Roles of Diagrammatic Information for the Discovery of Geometrical Theorems

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Abstract. Knowledge discovery is one of the humans' most creative activities. As is often pointed out, humans often draw figures in the process of reasoning and problem solving. Diagrammatic information from a figure gives us numerical and relational data for reasoning, and it also gives us cues for controlling reasoning. The author has developed four discovery systems in the domain of plane geometry. These systems automatically draw several forms of figures and observe the figures in order to acquire geometrical data. The data are used for extracting numerical expressions needed for discovering theorems, controlling discovery processes in order to avoid combinatorial explosion, and evaluating the usefulness of generated numerical expressions. As an approach of automated scientific discovery, this paper discusses the roles of diagrammatic information in the process of knowledge discovery.

Keywords: Automated scientific discovery, discovery systems, diagrammatic reasoning, geometrical theorems.

1 Introduction

Knowledge discovery is one of the humans' most creative activities. There are two main approaches for machine discovery: data mining and automated scientific discovery. Although the former is intensively studied by many researchers, the latter is also important for clarifying the process of humans' intelligent activities.

As the research for the latter approach, the author has developed four discovery systems in the domain of plane geometry. These systems automatically draw several forms of figures and observe the figures in order to acquire geometrical data. The data are used for extracting numerical expressions needed for discovering theorems, controlling discovery processes in order to avoid combinatorial explosion, and judging the usefulness of generated numerical expressions. This paper discusses the roles of diagrammatic information in the process of knowledge discovery.

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2 Discovery Systems for Geometrical Theorems

Discovery systems that the author has developed are briefly explained below:

- DST[1]: The system draws figures by adding lines on a triangle and by giving constraints to sides or angles. By observing the figures, relation of sides and angles are extracted in the form of numerical expressions. Well-known basic geometrical theorems such as Pythagorean theorem are rediscovered as the result of transformation of observed numerical expressions.
- PLANET: The system accepts an input figure and observes similarity or congruence of triangles in the figure. Numerical expressions extracted from the figure are then combined to generate new expressions by substitutions. Expressions of closely located sides or angles are regarded as theorems. The system rediscovers theorems such as Menelaus's theorem and addition theorem of trigonometric functions.
- EXPEDITION[2]: The system draws figures by adding lines on a circle. Numerical values of the length of sides or measure of angles are observed from the figures. Theorems such as Power theorems and Thales' theorem are rediscovered inductively from the numerical values.
- DIGEST[3]: The system accepts an input figure and observes the relations of areas of triangles in the figure. Numerical expressions extracted from the figure are then combined to generate new expressions by substitutions. Expressions that do not contain terms of areas represent relation of sides, which are regarded as theorems. The system rediscovers theorems such as Ceva's theorem or Menelaus's theorem.

As an example of above discovery systems, explanation of EXPEDITION is described below. The system draws figures by adding lines on a circle as shown in Fig. 1. By observing each figure, numerical data such as the length of line segments and the measure of angles are acquired. From two entries of approximately equal numerical values, a candidate for theorem is obtained as shown in Fig. 2.

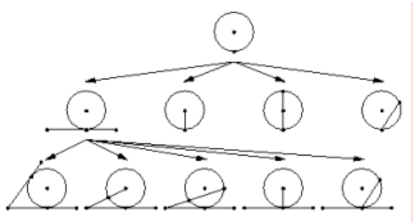

Fig. 1. Generation of Fig.s by drawing lines on a circle

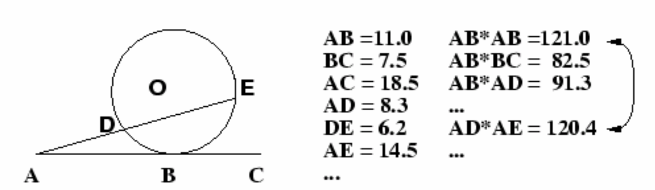

Fig. 2. A candidate for theorem

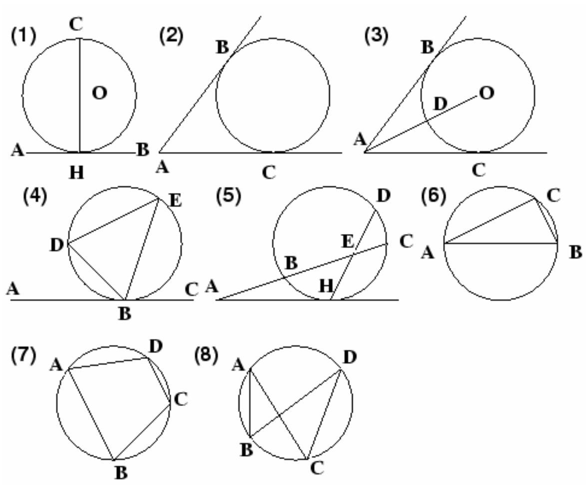

Fig. 3. Figures for rediscovering theorems

Fig. 3. shows figures that EXPEDITION actually generates. Many well-known geometrical theorems are rediscovered from these figures, such as Power theorem ($AB*AC=AH^2$) from figure (5), and Thales' theorem (angle ACB = 90^o) from figure (6).

3 Roles of Diagrams for the Discovery of Geometrical Theorems

In the case of the above discovery systems, processing of diagrammatic information can be divided into two: 1) generation of figures by drawing lines or giving constraints, and 2) observing data from them. The latter data are divided further: 2a) data that represent relations of sides or angles that are basis of theorem discovery, 2b) data for evaluating the usefulness of generated numerical expressions, and 2c) data for controlling transformation of numerical expressions.

In case 1, the systems make use of emergent property for the acquisition of diagrammatic data. Fig.s often clarify relations that are not explicitly given. By observing a figure that is drawn based on a few explicit constraints, much data about implicit relations are also obtained. In case 2a, data obtained from diagrams are both relational (equations among sides or angles) and numerical (numerical data of sides or angles). The former data is used for deductive reasoning (transformation by substitution), and the latter data is used for inductive reasoning (verification of theorematic hypothesis). In case 2b, the systems make use of emergent property as the criteria for discovery. Evaluating the usefulness of numerical expressions is not an easy task in general. In the above systems, numerical expressions about closely located sides or angles are regarded as useful theorems. The systems utilize figures for both data for discovering theorems, and criteria for evaluating them. There is another criterion for evaluating usefulness based on syntactic form of numerical expression, such as symmetry or simplicity. Combination of both criteria (one based on diagrammatic information and the other based on syntactic form) is one of the most interesting and challenging topic, which are not yet tackled in the above systems.

Another important role of diagrams is to control the direction of reasoning process (case 2c). It is often pointed out that humans often take the strategy of choosing objects that are closely located to current viewpoint as the next target for reasoning. The strategy is based on an assumption that related objects are often closely located in a figure. In the above discovery systems, DST utilizes diagrammatic information as the strategy for transforming numerical expressions; the system tries to eliminate terms that are newly generated by drawing additional lines. Resultant expressions are about sides and angles that exist before drawing additional lines.

4 Concluding Remark

This paper briefly discusses the roles of diagrammatic information based on the development of discovery systems for geometrical theorems. Discovery of geometrical theorem is one of the most intelligent activities, and it requires reasoning using both numerical expressions and diagrammatic information. Development of the above systems is expected to help the understanding of humans' diagrammatic reasoning processes. Discussion of this paper is the first step for clarifying the roles of diagrammatic information for humans' intelligent activities.

References

[1] Murata, T., Mizutani, M., and Shimura, M.: A Discovery System for Trigonometirc Functions, Proc. of the Twelfth National Conference on Artificial Intelligence (AAAI-94), (1994) 645–650

[2] Murata, T., and Shimura, M.: Machine Discovery Based on Numerical Data Generated in Computer Experiments, Proc. of the Thirteenth National Conference on Artificial Intelligence (AAAI-96) , (1996) 737–742

[3] Murata, T., and Shimura, M.: A Method of Machine Discovery Based on Geometrical Structure, Systems and Computers in Japan, Vol. 29, No. 6, (1998) 74–81