What is a Wavefunction?

Wayne C. Myrvold
Department of Philosophy
The University of Western Ontario
wmyrvold@uwo.ca

June 13, 2014

Abstract

Much of the the discussion of the metaphysics of quantum mechanics focusses on the status of wavefunctions. This paper is about how to think about wavefunctions, when we bear in mind that quantum mechanics —that is, the nonrelativistic quantum theory of systems of a fixed, finite number of degrees of freedom—is not a fundamental theory, but arises, in a certain approximation, valid in a limited regime, from a relativistic quantum field theory. We will explicitly show how the wavefunctions of quantum mechanics, and the configuration spaces on which they are defined, are constructed from a relativistic quantum field theory. Two lessons will be drawn from this. The first is that configuration spaces are not fundamental, but rather are derivative of structures defined on ordinary spacetime. The second is that wavefunctions are not as much like classical fields as might first appear, in that, on the most natural way of constructing wavefunctions from quantum-field theoretic quantities, the value assigned to a point in configuration space is not a local fact about that point, but rather, depends on the global state.

1 Introduction

This paper is about how we should think about quantum mechanics—that is, the nonrelativistic theory of systems of a fixed, finite number of degrees of freedom—in light of the fact that it is not a fundamental theory, but should, rather, be regarded as a low-energy, nonrelativistic approximation to a relativistic quantum field theory (which itself might be a low-energy approximation to something else). With this in mind, we will take a look at how quantum-mechanical wavefunctions arise from a quantum field theory. We will find that doing so sheds light on the ontological status of wavefunctions.

Wavefunctions are often thought of as analogous to classical fields. This seems most natural in the single-particle case, in which a wavefunction, assigns, fot any given time, values to points in ordinary space. For two particles, we need to specify two points in space to specify the value of a wavefunction; we say that the wavefunction is defined on a 6-dimensional *configuration space* for the pair of particles. And so on, for n-particle wavefunctions, which require a $3n$-dimensional configuration space.

If one wants to continue to think of a wavefunction as analogous to a classical field, then this, it seems, involves taking the space on which they are defined, a configuration space of extraordinarily high dimension ($3n$, where n is the number of fundamental particles in the universe) as the arena in which events take place, and our familiar three-dimensional space as somehow derivative of structures on this space. And indeed, some are willing to bite this bullet. For a particularly emphatic expression of this view, see Albert (1996); see also Loewer (1996), Lewis (2004), Ney (2012, 2013a,b), and North (2013). This position has come to be known, somewhat misleadingly, as "wavefunction realism." This is perhaps not the best terminology, as one can accept wavefunctions as real without regarding them as fundamental (which is the position adopted in this paper), and, even if wavefunctions are part of the fundamental ontology, it is a further step to insist that a wavefunction must be analogous to a classical field.

A field, as ordinarily construed, is an assignment of field values (numbers or vectors, or something more complicated) to spacetime points. These values are thought of as local properties of the spacetime points to which they are assigned. *Prima facie*, a single-particle wavefunction is like a field, and it is this thought that underlies the construal of n-particle wavefunctions as fields on an $3n$-dimensional space. Thinking of wavefunctions as derivative structures in a quantum field theory gives us incentive to think of them in a different way. In section 4 we will show how to construct wavefunctions from a quantum field theory. As we shall see, some consequences of this construction are that configuration spaces are not fundamental, and that wavefunctions are relevantly unlike fields, in that the value of a wavefunction assigned to a point in configuration space is not a local property of that point.

In a quantum field theory, the particle concept is not fundamental. Nonetheless, under certain circumstances—for free (that is, noninteracting) field theories, and, for interacting theories, within certain approximations in a limited regime—the energy states will differ by discrete field quanta, which act somewhat like particles and underwrite particle talk in the sorts of contexts in which particle talk is used (*e.g.*, in the context of incoming and outgoing states of scattering experiments). Even in the free field case, states of a fixed, finite number of field quanta are exceptional states; a general state will be a superposition of n-particle states, for arbitrarily large n. For such states, we can specify the state by providing, for each n, an n-particle wavefunction, defined on a $3n$-dimensional configuration space.

These configuration spaces are constructed from field operators defined on ordinary spacetime. Moreover, talk of such configuration spaces is not available for general states of an interacting field theory, but is limited to regimes that license talk of the particle content of the states. For this reason, we cannot think of the configuration spaces as more fundamental than ordinary spacetime.

Moreover, though we can (for some states, making certain approximations) construct $3n$-dimensional configuration spaces on which to define n-particle wavefunctions, these wavefunctions will not be field-like, in anything like the ordinary sense of field. This is because they are not assignments of *local* quantities to points in their respective configuration spaces. This is easiest to see in the cases that would seem to be most conducive to that interpretation, namely, in single-particle states. On the most natural way of defining wavefunctions in the context of a quantum field theory, if a state has no single-particle component—that is, if, when writing the state as a superposition of n-particle states, the single-particle term is zero—then its single-particle wavefunction is identically zero.

This is unsurprising, but it has a consequence that is perhaps a bit surprising: the value of a single-particle wavefunction at a point in space carries consequences for the state arbitrarily far from that point. Start with a single-particle state (or any other state) in which the single-particle wavefunction has a nonzero value at some point x. Consider a second state, which is just like the first in a neighborhood of x, but differs from it in that there is a particle confined to a region R, far from x. In this second state, the probability that an array of detectors spread through space will report a detection at *x and nowhere else* is zero, and this means that the single-particle wavefunction for this state has the value zero at x.[1] A nonzero value of a single-particle wavefunction at x is incompatible with there being a particle definitely located in the region R, no matter how far R is from x.

It is sometimes said that the conclusions about the dimensionality of spacetime are unaffected by a move to a quantum field theory, except that they become infinitely more radical, in that quantum field theory leads to the conclusion that the dimension of spacetime is infinite (see, *e.g.* Ney 2013a, 48–49). It is difficult to evaluate this suggestion without an explicit proposal of what we are to take the space that is meant to be the arena of such a theory to be. The Hilbert space of our theory will be infinite-dimensional, to be sure, and this means that, if we choose a basis for the space, we can represent a state vector by a sequence of numbers, which are the expansion coefficients in terms of this basis, and this sequence will be an infinite sequence. But this is not the same as having a theory in which the basic structures are fields on an infinite-dimensional space. The prospects for formulating a theory in which states are represented by something field-like on an infinite-dimensional space are briefly discussed in section 4.3.

[1]In the context of a relativistic theory, this story needs to be qualified with talk of approximate localization of a particle in R and the single-particle wavefunction being close to zero in the second state, but the conceptual point remains. See §**??**.

The guiding principle adopted in this paper is that ontological conclusions drawn from quantum mechanics should be compatible with the idea that quantum mechanics is not fundamental, but an approximation, valid in a limited regime, to a more fundamental theory. This, I claim, should be our attitude to *all* metaphysical conclusions drawn from physics; no matter how good our current theories are, we should be prepared to learn that they are approximations to something more fundamental. This does not vitiate the project of drawing ontological conclusions from physical theory. The objects of which classical physics speaks are real, even if they aren't exactly as imagined in classical physics. Quantum wavefunctions are real, though they are not fundamental. We should assume that our current quantum field theories are approximations to some more fundamental theory, which might be a theory (as envisioned by some workers in quantum gravity) in which spacetime structure is not fundamental, but emergent. Nothing in what follows presumes the fundamentality of quantum field theory. We will assume, however, that in the relevant regime it makes sense to talk of spacetime structure, and also that, whatever our ontology, it is realist about quantum states.

2 Some preliminary terminological remarks.

For the purposes of this paper, a wavefunction for a quantum mechanical system with N spatial degrees of freedom is a function ψ, obeying the Schrödinger equation or some other appropriate wave equation, that, for each time, assigns a complex number (or spinor, or something more complicated, if there are internal degrees of freedom) to each point in the configuration space of the system, such that the integral of $\psi^*\psi$ (summing over internal degrees of freedom, if any) over a measurable subset of the system's configuration space yields the probability of finding the configuration of the system in that set. We will not take "wavefunction" to be synonymous with "quantum state," as there are quantum theories—notably, quantum field theories—in which the quantum state is not represented by a wavefunction as we have described it.

Alyssa Ney captures what seems to be common usage in the literature on the metaphysics of quantum mechanics when she says, "The view that the wave function is a fundamental object and a real, physical field on configuration space is today referred to as 'wave function realism' " (Ney, 2013a, p. 37). This is a conjunction of three distinct claims: that the wavefunction represents something real, that this is a fundamental object, and this object is a field on configuration space. This is worth mentioning because the position in this paper is that wavefunctions represent something real but they are not fundamental and they are not fields on configuration space.[2]

[2]Since one can believe in the reality of wavefunctions without accepting the other two conjuncts, the name is misleading. At a recent workshop on Spacetime and the Wavefunction, held in Barcelona,

Quantum state monism is the view that all there is to the world is whatever is represented by the quantum state. When the quantum state can be represented by a wavefunction, we might express this as the view that the wavefunction is everything,[3] as long as we regard this as elliptical for the claim that reality is exhausted by what is represented by the wavefunction; taken literally, the assertion that the wavefunction is everything is a category mistake; the wavefunction is a mathematical entity, not a piece of physical reality.

We will say that a physical quantity is *intrinsic* to a spacetime region if the fact the quantity has the value that it has carries no implications about states of affairs outside the region. A *local beable*, as we understand it, is one that can be regarded as an intrinsic property of a bounded spacetime region, and will be said to be local to that region. These are to be distinguished from quantities, such as the center of mass of an extended distribution of masses, with which a location is associated, though the quantity is not local to small neighborhoods of its location.

A state description is *separable* if the state of the world supervenes on assignments of local beables to elements of arbitrarily fine coverings of spacetime (see Myrvold 2011 for discussion); *nonseparable* if not.

3 Wavefunctions and dimensionality

The *locus classicus* of claims that quantum mechanics has radical implications for the dimensionality of space is David Albert's "Elementary Quantum Metaphysics" (1996). The reasoning therein begins, explicitly, from the premise that wavefunctions are "(plainly) fields." These are "thought of (as with all fields) as intrinsic properties of the points in configuration space with which they are associated" (278). If a field regarded as part of the fundamental ontology of the theory, the argument continues, then its substratum, the space on which it is defined, should be regarded as the fundamental space of the theory, the arena in which events transpire.

The notion of physical space at work, for Albert, is that of

> a stage on which whatever theory we happen to be entertaining at the moment depicts the world as unfolding: a space (that is) in which a specification of the local conditions at every address at some particular time (but not at any proper subset) of them amounts to a complete specification of the world, on that theory, at that time.

Note that Albert builds separability of the state description into this conception; a specification of the state of the world must be a specification of *local* conditions at every address at some particular time.

configuration space fundamentalism was suggested as a less misleading alternative.

[3]As does Bell, in his well-known remark,"Either the wavefunction, as given by the Schrödinger equation, is not everything, or it is not right" (1987a, 201).

Quantum states are nonseparable (though that does not prevent us from thinking of Minkowski spacetime or Galilean spacetime as the arena on which they unfold). If we are to insist that a complete specification of the world consist of specifications of local conditions at each point, then, clearly, a quantum state description on ordinary spacetime will not satisfy this demand; this is what motivates the move to configuration space as the arena of events. Similar motivations are at work for Barry Loewer (1996); there the concern is to rescue Lewis's doctrine of Humean Supervenience from the threat of quantum nonseparability.

The picture sketched by Albert is one on which we have events unfolding on a high-dimensional space that has no built-in relation to our familiar spacetime; our three-dimensional space is meant to be something that emerges from the sorts of interactions that characterize a world like ours. The Hamiltonian (or Lagrangian) contains a potential term that depends on certain functions on configuration space—Albert calls them "interaction distances"—that happen to mesh in such a way that is consistent with these functions being pairwise distances of n particles in a 3-dimensional space. This is meant to be contingent on the sorts of interactions that actually obtain; other interactions would lead to a picture of objects in a space of different dimension, or, generically, to no space of dimension lower than $3n$.

Note that the claim about the high-dimensionality of spacetime is not meant to follow merely from the appearance of a high-dimensional space in the theory. We are used to theories in which the set of states of a theory—which we refer to as the "state space" of the theory—has the structure of some high-dimensional or infinite-dimensional space. Consider, for example, the theory of a classical field. A state of the field is an assignment of field values to points in ordinary physical space. The state space of the theory—that is, the set of all possible field configurations—is a vector space, by which we mean nothing more than (i) we can multiply any field configuration by a real number and get another possible field configuration, and (ii) we can add field configurations to get other field configurations. There is no finite set of field configurations that yield all possible field configurations via linear combinations, and so the vector space is an infinite-dimensional one. This is not (I hope) thought by anyone to have implications for the structure of spacetime. The substratum, the space on which the field is defined, is three-dimensional.

Every element of the state space of a physical theory depicts a way that the world (or fragment of it that the theory is about) could be at a time. As the state of the world changes, the element of state space that represents the state of the world changes. We could, metaphorically, refer to this as the state-point moving about in state-space. But it would be a mistake to say that the theory is *about* a state-point moving about in state space, except as a somewhat roundabout of saying that the state of the world represented, at any given time, by the state-point at that time, changes, as time goes on, from one possibility to another. Let us call this mistake the *State Space Substitution*: the substitution of the state space of a theory (a theory whose states are states of affairs in some spacetime arena) for the spacetime arena. That this is a fallacy perhaps goes without saying, but it is worth mentioning, because sometimes people *do* talk about quantum mechanics as a theory that is about the motion of a state vector in a Hilbert space. This is a misleading way of putting things; the theory only becomes a physical theory via an association of certain operators on that Hilbert space with dynamical variables belonging to the system of interest, and it is only via such an association that a vector in a Hilbert space can represent anything physical.[4]

Radical as the idea of taking configuration space as our fundamental space may seem, it is not clear that even this is available to the would-be wavefunction monist. As Tim Maudlin (2010) has argued, if the wavefunction is all that we have to represent the world, then it is unclear that we have a right to talk of configuration space at all. In a classical theory of point particles, we know what it means to talk of the instantaneous configuration of these particles. The de Broglie-Bohm pilot-wave theory retains particles whose configuration is given by a point in configuration space. But on a wavefunction monist ontology, there is nothing, apparently, for the points in configuration space to be configurations *of*. Without such configurations, the relation of points in configuration space to points in physical space is obscured, configuration space threatens to come unmoored from spacetime, and it begins to look as if there is no way for a function on this $3n$-dimensional space, whose spatiotemporal significance is unclear, to represent ordinary three-dimensional objects such as tables, chairs, and Stern-Gerlach devices.

It seems to me that *this* worry about wavefunction monism is misplaced. Wavefunction monism faces an interpretational problem of interpreting the wavefunction as representing a world of objects, but this is a problem of finding objects in the wavefunction, not a problem of the relation of the wavefunction to ordinary space. On the most natural way of thinking about quantum states, they have a built-in relation to regions of spacetime, whether or not we have configuration spaces available.

4 Wavefunctions from quantum field theory

In this section, we will explicitly construct wavefunctions from quantum field theory. It will instructive to do this first for a quantum field theory in Galilean spacetime, because things are more straightforward in this context. This will be of

[4]And it is the need for a choice of an association of operators with dynamical variables that makes it possible to have two common pictures of state evolution. On one—the Schrödinger picture—we keep fixed the association of operators with physical degrees of freedom, and depict a change of physical state via a change in state vector. On the other, the Heisenberg picture, we keep fixed the Hilbert-space vector that represents the state, and change the operator associated with a physical degree of freedom. Both of these depict a changing physical state.

service in making the transition from a quantum field theory on Minkowski spacetime to quantum mechanics on Galilean spacetime, as it will be convenient to first choose a foliation of spacetime and construct a Galilean invariant quantum field theory on it.

The basic structures of quantum field theories are field operators associated with spacetime points,[5] from which the observables of the theory are constructed, and quantum states. A quantum state is a positive linear functional on the algebra of operators constructed from these field operators, that is, a function ρ that assigns to each operator $\hat{A}$ a complex number $\rho(\hat{A})$, such that, for all operators $\hat{A}$, $\hat{B}$ and all complex numbers α, β.

i). $\rho(\hat{A}^\dagger \hat{A}) > 0$;

ii). $\rho(\alpha\hat{A} + \beta\hat{B}) = \alpha\rho(\hat{A}) = \beta\rho(\hat{B})$.

For an operator that represents an observable, the expectation value, in state ρ, of a result of a measurement of that observable is given by

$$\langle \hat{A} \rangle_\rho = \rho(\hat{A})/\rho(\hat{I}), \tag{1}$$

where $\hat{I}$ is the identity operator. It is usually convenient to normalize ρ so that $\rho(\hat{I}) = 1$.

It is a theorem (the GNS theorem) that we can always construct a Hilbert space in which a state ρ is represented by Hilbert-space vector $|\Psi\rangle$ on which the operators act:

$$\rho(\hat{A}) = \langle \Psi|\hat{A}|\Psi\rangle. \tag{2}$$

The quantum state is usually thought of representing something real, whether supplemented by additional ontology or not. The restriction of the quantum state to a local algebra associated with some spacetime region yields the state of that region. This picture is either implicit or explicit in most discussions of the ontology of quantum field theory (*e.g.*, Myrvold 2003), and has recently been dubbed *Spacetime State Realism* by Wallace and Timpson (2010). Among the quantities assigned expectation values by quantum states are quantum analogues of mass density, energy density, charge density, *etc.*, and this gives us a start in construing the quantum state (perhaps supplemented by additional ontology) as representing a world of objects of the familiar sort.

[5]Strictly speaking, these are *operator-valued distributions*, which yield operators when smeared with appropriate test functions. But it is common, in the physics literature, and especially in textbooks, to call them field operators, and we will adopt this parlance. One should bear in mind, however, that $|\mathbf{x}, \mathbf{t}\rangle$, as defined by (21), is not a vector in our Hilbert space, though, for any square-integrable function f,

$$\int d^3\mathbf{x}\; f(\mathbf{x})|\mathbf{x}, t\rangle$$

is. We can think of $|\mathbf{x}, t\rangle$ as a mapping from functions f to Hilbert space vectors.

A start, but this cannot be the whole story. The familiar quantum measurement problem arises from the fact that typical states, including those that result from assuming the usual unitary evolution in the context of an experiment, are not eigenstates of the sorts of quantities—pointer positions, and the like—that one would expect to have definite values. None of this is affected by the move to quantum field theory, and, in fact, will be exacerbated by it, as it is a consequence of the Reeh-Schlieder theorem that no state of bounded energy will be an eigenstate of any local observable belonging to a bounded spacetime region. Common approaches to the problem of finding a world (or worlds) in quantum theory are hidden-variables theories (such as the de Broglie-Bohm theory), dynamical collapse theories, and Everettian approaches. In what follows, it is assumed that some satisfactory extension of such approaches to the context of quantum field theory is available. Exactly how these will work won't be discussed in this paper, but what is said is meant to be applicable to any viable resolution of the problem.

4.1 Galilean invariant quantum field theory

Consider a classical complex-valued field $\psi(\mathbf{x}, t)$, satisfying the Schrödinger equation,

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V(\mathbf{x}, t)\psi, \tag{3}$$

where $V(\mathbf{x}, t)$ is some potential energy function. (Though no fields like this appear in classical physics, this does not prevent us from considering the classical theory of such a field). This field equation can be derived from a Lagrangian density,[6] which gives rise to a Hamiltonian density (that is, energy density),

$$\mathcal{H}(\mathbf{x}, t) = \frac{\hbar^2}{2m} \nabla \psi^*(\mathbf{x}, t) \cdot \nabla \psi(\mathbf{x}, t) + V(\mathbf{x}, t) |\psi(\mathbf{x}, t)|^2. \tag{4}$$

Integrating this over all space, we get the total energy

$$H = \int d^3\mathbf{x}\, \mathcal{H}(\mathbf{x}, t) = \int d^3\mathbf{x}\, \psi^*(\mathbf{x}, t) \left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{x}, t) \right) \psi(\mathbf{x}, t). \tag{5}$$

Suppose we want now to construct a quantum theory. In the classical theory, the basic dynamical variables are the field values $\psi(\mathbf{x}, t)$; in the quantum theory, these become field operators $\hat{\psi}(\mathbf{x}, t)$ operating on an appropriately constructed Hilbert space. The standard procedure for passing from a classical theory to the

[6] If you don't know what this means, see the early chapters of almost any introductory quantum field theory text; one textbook that begins with Galilean invariant field theory is Greiner and Reinhardt (1996).

corresponding quantum theory, known as canonical quantization, leads to equal-time commutation relations,

$$\begin{aligned} [\hat{\psi}(\mathbf{x},t), \hat{\psi}^\dagger(\mathbf{x}',t)] &= i\hbar\, \delta^3(\mathbf{x}-\mathbf{x}')\, I. \\ [\hat{\psi}(\mathbf{x},t), \hat{\psi}(\mathbf{x}',t)] &= [\hat{\psi}^\dagger(\mathbf{x},t), \hat{\psi}^\dagger(\mathbf{x}',t)] = 0. \end{aligned} \tag{6}$$

We can form Hamiltonian density field operators,

$$\hat{\mathcal{H}}(\mathbf{x},t) = \frac{\hbar^2}{2m} \nabla\hat{\psi}^\dagger(\mathbf{x},t)\cdot\nabla\hat{\psi}(\mathbf{x},t) + V(\mathbf{x},t)\hat{\psi}^\dagger(\mathbf{x},t)\hat{\psi}(\mathbf{x},t), \tag{7}$$

which, when integrated over all space, yield the Hamiltonian operator,

$$\hat{H} = \int d^3\mathbf{x}\, \hat{\psi}^\dagger(\mathbf{x},t) \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x},t) \right) \hat{\psi}(\mathbf{x},t). \tag{8}$$

Now let $\{u_i(\mathbf{x})\}$ be a set of functions that are orthonormal,

$$\int d^3\mathbf{x}\, u_i^*(\mathbf{x})\, u_j(\mathbf{x}) = \delta_{ij}, \tag{9}$$

and satisfy the completeness condition,

$$\sum_i u_i^*(\mathbf{x}')\, u_i(\mathbf{x}) = \delta^3(\mathbf{x}-\mathbf{x}'). \tag{10}$$

Then we can define new operators, $\hat{a}_i(t)$, via

$$\hat{\psi}(\mathbf{x},t) = \sum_i u_i(\mathbf{x})\, \hat{a}_i(t). \tag{11}$$

Now suppose that the potential V is time-independent. We can take the functions $\{u_i(\mathbf{x})\}$ to be a complete set of solutions to the eigenvalue equation:[7]

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x},t) \right) u_i = \varepsilon_i\, u_i. \tag{12}$$

With this choice, the Hamiltonian operator is related to the operators $\hat{a}_i$, $\hat{a}_i^\dagger$ in a simple way:

$$\hat{H} = \sum_i \varepsilon_i\, \hat{a}_i^\dagger(t)\hat{a}_i(t). \tag{13}$$

[7] We write this with a discrete index. But treating cases, such as the case of zero potential, in which we have a continuum of energy eigenvalues, proceeds analogously; in the next section we will consider the relativistic field with no external potential.

This gives us commutation relations

$$[\hat{H}, \hat{a}_i(t)] = -\varepsilon_i\, \hat{a}_i(t)$$
$$[\hat{H}, \hat{a}_i^\dagger(t)] = \varepsilon_i\, \hat{a}_i^\dagger(t). \tag{14}$$

Solving the Heisenberg equations of motion for the operators $\hat{a}_i$,$\hat{a}_i^\dagger$, we get

$$\hat{a}_i(t) = e^{-i\varepsilon_i t/\hbar}\, \hat{a}_i(0) \qquad \hat{a}_i^\dagger(t) = e^{i\varepsilon_i t/\hbar}\, \hat{a}_i^\dagger(0). \tag{15}$$

We will henceforth write $\hat{a}_i$ and $\hat{a}_i^\dagger$ for $\hat{a}_i(0)$ and $\hat{a}_i^\dagger(0)$.

The commutation relations (14) have the consequences that,

i). If a state $|\Psi\rangle$ is an eigenstate of the Hamiltonian $\hat{H}$ with eigenvalue E, then $\hat{a}_i^\dagger|\Psi\rangle$ is also an eigenstate of $|H\rangle$, with eigenvalue $E + \varepsilon_i$.

ii). If a state $|\Psi\rangle$ is an eigenstate of the Hamiltonian $\hat{H}$ with eigenvalue E, then $\hat{a}_i|\Psi\rangle$ is also an eigenstate of $|H\rangle$, with eigenvalue $E - \varepsilon_i$.

This means that the operators $\hat{a}_i^\dagger$ and $\hat{a}_i$ can be thought of as creation and annihilation operators for field quanta of energy ε_i. This means: the state $\hat{a}_i^\dagger|\Psi\rangle$ differs from the state $|\Psi\rangle$ by containing one more field excitation of energy ε_i; the state $\hat{a}_i|\Psi\rangle$, one less.

We want to form a Hilbert space on which the operators $\hat{\psi}(\mathbf{x}, t)$, $\hat{\psi}^\dagger(\mathbf{x}, t)$, and all operators that can be formed from them, such as $\hat{H}$, $\hat{a}_i$, $\hat{a}_i^\dagger$, *etc.*, act. Assume that all of the ε_i are positive. Then, since, for any vector $|\Psi\rangle$ in our Hilbert space, $\hat{a}_i|\Psi\rangle$ must also be a vector in the Hilbert space, the only way for there to be a ground state, that is, a state of lowest energy, is for there to be a vector $|0\rangle$ such that

$$\hat{a}_i|0\rangle = 0 \tag{16}$$

for all i. We will take as our Hilbert space the smallest Hilbert space that contains the energy ground state vector $|0\rangle$ and is closed under the algebra of operators formed from the field operators $\hat{\psi}(\mathbf{x}, t)$, $\hat{\psi}^\dagger(\mathbf{x}, t)$.

Energy eigenstates are those states that are formed from action of the creation operators $\hat{a}_i^\dagger$ on the vacuum state. States of the form $(\hat{a}_i^\dagger)^{n_i}|0\rangle$ contain n_i quanta of energy ε_i, for total energy $n_i\,\varepsilon_i$; state of the form $(\hat{a}_i^\dagger)^{n_i}(\hat{a}_j^\dagger)^{n_j}|0\rangle$ contain n_i quanta of energy ε_i and n_j quanta of energy ε_j, for total energy $n_i\varepsilon_i + n_j\varepsilon_j$, and so on. We can define number operators

$$\hat{n}_i = \hat{a}_i^\dagger \hat{a}_i|0\rangle. \tag{17}$$

Then the state $|0\rangle$ is a zero eigenstate of all the number operators, and $\hat{a}_i^\dagger$ and $\hat{a}_i$ are raising and lowering operators for $\hat{n}_i$. We can also define a total number operator

$$\hat{N} = \sum \hat{n}_i. \tag{18}$$

We find that the operators $\hat{\psi}^\dagger(\mathbf{x}, t)$ are raising operators for the total number operator; if $|\Psi\rangle$ is a an eigenvector of $\hat{N}$ with eigenvalue n, then $\hat{\psi}^\dagger(\mathbf{x}, t)|\Psi\rangle$ is an eigenvector of $\hat{N}$ with eigenvalue $n + 1$. Moreover, for any t, we can write the total number operator as

$$\hat{N} = \int d^3\mathbf{x} \, \hat{\psi}^\dagger(\mathbf{x}, t)\hat{\psi}(\mathbf{x}, t). \tag{19}$$

This suggests the interpretation of $\hat{\psi}^\dagger(\mathbf{x}, t)$ as a creation operator for a field excitation located, at time t, at the point $\mathbf{x}$, and the operators

$$\hat{N}(\Delta; t) = \int_\Delta d^3\mathbf{x} \, \hat{\psi}^\dagger(\mathbf{x}, t)\hat{\psi}(\mathbf{x}, t) \tag{20}$$

as local number operators; an eigenvector of $\hat{N}(\Delta; t)$ with eigenvalue n represents a state in which exactly n field quanta are located, at time t, in the region Δ. These localizable field quanta are the field-theoretic counterpart of quantum-mechanical particles, and we will henceforth refer to them as such.

We will write

$$|\mathbf{x}, t\rangle = \hat{\psi}^\dagger(\mathbf{x}, t)|0\rangle. \tag{21}$$

for the state in which there is a single particle that, at time t, is located at $\mathbf{x}$. From the commutation relations (6) it follows that two single-particle states with the particle located at distinct locations at a given time t are orthogonal to each other:

$$\langle \mathbf{x}', t|\mathbf{x}, t\rangle = \delta^3(\mathbf{x} - \mathbf{x}'). \tag{22}$$

There are also 2-particle localized states,

$$|\mathbf{x}, t; \mathbf{x}', t'\rangle = \hat{\psi}^\dagger(\mathbf{x}', t') \, \hat{\psi}^\dagger(\mathbf{x}, t)|0\rangle, \tag{23}$$

and so on, for n-particle states.

A general one-particle state—that is, an eigenstate of $\hat{N}$ with eigenvalue 1—can be expanded in terms of the localized states $|\mathbf{x}, t\rangle$.

$$|\Psi\rangle = \int d^3\mathbf{x} \, \psi^{(1)}(\mathbf{x}, t) \, |\mathbf{x}, t\rangle, \tag{24}$$

where

$$\psi^{(1)}(\mathbf{x}, t) = \langle \mathbf{x}, t|\Psi\rangle = \langle 0|\hat{\psi}(\mathbf{x}, t)|\Psi\rangle \tag{25}$$

The function $\psi^{(1)}(\mathbf{x}, t)$ is the single-particle wavefunction for the state $|\Psi\rangle$. Similarly, for a general two-particle state, we can write

$$|\Psi\rangle = \int d^3\mathbf{x} \, d^3\mathbf{x}' \, \psi^{(2)}(\mathbf{x}, \mathbf{x}', t)|\mathbf{x}, t; \mathbf{x}', t\rangle, \tag{26}$$

where

$$\psi^{(2)}(\mathbf{x}, \mathbf{x}', t) = \langle \mathbf{x}, t; \mathbf{x}', t|\psi\rangle = \langle 0|\hat{\psi}(\mathbf{x}', t) \, \hat{\psi}(\mathbf{x}, t)|\Psi\rangle. \tag{27}$$

n-particle wave-functions, for any positive n, are defined analogously.

Because the field operators $\hat{\psi}(\mathbf{x},t)$ satisfy the operator version of the Schrödinger equation,

$$i\hbar\frac{\partial\hat{\psi}}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\hat{\psi} + V\hat{\psi}, \tag{28}$$

the single-particle wavefunctions $\psi^{(1)}(\mathbf{x},t)$ satisfy the Schrödinger equation

$$i\hbar\frac{\partial\psi^{(1)}}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi^{(1)} + V\psi^{(1)}. \tag{29}$$

Similarly, the two-particle wavefunctions $\hat{\psi}^{(2)}(\mathbf{x},t;\mathbf{x}',t)$ satisfy[8]

$$i\hbar\frac{\partial\psi^{(2)}}{\partial t} = -\frac{\hbar^2}{2m}\left(\nabla^2 + \nabla'^2\right)\psi^{(2)} + \left(V(\mathbf{x}) + V(\mathbf{x}')\right)\psi^{(2)}, \tag{30}$$

and n-particle wave-functions satisfy the n-particle Schrödinger equation.

For states that are not single-particle states, we can choose to leave the single-particle wavefunction $\psi^{(1)}$ undefined, or else, since (25) makes sense for any state, not just a single-particle state, we can define, for any state, a single-particle wavefunction, and also a two-particle wavefunction, and, for any n, an n-particle wavefunction. It is this latter convention that we will adopt. A general state vector, which will be a superposition of n-particle states, can be represented as

$$|\Psi\rangle = c_0\,|0\rangle + \int d^3\mathbf{x}\;\psi^{(1)}(\mathbf{x},t)\,|\mathbf{x},t\rangle + \int d^3\mathbf{x}\int d^3\mathbf{x}'\;\psi^{(2)}(\mathbf{x},t;\mathbf{x}',t)\,|\mathbf{x},t;\mathbf{x}',t\rangle + ... \tag{31}$$

The integral of $|\psi^{(1)}(\mathbf{x},t)|^2$ over all space gives the squared norm of the projection of the state onto the single-particle subspace, that is, the probability, in such a state, that, of an array of detectors spread out over all space, exactly one will fire. Similarly, the integral of $|\psi^{(2)}(\mathbf{x},t;\mathbf{x}',t)|^2$ gives the square of the norm of the projection of the state onto the two-particle subspace. For a two-particle state, or any other state that contains no one-particle component, the single-particle wavefunction will be identically equal to zero.

As mentioned in the introduction, our definition of the wavefunction has the consequence that the value of a single-particle wavefunction at a point x in space-time is not a local property of that point. Recall that a local property of a point x is meant to be one that carries no implications for states of affairs outside arbitrarily small neighborhoods of x. For any state in which there is one (or more) particle definitely located in a region R, the single-particle wavefunction is zero at

[8]Note that, since we started with a single, non-self-interacting field, subject only to external potentials, the potential energy for two particles is additive and contains no interaction term. If we had started with the classical theory of two or more interacting fields, we would have obtained creation operators for quanta of each of these fields, and the wavefunction corresponding to a state containing one of each type of field quanta would contain an interaction term.

any point x outside of R. A nonzero value of a single-particle wavefunction at a point x is incompatible with there being a particle definitely located in a region R not containing x.

Lest this seem counterintuitive,[9] consider the following. Think of an entangled state of a pair of spin-1/2 particles, located a large distance from each other, say,

$$c_1|z^+\rangle_A|z^-\rangle_B + c_2|z^-\rangle_A|z^+\rangle_B. \tag{32}$$

The coefficients c_1 and c_2 are not naturally thought of as associated with either one of the systems, or as local to either one of the regions in which a particle is located. Now consider a particle that can be in one of two boxes, A and B, and let $|0\rangle_A$ be a state in which box A is empty, and let $|1\rangle_A$ be a state in which it contains one particle, and similarly for box B. Suppose the state is

$$c_1\,|1\rangle_A|0\rangle_B + c_2\,|0\rangle_A|1\rangle_B. \tag{33}$$

This is, as far as the states of the two boxes are concerned, a single-particle state, with a wavefunction proportional to c_1 in box A, and to c_2 in box B. It is also a state in which there is entanglement across the regions A and B, manifested in anticorrelation of experimental results: if a particle-detection experiment yields a particle detected in region A, a particle detection in region B is guaranteed to yield a negative result, and *vice versa*. It is, in fact, analogous to the entangled spin-state (32), and we should no more be inclined to think of the coefficients c_1, c_2 as properties local to the boxes than we are in the case of the two entangled spins. Thinking of single-particle states in this way, as involving entanglement between states of spatially separated regions, helps us see Einstein's Boxes thought experiment and his remarks at the 1927 Solvay conference as precursors to the EPR argument and its kin.[10]

Quantum mechanics is the quantum theory of systems with a fixed, finite number of degrees of freedom. We obtain n-particle quantum mechanics from a Galilean quantum field theory by restriction to the space of n-particle states. Consideration of single-particle quantum mechanics as a sector of a theory that includes states of larger numbers of particles has led us to the conclusion: the value of a single-particle wavefunction is not a local beable. The reasoning extends, of course, to n-particle wavefunctions, thought of as defined on a $3n$-dimensional configuration space. For any n, a nonzero value of a n-particle wavefunction at a point in configuration space is not local in configuration space.

Now, let's think about how things stand with configuration spaces. A two-particle wavefunction, defined by

$$\psi^{(2)}(\mathbf{x}, \mathbf{x}', t) = \langle 0|\hat{\psi}(\mathbf{x}', t)\,\hat{\psi}(\mathbf{x}, t)|\Psi\rangle \tag{34}$$

[9] Not that counterintuitiveness would necessarily be objectionable!

[10] For discussion of these arguments, include the analogy with the EPR argument, see Norsen (2005) and Norton (2011).

takes two spatial points as arguments. The ingredients that go into defining it are the quantum state $|\Psi\rangle$ (a global, nonseparable state, not supervening on local parts), the vacuum state $|0\rangle$, and field operators $\hat{\psi}(\mathbf{x}, t)$, which are associated with points of spacetime. The double occurrence of a spatial argument on the left corresponds to the appearance of two field operators on the right, defined at different points. We can construct a $6D$ space out of the set of pairs of points in $3D$ space. But there is no temptation to think of this space as more fundamental than the $3D$ space from which it is constructed; a point in this 6D space appears as an argument of the wavefunction via occurrence of two spatial indices in the local field operators $\hat{\psi}$. Nor is there any threat of wavefunctions coming unmoored from ordinary spacetime; a point in the domain of a two-particle wavefunction retains its association with pairs of spacetime points, via the associate of the operators $\hat{\psi}(\mathbf{x}, t)$ with spacetime points.

Though we can form a configuration space whose elements are ordered pairs $(\mathbf{x}, \mathbf{x}')$ of points in Euclidean space, and represent a two-particle state by such a wave-function on this space, this is not the most perspicuous representation of the state, as it introduces apparent distinctions that correspond to no differences.

Recall that, at equal times, field operators commute:

$$\hat{\psi}(\mathbf{x}', t)\, \hat{\psi}(\mathbf{x}, t) = \hat{\psi}(\mathbf{x}, t)\, \hat{\psi}(\mathbf{x}', t). \tag{35}$$

It follows from this that $\psi^{(2)}$ is symmetric in its two arguments.

$$\psi^{(2)}(\mathbf{x}, \mathbf{x}', t) = \psi^{(2)}(\mathbf{x}', \mathbf{x}, t). \tag{36}$$

A two-particle wavefunction, defined by (34), really takes as its arguments *unordered pairs* of points in space.

One way to deal with this is to construct a configuration space consisting of ordered pairs of points, and impose the symmetrization condition (36) on wave-functions. But this move first introduces physically meaningless structure and then removes it by *fiat*.

Questions about the metaphysical status of particles (are they entities that lack haecceity?) arise from a misleading representation of the quantum state via a wavefunction on a two-particle configuration space consisting of ordered pairs of points. The appropriate configuration space is the quotient space of this configuration space under permutations of the two points, or, to put it more simply, the space consisting of unordered pairs of points in space.

If we had started with a fermionic field, that is, a field obeying the anticommutation relations

$$\psi(\mathbf{x}, t)\psi(\mathbf{x}', t) + \psi(\mathbf{x}', t)\psi(\mathbf{x}, t) = 0, \tag{37}$$

then the definition (34) would have the consequence that

$$\psi^{(2)}(\mathbf{x}, \mathbf{x}', t) = -\psi^{(2)}(\mathbf{x}', \mathbf{x}, t). \tag{38}$$

There is a deep relation between the spin associated with a field and whether the field obeys the equal-time commutation relations (35) or the anticommutation relations (37). Fields with integer spin commute at spacelike separation; those with integer spin commute. The connection is seen most perspicuously in the context of relativistic quantum field theory (see, *e.g.*, Streater and Wightman 2000, Haag 1996).

4.2 Relativistic quantum field theory

We now want consider things from the point of view of a relativistic quantum field theory. For simplicity, we will consider the theory of a free, spinless Klein-Gordon field. Things do not change essentially if we add in external potentials. We will find that the picture, sketched in the previous section, of a theory that includes states of a definite number of localizable field quanta, is not available, although it *is* a reasonable approximation in a nonrelativistic regime. If (as actual fields are), the fields are interacting fields, then this complicates the picture of field quanta even further (see Fraser 2008 for discussion), and further restricts the applicability of the picture. The ontology of localizable field quanta is not fundamental, though it can be useful for certain purposes.

By a nonrelativistic regime, we will mean that we are dealing with time and distance scales on which the relativity of simultaneity is negligible. If two objects are (with respect to some reference frame) located a distance d apart, then, for any point p on one object's worldline, the events on the worldline of the other that could, for some reference frame, count as simultaneous with p encompass a time interval of duration $2d/c$. As long as d is not too large (that is, we are not dealing with processes that are spread out too far in space), and the temporal resolution with which we are concerned is not too small, then this ambiguity of simultaneity will be negligible. Under such conditions, differences between ways of slicing up spacetime into spacelike hypersurfaces will be negligible, and we may arbitrarily choose one such slicing, and use it, assured that others will not give us appreciably different results.

By a low-energy regime, we will mean that we are dealing with processes whose total energy E is dominated by the rest energy $m_0 c^2$. That is,

$$\frac{E - m_0 c^2}{m_0 c^2} \ll 1. \tag{39}$$

We begin with a classical field satisfying the Klein-Gordon equation,

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \phi(x) + \mu^2 \phi(x) = 0, \tag{40}$$

where $\mu = m_0 c/\hbar$. The equation has plane-wave solutions

$$\phi_k(x) = e^{-ikx}, \tag{41}$$

for any four-vector k satisfying the mass-shell condition

$$k^2 = k_0^2 - \mathbf{k}^2 = \mu^2. \tag{42}$$

We will write

$$\omega_{\mathbf{k}} = c\sqrt{\mu^2 + \mathbf{k}^2}. \tag{43}$$

Then the mass-shell condition can be written

$$c\,k_0 = \pm\,\omega_{\mathbf{k}}. \tag{44}$$

Any solution of the Klein-Gordon equation can be given a Fourier decomposition in terms of these plane waves,

$$\phi(x) = \int \frac{d^4k}{(2\pi)^4}\; f(k)\; e^{-ikx} \tag{45}$$

where the integral is taken over all k satisfying the mass-shell condition. This can be written (see, *e.g.* Ryder (1996, 127, 135) for details) as,

$$\phi(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3 2\omega_{\mathbf{k}}} \left(a(\mathbf{k})\; e^{-i(\omega_{\mathbf{k}}t - \mathbf{k}\cdot\mathbf{x})} + b^*(\mathbf{k})\; e^{i(\omega_{\mathbf{k}}t - \mathbf{k}\cdot\mathbf{x})} \right). \tag{46}$$

We can associate with the Klein-Gordon equation a locally conserved charge density,

$$q(x) \propto \frac{i}{c^2} \left(\phi^*(x)\, \frac{\partial\phi(x)}{\partial t} - \phi(x)\, \frac{\partial\phi^*(x)}{\partial t} \right), \tag{47}$$

with current density

$$\mathbf{j}(x) \propto -i\left(\phi^*(x)\nabla\phi(x) - \phi(x)\nabla\phi^*(x)\right). \tag{48}$$

These satisfy the continuity equation

$$\frac{\partial q}{\partial t} + \nabla\cdot\mathbf{j} = 0. \tag{49}$$

We also have energy density

$$\mathcal{H}(x) = \frac{1}{c^2}\frac{\partial\phi^*}{\partial t}\;\frac{\partial\phi}{\partial t} + \nabla\phi^*\cdot\nabla\phi + \mu^2|\phi|^2. \tag{50}$$

We now construct a quantum theory by forming field operators $\hat{\phi}(x)$; the functions $a(\mathbf{k})$, $b(\mathbf{k})$ become operators also. Canonical quantization leads to commutation relations

$$\begin{aligned} &[\hat{a}(\mathbf{k}), \hat{a}^\dagger(\mathbf{k}')] = [\hat{b}(\mathbf{k}), \hat{b}^\dagger(\mathbf{k}')] = \hbar c^2\,(2\pi)^3 2\omega_{\mathbf{k}}\,\delta^3(\mathbf{k} - \mathbf{k}'), \\ &[\hat{a}(\mathbf{k}), \hat{a}(\mathbf{k}')] = [\hat{b}(\mathbf{k}), \hat{b}(\mathbf{k}')] = [\hat{a}(\mathbf{k}), \hat{b}(\mathbf{k}')] = [\hat{a}(\mathbf{k}), \hat{b}^\dagger(\mathbf{k}')] = 0. \end{aligned} \tag{51}$$

We can define operators corresponding to energy-momentum and charge-current four-vectors. The t-component of the energy-momentum vector gives us, on the spacelike hyperplane of constant t, energy density operators,[11]

$$\hat{\mathcal{H}}(\mathbf{x},t) = :\frac{1}{c^2}\frac{\partial\hat{\phi}^\dagger}{\partial t}\,\frac{\partial\hat{\phi}}{\partial t} + \nabla\hat{\phi}^\dagger\cdot\nabla\hat{\phi} + \mu^2\hat{\phi}^\dagger\hat{\phi}: \tag{52}$$

This gives a Hamiltonian operator

$$\hat{H} = \int d^3\mathbf{x}\,\hat{\mathcal{H}}(\mathbf{x},t). \tag{53}$$

The charge density operators are

$$\hat{q}(\mathbf{x},t) = :\frac{i}{c^2}\left(\hat{\phi}^\dagger(x)\,\frac{\partial\hat{\phi}(x)}{\partial t} - \hat{\phi}(x)\,\frac{\partial\hat{\phi}^\dagger(x)}{\partial t}\right):, \tag{54}$$

which yields total charge

$$\hat{Q} = \int d^3\mathbf{x}\,\hat{q}(\mathbf{x},t). \tag{55}$$

We also have current density operators

$$\hat{\mathbf{j}}(x) = -:i\left(\hat{\phi}^\dagger(x)\,\nabla\hat{\phi}(x) - \hat{\phi}^\dagger(x)\nabla\hat{\phi}(x)\right): \tag{56}$$

Expressed in terms of $\hat{a}(\mathbf{k})$, $\hat{b}\mathbf{k})$, the total charge and total energy operators take on simple forms

$$\hat{Q} = \int \frac{d^3\mathbf{k}}{(2\pi)^3 2\omega_\mathbf{k}}\left(\hat{a}^\dagger(\mathbf{k})\hat{a}(\mathbf{k}) - \hat{b}^\dagger(\mathbf{k})\hat{b}(\mathbf{k})\right) \tag{57}$$

$$\hat{H} = \int \frac{d^3\mathbf{k}}{2c^2(2\pi)^3}\left(\hat{a}^\dagger(\mathbf{k})\hat{a}(\mathbf{k}) + \hat{b}^\dagger(\mathbf{k})\hat{b}(\mathbf{k})\right) \tag{58}$$

These have the consequences that $\hat{a}^\dagger(\mathbf{k})$ and $\hat{b}^\dagger(\mathbf{k})$, acting on a state vector, increase the total energy, and $\hat{a}(\mathbf{k})$ and $\hat{b}(\mathbf{k})$ decrease it, by $\hbar\omega_\mathbf{k}$, whereas $\hat{a}^\dagger(\mathbf{k})$ increases the total charge and $\hat{a}(\mathbf{k})$ decreases it by one unit, and $\hat{b}^\dagger(\mathbf{k})$ decreases the total charge and $\hat{b}(\mathbf{k})$ increases it by one unit. This means that $\hat{a}^\dagger(\mathbf{k})$ and $\hat{b}^\dagger(\mathbf{k})$ are creation operators for field quanta with opposite charge—a particle and its antiparticle, and $\hat{a}(\mathbf{k})$ and $\hat{b}(\mathbf{k})$ the corresponding annihilation operators.

Defining number operators

$$\hat{n}_a(\mathbf{k}) = \frac{1}{\hbar c^2(2\pi)^3 2\omega_\mathbf{k}}\,\hat{a}^\dagger(\mathbf{k})\hat{a}(\mathbf{k}), \quad \hat{n}_b(\mathbf{k}) = \frac{1}{\hbar c^2(2\pi)^3 2\omega_\mathbf{k}}\,\hat{b}^\dagger(\mathbf{k})\hat{b}(\mathbf{k}), \tag{59}$$

[11]The colons : : indicate normal, or Wick ordering, in which annihilation operators $\hat{a}(\mathbf{k})$, $\hat{b}(\mathbf{k})$ occur to the right of creation operators $\hat{a}^\dagger(\mathbf{k})$, $\hat{b}^\dagger(\mathbf{k})$.

we can write the total charge and total energy as

$$\hat{Q} = \int d^3\mathbf{k}\,(\hat{n}_a(\mathbf{k}) - \hat{n}_b(\mathbf{k})) \tag{60}$$

$$\hat{H} = \int d^3\mathbf{k}\;\hbar\omega_{\mathbf{k}}\,(\hat{n}_a(\mathbf{k}) + \hat{n}_b(\mathbf{k})) \tag{61}$$

The operators $\hat{\phi}^\dagger(\mathbf{x}, t)$ and $\hat{\phi}(\mathbf{x}, t)$ raise and lower the total charge by one unit, respectively. Moreover, the charge created by $\hat{\phi}^\dagger(\mathbf{x}, t)$ is localized, on the hyperplane σ_t of constant t, at the point $\mathbf{x}$. This suggests that we think of the $\hat{\phi}^\dagger(\mathbf{x}, t)$ as a creation operator for a field quantum localized, on σ_t, at $\mathbf{x}$, and that we proceed with defining wavefunctions for these field quanta as we did in the nonrelativistic case. With this in mind, let us define,

$$|\mathbf{x}, t\rangle = \phi^\dagger(\mathbf{x}, t)|0\rangle. \tag{62}$$

There's a hitch, however. The states $|\mathbf{x}, t\rangle$ and $|\mathbf{x}', t\rangle$ are not orthogonal for distinct $\mathbf{x}$, $\mathbf{x}'$. Their inner product falls off rapidly with distance, however; for separations much larger than the Compton wavelength $\mu^{-1} = \hbar/m_0 c$, $\langle\mathbf{x}', t|\mathbf{x}, t\rangle$ decays exponentially as $e^{-\mu|\mathbf{x}-\mathbf{x}'|}$.[12]

What this means is this. Suppose we take a bounded spatial region Δ in the time-slice σ_t. We want to construct a state in which there is one field quantum definitely located in Δ, and none, anywhere else. We might try the state

$$|\Delta\rangle = \int_\Delta d^3\mathbf{x}\;|\mathbf{x}, t\rangle. \tag{63}$$

But any experiment that is guaranteed to give a positive result in this state has a nonzero probability of yielding a positive result in a state that results from applying a spatial shift to $|\Delta\rangle$, even if the shift is a large one. It is, however, possible for this probability to decay exponentially with distance from Δ.

Can we do better? Can we construct a relativistic theory of localizable field quanta, that mimics our Galilean field theory, with relativistically invariant creation

[12] To get a sense of the magnitude of Compton wavelengths, for an electron the Compton wavelength is about 1/137th of the Bohr radius of the Hydrogen atom, and for a proton, it is smaller by a factor of 18 million.

For those who care, the quantity

$$D(\mathbf{x} - \mathbf{x}') = \langle\mathbf{x}', t|\mathbf{x}, t\rangle$$

can be calculated explicitly, and is given by

$$D(\mathbf{x} - \mathbf{x}') = \frac{\mu}{4\pi^2 r} K_1(\mu r),$$

where K_1 is a modified Bessel function of the second kind, and $r = |\mathbf{x} - \mathbf{x}'|$. See Weinberg (1995, p. 202).

operators yielding localized states that, at spacelike separation, are orthogonal to each other? The answer is no; this is the content of the Malament no-go result and related results (see Malament 1996; Fleming and Butterfield 2000; Halvorson 2001; Halvorson and Clifton 2002). In order to make the transition from quantum field theory to quantum mechanics, we're going to have to go nonrelativistic.

As already mentioned, the nonrelativistic regime in which we will be interested is one in which considerations of relativity of simultaneity can be ignored. Let us, then, arbitrarily choose a family $\{\sigma_t\}$ of nonintersecting spacelike hyperplanes, indexed by a time coordinate t, and let us coordinatize each of these by spatial coordinates $\mathbf{x}$, in such a way that lines of constant $\mathbf{x}$ form a set of parallel timelike lines. We will use these to construct a quantum field theory on this foliation of hypersurfaces, and from there get to nonrelativistic quantum mechanics.

It turns out that we *can* construct a set of states indexed by points $\mathbf{x}$ on the hypersurface σ_t, that are orthogonal to each other for distinct $\mathbf{x}$, $\mathbf{x}'$. These states are created by smearing $\hat{\phi}^\dagger(\mathbf{x}, t)$ over the entire hypersurface σ_t. They are the Newton-Wigner states

$$|\mathbf{x}, t\rangle_{NW} = \hat{\psi}^\dagger_{NW}(\mathbf{x}, t)|0\rangle, \tag{64}$$

where

$$\hat{\psi}_{NW}(\mathbf{x}, t) = \int \frac{d^3\mathbf{k}}{(2\pi)^3\sqrt{2\omega_\mathbf{k}}}\, \hat{a}(\mathbf{k}) e^{-i(\omega_\mathbf{k} t - \mathbf{k}\cdot\mathbf{x})}. \tag{65}$$

From the commutation relations (51) it follows that the Newton-Wigner operators obey the equal-time commutation relations

$$[\hat{\psi}_{NW}(\mathbf{x}, t), \hat{\psi}^\dagger_{NW}(\mathbf{x}', t)] = \delta^3(\mathbf{x} - \mathbf{x}'), \tag{66}$$

from which it follows that $|\mathbf{x}, t\rangle_{NW}$ and $|\mathbf{x}', t\rangle_{NW}$ *are* orthogonal to each other for distinct $\mathbf{x}, \mathbf{x}'$.

The state $|\mathbf{x}, t\rangle_{NW}$ is not a strictly localized state on σ_t, in the sense of being just like the vacuum for every region of this hypersurface not containing $\mathbf{x}$. There are local observables, such as smearings of the charge density, whose expectation values differ, in this state, from their vacuum expectation values, everywhere on the hypersurface. However, they rapidly approach their vacuum expectation values as we move away from the point $\mathbf{x}$. A Newton-Wigner particle "located" at $(\mathbf{x}, t)$ can be thought of as a disturbance of the vacuum that, on the hypersurface σ_t, is centered at $\mathbf{x}$, and is non-negligible only in a region of size comparable to the Compton wavelength μ^{-1}.

Given a state $|\Psi\rangle$, we define a one-particle Newton-Wigner wavefunction $\psi^{(1)}_{NW}(\mathbf{x}, t)$ as the inner product of $|\mathbf{x}, t\rangle_{NW}$ with $|\Psi\rangle$.

$$\psi^{(1)}_{NW}(\mathbf{x}, t) = \langle 0|\hat{\psi}_{NW}(\mathbf{x}, t)|\Psi\rangle. \tag{67}$$

Similarly, we define a two-particle Newton-Wigner wavefunction by

$$\psi^{(2)}_{NW}(\mathbf{x}, \mathbf{x}', t) = \langle 0|\hat{\psi}_{NW}(\mathbf{x}, t)\hat{\psi}_{NW}(\mathbf{x}', t)|\Psi\rangle. \tag{68}$$

The single-particle Newton-Wigner wavefunctions obey the Klein-Gordon equation.

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\psi^{(1)}_{NW}(\mathbf{x},t) + \mu^2\,\psi^{(1)}_{NW}(\mathbf{x},t) = 0, \tag{69}$$

To get operators, and hence wavefunctions, satisfying the Schrödinger equation, we employ the following recipe, standard in textbook presentations.

In a low-energy regime, most of the energy consists of the constant term m_0c^2. Since the temporal evolution of our field operators depends on the energy

$$\varepsilon(\mathbf{k}) = \hbar\,\omega_{\mathbf{k}} = m_0c^2\sqrt{1+\mathbf{k}^2/\mu^2}, \tag{70}$$

and this is, in the low-energy regime, approximately equal to m_0c^2 and hence approximately constant (independent of $\mathbf{k}$), it is useful (though not necessary), in considering the evolution of states in a low-energy regime, to separate out this constant part, defining the nonrelativistic energy $\varepsilon_{nr}(\mathbf{k})$ by

$$\varepsilon(\mathbf{k}) = m_0c^2 + \varepsilon_{nr}(\mathbf{k}). \tag{71}$$

Then we can rewrite the Newton-Wigner operators as

$$\hat{\psi}_{NW}(\mathbf{x},t) = e^{-im_0c^2t/\hbar}\int\frac{d^3\mathbf{k}}{(2\pi)^3\sqrt{2\omega_{\mathbf{k}}}}\;\hat{a}(\mathbf{k})e^{-i(\varepsilon_{nr}(\mathbf{k})t/\hbar-\mathbf{k}\cdot\mathbf{x})}. \tag{72}$$

Define new operators $\hat{\psi}(\mathbf{x},t)$ by

$$\hat{\psi}_{NW}(\mathbf{x},t) = e^{-im_0c^2\,t/\hbar}\;\hat{\psi}(\mathbf{x},t). \tag{73}$$

These operators satisfy the same equal-time commutation relations as the Newton-Wigner operators,

$$[\hat{\psi}(\mathbf{x},t),\hat{\psi}^\dagger(\mathbf{x}',t)] = \delta^3(\mathbf{x}-\mathbf{x}'). \tag{74}$$

Thus, $\hat{\psi}^\dagger(\mathbf{x},t)$ and $\hat{\psi}(\mathbf{x},t)$ are suited to play the role of creation and annihilation operators for localized particles in a nonrelativistic quantum field theory.

Since the operators $\hat{\psi}_{NW}(\mathbf{x},t)$ satisfy the Klein-Gordon equation, the operators $\hat{\psi}(\mathbf{x},t)$ satisfy

$$i\hbar\frac{\partial}{\partial t}\hat{\psi} = -\frac{\hbar^2}{2m}\nabla^2\hat{\psi} + \frac{\hbar^2}{2mc^2}\frac{\partial^2\hat{\psi}}{\partial t^2}. \tag{75}$$

If we now define a single-particle wavefunction $\psi^{(1)}(\mathbf{x},t)$ by

$$\psi^{(1)}(\mathbf{x},t) = \langle\mathbf{x},t|\Psi\rangle = \langle 0|\hat{\psi}(\mathbf{x},t)|\Psi\rangle, \tag{76}$$

then this will satisfy

$$i\hbar\frac{\partial\psi^{(1)}}{\partial t} = -\frac{\hbar^2}{2m_0}\nabla^2\psi^{(1)} + \frac{\hbar^2}{2m_0c^2}\frac{\partial^2\psi^{(1)}}{\partial t^2}. \tag{77}$$

For any state $|\Psi\rangle$ whose expansion in terms of the states $|\mathbf{k}\rangle = \hat{a}^\dagger(\mathbf{k})|0\rangle$ contains non-negligible contributions only for $|\mathbf{k}| \ll \mu$, the last term will be negligible compared to the others, and we will have

$$i\hbar \frac{\partial \psi^{(1)}}{\partial t} \approx -\frac{\hbar^2}{2m_0} \nabla^2 \psi^{(1)}. \tag{78}$$

That is, the wavefunction $\psi^{(1)}(\mathbf{x}, t)$ will approximately satisfy the Schrödinger equation.[13]

Defining $\hat{\psi}$ in terms of Newton-Wigner operators has the consequence that we obtain equal-time commutation relations (74) appropriate for particle creation and annihilation operators. An alternative route would be to define $\hat{\psi}$ by

$$\hat{\psi}(x) = e^{im_0 c^2 t/\hbar}\, \hat{\phi}_+(x), \tag{79}$$

where

$$\hat{\phi}_+(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3 2\omega_\mathbf{k}} \hat{a}(\mathbf{k})\; e^{-i(\omega_\mathbf{k} t}. \tag{80}$$

On this route, we would have, instead of the commutation relations (74), commutation relations

$$[\hat{\psi}(\mathbf{x}, t), \hat{\psi}^\dagger(\mathbf{x}', t)] = [\hat{\phi}_+(\mathbf{x}, t), \hat{\phi}_+^\dagger(\mathbf{x}', t)] = D(\mathbf{x} - \mathbf{x}'). \tag{81}$$

The right hand side of this equation is not a δ-function, but a function centred at the origin, which in nonegligible only within a few Compton wavelengths of the origin.

4.2.1 The value of a wavefunction is not a local beable

The value of a Newton-Wigner wavefunction depends not only on the spacetime point x, but also on a hypersurface of simultaneity σ_t. Given a state vector $|\Psi\rangle$, we can also define a covariant wavefunction $\phi^{(1)}(x)$, defined by

$$\phi^{(1)}(x) = \langle 0|\hat{\phi}(x)|\Psi\rangle. \tag{82}$$

The covariant wavefunction depends only on the state and the spacetime point x. In a low-energy regime, $\omega_\mathbf{k}$ is approximately equal to μc, independent of $\mathbf{k}$, and so we will have

$$\psi^{(1)}_{NW}(x) \approx \sqrt{2\mu c}\, \phi^{(1)}(x). \tag{83}$$

Whether we define the wavefunction $\psi^{(1)}(x)$ using the Newton-Wigner wavefunction or the covariant wavefunction, it will not be a local property of the point x. An argument similar to that given for the nonrelativistic case applies.

[13] Note that we did not take a counterfactual limit, such as $c \to \infty$, of the sort that worries philosophers, nor did we consider any idealization. We employed approximations valid for certain regimes.

Consider a one-particle state $|\Psi\rangle$, and a point x, and let N be some bounded neighbourhood of x, and let N' be the causal complement of N, that is, the set of all points spacelike separated from N. Let $|\Psi'\rangle = U|\Psi\rangle$, where U is a unitary operator associated with N', and commutes with all observables associated with N. All such observables have the same expectation values in the two states, and, in this sense, within the neighbourhood N, the state $|\Psi'\rangle$ is just like $|\Psi\rangle$. We can, by appropriate choice of U, make the probability of detecting more than one particle at the same time as high as we like, and this means that we can make the state $|\Psi'\rangle$ as close to being orthogonal to all single-particle states as we like, and so, we can make the value of $\psi^{(1)}(x)$ as close to zero as we like, without changing the state within N. Wavefunctions, as we have defined them, are not assignments of local properties to spacetime points.

4.3 Reconfiguring quantum theory?

As we have seen, on the most natural way of construing them as emergent from a quantum field theory, wavefunctions are not field-like, and the spaces on which they are defined are not more fundamental than ordinary spacetime.

However, suppose one were driven by the conviction that any credible physical theory *has* to satisfy the condition of separability, and hence driven to seek out some theory, more fundamental than quantum field theories as ordinarily conceived, that would satisfy the condition, and, moreover be such that our familiar spacetime, rather than being part of the fundamental structure of the theory, is contingent on the form of the dynamical laws (emerging, perhaps, from symmetries of the Lagrangian).

This raises the questions:

i). Could there be a theory that mimics our quantum field theories, according to which the state of the universe *is* represented by an assignment of local properties to points in some space (if need be, an infinite-dimensional one)?

ii). Moreover, could we formulate a dynamics of the theory in a way that does not presuppose any relation of this space to our familiar four-dimensional spacetime, so that any such relation would be a consequence of certain sorts of interactions?

Two routes suggest themselves in answering the first question, which lend some plausibility to the conjecture that the answer is affirmative, though, it must be admitted, we have at best a promissory note from the advocates of the view, rather than a well-defined proposal. I call these *the lattice maneuver* and the *revisionist wavefunctions* approach.

4.3.1 The Lattice Maneuver

Any experiment takes place in a bounded region of spacetime and, due to energy limitations, only probes distances up to a finite degree of resolution. For this reason (though this involves a bit of hand-waving), we usually take it that the predictions of a quantum field theory can be approximated by a theory that replaces a continuous spacetime with a finite lattice, with each lattice point having a degree of freedom corresponding to a field value at that point.

Such a theory is formally equivalent to a quantum theory of point particles with finitely many spatial degrees of freedom. We can construct a configuration space whose points represent "field configurations"—specifications of field values at each of the lattice points—and, in principle, represent the states of this lattice field theory via wavefunctions on this space of configurations (though, in practice, this is not the most convenient or perspicuous representation of the state, and we never, in fact, explicitly write down such wavefunctions for the states of such a lattice field theory). To treat this space of configurations—each point of which represents an assignment of field values to a point in ordinary space—as the arena in which events take place would be a blatant case of what we have earlier called the State Space Substitution. But let us set aside the issue of motivation and continue to ask whether we can construct a space such the states of a quantum field theory are assignments of beables that are local in the topology of that space.

We can wave our hands a bit more and say that, if we take a limit in which the spacetime region of concern expands without limit and the lattice spacing decreases without limit (and hence the number of lattice points increases without limit), we can obtain, in the limit, a continuum field theory whose states can be represented by wavefunctions on a space of field configurations.

Can the hand-waving be dispensed with? Can this maneuver produce a viable arena for the proponent of wavefunction realism? An exercise for the would-be wavefunction realist: define a space that is the product of infinitely many configuration spaces, with an appropriate topology, and show that states of a quantum field theory can be represented by a function on that space. Problem: it is not known whether any quantum field theory with nontrivial interactions is a continuum limit of a quantum field theory on a lattice. If it can be done, it will be a highly nontrivial task.

4.3.2 Revisionist Wavefunctions

Though wavefunctions, the way they are usually defined, are not assignments of local beables to points in configuration space, one can ask whether there might be other quantities that are suited to act as stand-ins for them and which *can* be regarded as local properties of points in configuration space.

The field operator $\hat{\phi}^\dagger(x)\hat{\phi}(x)$ yields a local observable when smeared, as do the charge-current density operators (54), (56). Therefore, the expectation values of these operators can be regarded as local beables, dependent only on the quantum state in arbitrarily small neighbourhoods of x.

Suppose that we write the complex-valued covariant single-particle wavefunction $\phi^{(1)}(x)$ in polar form, in terms of real-valued functions $R(x)$, $S(x)$:

$$\phi^{(1)}(x) = R(x)e^{iS(x)} \tag{84}$$

It is easy to show that, for a single-particle state $|\Psi\rangle$,

$$|\phi^{(1)}(x)|^2 = R(x)^2 = \langle\Psi|\hat{\phi}^\dagger(x)\hat{\phi}(x)|\Psi\rangle, \tag{85}$$

and thus $R(x)$ can be recovered from the expectation value of a local operator. Note that this relation holds for single-particle states, but not for general states. If we start with a single-particle state $|\Psi\rangle$, and operate on it with a unitary operator U that commutes with all observables associated with a neighbourhood N of x, we will leave the value $\langle\Psi|\hat{\phi}^\dagger(x)\hat{\phi}(x)|\Psi\rangle$ unchanged, but, if the resulting state is not a single-particle state, we will disrupt the relation between this quantity and the single-particle wavefunction $\phi^{(1)}(x)$, so that (85) no longer holds.

For a one-particle state $|\Psi\rangle$, we also have

$$\langle\Psi|\hat{q}(x)|\Psi\rangle = -\frac{2}{c^2}\,R(x)^2\,\frac{\partial S(x)}{\partial t}; \tag{86}$$

$$\langle\Psi|\,\hat{\mathbf{j}}(x)\,|\Psi\rangle = 2R(x)^2\,\nabla S(x). \tag{87}$$

Thus, we can recover S, up to an additive constant, from expectation values of local quantities. The wavefunction of a single-particle state, up to an irrelevant phase factor, can be recovered from the expectation values of local operators.

Something similar can be said for two-particle states. There we need to invoke expectation values of products of local observables defined at two points, x, x'. For any neighborhoods M of x and N of x', these expectation values depend only on the state of $M \cup N$, and hence can be thought of assignments to local beables in a two-particle configuration space.

More generally: any quantum state can be completely specified by giving, for arbitrarily high n, the values of n-fold products of local observables. One can imagine a theory in which all of these functions are assignments of local beables to points of some space isomorphic to the infinite-dimensional Cartesian product $\mathbb{R}^\infty$. Thus, though this is nothing like what is suggested by our quantum field theories in anything like their current form, it is plausible that a sufficiently determined adherent of separability could construct a formalism in which quantum state can be represented as assignments of local quantities to points in some infinite-dimensional space.

4.3.3 The problem of dynamics

Suppose we manage, via one or the other of these two approaches, to construct an infinite-dimensional space such that a quantum state can be represented by a field on that space. This still leaves us with our second question: can we formulate the dynamics of the theory without any commitment to our familiar spacetime, in such a way that four-dimensional spacetime structure could be an emergent consequence of certain sorts of interactions?

The reason that this is a difficulty is that our formulations of quantum theories make heavy use of background spacetime structure. For one thing, it is a fundamental principle of relativistic quantum field theory that operators corresponding to observables commute at spacelike separation, and formulating this condition requires a background causal structure. Would the theory imagined have a background causal structure, or would the causal structure depend on the particular Lagrangian?

Another difficulty is that we use spacetime symmetry to pick out a privileged vacuum state. Quantum field theory can be formulated in curved spacetimes, including spacetimes that lack sufficient symmetry to pick out such a state, but this makes things much more complicated.[14]

This leads us to ask: what sort of spacetime structure is to be presumed, for the infinite-dimensional space that would be the arena of the theory envisaged by proponents of the view known as "wave-function realism"? Does it come pre-equipped with a privileged group of symmetry transformations, isomorphic to the Lorentz group or Galilei group or some other low-dimensional symmetry group, which all interactions are obliged to respect? If so, then it is hard to make sense of the claim that the low-dimensional spacetime structure is not fundamental. If not, then how do we formulate the theory? Some workers in quantum gravity hope, someday, to formulate quantum theory in a manner that is independent of background spacetime structure. And perhaps this can be done. But one thing is clear: the sort of theory that would mimic a quantum field theory and admit of an interpretation such as is envisaged by Albert (1996) remains hypothetical, not something that we have in hand.

5 Conclusions

What have we learned from this exercise? For one thing, it should be clear that the wavefunctions of quantum mechanics are *not* part of the fundamental ontology of the world. They emerge, via certain approximations, in a low-energy, nonrelativistic regime. Nor are configuration spaces more fundamental than ordinary spacetime. Our quantum field theory is a theory on Minkowski spacetime. For certain states,

[14] See Wald (1994) for the classical overview.

namely, states of a definite particle number n, and for low-energy regimes, we can represent the state via a function on a $3n$-dimensional space, but this representation is not available for arbitrary states.

Moreover, wavefunctions, obtained in the most natural way from a quantum field theory, are not assignments of local beables to points in configurations space, even in the single-particle case. This is not to say that an advocate of separability could not, with some effort, reconstrue things so as to represent quantum states via assignments of local beables to points in some appropriately constructed space, but it is clear that this would be an *imposition* of separability on the theory, and can by no means be regarded as the default position on the ontology of quantum theories. What quantum theory suggests is that we accept nonseparability of state descriptions.

6 Acknowledgments

I would like to thank David Albert, Erik Curiel, Sheldon Goldstein, Carl Hoefer, Douglas Kutach, Joshua Luczak, Kerry McKenzie, Allysa Ney, Travis Norsen, and Albert Solé for helpful discussions and comments, as well as members of the audience at the Southwestern Ontario Philosophy of Physics colloquium, at PSA 2012, and at the Second PERSP Workshop on Space-Time and the Wave Function, Barcelona 2014. This work was funded by a grant from the Foundational Questions Institute (FQXi).

References

Albert, D. Z. (1996). Elementary quantum metaphysics. In J. T. Cushing, A. Fine, and S. Goldstein (Eds.), *Bohmian Mechanics and Quantum Mechanics: An Appraisal*, pp. 277–284. Dordrecht: Kluwer.

Bell, J. S. (1987a). Are there quantum jumps? In C. Kilmister (Ed.), *Schrödinger: Centenary celebration of a polymath*, pp. 41–52. Cambridge: Cambridge University Press. Reprinted in Bell (1987b, 201–212).

Bell, J. S. (1987b). *Speakable and Unspeakable in Quantum Mechanics*. Cambridge: Cambridge University Press.

Fleming, G. and J. Butterfield (2000). Strange positions. In J. Butterfield and C. Pagonis (Eds.), *From Physics to Philosophy*, pp. 108–165. Cambridge: Cambridge University Press.

Fraser, D. (2008). The fate of 'particles' in quantum field theories with interactions. *Studies in History and Philosophy of Modern Physics 39*, 841–859.

Greiner, W. and J. Reinhardt (1996). *Field Quantization*. Berlin: Springer.

Haag, R. (1996). *Local Quantum Physics: Fields, Particles, Algebras. 2nd edition*. Berlin: Springer.

Halvorson, H. (2001). Reeh-Schlieder defeats Newton-Wigner: On alternative localization schemes in relativistic quantum field theory. *Philosophy of Science 68*, 111–133.

Halvorson, H. and R. Clifton (2002). No place for particles in relativistic quantum theories? *Philosophy of Science 69*, 1–28.

Lewis, P. J. (2004). Life in configuration space. *The British Journal for the Philosophy of Science 55*, 713–729.

Loewer, B. (1996). Humean supervenience. *Philosophical Topics 24*, 101–127.

Malament, D. B. (1996). In defense of dogma: Why there cannot be a relativistic quantum mechanics of (localizable) particles. In R. Clifton (Ed.), *Perspectives on Qauntum Reality: Non-Relativistic, Relativistic, and Field-Theoretic*, pp. 1–10. Dordrecht: Kluwer Academic Publishers.

Maudlin, T. (2010). Can the world be only wavefunction? In S. Saunders, J. Barrett, A. Kent, and D. Wallace (Eds.), *Many Worlds? Everett, Quantum Theory, and Reality*, pp. 121–143. Oxford: Oxford University Press.

Myrvold, W. C. (2003). Relativistic quantum becoming. *The British Journal for the Philosophy of Science 53*.

Myrvold, W. C. (2011). Nonseparability, classical and quantum. *The British Journal for the Philosophy of Science 62*, 417–432.

Ney, A. (2012). The status of our ordinary three dimensions in a quantum universe. *Noûs 46*, 525–560.

Ney, A. (2013a). Introduction. In Ney and Albert (2013), 1–51.

Ney, A. (2013b). Ontological reduction and the wave function ontology. In Ney and Albert (2013), 168–183.

Ney, A. and D. Z. Albert (Eds.) (2013). *The Wave Function: Essays on the Metaphysics of Quantum Mechanics.* Oxford University Press.

Norsen, T. (2005). Einstein's boxes. *American Journal of Physics 73*, 164–176.

North, J. (2013). The structure of a quantum world. In Ney and Albert (2013), 184–202.

Norton, J. D. (2011). Little boxes: A simple implementation of the Greenberger, Horne, and Zeilnger result for spatial degrees of freedom. *American Journal of Physics 79*, 182–188.

Ryder, L. H. (1996). *Quantum Field Theory* (2nd ed.). Cambridge University Press.

Streater, R. F. and A. S. Wightman (2000). *PCT, Spin and Statistics, and All That.* Princeton: Princeton University Press.

Wald, R. M. (1994). *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics.* Chicago: University of Chicago Press.

Wallace, D. and C. G. Timpson (2010). Quantum mechanics on spacetime I: Spacetime state realism. *The British Journal for the Philosophy of Science 61*, 697–727.

Weinberg, S. (1995). *The Quantum Theory of Fields, Vol. I.* Cambridge University Press.