Benedykt Bornstein] (1914). In turn Bornstein wrote an article ‘W sprawie recenzji p. Stanisława Leśniewskiego rozprawy mojej pt. “Podstawy filozoficzne teorji mnogości”’ [On Mr Stanisław Leśniewski’s Review of My Dissertation ‘The Philosophical Foundations of Set Theory’] (1915). Thus the polemic ended.

We cannot discuss here the technical details of the polemic and more, the polemic did not bring about any effects. However, some arguments of both thinkers are worth mentioning.

Let us begin by stating that in his work (1914) Bornstein notices that the source of antinomy in set theory is its erroneous philosophical justification. He concludes that a set of individually existing elements can be only finite. In addition, he bases his thesis concerning the existence of finite and infinite sets having individually existing elements on the following three lemmata (cf. 1914, pp. 183–185):

- The same number corresponds to two equivalent sets with individually existing elements,

[7] Observe that Bornstein’s idea of an infinite set is not identical with Cantor’s one. Cantor—following Platonizm—did not distinguish between existing and actually given elements.

- in a set of elements, existing individually, the same number cannot correspond to the proper part of this set in the same way as to the whole,
- a set of elements, existing individually, cannot be equivalent to its own part.

He explains the used terms in the following way:

> If a plurality of elements, each existing individually, i.e. as a different unit, is analysed *only as a plurality of units*, we analyse it from the point of view of quantity; at the same time, this plurality of units constitutes *the quantity, relatively, its number* of individually existing elements of the given plurality. […] between the plurality of elements, existing individually, and the plurality of units, constituting its quantity, relatively its number, there is one–one correspondence; these pluralities are, as we say, equivalent or of equal power. […] since quantity is a real feature of the plurality of elements, existing individually, whereas the number is a notional equivalent of this feature (1914, p. 183).[8]

Omitting the technical details of Bornstein's reasoning we must say that he made the error of *quaternio terminorum*, i.e. the use of the same term in two different meanings—in this case it is the term 'the same number.'

Assuming the existence of an infinite set of natural numbers Bornstein shows the essential nature of infinite pluralities. Now, in the infinite plurality of natural numbers only their finite quantity—in his opinion—can be considered individually. Therefore, there can be infinite pluralities without any possibile individual content. He writes:

> […] here we have a perfect example, showing the essential nature of infinite pluralities, consisting in their full independence from the matters of actualising (individualising, materialising) the elements of plurality. Here we have an example of a pure form in ideal perfectness (1914, p. 190).[9]

He also concludes that the well-ordering theorem (equivalent to the axiom of choice) 'applying in general to all kinds of plurality is wrong; whereas applying to the plurality of elements, existing individually, physically, is an obvious truth' (Bornstein 1914, p. 190).[10]

Leśniewski began his criticism of Bornstein's work (1914) with the following words:

[8] 'Jeżeli mnogość elementów, z których każdy istnieje indywidualnie, tj. jako różna od innych jednostka, rozpatrujemy *tylko jako mnogość jednostek*, to rozpatrujemy ją z punktu widzenia ilości, przy czym ta mnogość jednostek stanowi właśnie *ilość, względnie liczbę* istniejących indywidualnie elementów danej mnogości. […] miedzy mnogością elementów, istniejących indywidualnie, a mnogością jednostek, stanowiącą jej ilość, względnie liczbę, istnieje odpowiedniość jedno-jednoznaczna; mnogości te są, jak mówimy, równoważne lub równej mocy. […] ilość bowiem jest cechą rzeczywistą mnogości elementów istniejących indywidualnie, liczba zaś jest odpowiednikiem pojęciowym tej cechy.' .

[9] '[…] mamy tu doskonały przykład, wykazujący istotną naturę mnogości nieskończonych, polegającą na ich zupełnej niezależności od spraw zaktualizowania (zindywidualizowania, zmaterializowania) elementów mnogości. Mamy tu przykład czystej formy w idealnej doskonałości.'

[10] 'w zastosowaniu do wszelkiej mnogości w ogóle jest błędne; w zastosowaniu natomiast do mnogości elementów, istniejących indywidualnie, aktualnie, jest prawdą oczywistą.'

> Dr Benedykt Bornstein wrote a treatise in which he tried to provide set theory with 'philosophical foundations'; he thought that certain contradictions, which can be seen in set theory, are not caused by set theory but by its wrong philosophical justification, and this view of the problems, prevailing in set theory, must have been the origin of the author's desire to add to this science some thoughts, which could justify it 'philosophically' (Bornstein 1914, p. 488).[11]

Further, Leśniewski analyses Bornstein's formal argumentations—ignoring the ontological questions, which were so important to the latter. In particular, Leśniewski criticises Bornstein's terms 'existing individually' and 'existing formally,' accusing him of not giving any precise definition of the concept of 'unit.' In addition, he proposes to replace the term 'unit' by the term 'object,' which, however, as seen in Bornstein's response (1915) does not satisfy the latter. Leśniewski also criticises Bornstein's interpretation of Zermelo's well-ordering theorem.

Avoiding any complicated (and devoid of deeper meaning now) technical questions concerning the polemic between Leśniewski and Bornstein it would be sufficient to say that their levels of discourse were entirely different. Leśniewski defended the standard approach towards set theory (which he then refuted for the cause of mereology) against Bornstein's criticism flowing from philosophical motives. As Śleziński (2010) notices 'for Leśniewski the formal analyses are binding whereas for Bornstein the argumentations, apart from formal correctness, must refer to the objective layer of the problems under consideration' (p. 110).[12] Leśniewski summarised his critical review of Bornstein's words in the following way:

> The work of Mr Bornstein has no value for the 'foundations' of set theory. It does not remove any 'contradictions' from set theory as Mr Bornstein seems to be claiming; on the contrary, he creates them to a much bigger extent; he does not justify them 'philosophically' and in no other way does he justify even one theorem of set theory; since one cannot justify something with the help of 'definitions' and 'lemmata' that are full of errors and contradictions; he explains nothing because the seemingly devised conceptions of something, for example the conception of 'capacity,' are inconsistent and unclear (1914, p. 507).[13]

[11] 'Dr Benedykt Bornstein napisał rozprawę, w której starał się zaopatrzyć teorię mnogości w „podstawy filozoficzne"; uważał on, iż do pewnych sprzeczności, które dają się widzieć w teorii mnogości, prowadzi nie sama teoria mnogości, lecz błędne jej uzasadnienie filozoficzne, a pogląd taki na stan rzeczy, panujący w teorii mnogości, stanowił właśnie zapewne genezę pragnienia autora, by przysporzyć tej nauce trochę myśli, które by ją mogły „filozoficznie" uzasadnić.'

[12] 'dla Leśniewskiego wiążące są analizy formalne, a dla Bornsteina rozumowania, oprócz poprawności formalnej, muszą odnosić się do warstwy przedmiotowej badanych problemów naukowych.'

[13] 'Praca p. Bornsteina nie ma żadnej w ogóle wartości dla „podstaw" teorii mnogości. Nie usuwa ona żadnych „sprzeczności" z teorii mnogości, jak się to zdaje p. Bornsteinowi, lecz je przeciwnie w wielkiej obfitości stwarza; nie uzasadnia „filozoficznie" ani też w żaden inny sposób ani jednego twierdzenia teorii mnogości, nie można bowiem uzasadnić czegoś za pomocą „definicji" i „lematów", pełnych błędów i sprzeczności; nie wyjaśnia nic, bo obmyślone niby czegoś koncepcje, jak np. koncepcje „pojemności", są sprzeczne i niejasne.'

In his response (Bornstein 1915) to Leśniewski's criticism Bornstein tried to specify his conception of set theory. He also saw certain inconsistencies in Leśniewski's arguments. He was not convinced about the validity of the accusations and concluded his answer:

> Facing the foregoing arguments it seems to me that I will be impartial responding to Mr Leśniewski's review: *primo*—it does not show, even to the slightest extent, any contradictions which are to be stuck in the concepts I have used, and *secundo*—it is an example of Mr Leśniewski's extremely careless disregard of the elementary principles of logic (Bornstein 1915, pp. 139–140).[14]

As we have seen both debaters remained on different planes. Leśniewski conducted his argumentation and analyses in the spirit preferred by the Lvov–Warsaw School, i.e. using the apparatus of mathematical logic and focusing on formal matters, whereas Bornstein favoured ontological questions and worked in the spirit of the concept of the mathematics of quality, which he was developing himself. In particular, the latter might have been the reason why there were no polemics (except the one held by Leśniewski) with Bornstein's later works—in fact, the concept of the mathematics of quality was so different from the universally accepted tendencies and styles of thinking that it was difficult to find any common points. On the other hand, Bornstein criticised the widespread practice of treating mathematics as the science on quantity and magnitude, number and measure—in his opinion there is also qualitative mathematics, especially qualitative algebra or geometry. This qualitative mathematics deals not only with order, in particular with order between qualities. It should serve a mathematization of the philosophy and the construction of a qualitative-mathematical model of the world. Let us add that details of Bornstein's attempts to develop the qualitative mathematics are not quite clear.

Let us proceed to the next idea of Bornstein, namely, his conception of the geometrisation of logic, i.e. geometrical logic. Referring to Leibniz, who was always closer to the intensional than the extensional conception of logical forms and who wanted to construct logic based on the content of expressions and not only on the extensions of concepts, Bornstein tried to create a new logic—namely the logic of content. Since he thought that the content of a concept sets out its extension, and thus the exactness and definiteness of the content determine the precision and definiteness of the extensions and in general, of the classes.

Bornstein divided concepts and judgements into those which were set out objectively and those which were set out logically. The former parallel objects in reality and the latter gain their meaning through definitions. In addition, Bornstein distinguishes between nominal and real definitions. In nominal definitions the definiendum as if synthetises the essence of words constituting the definiens. In real definitions we have the reverse process—the definiendum is divided into a

[14] 'Wobec powyższego wydaje mi się, że będę obiektywnym, gdy o recenzji w mowie będącej p. Leśniewskiego powiem: *primo*—że w najmniejszym nawet stopniu nie wykazuje sprzeczności, tkwić mających w używanych przeze mnie pojęciach, i *secundo*—że jest przykładem niebywale lekkomyślnego nieliczenia się p. Leśniewskiego z elementarnymi zasadami logiki.'

combination of simpler constituents occurring in the definiens. However, both types of definition are definitions *per genus proximum et differentiam specificam*. Likewise, we have judgements set out objectively and judgements set out logically. At the same time, Bornstein assumes that all judgements have subject-predicative structures.

Bornstein, following the conceptions of Edward Vermilye Huntington (1904), proposed his own system of the algebra of logic, which he formulated as categorial. He accepted three logical operators: negation, addition and multiplication. Addition consists in integrating the contents of concepts whereas multiplication sets out the biggest common element of concepts. Here two constants appear: 0 and 1, where 0 is the lower bound of any content and 1 is the upper bound of any content. Element 0 has the weakest logical content since when added to any element it does not change the content of it. Element 0 expresses the content of the concept of 'something' or 'the object in general' whereas element 1 presents the substantially strongest concept, concept with the richest content, 'whole' and 'everythingness.' The element 1 is the upper limit of all concepts whereas 0 is the lower limit of all concepts. Moreover, there is a relation of the subordinance of content marked as <, but it does not have the property of connectedness.

Furthermore, Bornstein tried to give a geometrical interpretation to his categorial logic of content.[15] His first attempts can be found in 'Zarys architektoniki i geometrji świata logicznego' [Outline of Architectonics and Geometry of the Logical World] (Bornstein 1922), and then in his more mature work 'Geometrja logiki kategorialnej i jej znaczenie dla filozofii' [Geometry of Categorial Logic and Its Importance for Philosophy] (Bornstein 1926).[16] However, we cannot get entangled in complicated (and not always clear) technical details. Suffice it to say that Bornstein refers to projective geometry stressing its qualitative character. He shows the structure of his logic of content through various diagrams, both two-dimensional and three-dimensional. Thus he refers to the works of the previous authors who used a geometrical exposition of certain logical dependencies, for example Euler's wheels, the diagrams of Venn and Haase or certain conceptions of Leibniz, Peirce and Grassmann.

The analyses on logic and the use of geometrical interpretations led Bornstein to the conclusion that both domains could be linked and thus a qualitative-categorial geometrical logic could be created. This logic can help us discover and reveal the universal structures of reality. In his work *La logique géométrique et sa portée philosophique* [Geometrical Logic and Its Meaning for Philosophy] (Bornstein 1928) he tried to show the similarity of the domain of thought and the domain of space objects. He tried to unite both of his systems: algebraic logic and geometrical logic in one system called topologic (Polish: topologika).

Bornstein's system of qualitative-categorial geometrical logic is not quite clear—therefore we cannot go into details. Let us say only that he used in his system some ideas of projective geometry. He considered two-dimensional and three-dimensional

[15] Add that the adjective "categorial" means here something else than in Ajdukiewicz' „categorial grammar". .

[16] Cf. also his unpublished works (a), (b) and (c).

categorial logic. The two-dimensional logico-geometrical space was spread by him on two categories: *genus proximus* and *differentia specifica* whereas the three-dimensional one on those two categories and additionally on the category of individualization (individual determination).

Bornstein generalised his system of logic as a dialectical geometrical logic and presented it in his unpublished work *Zarys teorii logiki dialektycznej* [Outline of the Theory of Dialectical Logic] (Bornstein 1946). Unfortunately his explanations were not clear enough. It should be stressed that he assumed the possibility of various degrees of dialecticality and consequently, various kinds of dialectical logics. In his opinion traditional logic is the least dialectical one whereas mathematical logic is partially-dialectical. In the quoted work he wanted to show that dialectical logic could be treated in a mathematical way, could be axiomatized and given a geometrical interpretation. However, the problem of the consistency of dialectical logic appeared. The need to show consistency was very essential and more, this logic was to help examine the real world. The sought-after proof of consistency would refute the accusation of the irrationality of this logic. Unfortunately, Bornstein did not give such a proof—he gave only certain arguments supporting consistency but they were disputable.

Bornstein's considerations were based on his conviction that there existed a harmony between the world of non-spatial thoughts and the world of spatial beings. He thought that mathematics and the logic of quality were objectively grounded in the real world. At the same time, he treated mathematics as an auxiliary domain of philosophy. Bornstein wanted to construct a philosophical system using mathematical concepts. He thought after certain universal structures and principles of the real world; besides the quantitative aspect he looked for the qualitative aspect. In his opinion the order of the world concerns both of these aspects. Thus he spoke about the mathematics of quantity and the mathematics of quality. Mathematics is not only the science of quantity and measure but of order, in particular the order between qualities. For Bornstein metrical geometry was an example of the mathematics of quantity whereas projective geometry—the mathematics of quality. Philosophy should look for the qualitative structures of the world—its starting point should be qualitative mathematical logic.

Bornstein's conceptions did not win recognition and acceptance of his contemporaries. The reasons for this included the lack of clarity and precision of his ideas. Moreover, they were not completely worked out. Bornstein's investigations did not follow the main trend of research. The mathematical and logical tools he constructed were to create a metaphysical system and not to serve analyses, which was decidedly different from the style of philosophy accepted and developed in the Lvov–Warsaw School.

Acknowledgments The following footnote should be added here: The financial support of the National Center for Science [Narodowe Centrum Nauki] (grant no N N101 136940) is acknowledged. The paper is based on my book Philosophy of Logic and Mathematics in Poland in the 1920s and 1930s (Murawski 2014) that will be published by Birkhäuser Verlag, Basel. I would like to thank Professor Jan Woleński for helpful hints concerning Leśniewski's philosophy.

Open Access This article is distributed under the terms of the Creative Commons Attribution License which permits any use, distribution, and reproduction in any medium, provided the original author(s) and the source are credited.

References

Bornstein, B. (1912). *Prolegomena filozoficzne do geometryi* [Philosophical Prolegomena to Geometry], Skład Główny w Księgarni E. Wende i S-ka (T. Hiż and A. Turkuł), Warszawa

Bornstein B (1913) Problemat istnienia linji geometrycznych [The Problem of the Existence of Geometrical Lines]. Przegląd Filozoficzny 16:64–73

Bornstein B (1914) Podstawy filozoficzne teorji mnogości [The Philosophical Foundations of Set Theory]. Przegląd Filozoficzny 17:183–193

Bornstein B (1915) W sprawie recenzji p. Stanisława Leśniewskiego rozprawy mojej pt. "Podstawy filozoficzne teorji mnogości" [On Mr Stanisław Leśniewski's Review of My Dissertation 'The Philosophical Foundations of Set Theory']. Przegląd Filozoficzny 18:121–140

Bornstein, B. (1916). *Elementy filozofii jako nauki ścisłej* [Elements of Philosophy as an Exact Science], Skład Główny w Księgarni E. Wendego i S-ka, Warszawa

Bornstein B (1922) Zarys architektoniki i geometrji świata logicznego [Outline of Architectonics and Geometry of the Logical World]. Przegląd Filozoficzny 25:475–490

Bornstein B (1926) Geometrja logiki kategorialnej i jej znaczenie dla filozofii [Geometry of Categorial Logic and Its Importance for Philosophy]. Przegląd Filozoficzny 29:173–194

Bornstein B (1928) La logique géométrique et sa porte philosophique. Bibliotheca Universitatis Liberae Polonae, Warszawa

Bornstein, B. (1946). Zarys teorii logiki dialektycznej [Outline of the Theory of Dialectical Logic], unpublished, hand-written copy in the Jagiellonian Library, Kraków

Bornstein, B. (a). O kategorialnej geometrii algebraicznej [On Categorial Algebraic Geometry], unpublished, hand-written copy in the Jagiellonian Library, Kraków

Bornstein, B. (b). Co to jest kategorialna geometria algebraiczno-logiczna [What Is the Categorial Algebraic-Logical Geometry], unpublished, hand-written copy in the Jagiellonian Library, Kraków

Bornstein, B. (c). Geometria analityczna Descartes'a a geometria filozoficzna [Descartes' Analytic Geometry vs. Philosophical Geometry], unpublished, hand-written copy in the Jagiellonian Library, Kraków

Leśniewski, S. (1914). Teoria mnogości na "podstawach filozoficznych" Benedykta Bornsteina [Set Theory on the 'Philosophical Foundations' of Benedykt Bornstein]. *Przegląd Filozoficzny* 17, 488–507. Reprint: O podstawach filozoficznych teorii mnogości. *Filozofia Nauki* 2(22), 1998, 123–139

Murawski R (2014) Philosophy of logic and mathematics in Poland in the 1920s and 1930s. Birkhäuser Verlag, Basel (to appear)

Śleziński, K. (2009). *Benedykta Borsteina koncepcja naukowej metafizyki i jej znaczenie dla badań współczesnych* [Benedykt Bornstein's Conception of Scientific Metaphysics and Its Meaning For the Contemporary Investigations], Wydawnictwo "scriptum", Kraków

Śleziński K (2010) Bornsteinowska koncepcja podstaw teorii mnogości. Krytyka Stanisława Leśniewskiego i polemika Benedykta Bornsteina [Bornstein's Conception of the Foundations of Set Theory. Stanisław Leśniewski's Critique and Benedykt Bornstein's Polemics]. Studia z Filozofii Polskiej 5:97–112

Twardowski, K. (1894). *Zur Lehre vom Inhalt und Gegenstand der Vorstellungen. Eine psychologische Untersuchung*, Alfred Hölder, Wien. Second edition: Philosophia Verlag, München-Wien 1982. English translation: *On the Content and Object of Presentations*, Nijhoff, The Hague 1976

Woleński J (1989) Logic and philosophy in the Lvov–Warsaw School. Kluwer Academic Publishers, Dordrecht