



Definition of Intuitive Set Theory

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September 25, 2001

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Introduction

In two earlier papers [1, 2], intuitive set theory was defined as the theory we get when we add the two axioms, *monotonicity* and *fusion*, to ZF theory. Here we attempt to replace these two axioms with, Axiom of Combinatorial sets and Axiom of Infinitesimals, the motivation being that they are simpler to state and possibly easier to accept.

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Statement of the Axioms

If k is an ordinal, we will write $\binom{\aleph_\alpha}{k}$ for the cardinality of the set of all subsets of \aleph_α with the same cardinality as k .

Axiom of Combinatorial Sets:

$$\aleph_{\alpha+1} = \binom{\aleph_\alpha}{\aleph_\alpha}.$$

We will accept the fact that every number in the interval $(0, 1]$ can be represented *uniquely* by an *infinite* nonterminating binary sequence. For example, the infinite binary sequence

.10111111...

can be recognized as the representation for the number $3/4$ and simi-

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larly for other numbers. This in turn implies that an *infinite recursive* subset of positive integers can be used to represent numbers in the interval $(0, 1]$. It is known that the cardinality of the set R of such recursive subsets is \aleph_0 . Thus, every $r \in R$ represents a real number in the interval $(0, 1]$.

We will write

$$\binom{\aleph_\alpha}{\aleph_\alpha}_r$$

to represent the cardinality of the set of all those subsets of \aleph_α of cardinality \aleph_α which contain r , and also write

$$\left\{ \binom{\aleph_\alpha}{\aleph_\alpha}_r \mid r \in R \right\} = \binom{\aleph_\alpha}{\aleph_\alpha}_R.$$

We will define a *bonded set* as a set which can appear only on the left side of the binary relation \in and not on the right side. What this means is that a bonded set has to be considered as an integral unit from which not even the axiom of choice can pick out an element. The definition of a bonded set is very much similar to that of an urelement.

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Axiom of Infinitesimals:

$$(0, 1] = \left(\begin{matrix} \aleph_\alpha \\ \aleph_\alpha \end{matrix} \right)_R$$

where $\left(\begin{matrix} \aleph_\alpha \\ \aleph_\alpha \end{matrix} \right)_r$ is a bonded set.

The axiom of infinitesimals makes it easy to visualize the unit interval $(0, 1]$.

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Generalized Continuum Hypothesis

We derive the generalized continuum hypothesis from the axiom of combinatorial sets as below:

$$2^{\aleph_\alpha} = \binom{\aleph_\alpha}{0} + \binom{\aleph_\alpha}{1} + \binom{\aleph_\alpha}{2} + \cdots + \binom{\aleph_\alpha}{\aleph_0} + \cdots + \binom{\aleph_\alpha}{\aleph_\alpha}.$$

Note that $\binom{\aleph_\alpha}{1} = \aleph_\alpha$. Since, there are \aleph_α terms in this addition and $\binom{\aleph_\alpha}{k}$ is a monotonically nondecreasing function of k , we can conclude that

$$2^{\aleph_\alpha} = \binom{\aleph_\alpha}{\aleph_\alpha}.$$

Using axiom of combinatorial sets, we get

$$2^{\aleph_\alpha} = \aleph_{\alpha+1}.$$

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Infinitesimals

The axiom of infinitesimals allows us to visualize the unit interval $(0, 1]$ as a set of bonded sets, with cardinality \aleph_0 . Thus, $\left(\frac{\aleph_\alpha}{\aleph_\alpha}\right)_r$ represents an infinitesimal corresponding to the number r in the interval $(0, 1]$.

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Conclusion

In view of the fact that we can derive the generalized continuum hypothesis from the axiom of combinatorial sets, it is easy to see that we can use the axiom of combinatorial sets and the axiom of infinitesimals for defining intuitive set theory.

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References

- [1] K. K. Nambiar, *Intuitive Set Theory*, Computers and Mathematics with Applications **39** (1999), no. 1-2, 183–185.
- [2] K. K. Nambiar, *Real Set Theory*, Computers and Mathematics with Applications **38** (1999), no. 7-8, 167–171.



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