

A defense of contingent logical truths

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Abstract A formula is a contingent logical truth when it is true in every model M but, for some model M , false at some world of M . We argue that there are such truths, given the logic of actuality. Our argument turns on defending Tarski's definition of truth and logical truth, extended so as to apply to modal languages with an actuality operator. We argue that this extension is the philosophically proper account of validity. We counter recent arguments to the contrary presented in Hanson's 'Actuality, Necessity, and Logical Truth' (Philos Stud 130:437–459, 2006).

In a paper of 1988, Edward Zalta argues that there are logical truths and analytic truths that are not necessary. Primary examples are instances of the schema

$$LA1 : \mathcal{A}\phi \rightarrow \phi$$

where ϕ is some contingent claim, and \mathcal{A} is the actuality operator. Take as an example the sentence 'If it is actually the case that Obama is president, then Obama is president'. This sentence is contingent. The justification for this is straightforward. Given the contingency of the proposition [Obama is president], there is a world at which it is false; i.e., a world, say w_1 , where Obama is not president. At w_1 , the proposition [It is actually the case that Obama is president] is true, as that proposition is a proposition not about w_1 but the actual world, where Obama is president. But then, at w_1 , the left side of our conditional is true and the right side false, making the conditional itself false at w_1 and so only contingently true at the

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actual world. All sides of the debate on which we are about to embark agree that there are contingent instances of LA1.

What is controversial is whether or not all instances of LA1 are analytic and logically true. Zalta argues that they are (and hence that we need to learn to live with contingent logical and analytic truths). In his 2006, Hanson argues to the contrary, defending the traditional connection between logical truth, analyticity, and necessity. In this paper we consider Hanson's case and argue that LA1 is indeed a logical and analytic truth.

1 Formal preliminaries and Zalta's argument

Whether or not all instances of LA1 are logically true depends on the notion of logical truth one adopts. In their pioneering work on the logic of the actuality operator, Crossley and Humberstone (1977) distinguish two distinct notions of validity, which they dub *general* and *real-world* validity. We shall first characterize these competing notions, relate the choice between these notions to the logical status of LA1, and then turn to the matter of defending a choice of real-world validity over general validity.

Both definitions assume that a model M of modal logic is a structure of the form $\langle W, w_x, V \rangle$, where W is a set of possible worlds, w_x is a distinguished world in W , and V is a valuation function that assigns a set of worlds to each atomic sentence of the language.¹ In what follows, we shall assume an S5 propositional modal logic and so we need not include an accessibility relation, which will play no role in what follows. Moreover, we shall use 'model' and 'interpretation' interchangeably, since some authors use the former while others use the latter in talking about the same set-theoretic structure.

Let us assume that the definition of truth $_M$ of a formula ϕ at a world w (i.e., $M, w \models \phi$) has been given recursively in the standard way.² Then the notion of real-world validity is defined in terms of the notion of truth in a model: ϕ is true $_M(M \models \phi)$ just in case ϕ is true at the distinguished world w_x in M (i.e., $M \models \phi$ iff $M, w_x \models \phi$). We now define real-world validity as: a formula ϕ is *R*-valid just in case ϕ is true in every model M , i.e., $\forall M(M \models \phi)$.

By contrast, the definition of general validity bypasses the notion of truth in a model and defines validity directly from the notion of truth at a world in the model, as follows: ϕ is *G*-valid just in case, for every model M and every world $w \in W_M$,

¹ We shall not be discussing those modal logicians who eliminate altogether the actual world from the models of modal logic. Those logicians face two further problems, namely, that there seems to be no way for them to introduce an actuality operator and there is no way to define the notion of truth in a model if no reference to a distinguished actual world can be made (as done below).

² I.e., as follows:

1. When ϕ is an atomic sentence letter p , $M, w \models \phi$ iff $w \in V_M(p)$
2. When ϕ has the form $\neg\psi$, $M, w \models \phi$ iff it is not the case that $M, w \models \psi$
3. When ϕ has the form $\psi \rightarrow \chi$, $M, w \models \phi$ iff either it is not the case that $M, w \models \psi$ or $M, w \models \chi$
4. When ϕ has the form $\Box\psi$, then $M, w \models \phi$ iff for every world $w \in W, M, w \models \psi$.

We can then define the other truth-connectives and \diamond in the standard ways.

ϕ is true_{*M*} at w , i.e., iff $\forall M \forall w (M, w \models \phi)$. Thus, *G*-validity is not defined in terms of the notion of truth in a model ($M \models \phi$).

Intuitively and informally, a formula is *R*-valid just in case, for every model, it is true at the distinguished world of the model. A formula is *G*-valid, on the other hand, just in case, for every model, it is true in every world of the model. So, whereas the *G*-validity of a formula involves its truth value at counterfactual worlds of the models, its *R*-validity does not.

The choice between these competing definitions of validity determines the logical status of instances of LA1. While every instance of LA1 is true in every model *M*, for every contingent instance ψ of LA1, there is a model *M* and world $w \in W_M$, where w is not the distinguished world of *M*, such that ψ is false_{*M*} at w . Hence, while every instance of LA1 is *R*-valid, there are instances of LA1 that are not *G*-valid. This follows from the definition of the two notions of validity and the fact that there are contingent instances of LA1.

Zalta argues that the proper way to understand the notion of logical truth excludes *G*-validity as a genuine kind of logical truth. The argument turns on the role that the notion of truth in a model plays in our intuitive conception of logical truth. It is argued that only the notion of *R*-validity respects this role.³ The idea is that the most fundamental semantic notion is that of truth in a model. Logical truth is then explicated as truth in every model. Truth invariance under permutation of the interpretation of the nonlogical vocabulary is what it is to be logically true. This establishes an intimate connection between truth in a model and logical truth. But the notion of truth in a model does not play this foundational role in the definition of *G*-validity. A formula is *G*-valid, recall, just in case it is true for every model *M* and world w . The notion of *G*-validity thus by-passes the notion of truth in a model and thus does not respect the foundational role this notion plays. So *G*-validity does not characterize a kind of genuine logical truth.

2 Hanson's first two problems

Hanson raises two problems with Zalta's argument for the primacy of the notion of *R*-validity. First, he challenges the claim that Tarski's (1936) definition of logical truth, which is in terms of an extensional (i.e., nonmodal) language, carries over to a nonextensional (i.e., modal) language (Hanson 2006, p. 442).⁴ Second, he claims that, with a minor modification, the proponent of *G*-validity can explicate the notion of logical truth in terms of truth in a model. We consider each claim in turn.

It is true that Tarski's work on logical truth was carried out in the context of a nonmodal language and so when we move to a modal language certain changes and additions are necessary. But we claim that Tarski provided us with a proper

³ See Zalta (1988, p. 66).

⁴ For the purposes of this paper, we will consider the language of propositional logic to be extensional and the language of propositional modal logic to be nonextensional. But see Zalta (1993), where it is argued that when propositions are assigned as the denotations of the sentence letters in propositional modal logic, the language of propositional modal logic becomes *extensional*.

understanding of the notion of logical truth as truth under every interpretation.⁵ This basic insight should not be altered, no matter what new languages we go on to consider. What will change is what constitutes an interpretation of the language in question. The basic conception of logical truth as truth across all interpretations should remain the same, even when we move from a nonmodal to a modal language. Our response to Hanson's first claim, then, is simply this. We agree that one should expect there to be changes in the semantic definitions when we move from nonmodal to modal languages, but we think those changes should take place in the definition of an interpretation and not the definition of logical truth. It does not follow from the fact that interpretations of modal languages are more complex structures that the definition logical truth has to change. Thus, Hanson has not offered a reason to think that the concept of logical truth changes when we move from a nonmodal to a modal language.

Hanson's second objection grants for the sake of argument that preserving Tarski's definition of logical truth is a virtue. He then argues that it is a virtue that the proponent of *G*-validity can have as well by a simple addition of an extra parameter to a model. Classical models of modal logic are set-theoretic triples consisting of a set of worlds, a distinguished actual world, and a valuation function (omitting accessibility relations). Hanson suggests adding a new element, namely, a "designated member of *W*" named w^* (442), which plays one of the roles the distinguished actual world plays in the original model theory. Models become defined as quadruples of the form $\langle W, w_\alpha, w^*, V \rangle$. The semantics of the actuality operator \mathcal{A} is still sensitive to the distinguished actual world of the model. But the notion of truth in a model, which the proponent of *R*-validity identifies as truth in the model at the distinguished actual world w_α of the model, is now redefined in terms of the newly added designated world parameter w^* . With this new element in place, Hanson offers the following redefinition of truth in a model:

ϕ is true in a model M just in case $w^* \in V(\phi)$.

Now that he has a notion of "truth in a model," he can then go on to employ the Tarskian definition of logical truth as truth in every model. But, because the distinguished actual world w_α need not be identical to the designated world w^* , there will be models where instances of LA1 are false. For example, let M be a model with two worlds, w_1 and w_2 , where w_1 is the distinguished world and w_2 the designated world, and let $w_1 \in V(p)$ and $w_2 \notin V(p)$. Then $\mathcal{A}p \rightarrow p$ is false in M . So, with this simple addition, the proponent of *G*-validity can accept the Tarskian definition of logical truth as truth in every model and still deny all instances of LA1 the status of logical truth. Zalta's Tarskian argument in favor of *R*-validity, Hanson concludes, fails.

This response does not satisfy. The Tarskian definition of logical truth is philosophically illuminating. The desire to respect and retain that philosophical insight is what motivated Zalta's original argument in favor of *R*-validity.

⁵ For the purposes of this paper, we won't take a stand on what the primary bearers of truth are. We think there are good reasons to suppose that an appeal to propositions should be made. This is something Tarski doesn't do, and so, our assessment of Tarski's notion as 'proper' should be understood modulo the assumption, which we reject, that sentences or formulae are the primary bearers of truth.

Furthermore, the presence of a distinguished actual world in a Kripke model for modal languages is philosophically illuminating. We have an idea of what role the distinguished actual world plays and modal logicians have seen a need for something to play that role. But Hanson's addition of a 'designated' world seems to us a parameter without intuitive motivation and grounding, especially when we are told that this floats free from the distinguished actual world. The philosophical insight seems lost.

It is natural to think that what is the case is what is true *simpliciter*. It is simply true that snow is white. The notion of truth in a model is meant to mimic this natural thought, supposing the (intended) model to represent what is the case. But what is the case is what is actually the case. This demonstrates that the two roles the proponent of *R*-validity assigns the distinguished actual world—namely, being the element of the model to which the \mathcal{A} -operator is sensitive and being the element of the model in terms of which the notion of truth in a model is defined—should be played by a single object. But, of course, if the distinguished actual world and designated world of a model are always the same, then LA1 is once again reinstated as a logical truth, even with Hanson's modification. Hanson's proposal delivers the result of allowing the proponent of *G*-validity to operate with a Tarskian definition of logical truth only by undermining the philosophical insight that that definition promises. We conclude, then, that Hanson's second objection to Zalta's argument fails.

3 Hanson's defense of *G*-validity

So far we have discussed Hanson's objections to Zalta's arguments for preferring *R*-validity. We now turn to Hanson's positive argument in favor of *G*-validity, offered in Sect. 2 of his paper.

In his monograph of 1989, Kaplan argued that the logic of demonstratives gives rise to contingent logical truths. His favorite example is 'I am here now'. Zalta argued that such examples are not the most basic examples of contingent logical truths. Every genuinely logical truth is analytic, in a strict sense of being true in virtue of its meaning alone. But the meaning of the sentence 'I am here now' does not, by itself, determine a truth value. It is only relative to a context that the sentence can be said to be true or false, on Kaplan's view. Zalta argues that this is, at best, an extended notion of analyticity.

We cannot consider the truth of the sentence ['I am here now'] without appealing to some context, and so we cannot simply say that it has the property that traditional analytic truths have, namely, being true in virtue of the meanings of its words. Rather, it has the property of being true in all contexts in virtue of the meanings of its words relative to such contexts. (1988, p. 71)

Hanson tries to turn this reasoning against Zalta. He presents an analogous argument against the validity of LA1:

We cannot consider the truth of the sentence without appealing to some actual-world candidate, and so we cannot simply say that it has the property that traditional analytic truths have, namely, being true in virtue of the meanings of its words. Rather, it has the property of being true in all actual-world candidates in virtue of the meanings of its words relative to such actual-world candidates.

In summary, Hanson claims that Zalta's own reasons for doubting that 'I am here now' is a genuine logical truth carry over to instances of LA1.

But we think that there is an important difference between requiring a context and requiring a distinguished actual world. Consider a paradigm example of a logically true sentence in a formal, nonmodal propositional language: $p \rightarrow p$, for example. Like any formula, this formula is only true relative to a model. Similarly, the formula $Ap \rightarrow p$ is only true relative to a model. But, unlike Kaplan's 'I am here now', *all* that we need to supply is a model; we don't need anything other than an interpretation (which we believe requires a distinguished actual world if truth for modal claims is to be definable). So the claim $Ap \rightarrow p$ is true in virtue of the meaning of its words in exactly the same sense as classical examples of logical truths are true in virtue of the meanings of their words. Hanson's argument fails to establish an analogy as it fails to appreciate the difference between the need to supply an interpretation and the need to supply an interpretation and a context.

4 Considering a counterfactual world as actual

We take ourselves to have answered Hanson's objections to Zalta's original argument in favor of *R*-validity and objected to Hanson's positive arguments in favor of *G*-validity. We'd like to conclude with a discussion of a notion that he employs in his defense of *G*-validity that we find problematic. It is the notion of "truth at w from the point of view of w' " or "truth at counterfactual world w' when considering w' as actual." (This notion is widely used in the literature on two-dimensional modal logic but we won't be discussing that literature explicitly here.) We first discuss the ways in which Hanson employs this notion and then argue that it is problematic. The upshot of these criticisms is that there is only one way to "consider" a counterfactual world and that is as counterfactual. Insofar as Hanson's defense of *G*-validity requires otherwise, we maintain that to be further evidence against that defense.

Hanson tells us (444) that the notion of *R*-validity is based on Evans's (1982) notion of *truth in* a possible situation and the notion of *G*-validity is based on Evans's notion of *truth with respect to* a possible situation. Evans's notion of truth in a possible situation w is glossed in terms of what would have been true were w actual. His notion of truth with respect to a situation is "purely internal to the semantic theory ... need[ing] no independent explanation" and is that in terms of which modal operators and ordinary counterfactuals are explicated (Evans 1979, p. 207). To capture the distinction, Hanson introduces the notion of a point of view. The idea is supposed to be that when we evaluate a formula at a world w , we can

evaluate it from the point of view of w itself or, supposing w to be a counterfactual world, from the point of view of the actual world. If the formula is sensitive to which world is actual, then this can make a difference. For example, suppose $w \notin V_M(p)$ and $w_\alpha \in V_M(p)$. Then the formula $\mathcal{A}p$ is false in w considered as actual (or “from the point of view” of w) and yet true with respect to w considered as counterfactual (or “from the point of view” of w_α). (Similarly, if $w \in V_M(p)$ and $w_\alpha \notin V_M(p)$, then the formula $\mathcal{A}p$ is true in w considered as actual and false with respect to w considered as counterfactual.) Intuitively, when we evaluate the formula in w considered as actual, we are allowing \mathcal{A} to “pick out” w , in the sense that the truth value of $\mathcal{A}\phi$ in w is dependent on the truth value of ϕ in w . When we evaluate that formula with respect to w considered as counterfactual, on the other hand, we keep \mathcal{A} fixed on w_α , in the sense that the truth value of $\mathcal{A}\phi$ is dependent on the truth value of ϕ in w_α .

The above distinction between two ways of evaluating a formula with respect to a world gives rise to “two notions” of necessity: what Evans calls *deep* and *superficial necessity*. A formula ϕ is deeply necessary just in case it is *true in every possible world*, in the sense that, for every world w , ϕ is true at w considered as actual. A formula ϕ is superficially necessary just in case it is *true with respect to every possible world*, in the sense that, for every world w , ϕ is true at w considered as counterfactual. Suppose again that $w \notin V_M(p)$ and $w_\alpha \in V_M(p)$. Then $\mathcal{A}p$ is superficially necessary because it is true at w_α and at w considered as counterfactual. However, $\mathcal{A}p$ is not deeply necessary, because it is *false at w considered as actual* and so it is not true at all worlds considered as actual. On Evans’s view, the “necessary” truth of $\mathcal{A}p$, given p ’s actual but contingent truth, is superficial as it depends on a contingent feature of reality; namely, the truth value of p . A deeply necessary truth, on the other hand, is free of such contingency. Evans thought that this fact is captured by claiming that these truths are true no matter which world were actual.⁶

We can now return to Hanson’s employment of these notions. Hanson grounds his understanding of “point of view” in his conception of ‘truth at world’. He says:

The basic semantic clauses for the necessity, possibility, and actuality connectives are the same under both real world and general validity, and both notions of validity make use of the idea that a sentence has a truth value at a world from the point of view of some other world. (445)

We think the second conjunct here is simply false; we see no reason to think that the semantics for modal logic outlined at the beginning of this paper makes use of the idea that a sentence has a truth value at a world from the point of view of some other world. By inspection one can see that the semantics defines only the notion of *truth at* (i.e., $M, w \models \phi$), and it does not follow from the fact that the second index is a variable ranging over worlds that a sentence has a truth value at a world from the point of view of some other world, even when the variable w takes the distinguished

⁶ In a very influential paper, Davies and Humberstone (1980) propose to formalize this distinction in terms of two modal operators— \square (superficial necessity) and their proposed $\mathcal{F}\mathcal{A}$ (read ‘fixedly-actual’, for deep necessity), as we shall discuss below.

world as its value. The semantics only employs truth at a counterfactual world considered as counterfactual.

Hanson then goes on to attempt to align *R*-validity with Evans's truth in a situation. For example, he says: (444)

Indeed making use of the notion of point of view, it is easy to see that the fundamental property on which the definition of real world validity is based is the following

(R) Truth at a world w from the point of view of w as actual.

This also strikes us as mistaken. What drives the proponent of *R*-validity is the idea that a formula is true in a model just in case it is true in the distinguished actual world of that model. There is no need for two world indices and no need for the notion of a point of view. If there is such a thing as a point of view, it is provided by the model as a whole and does not shift within the model. That is to say, the proponent of *R*-validity is interested in a cross-model feature—namely, whether or not the formula is true at the distinguished world of every model. But Hanson's notion of truth at a world from the point of view of that world does not capture a cross-model feature but rather an intra-model feature. So, it is a mistake to think that the proponent of *R*-validity has a use for the notion of truth at a counterfactual world considered as actual. It is only the the proponent of *G*-validity that has a use for such a notion.

Hanson's second use of the distinction between considering a world as actual and considering that world as counterfactual occurs in the third section of his paper, in an attempt to explain away the intuitions supporting *R*-validity. The proponent of *R*-validity is, as Hanson sees it, impressed by the fact that, for any world w , $\mathcal{A}\phi$ and ϕ have the same truth value in w considered as actual.⁷ Hanson claims to be able to accommodate this insight in his framework for *G*-validity by introducing a distinct kind of necessity operator—what Davies and Humberstone call the fixedly-actual operator \mathcal{FA} , defined as follows: $M, w \models \mathcal{FA}\phi$ just in case for every M' just like M except that w' is the distinguished actual world, $M', w' \models \phi$ (Davies and Humberstone 1980, p. 2). In other words, $M, w \models \mathcal{FA}\phi$ iff in every model M' that differs from M only by which world in the set W of M is identified as actual, ϕ is true $_{M'}$ at the distinguished actual world of M' . Now Hanson would render this clause as follows: $\mathcal{FA}\phi$ is true at w iff for every world w' , ϕ is true at w' when w' is considered as actual. While $\mathcal{A}\phi \equiv \phi$ is not necessary, it is fixedly-actual and so, claims Hanson, the basic insight supporting *R*-validity is accommodated within his framework for *G*-validity, enriched with Davies and Humberstone's \mathcal{FA} operator. Whereas the proponent of *R*-validity would say every instance of LA1 is logically true, Hanson would say that every instance involving contingent propositions is merely fixedly-actual.

We have seen some of the purposes to which Hanson puts the distinction between considering a counterfactual world as actual and considering a counterfactual world as counterfactual. It is time now to say more generally what is wrong with

⁷ We have rejected this characterization of the motivations of the proponent of *R*-validity in the previous paragraph.

employing this kind of discourse in the analysis of modality. While we do not take our discussion to be decisive, we do think that it raises a legitimate concern. The basic problem is that this kind of discourse fails to adequately separate the formal and material mode of speech, and that such a failure calls into question whether such discourse even makes sense.

This problem arises from what we take to be a normal understanding of the enterprise of modal logic. Basically, as we understand it, the analysis of modal discourse is an enterprise which assumes a certain target language and its regimentation: the target language is natural language (which contains modal locutions of various sorts) and its regimentation consists of systematic translations of natural language sentences into the formal language of modal logic. The enterprise also assumes that one may use a theoretical language (such as the language of set theory with the addition of quantifiers over possible worlds) which contains no modal locutions, for the purpose of indirectly interpreting the target discourse by directly interpreting its regimentation.⁸ But when Hanson (and others) talk about ‘considering a nonactual possible world as actual’, he is essentially importing modal notions into the modally-innocent theoretical language of interpretation. For the idea of considering a nonactual possible world w as actual can only be understood as a request to suppose what it *would* be like *were* w actual, and this is obviously just modal talk about worlds. So, on formal grounds alone, it seems illegitimate to appeal to the notion of ‘considering a world as actual’ in the semantics of modal logic.

To put the objection another way, note that the notion of considering a counterfactual world as actual requires that actuality be a contingent property of a world. But the framework of possible worlds was invoked, in part, to explain the modal notion of contingency. Thus it is not clear to us that it makes sense to apply contingency talk to the very entities that are used to explain contingency.

We think this concern at least shifts the burden of proof. For our argument suggests that those who insist on using the notion of considering a counterfactual world as actual should demonstrate that it makes sense to do so.

5 Conclusion

We have defended Zalta’s contention, that R -validity rather than G -validity is the proper notion of logical truth for modal languages, against Hanson’s objections. We argued that the Tarskian notion of logical truth as truth in every model is philosophically illuminating and can be carried over from nonmodal logic to modal logic simply by adding a distinguished world to interpretations along with the

⁸ Menzel (1990), and others (e.g., Ray 1996, Chihara 1998), have employed modal locutions in the semantics. For example, in Menzel (1990) there is a semantics for modal logic which goes roughly as follows: a modal statement like ‘It is possible that p ’ is true just in case there is a non-modal, set-theoretically described Tarski-model in which p is true which *might have been* a model of the actual world. Note here that Menzel is not applying modal talk to possible worlds, i.e., he is not applying modal talk to entities introduced for the purpose of explaining modal talk. Instead, he is applying it to things he believes we already accept, such as the applied set theory used in defining Tarski models.

domain of possible worlds. Furthermore, the proponent of G -validity cannot mimic this notion of logical truth, as Hanson suggests, by adding an extra element (i.e., a designated world) to an interpretation already containing a distinguished actual world. In addition to the arguments we developed by way of answering Hanson's objections, we also developed a positive argument against the idea that G -validity is the philosophically deeper notion of logical truth and gave voice to a concern about the distinction between considering a counterfactual world as counterfactual and as actual.

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