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A Formal Theory of Substances, Qualities, and Universals

Fabian NEUHAUS [1,2], Pierre GRENON [2], Barry SMITH [2,3]
[1] *Humboldt University, Berlin*
[2] *IFOMIS, Saarland University, Saarbrücken*
[3] *University at Buffalo, NY, USA*

Abstract. One of the tasks of ontology in information science is to support the classification of entities according to their kinds and qualities. We hold that to realize this task as far as entities such as material objects are concerned we need to distinguish four kinds of entities: substance particulars, quality particulars, substance universals, and quality universals. These form, so to speak, an ontological square. We present a formal theory of classification based on this idea, including both a semantics for the theory and a provably sound axiomatization.

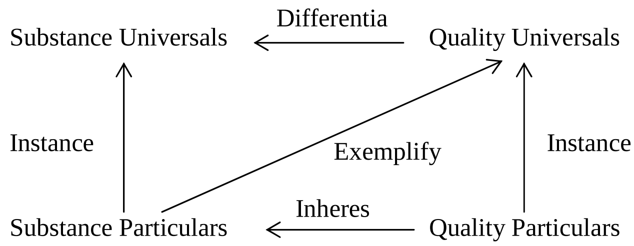
Introduction

Common sense recognizes distinct things in the world, things which may persist self-identically through time while sharing similar qualities. For instance, two footballs which have the same spherical shape, two coins which have the same 50 cents value. In each case, the two objects are said to be qualitatively identical in some respect: for example with respect to shape or value. In order to account for such examples, we need to develop a theory of the links between particulars and universals. Within the large repertoire of available theories [1], our perspective is drawn from the Aristotelian tradition of so-called immanent realism. [2,3]

Fundamental to the view articulated here is the distinction between particulars and universals. The reason why this distinction deserves a rigorous treatment in the context of ontology for information science is that it is precisely universals which provide the basis for the classification of particulars. Romeo *is a man* because he instantiates the universal *man*. The particular shape of this ball *is spherical* because it (but not the ball) instantiates the universal *sphericity*. On our view, a universal is an entity with a spatiotemporal existence which is yet distinct from its extension (the set of its instances) at any given time.[4] Universals exist when and where they are instantiated; they are generically dependent on their instances – which means that a universal can exist only if there is some appropriate instance. In turn, every particular is an instance of one or more universals. In contra-distinction to particulars, universals can exist at different places simultaneously, and they can also exist at different times without existing in the intervening interval.

A second distinction, orthogonal to the first, is that between qualities and their substantial bearers. Quality particulars cannot exist without substance particulars as their bearers; substance particulars cannot exist without bearing quality particulars of, for instance, shape and colour. Our term "quality particular" is used to refer to both monadic qualities, like the age of John, and relational qualities, such as the bound of friendship between John and Mary. John's age would not exist without John, and the marriage contract between John and Mary would not exist without John and Mary. In general any *d*-ary relational quality particular inheres in *d* substance particulars. The ball is a bearer of qualities, for instance shape or colour. The particular shape is existentially dependent on the ball. In addition, this particular is the shape of the ball. We say that the shape of the ball inheres in the ball. The ball and its shape are of different kinds: the one is a substance-particular, and the other a quality-particular. Moreover, they instantiate different universals, and these too are of different kinds: the one is a substance-universal, and the other is a quality-universal. As a result, there will be four basic kinds of entities under consideration in what follows, which form so to speak an ontological square: [5]

Figure 1. The Ontological Square



A substance particular may be related to a quality universal via a quality particular which inheres in the substance particular and instantiates the quality universal. In such cases we speak of *exemplification*; for example a ball exemplifies the quality universal red. The set of quality universals exemplified by all instances of a given substance universal is called differentia.

Universals are standardly treated in the context of a classification system or a taxonomy. There are various ways of structuring such systems. Here we will be interested in those taxonomies whose structure we deem to be ontologically well-founded. These taxonomies are so-called Porphyrean trees: they are true hierarchies, which means that they satisfy the JEPD criterion commonly accepted as the goal of scientific classification. This means that the extensions of the lower-level universals (species) of each given higher level universal (genus) are (i) jointly exhaustive of the extensions of the genus and (ii) pairwise disjoint (so that the taxonomy in question is a tree in the mathematical sense, so that it does not involve the phenomenon of multiple inheritance [3]). Note that the exhaustiveness criterion represents an ideal which our scientific classifications in practice often do not meet.

In sum, a taxonomy consists of finitely many universals arranged in a tree-like hierarchy. There are taxonomies of substance universals (for example biological species taxonomies) and of quality universals (for example taxonomies of colours or shapes).

In this paper we present a formal theory of the distinction between particulars and universals and of taxonomies as just described. We focus exclusively on continuant entities and their enduring qualities, leaving aside the treatment of those occurrent entities, such as events and processes, in which continuants participate. Our treatment can be generalized to apply to these entities and also to other categories of dependent endurant entities – such as functions, powers, dispositions, liabilities – along the lines sketched in [6].

The formal theory consists of a formal first-order semantics and a sound axiomatization of this semantics. This axiomatization is an extension of classical first-order predicate logic. The first part of the paper presents the syntax of our language and a semantics which accounts rigorously for the conditions set forth above. In the second part, we present the axiomatization of a theory governing the kinds of entities and links which form the ontological square. The third part contains an axiomatization of a theory of taxonomies.

1 Syntax and Semantics

1.1 Syntax

Let *L* be any first-order language, whose alphabet consists of finitely many individual constants and countably many individual variables, together with

• Three 2-place predicates:	=	(read: 'i_1 is identical to i_2')
	DifferentiaOf	('i_1 is a differentia of i_2')
	GenusOf	('i_1 is the genus of i_2')
• A 3-place predicate:	*InstAt*	('i_1 is an instance of i_2 at i_3')
• A multi-place predicate:	*Inheres*	('i_{d+1} inheres in the tuple $<i_1, ..., i_d>$')
• Three logical constants:	$\sim \ \rightarrow \ \forall$	($\vee$, $\wedge$, $\leftrightarrow$ and $\exists$ are defined as usual)
• Three auxiliary symbols:	,) (	

Note that *Inheres* as we conceive it is a multi-place predicate. This means that, for every natural number *d*, sentences of the form *Inheres*(i_1, ..., i_d, i_{d+1}) are admissible, each of which signifies: 'i_{d+1} inheres in the tuple $<i_1, ..., i_d>$'. The terms 'well-formed formula', 'closed formula', etc., and other logical constants are defined as usual. Hence *L* is a quite ordinary first-order language. Since the alphabet of *L* does not contain different types of individual variables, the individual variables range over every entity in the relevant universe of discourse. What sets *L* apart from more familiar languages is the fact that it contains only five predicates, all of which are relational. With their help it is possible to define notions like universal, particular, quality, etc. along lines to be set forth below. Part of the job standardly performed by monadic or relational predicates in first-order logic is here performed by terms (names) for universals. Since we quantify over these terms, our framework simulates some aspects of second-order logic while remaining of first order. [7, 8, 9, 10]

1.2 Universal-Particular Structure

According to the Aristotelian perspective, change is the succession of distinct quality particulars inhering in the same substance particular. Hence a formal theory about substance and quality particulars is essentially incomplete if it does not address time. In this paper we assume only that there are times and we do not make any specific assumption about the nature of time or its ordering relation.

Since the formal theory is going to be an extension of classical first-order logic, the universe of discourse *U* has to be non-empty (1). (Numbers in parentheses refer to the corresponding clauses in the definition below.) *U* is partitioned into the set of times *T*, the set of substance particulars *IP*, the set of substance universals *IU*, the set of quality particulars *DP*, and the set of quality universals *DU* (2). Further, *DP* and *DU* are divided according to the arity of the entities at issue (3, 4). f_{time} is a function which determines the set of entities existing at a given time. Since particulars and universals are spatiotemporal entities, each particular or universal is such that there is at least one time at which it exists (5a). For the purpose of simplifying the theory, we assume that at each time there is something which exists at that time (5b).

The function f_{inst} determines the extension of a universal (i.e. the set of its instances at a given point in time). If a universal does not exist at time *t*, then it is not instantiated at *t* (6a). If a substance universal does exist at *t*, then it is instantiated by substance particulars which also exist at *t* (6b). Similarly for *d*-ary quality universals and *d*-ary quality particulars

(6c). Every substance particular is an instance of at least one substance universal (6d), and every *d*-ary quality particular is an instance of at least one *d*-ary quality universal (6e). Clause (6f) expresses a weak kind of extensionality: if two universals never differ with regard to their instances, then they are identical. (6g) prevents change with regard to the instantiation relation in the sense that it guarantees that if a particular is an instance of a universal, then it is at any time *t* an instance of this universal unless it does not exist at *t*. The function f_{inh} governs the inherence relation. It means that any *d*-ary quality particular inheres in *d* substance particulars (7a), which all exist whenever the quality particular exists (7b). Moreover, there are no 'bare' substance particulars; substance particulars always bear quality particulars (7c). Finally, the relation of inherence is defined in such a way that only quality particulars inhere in other entities (7d).

Definition: A UPS (a Universal-Particular Structure) is a tuple $<U, T, IP, IU, DP, DU, f_{dp}, f_{du},, f_{time}, f_{inst}, f_{inh}>$, such that

1. $U \neq \emptyset$;
2. $\{IP, IU, DP, DU, T\}$ is a partition of U;
3. f_{dp} is a function from the positive natural numbers into the power set of DP, such that $f_{dp}(m) \cap f_{dp}(n) = \emptyset$, if $m \neq n$, and $\cup\{f_{dp}(1), f_{dp}(2), ... \} = DP$;
4. f_{du} is a function from the positive natural numbers into the power set of DU, such that $f_{du}(m) \cap f_{du}(n) = \emptyset$, if $m \neq n$, and $\cup\{f_{du}(1), f_{du}(2), ... \} = DU$;
5. f_{time} is a function from T into the power set of $U \setminus T$, such that
   (a) for all $u \in U \setminus T$ there is a $t \in T$ with $u \in f_{time}(t)$;
   (b) for $t \in T$: $f_{time}(t) \neq \emptyset$;
6. f_{inst} is a two-placed function from $IU \cup DU \times T$ into the power set of $IP \cup DP$, such that
   (a) for all $u \in IU \cup DU$ and all $t \in T$: if $u \notin f_{time}(t)$, then $f_{inst}(u, t) = \emptyset$;
   (b) for all u and all $t \in T$: if $u \in IU \cap f_{time}(t)$, then $f_{inst}(u, t) \subseteq IP \cap f_{time}(t)$ and $f_{inst}(u, t) \neq \emptyset$;
   (c) for all $d \geq 1$ and for all $t \in T$ and u: if $u \in f_{du}(d) \cap f_{time}(t)$, then $f_{inst}(u, t) \subseteq f_{dp}(d) \cap f_{time}(t)$ and $f_{inst}(u, t) \neq \emptyset$;
   (d) for all $t \in T$ and all u: if $u \in IP \cap f_{time}(t)$, then there is a $u_1 \in IU \cap f_{time}(t)$, such that $u \in f_{inst}(u_1,t)$;
   (e) for all $t \in T$ and all u: if $u \in DP\cap f_{time}(t)$, then there is a $u_1 \in DU \cap f_{time}(t)$, such that $u \in f_{inst}(u_1,t)$;
   (f) for all $u_1, u_2 \in IU \cup DU$: if $f_{inst}(u_1, t) = f_{inst}(u_2, t)$ for all $t \in T$, then $u_1 = u_2$;
   (g) for all $u_1, u_2 \in U$ and $t_1, t_2 \in T$: if $u_1\in f_{inst}(u_2, t_1)$ and $u_1\in f_{time}(t_2)$, then $u_1\in f_{inst}(u_2, t_2)$;
7. f_{inh} is a one-place function, such that for all $u \in U$, for all $t \in T$ and for all $d \geq 1$:
   (a) if $u \in f_{dp}(d)$, then there are $u_1, ..., u_d \in IP$, such that $f_{inh}(u) = <u_1, ..., u_d>$;
   (b) if $u \in f_{dp}(d) \cap f_{time}(t)$ and $f_{inh}(u) = <u_1, ..., u_d>$, then $u_1, ..., u_d \in f_{time}(t)$;
   (c) if $u \in IP \cap f_{time}(t)$, then there is a $u_1 \in f_{dp}(1) \cap f_{time}(t)$, such that $f_{inh}(u_1) = u$;
   (d) if $u \notin DP$, then $f_{inh}(u)$ is not defined.

1.3 UPS-model

Universal-Particular Structures deal with the four kinds of entities represented in the ontological square and with their interrelations. A UPS is enlarged to a UPS-model through the adding of a specific taxonomy. As explained in the introduction, a taxonomy as we conceive it consists of a finite set of universals, which we call the *classes* of the taxonomy, arranged in a tree-like hierarchy. In this section we use the terms 'class', 'genus', and ‘species’ in the meta-language in order to motivate the definition of UPS-models. These notions will be defined in the object-language later.

A UPS-model includes a UPS (1) together with a set of classes *C*, a non-empty set of universals (2) which together constitute the taxonomy in which we are interested. For example the classes that constitute a biological species taxonomy are substance universals, and the classes that constitute a taxonomy of colours are unary quality universals. Each taxonomy has a unique root c_r, which is the most general universal within this taxonomy (3). Further there is a non-empty relation *R* (4a), which can be read ‘is the genus of’. It is such that: (i) $<C, R, c_r>$ is a tree; (ii) the tree branches at each node (4c); (iii) the instances of each species in the tree are instances of a corresponding genus (the next highest class in the tree) (4d); (iv) the extensions of the species of a genus are pairwise disjoint (4e); (v) the union of the extensions of the species is the extension of the genus (4f). In a taxonomy each class is related to one or more qualities, which its instances exemplify. The set of these quality universals is called the differentia of the species. Formally, in a taxonomy every class *c* is related to a set of quality universals *dif*(*c*), which is a superset of the set of quality universals linked to its genus (if there is any) (5a). Every instance of a given class *c* exemplifies all qualities in *dif*(*c*) (5b). Since there might be taxonomies without differentia, we do not exclude the possibility that *dif*(*c*) might be empty for some *c*. *I* is the usual interpretation function, but with the extra condition that all classes in a taxonomy have names (6).

Definition. A *UPS*-model for *L* is a tuple $<ups, C, R, dif, c_r, f_{typ}, I>$ such that

1. *ups* is a *UPS* $<U, T, IP, IU, DP, DU, f_{dp}, f_{du},, f_{time}, f_{inst}, f_{inh}>$;
2. *C* is a finite subset of either *IU* or $f_{du}(d)$ for some $d \geq 1$;
3. $c_r \in C$;
4. *R* is a binary relation on *C*, such that
   (a) $R \neq \varnothing$;
   (b) for all $c \in C \backslash \{c_r\}$, there is a unique finite sequence $c_1, c_2, ..., c_n$, such that $c_r = c_1$ and $c_1 R c_2$ and ... and $c_{n-1} R c_n$ and $c_n = c$;
   (c) if $c_1 R c_2$, then there is a c_3, such that $c_2 \neq c_3$ and $c_1 R c_3$;
   (d) if $c_1 R c_2$, then for all $t \in T$: $f_{inst}(c_2, t) \subseteq f_{inst}(c_1, t)$;
   (e) if $c_1 R c_2$ and $c_1 R c_3$ and $c_2 \neq c_3$, then for all $t \in T$: $f_{inst}(c_2, t) \cap f_{inst}(c_3, t) = \varnothing$;
   (f) for all $t \in T$ and $c \in C$: if there is a c_1, such that cRc_1, then $f_{inst}(c, t) = \cup\{x \mid \text{there is a } c_2 \in C \text{ such that } cRc_2 \text{ and } x = f_{inst}(c_2, t)\}$;
5. *dif* is a one-place function from *C* into the power set of $f_{du}(1)$, such that for all $c, c_1, c_2 \in C$ and $t \in T$:
   (a) if $c_1 R c_2$, then $dif(c_1) \subseteq dif(c_2)$;
   (b) if $u_1 \in f_{inst}(c, t)$, then for all $u_2 \in dif(c)$ there is a u_3, such that $u_3 \in f_{inst}(u_2, t)$ and $f_{inh}(u_3) = u_1$;
6. *I* is a function from the set of all individual constants of *L* into *U*, such that for all $c \in C$, there is an individual constant *a*, such that $I(a) = c$.

1.4 Truth and Tautology

An assignment function *s* assigns to each variable *v* of *L* an element of the universe *U*. s_v is the name for an assignment, which differs from *s* at most in regard to *v*. Let *i* be an individual constant or an individual variable, $<ups, C, R, dif, c_r, f_{typ}, I>$ a *UPS*-model for *L*, and *s* an assignment function. Then $r(i) = s(i)$, if *i* is an individual variable; $r(i) = I(i)$, if *i* is an individual constant.

Definition. (This defines what it is for a formula A to be L-true with respect to a *UPS*-model $M = <ups, C, R, dif, c_r, f_{typ}, I>$ for L and an assignment s, for short: $M, s \Rightarrow A$.)

M, $s \Rightarrow i_1 = i_2$ iff $r(i_1) = r(i_2)$;
M, $s \Rightarrow DifferentiaOf(i_1, i_2)$ iff $r(i_1) \in dif(r(i_2))$;
M, $s \Rightarrow GenusOf(i_1, i_2)$ iff $<r(i_1), r(i_2)> \in R$;
M, $s \Rightarrow InstAt(i_1, i_2, i_3)$ iff $r(i_1) \in f_{inst}(r(i_2), r(i_3))$;
M, $s \Rightarrow Inheres(i_1, ..., i_d, i)$ iff $f_{inh}(r(i))$ is defined and $f_{inh}(r(i)) = <r(i_1), ..., r(i_d)>$;
M, $s \Rightarrow \sim A$ iff not M, $s \Rightarrow A$;
M, $s \Rightarrow (A \rightarrow B)$ iff not M, $s \Rightarrow A$ or if M, $s \Rightarrow B$;
M, $s \Rightarrow \forall v\, A$ iff M, $s_v \Rightarrow A$, for all s_v.

Definition. A formula A is an L-tautology if and only if it is L-true with respect to all *UPS*-models for L and all assignments.

2 Axioms for Universals and Particulars

The following axiomatization is an extension of classical first-order logic. Since we did not specify a particular first-order language L, we use $t, t_1, t_2, v, v_1, v_2, \ldots$ for the individual variables of L in alphabetic order and $k, k_1, k_2, \ldots, k_n$ for the individual constants of L in alphabetic order. We presuppose a standard axiomatization of first-order logic with identity and discuss only the additional axioms and axiom schemata. In order to improve readability, these are grouped together with the relevant definitions. Important theorems are mentioned in the text.

2.1 Universals and Particulars

The predicates *Univer*, *Partic*, *Time*, and *Exist* are defined in terms of the *InstAt* relation. They are read: 'is a universal', 'is a particular', 'is a time', and 'exists at'.

Def 1. $Univer(i) =_{df} \exists v_1 \exists v_2\, InstAt(v_1, i, v_2)$
Def 2. $Partic(i) =_{df} \exists v_1 \exists v_2\, InstAt(i, v_1, v_2)$
Def 3. $Time(i) =_{df} \exists v_1 \exists v_2\, InstAt(v_1, v_2, i)$
Def 4. $Exists(i_1, i_2) =_{df} \exists v\, InstAt(v, i_1, i_2) \vee \exists v\, InstAt(i_1, v, i_2)$

A1. $\forall v\, (\sim Univer(v) \leftrightarrow (Partic(v) \vee Time(v)))$
A2. $\forall v\, (\sim Partic(v) \leftrightarrow (Univer(v) \vee Time(v)))$
A3. $\forall v\, (\sim Time(v) \leftrightarrow (Univer(v) \vee Partic(v)))$
A4. $\forall v \forall v_1 (Univer(v) \rightarrow (\forall v_2 \forall t (InstAt(v_2, v, t) \leftrightarrow InstAt(v_2, v_1, t)) \rightarrow v = v_1))$
A5. $\forall v \forall v_1 \forall t_1 \forall t_2 ((InstAt(v, v_1, t_1) \wedge Exists(v, t_2)) \rightarrow InstAt(v, v_1, t_2))$

The axioms A1–A3 ensure that the universe is strictly divided into particulars, universals and times. Additionally we have a temporalized version of the axiom of extensionality. If two universals are instantiated by the same particulars at all times, then they are identical (A4). Further, instantiation does not tolerate change: if v is an instance of v_1 at t_1 and v exists at t_2, then v is an instance of v_1 at t_2 (A5).

It follows from these axioms that everything is either a universal or a particular or a time. If v instantiates v_1 at t, then v is a particular, v_2 is a universal and t is a time; further v and v_1 exist at t. If v exists at t, then t is a time and v is a universal or a particular. If a universal v exists at t, then there is a particular v_1 which exists at t and instantiates v at t. If a particular v exists at t, then there is a universal v_1 which exists at t and is instantiated by v at t.

2.2 Inherence, Qualities, and Substances

We use the terms 'quality' and 'substance', and the corresponding predicates *Qua* and *Subst* in this section to refer to both particulars and universals, disambiguating only when occasion requires. The inherence relation can be used to define the notions of d-place quality, quality, and substance. A d-place quality is an entity which inheres in d other entities or which is instantiated by an entity inhering in d other entities (Def 5). (If the former it is a particular and if the latter it is a universal.) An entity is a quality if and only if it is a d-place quality, for some d (Def 6). An entity is a substance if and only if it is neither a time nor a quality (Def 7).

Def 5. $Qua^d(i) =_{df} \exists v_1 \ldots \exists v_d\, Inheres(v_1, \ldots, v_d, i) \vee \exists v_1 \ldots \exists v_{d+2}\, (Inheres(v_1, \ldots, v_d, v_{d+1}) \wedge InstAt(v_{d+1}, i, v_{d+2}))$

Def 6. $Qua(i) =_{df} Qua^d(i)$, for a $d \geq 1$

Def 7. $Subst(i) =_{df} {\sim}Time(i) \wedge {\sim}Qua(i)$

If v inheres in other entities, then v is a particular and it inheres in exactly one tuple of entities (A6, A7). If v inheres in $v_1, \ldots, v_d$ and v exists at t, then $v_1, \ldots, v_d$ are substance particulars which exist at t (A8). There is no substance without qualities. Hence if a substance particular v exists at t, then there exists at t a 1-place quality which inheres in v (A9). If v instantiates v_1, then v is a d-place quality if and only if v_1 is a d-place quality (A10). Further, the arity of a quality is determinate (A11).

A6. $\forall v \forall v_1 \ldots \forall v_d\, (Inheres(v_1, \ldots, v_d, v) \rightarrow Partic(v))$

A7. $\forall v \forall v_1 \ldots \forall v_d\, \forall w_1 \ldots \forall w_d\, ((Inheres(v_1, \ldots, v_d, v) \wedge Inheres(w_1, \ldots, w_d, v)) \rightarrow (v_1 = w_1 \wedge \ldots \wedge v_d = w_d))$

A8. $\forall v \forall v_1 \ldots \forall v_d \forall t((Inheres(v_1, \ldots, v_d, v) \wedge Exists(v, t)) \rightarrow (Subst(v_n) \wedge Partic(v_n) \wedge Exists(v_n, t)))$, for all n $(1 \leq n \leq d)$

A9. $\forall v \forall t\, (Subst(v) \wedge Partic(v) \wedge Exists(v, t) \rightarrow \exists v_1(Qua^1(v_1) \wedge Inheres(v, v_1) \wedge Exists(v_1, t)))$

A10. $\forall v\, \forall v_1(\exists t\, InstAt(v, v_1, t) \rightarrow (Qua^d(v) \leftrightarrow Qua^d(v_1)))$, for all $d \geq 1$.

A11. $\forall v\, (Qua^{d1}(v) \rightarrow {\sim}Qua^{d2}(v))$, where $d_1 \neq d_2$.

It follows that an entity is a universal or a particular if and only if it is a quality or a substance. Nothing is both a substance and quality. If v is a quality or a substance, then it exists at some time t. Each quality has a definite number of places, and every d-place quality is a quality.

Further it is provable that every instance of a substance universal is a substance particular, and every instance of a quality universal is a quality particular. If v is a d-place quality, then v inheres in d entities or there is an entity which inheres in d entities and is an instance of v. Hence if v is an instance of v_1, then v and v_1 are either substances (a particular and a universal respectively) or qualities (a particular and a universal respectively).

In addition, if v inheres in d other entities, then v is a d-place quality particular. And vice versa: if v is a d-place quality particular, then v inheres in d other entities. If v inheres in $v_1, \ldots, v_d$ and v exists at t, then $v_1, \ldots, v_d$ exist at t also. If a quality particular v exists at t, then there are substance particulars $v_1, \ldots, v_d$, such that v inheres in $v_1, \ldots, v_d$ and $v_1, \ldots, v_d$ exist at t.

Last but not least, we can prove that there are times, quality particulars, quality universals, substance particulars, and substance particulars. Roughly speaking, this follows from the fact that in classical logic the domain of discourse is not empty and that in our axiom system the existence of an entity of one type implies the existence of entities of all other types. For example assume that there is a substance universal. It follows immediately from the definitions that there is a time and a substance particular which are related to the universal via the instantiation relation. All substance particulars bear quality particulars, which are instances of quality universals.

3 Axioms for Taxonomies

3.1 Classes, Genus, Species

A taxonomy is a classification system that is used to classify entities in some portion of the world. It consists of finitely many named universals (classes) arranged in a tree-like hierarchy. At the top of the tree there is the highest genus or the root of the taxonomy. It is connected to every lowest species via chains of other classes. The relation *GenusOf* can be used to define the notions important to this structure. A genus is a genus of something (Def 8), and a species is an entity which has a genus (Def 9). An entity is a *class* if and only if it is a *genus* or a *species* (Def 10). A highest genus is a class without a genus (Def 11); a lowest species is a class without a species (Def 12). [11]

Def 8. $Genus(i) =_{df} \exists v\ GenusOf(i, v)$
Def 9. $Species(i) =_{df} \exists v\ GenusOf(v, i)$
Def 10. $Class(i) =_{df} Genus(i) \vee Species(i)$
Def 11. $Highestgenus(i) =_{df} Class(i) \wedge {\sim}Species(i)$
Def 12. $Lowestspecies(i) =_{df} Class(i) \wedge {\sim}Genus(i)$

Every class of a given taxonomy is a universal (A12), and it is named in *L* (A13). Note that according to our stipulations $k_1 \ldots k_n$ are the individual constants of *L* in alphabetic order. Because there are only finitely many individual constants in *L*, there are only finitely many classes. It follows from A14 that all classes in a given taxonomy are universals of the same type.

A12. $\forall v\ (Class(v) \rightarrow Univer(v))$
A13. $\forall v\ (Class(v) \rightarrow v = k_1 \vee \ldots \vee v = k_n)$
A14. $\forall v \forall v_1\ (Class(v) \wedge Class(v_1) \rightarrow (Qua^d(v) \leftrightarrow Qua^d(v_1)))$

There is exactly one highest genus (A15, A16). Every class is at most a species of one genus (A17), and every genus has at least two species (A18). If there is a finite path between two classes, then it is unique (A19, A20).

A15 $\exists v\ Highestgenus(v)$
A16. $\forall v \forall v_1\ ((Highestgenus(v) \wedge Highestgenus(v_1)) \rightarrow v = v_1)$
A17. $\forall v\ \forall v_1 \forall v_2\ ((GenusOf(v_1, v) \wedge GenusOf(v_2, v)) \rightarrow v_1 = v_2)$
A18. $\forall v\ (Genus(v) \rightarrow \exists v_1 \exists v_2 (GenusOf(v, v_1) \wedge GenusOf(v, v_2) \wedge v_1 \neq v_2))$
A19. $\forall v \forall v'\ (\exists v_1 \ldots \exists v_n (GenusOf(v, v_1) \wedge \ldots \wedge GenusOf(v_n, v')) \rightarrow {\sim} \exists w_1 \ldots \exists w_m (GenusOf(v, w_1) \wedge \ldots \wedge GenusOf(w_m, v')))$, if $n \neq m$, $0 \leq n$, $0 \leq m$

A20. $\forall v \forall v' \forall v_1 ... \forall v_n \forall w_1 ... \forall w_n((GenusOf(v, v_1) \wedge ... \wedge GenusOf(v_n, v') \wedge GenusOf(v, w_1) \wedge ... \wedge GenusOf(w_n, v')) \rightarrow (v_1 = w_1 \wedge ... \wedge v_n = w_n))$, $0 \leq n$

A13 and A15–A20 together ensure that a taxonomy is a finite tree. It follows that the relation *GenusOf* holds exclusively between classes, and that it is irreflexive, asymmetric and anti-transitive. Further, every class is either a species or the highest genus, and every class is either a genus or a lowest species. Every class which is not itself the highest genus is connected to the highest genus via a finite chain of other classes.

3.2 Instances and the is_a relation

Up to now we have concentrated only on the relations among classes themselves as these exist within a taxonomy. The instances of such classes must now also be considered. All instances of a species are instances of the corresponding genus (A21). All instances of a genus are instances of at least one species (A22). A22 corresponds to the condition we referred to above as 'exhaustiveness'. To capture systems, really existing in areas here exhaustiveness remains an unachieved goal, A22 would need to be skipped. An instance of a species is not an instance of another species of the same genus (A23).

A21. $\forall v \forall v_1 (GenusOf(v, v_1) \rightarrow \forall v_2 \forall t(InstAt(v_2, v_1, t) \rightarrow InstAt(v_2, v, t)))$
A22. $\forall v \forall v_1 \forall t ((InstAt(v, v_1, t) \wedge Genus(v_1)) \rightarrow \exists v_2(GenusOf(v_1, v_2) \wedge InstAt(v, v_2, t)))$
A23. $\forall v \forall v_1 \forall v_2 ((GenusOf(v, v_1) \wedge GenusOf(v, v_2) \wedge v_1 \neq v_2) \rightarrow \forall v_3 \forall t (InstAt(v_3, v_1, t) \rightarrow \sim InstAt(v_3, v_2, t)))$

GenusOf relates a genus and its (direct) species. Sometimes it is useful to use a notion of (indirect) subordination, which is transitive and reflexive: the so-called *is_a* relation.

Def 13. i_1 *is_a* $i_2 =_{df} \exists v_1 \exists v_2 ... \exists v_n (GenusOf(i_2, v_n) \wedge GenusOf(v_n, v_{n-1}) \wedge ... \wedge GenusOf(v_2, v_1) \wedge GenusOf(v_1, i_1)) \vee (i_1 = i_2 \wedge Class(i_1))$ (for some $n \geq 0$)

The *is_a* relation holds between classes. It is reflexive (with regard to classes), antisymmetric and transitive. If v_1 is connected by *is_a* to v_2 and v_3, then either v_2 is related by *is_a* to v_3 or v_3 is related by *is_a* to v_2. The *is_a* relation holds between every class v_1 and the highest genus, and the highest genus itself is connected by *is_a* only to itself.

It follows that if the *is_a* relation holds between two classes v_1, v_2 then all instances of v_1 at any given time t are instances of v_2 at t. If two classes v_1, v_2 share an instance at a time, then v_2 is related to v_1 or v_1 is related to v_2 by the *is_a* relation. If a genus and a lowest species share an instance at a time, then the lowest species is related to the genus by the *is_a* relation. If v is the genus of v_1, then there is an entity v_2 at some time t, such that v_2 is an instance of v and v is not an instance of v_1 at t.

3.3 Additional Notions

It is possible to define other notions in order to extend further the Aristotelian picture. We restrict ourselves here to a few examples. The substance particulars i_1, ..., i_{n-2} ($n \geq 3$) *exemplify* the quality universal i_{n-1} at the time i_n if and only if there is a quality particular which inheres in i_1, ..., i_{n-2} and is an instance of i_{n-1} at i_n (Def 15). A quality universal i_1 is a *proprium* of a substance universal i_2 if and only if: i_2 is a class, i_1 is not a differentia of i_2, and all instances of i_2 exemplify i_1 (Def 16). For example the ability to learn a language is a proprium of the universal human.

Def 14. $Differentia(i) =_{df} \exists v\ DifferentiaOf(i, v)$
Def 15. $Exemplify(i_1, ..., i_n) =_{df} \exists v(Inheres(i_1, ..., i_{n-2}, v) \wedge InstAt(v, i_{n-1}, i_n))$ (for $n \geq 3$)
Def 16. $Propria\_of(i_1, i_2) =_{df} \forall v_1 \forall v_2\ (InstAt(v_1, i_2, v_2) \rightarrow Exemplify(v_1, i_1, v_2)) \wedge {\sim}DifferentiaOf(i_1, i_2) \wedge Class(i_2)$

If v is a differentia of v_1, then v is a unary quality universal and v_1 is a class (A24). All instances of a class exemplify the differentiae of this class (A25). The differentiae of a genus are inherited also by the species (A26).

A24. $\forall v \forall v_1\ (DifferentiaOf(v, v_1) \rightarrow (Class(v_1) \wedge Qua^1(v) \wedge Univer(v)))$
A25. $\forall v \forall v_1 \forall v_2 \forall t\ ((DifferentiaOf(v, v_1) \wedge InstAt(v_2, v_1, t)) \rightarrow Exemplify(v_2, v, t))$
A26. $\forall v \forall v_1 \forall v_2\ ((GenusOf\ (v, v_1) \wedge DifferentiaOf(v_2, v)) \rightarrow DifferentiaOf(v_2, v_1))$

It follows that a differentia is a unary quality universal. Further all instances of a class v_2 exemplify a quality universal v_1 if and only if v_1 is a differentia of v_2 or v_1 is a propria of v_2.

4 Conclusion

As it stands, the formal theory presented here is a rigorous rendering of the neo-Aristotelian perspective on the distinctions between substances and qualities and between universals and particulars presented in the informal parts of this paper. Since the theory allows us to quantify over variables that may denote *d*-ary quality universals, it has a second-order flavour while remaining strictly first-order. It is easy to show that the axioms A1–A26 are sound (with regard to the semantics in section 1) and consistent. Completeness remains to be proved.

The neo-Aristotelian perspective leads to a logic that is more complicated than a usual first-order predicate logic in the sense that it contains a large number of axiom schemata. For this reason it will be difficult to come up with an effective inference mechanism for the formal theory. However, it has the benefit of distinguishing between the different ontological categories which are conflated in first-order logic. Additional complexity is compensated for by ontological expressivity, and our methodology is to captive the relations in question as accurately as possible and only then to advance to the point where we consider the issue of constructing computationally effective representations which will approximate as closely as possible.

The informal theory involves a number of simplifications (for example equating the notions of particulars and of instances, and disallowing the possibility of a single particular instantiating different universals at different times). However, we believe that it nonetheless represents an important step towards a robust ontology of universals and particulars.

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