Foundations of Computer Science

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Introduction

Teaching computer science for several years convinces me that many of the concepts in it can be simplified to a considerable extent. I believe that

- it is possible to define a machine equivalent to Turing machine, that is more intuitive in its working,
- if three more derivation rules are added to Elementary Arithmetic of Gödel, his Incompleteness Theorems can be proved without using any metalanguage,
- adding two more axioms to Zermelo-Fraenkel set theory, will allow us to derive Continuum Hypothesis and split the unit interval into infinitesimals.

The purpose of the presentations here is to give the basis for my beliefs and, hopefully, assist computer science students.

NuMachine and NuAlgebra

(Mechanical foundations of computer science)

Turing, long time ago, told us very clearly what we mean by computation. He described a well-defined machine and said that whatever that machine can do is what we should take as the definition of computation. Computer scientsts constructed models of Turing machines, with the confident knowledge that it will not be possible to excel their machines for all times to come.

There was only one problem for computer science undergraduates, they could not very easily program the clumsy movements of the Turing machine. Attempting to alleviate this difficulty, the NuMachine described here, whose movements follows the Peano postulates closely, provides an alternative to Turing machine as a model of computation.

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Sentient Arithmetic and Gödel's Theorems

(Logical foundations of computer science)

Investigating the mechanical and logical foundations of computer science, by and large, amounts to the same thing as looking into the foundations of mathematics. Since set theory forms the basis for all mathematics, it is clear that computer science forms a part of set theory. A theory we understand very well is the Elementary Arithmetic (EA) of Gödel, even though it is only a fragment of set theory.

Gödel has proved that there are formulas in Elementary Arithmetic, which will introduce contradictions, irrespective of whether we assume the formula itself or its negation. His proof is in metalanguage. Sentient Arithmetic (SA) adds three more derivation rules to EA and shows that that the proof for incompleteness of SA can be given in SA itself without using any metalanguage.

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Two Axioms to extend Zermelo-Fraenkel Theory

(Foundations of mathematics)

As Hilbert says, “The infinite! No other question has ever moved so profoundly, the spirit of man”. The trouble with humans is that they are not comfortable with any thing which does not have an end, and they find themselves embedded in a space which do not have an end. Attempting to get out of this dilemma, mathematicians have invented set theory, with a multiplicity of infinities of increasing complexity. Intuitive Set Theory (IST) attempts to explain a set theory in which we can have a reasonable visualization of our infinite universe.

Intuitive Set Theory is the theory we get when Axiom of Monotonicity (AM) and Axiom of Fusion (AF) are added to Zermelo-Fraenkel theory. AM makes the Continuum Hypothesis true, and AF splits the unit interval into infinitesimals. Skolem Paradox does not arise in IST, and also there are no sets which are not Lebesgue measurable.

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Conclusion

Our discussion about intuitive set theory (IST) leads us to some remarkable conclusions about any *significant theory* which satisfies all the axioms of ZF theory. An important one is that, we can always divide the statements of any significant theory into four mutually exclusive categories: F is a *theorem*, if a proof exists for F, but not for $\overline{F}$. F is a *falsehood*, if a proof exists for $\overline{F}$, but not for F. F is an *introversion*, if a proof exists for $\overline{F}$ when F is assumed, and a proof for F exists when $\overline{F}$ is assumed. F is a *profundity*, if a proof exists for neither F nor $\overline{F}$, and it is not an introversion.

Gödels first incompleteness theorem says that any significant theory has introversions in it. The second incompleteness theorem says that a consistency statement in any significant theory is an introversion. Note that generalized continuum hypothesis is a profundity in ZF theory, whereas it is a theorem in IST.

We will call a theory *profound*, if it contains a profoundity. Here is conjecture worth considering: *Every significant theory is profound*.