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Classes, Worlds and Hypergunk

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1. Testing the Possible

The question of what truths are necessary in the broadest possible sense is a difficult one to answer, as is the question of what the limits are to what is possible. (Most people would see these two questions as different sides of the same coin, of course, since many think the question of what is possible is just the question of what is not necessarily ruled out). We have three general sorts of strategies for determining whether something is necessary (or possible). We can identify it in a class that we were previously sure was a class of things that are necessary – we might show it is a theorem of a logical system that we have confidence in, or that the sentence appears to be true simply in virtue of the meanings of the words, or that it is a true statement involving names or about natural kinds of the "necessary *a posteriori*" sort discussed by Kripke and Putnam, and there are perhaps other classes of claims which we are prepared to accept are necessary if true.[1] Likewise, we might establish the possibility of something occurring by reference to a class of well-established or uncontroversial possibilities: e.g. we are inclined to think that it is possible (in the broadest sense) for an event to occur in the future if one of the same kind has occurred in the past.[2]

A second sort of strategy is to appeal to a general theory of modality (a theory of possibility, necessity, counterfactuals, possible worlds etc.). This can be useful both for questions about what is necessary and what is possible – such theories often have answers

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[1] There is of course controversy about which classes of statements should be treated in this way. The question of *why* a class of statements of one of these sorts should be thought to be necessary when true will be a matter of much greater controversy.

[2] There may be limited exceptions here – one might argue that even though "the death of the last Roman emperor" has occurred, it *cannot* occur again.

one way or the other about some difficult cases that we had little to say about pre-theoretically, or that we had mixed inclinations about. The advantages of having a systematic set of answers that have been justified in a non-haphazard way is one important reason for developing explicit theories, after all: though in many respects our theories of modality are much less well developed and supported than our theories of, for example, chemical composition or ancient history.

The third strategy is to apply some rough-and-ready tests to a case to see whether it is genuinely possible: these tests include seeing whether one can prove a contradiction from complete descriptions of the case; whether the case can be described without obvious violence to the language, and perhaps whether the case is conceivable (or has a certain sort of conceivability, since presumably in most disputes about what is possible both sides have a conception of the condition under dispute). It also helps if a possibility can be seen as an extension of cases agreed to be possible, or as capturing an option about the world that has some intuitive appeal, particularly an option that seems an open empirical conjecture (unless it is plausibly in conflict with an *a posteriori* necessity). If a putative possibility survives these tests, it has some call to be judged possible.

There is very little that is uncontroversial when it comes to modality, so we need to be able to test our modal theories and adjudicate between them. One way of testing theories of modality and possible worlds is to see how they stack up against our rough-and-ready tests for what is possible or necessary. If a theory of possible worlds rules that something is necessary, even though its negation appears coherent, does no violence to the meanings of expressions, and there is no natural assimilation of the case to other cases of the impossible, we might be suspicious. An example is that many people think it is an objection to David Lewis's theory of possible worlds that it says that it is impossible for there to be disconnected space-time regions[3] - not the objection that people think is most

[3] This is so with the usual "world bound" understanding of the existential quantifier in "there are": in the unbound sense, Lewis believes there *are* disconnected space-times, so is reasonably interpreted as thinking that in this broader sense there *could be* as well.

serious, typically, but one that is a cost to the theory of some sort (see Bigelow and Pargetter 1987, Bricker 2001).

In this paper I describe what I take to be a possible way an object with parts could be. It is apparently formally consistent, and answers to some extent to an intuitive conception of division. It turns out that it can be specified in relatively non-technical language, in a way that does not seem to me to suggest analytic falsehood. It seems, by the lights of many of our usual tests, to be a logical possibility, though perhaps one we would have little reason to suppose held in the actual world. However, if this apparently harmless, though remote, possibility is indeed a genuine possibility, then a whole swath of theories of possible worlds are false as they stand. I will describe the kind of object I have in mind, explain the conflict with theories of possible worlds, and conclude with a discussion of what should be done about this conflict.

2. Hypergunk

It is reasonably widely believed that it is possible for there to be stuff which is not made up of mereological atoms - that is, it is not ultimately made up of parts which do not themselves have further parts. David Lewis calls such stuff "Gunk" in his (1991, p. 20). Gunk is infinitely divisible, in that every part of it itself has proper parts. (One common example of gunk is to think of Euclidian space with open regions, but with no points. Every region has smaller and smaller sub-regions (which can be thought of as its parts), but there is no smallest region). In standard mereology, gunk can have as few as continuum-many parts, but it is presumably possible for pieces of gunk to have more parts than that: gunk with two to the power of the continuum many parts, for example (i.e. beth-two parts), or two to the power of that (beth-three) etc.[4] There is a hierarchy of possible kinds of gunk with increasingly many parts, as we race up through the higher

[4] Infinite numbers come in different sizes – and 2^K is always greater than K, no matter which infinite cardinality K is. (Or finite cardinality, for that matter).

and higher cardinalities.[5] As the cardinality of parts goes up, the kind of gunk we get seems more divisible than the one we had before.

Perhaps there can be a greater degree of divisibility still. One might think that it would be possible for there to be stuff that no matter how many of its parts you accounted for, you would not have accounted for all of its parts - for it would be able to be decomposed into strictly more parts than the ones you in fact decompose it into. I call such gunk "Hypergunk", where Hypergunk is gunk such that for any set of its parts, there is a set of strictly greater cardinality of its parts.[6] There would thus be too many pieces of hypergunk to form a set – for any sized set, there would be enough pieces of hypergunk to make up a larger set.

A piece of hypergunk would not *prima facie* seem to a contradictory object, and seems to have a formal model in a consistent system.[7] It seems to me that a claim about hypergunk existing does not violate any obvious analyticity either (it is not like a married bachelor or a colourless green idea). Indeed, the notion of hypergunk seems to be one disambiguation of the notion of something's being "divisible without limit", or "infinitely

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[5] There may not be able to be gunk of each cardinality size, but there will be for each size that is 2^K, for some infinite K.

[6] Perhaps this conception of hypergunk does not quite satisfy the conception of stuff which can be decomposed into strictly more parts no matter how many parts it is decomposed into – this may depend on whether one wants to say that proper classes have a size or not. In any case, it is a better approximation of this notion than the mere concept of gunk.

[7] I am reasonably confident that a model can be constructed which jointly models i) the axioms of ZFC, axiomatised so as not to preclude ur-elements, plus ii) mereological axioms ensuring the ur-elements obey Classical Extensional Mereology, plus iii) an axiom asserting the existence of a piece of hypergunk (with it and all of its parts serving as ur-elements). Such a model requires only the resources of ZFC plus two standard inaccessible cardinal axioms. Since hardly anyone seriously believes that ZFC plus two inaccessible cardinal axioms is formally inconsistent, this model would serve as a relative consistency proof for at least the formal consistency of postulating hypergunk. Space and the focus of the paper prevent inclusion of a specification of such a model here. Furthermore, my confidence that such a model can be constructed only stems from the "indirect method" of constructing proof-sketches and seeing how they would be interrelated, so I leave the formal construction of such a model to the interested reader.

divisible" in the traditional sense. After all, if the number of parts that one can divide something into is an indicator of how divisible it is (with increase in the size of the set of parts being proportional to degree of divisibility), then hypergunk will be as divisible as can be, and in particular would be much more divisible than the comparatively staid kind of gunk which has only continuum-many parts.[8] This thought would also apply to an object that was ultimately made of mereological indivisibles as well, but presumably gunk answers to our conception of limitless division better than something which is ultimately made up entirely of (mereological) atoms which are the limits of its division.[9]

The conception of divisibility captured here has some affinities with Charles Peirce's conception of the divisibility of the geometrical line (I follow the reconstruction of Peirce's view given in Putnam 1995). Peirce seems to have thought of points as parts of the line, and he also thought that the points in a geometrical line were a "multitude" greater than the cardinal size of any set (see Putnam 1995 p 11). I suspect he may have taken there to be "ultimate" points, not further resolvable: or alternatively that this infinite multitude of points was only "potential", so that while there was no last stage of resolution into points of a given cardinality, there was no resolution of points into a multitude that were too many to form a set. So I do not claim that Peirce anticipated hypergunk. However, that Peirce thought something (the geometrical line) *really does*

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[8] This may not be the only attractive metric of divisibility, but it is one of the attractive ones at least. And of course one might think that "infinite divisibility" is just a matter of the parts which something possesses being infinite in number, but another conception might be that there is no limit to the divisibility of a thing, not even a limit to a level of divisibility corresponding to one of the lower infinite cardinals.

[9] One topic to explore is different ways gunk and hypergunk could have a certain number of parts. For example I suspect it is consistent and possible to have a piece of hypergunk such that all of its parts where themselves hypergunk. There will be other sorts that are not like this: such as the object you get by fusing a proper class of distinct pieces of ordinary (continuum-many parts only) gunk. More exotic models can also be constructed. I take it that hypergunk such that all of its parts are themselves hypergunk would be closer to capturing our notion of unlimited division than some of these other varieties, but I will not pursue this here. Notice also that gunk with set-many pieces but with more than continuum-many pieces can have more variations in structure than mereologists have noticed – fusing distinct pieces of ordinary gunk is not the only way to produce gunk with more than continuum-many pieces.

have a structure which is very similar to hypergunk, however, is some evidence that hypergunk is something we should concede is at least a logical possibility (though perhaps weak evidence, if we think Peirce is a poor guide to what is ultimately coherent). It may also serve as some evidence that something like this kind of division has some claim on our imagination as one way (though not the only way) of spelling out what it would be for the to be no limit to the parts or divisions a thing has.

Perhaps it seems a little artificial to define hypergunk in terms of infinite cardinalities: we might doubt that our rough-and-ready tests for possibility apply so well to this sort of case. However a definition of hypergunk can be spelled out in relatively simple language. Consider the following:

Something is Hypergunk iff it is atomless, and for every set containing only its parts there is a strictly larger set containing only its parts.

If plural quantification only has the power of set theory, this can even be stated without talk of sets[10]:

Something is Hypergunk iff it is atomless, and whenever there are some of its parts, there are some others of its parts such that there are more of the second than there are of the first.

If it is possible to plurally quantify over things which do not form a set[11], then this paraphrase will be inadequate (since the definition is then inconsistent – all of its parts together (X) are not such that there are some of its parts (Y) are more than them (X)). I believe that with some plausible assumptions, it is possible to find other plurally

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[10] Some would argue that this employment of plural quantification just is to talk about sets, but even were this so, this way of putting it removes the suspicion that a formal, technical conception of sets is being employed. After all, users of ordinary language are not all secretly proponents of ZF or a rival thereof - most hardly have a clue about technical set theory, even despite New Math.

[11] As was argued in Boolos 1984, and which is a view that is being increasingly adopted in the literature.

quantified sentences that are consistent and do specify a condition something meets only if it is a piece of hypergunk, but this exploration is still in progress.

The claim that hypergunk exists seems to be formally consistent, analytically in good order, and a claim that can be made and considered in familiar language. It may also be a reasonably natural way of spelling out a natural conception of unlimited divisibility. I suspect that if we have reason at all to think that there is no such thing as hypergunk in the *actual* world, it is because of contingent-looking matters of scientific investigation: investigating what theory of space, fields and particles suit are observations of the microphysical world, for example. *Prima facie* we seem to have something here which is no doubt unusual or even bizarre, but seems possible, at least in the most generous sense of possible. However, if it is possible, a lot of theories of possible worlds are in trouble.

3. Possible Worlds and Sets

Among the accounts of possible worlds available in the literature, there are several varieties that take worlds to be sets, of one sort or another. Of these, the most common are those which take worlds to be sets of propositions (or abstract sentences) – these include Rudolf Carnap's theory of "state descriptions" (Carnap 1947), the "world books" of R.M. Adams 1974, and Hazen 1996 p 35. Others take worlds to be sets, or classes, or collections, of things other than propositions or sentences, like Brian Skyrms' 1981 which takes possible worlds to be "collections of compossible facts".[12] Others deny that possible worlds are sets, but are committed to each world containing only a set of possible objects, or at least only a set of contingent possible objects: one theorist who is

[12] There are many things left unexplained by Skyrms's paper, including the ontological status of non-actual facts and especially of the non-actual facts composed of the "new" objects and relations that appear in the fact-like objects produced in the "analogical phase". Armstrong 1990 p 46 clarifies Skyrms's position by reporting that Skyrms is happy for his view to be understood as a fictionalist view, according to which there are no non-actual facts, but we employ a fiction about them when engaged in possible-worlds discourse. Skyrms's fictionalism is not a target of this paper – but the position outlined in Skyrms 1981 itself remains in view.

explicit about this is David Lewis (1986 p 104 – all possible individuals form a set, so *a fortiori* all the individuals in a single world will).[13] Some non-realists about worlds who seek to retain some of the advantages of possible worlds also deny there could be more individuals than form a set: Gideon Rosen's modal fictionalism (Rosen 1990) is one (since it follows Lewis 1986 on this point), and a recent example is the basic version of the view offered in Sider 2002, though he does explore some options for modifying his view to accommodate the possibility of more than set-many objects (pp 307-309).

The problems that hypergunk raises for these sorts of views are cardinality problems: if hypergunk is possible, there is a case to be made that there are possibilities that cannot be exhaustively represented by a set of propositions (or correspond to those sets, or *be* those sets); and if hypergunk is possible, theories that insist that each world contains only a set of possible objects (or possible contingent objects) will be false, since when a piece of hypergunk exists in a world, there are more parts of that hypergunk in that world than can be contained in a set. Sets of atomic sentences, or of "compossible" atomic "facts", and the like, cannot handle hypergunk as a possibility. Neither can the theory of Lewis 1986, or relatives like Rosen 1990, that insist on a set of possible objects. There is a history of cardinality concerns about standard theories of possible worlds, but I think the cardinality problems raised by hypergunk are interestingly different. Let me say a little about why, before turning to discuss a theory specifically designed to respond in a principled manner to some previous cardinality objections: the theory of Hazen 1996.

Two sorts of common cardinality concerns are not about the size of individual worlds, but about the size of the set of all worlds. The first kind of concern manifests itself in arguments from a principle of recombination to the conclusion that for any set of worlds, there is a strictly larger set, and so (unless some revision is made) there cannot be a set of all worlds: arguments like this are offered by Forrest and Armstrong 1984, and Lewis 1986 pp 101-104 (Forrest and Armstrong draw the conclusion that we ought to reject

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[13] I briefly mentioned that hypergunk was trouble for Lewis's view in Nolan 1996 pp 258-259 and Nolan 2002 p 148.

Lewisian modal realism, Lewis draws the conclusion we should restrict the principle of recombination). The second sort of cardinality concerns surround what is known as the Peacocke/Kaplan paradox (which first appeared in print in Davies 1981, is discussed in Lewis 1986, and has been presented more recently in Kaplan 1995). If there are propositions corresponding to sets of worlds (or identical to sets of worlds), and if there is a way of mapping each proposition to a distinct world (e.g. by supposing there could be believers that differ in their beliefs by just one arbitrary proposition, or by more sophisticated methods), then there is a Cantorian paradox –there would be at least as many worlds as sets of worlds.

Both the recombination paradox and the Peacocke/Kaplan paradox have been addressed by various writers, but the only point I want to make about them here is that they rely on principles about the relationships between worlds (or worlds and sets of worlds) rather than intuitions about what goes on in a single possibility. Cardinality puzzles about the collection of worlds as a whole and the relationships between different worlds can shed useful light on questions about the nature of possible worlds and on some questions about modality, but I suspect in a different way from the sort of considerations that support the possibility of hypergunk.

Other cardinality arguments do not rely on principles about the relationships between worlds. Patrick Grim (1984) and Selmer Bringsjord (1985) have offered cardinality arguments that challenged conceptions of possible worlds as sets of propositions. Grim's argument, for example, can be paraphrased as follows: for any set which purports to be a set of all truths, there will be at least one truth corresponding to each of its subsets (for instance the truth that such-and-such a proposition does not belong to that subset). Since there are always strictly more subsets of a set than there are members of that set, there cannot be a set that contains all of the truths. This shows that not only is there no set of truths, but also that there cannot be: no set of propositions could be a maximal consistent set of propositions, so possible worlds cannot be construed in this fashion. Grim also points out that taking possible worlds as maximal sets of co-possible contingent truths will also be in trouble, since a contingent truth conjoined with a necessary truth yields a

contingent truth, and so given even one contingent truth, there will be at least as many propositions formed by conjoining it with necessary propositions as there are necessary truths.[14]

A "sets of propositions" conception of worlds can be constructed so as to avoid these objections. The most prominent attempt to do so is that of A.P. Hazen (1996). Hazen suggested that we construe worlds as specific sets of contingent propositions—a set of "basic" contingent propositions upon which all of the other propositions supervene (in particular, those contingent propositions obtained by conjoining a necessary proposition and a contingent one) (Hazen 1996 p 34). He illustrated this with the familiar example of Democritean worlds and the propositions necessary to describe them, and generalises to various more complicated cases. Hazen admits that his account of possible worlds as sets of "basic" contingent propositions relies on the hypothesis "that each world should contain a set (not a proper class) of 'physical' objects, and that they should be characterised by a set (not a proper class) of properties" (Hazen, 1996, p.35). Otherwise there might not be any set of even the "basic" propositions needed to be the supervenience base for all the propositions true at a given world. It should be noticed that Hazen's hypothesis that the objects are 'physical' should not be interpreted too narrowly - any contingent objects in worlds will need to form a set, whether they are Cartesian egos, Hegelian absolutes, Liebnizian monads, angels, or whatever. The restriction to 'physical' objects is presumably to indicate that necessarily existing abstracta, such as sets or numbers or propositions or whatever, do not need to respect this restriction. Hazen should also stipulate, I take it, that as well as there being a set of properties characterising the objects in each world, there should be only a set of relations as well, on pain of there being more than a set of truths about what is related to what at some world. (Even a world with two objects in it might not be able to be characterised

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[14] The argument of Bringsjord (1985), establishes the truths which are correlated one-one with the relevant sets of truths by pointing out that for any set there is the (true) proposition that that set is a set, and likewise draws the conclusion that there could be no set of all such truths, and so worlds could not be maximal consistent sets which contained all of these truths.

by a set of "basic" truths if there were a proper class of fundamental relations holding between the two objects). Let us then interpret his hypothesis in this way.

Hazen's approach faces several challenges, some of which are posed in Grim 1997. Traditionally, those who construed possible worlds as sets of propositions had a simple story about what it was for a proposition to be true according to such a world: it was for that proposition to be a member of that world. The relation between a proposition being true in a world and that world is not that simple for Hazen, since for him a proposition is true at a possible world (or according to a possible world) if either it is a member of that world, or it is "determined" by the members of that world or whose truth would supervene on the truth of those basic propositions. Grim objects that this determination or supervenience is problematic (Grim 1997 pp 148-149). I will suppose for the sake of the argument that these objections can be met satisfactorily.

Even with these objections satisfactorily dealt with, Hazen's system would rule out there being more than set-many 'physical' objects (or "basic" properties and relations of those objects).[15] If hypergunk were possible, as it seems, then Hazen's system would still be inadequate. (There might be other sorts of motivations for accepting the possibility of more than set-many 'physical' objects: Grim 1997 p 147, for example, suggests there might be too many 'physical' objects to form a set, in a world where there is a separate expert for each ordinal number.) Hazen does not simply assume this restriction on objects and their properties and relations, however: he offers a defence of it. Hazen's defence is basically that anything we could want to describe, or think is possible, is able to be described by set theory, because that is what set theory is designed to do. In Hazen's words, "Part of what is implied in taking set theory as foundational for mathematics is that every structure studied by mathematicians should be represented by a set, and the axioms of set theory have been chosen, historically, to ensure that this is so." (Hazen 1996 pp. 35-36). Hazen then offers a historical discussion of set theory's

[15] Perhaps it only rules out having more than set many "basic objects", if the existence of some objects, (e.g. composites), followed by supervenience from the existence and nature of other objects (e.g. simples). This loophole will be closed later in the paper.

evolution in response to realisations that various axiom systems did not prove the existence of various desired mathematical structures, such as the continuum, and he also claims that some of the impetus in these sorts of cases grew out of mathematician's desires to model actual phenomena, or idealisations of actual phenomena which would be useful for the sciences. Hazen concludes his historical discussion with the following extrapolation:

> We can predict with confidence, therefore, that should any fantasy-writer (or example seeking philosopher) succeed in conceiving a possible world too large and complex to be modelled in current set theory, and if the conception is clear and convincing enough for mathematicians to take it seriously, then it will be taken as *ipso facto* grounds for adopting a new set-theoretic axiom. (Hazen 1996 p. 37)

The implicit assumption is that any such conception will be susceptible to being adequately modeled in a set theory which is simply the traditional one extended with new set-theoretic axioms. Hazen also has a second defence, or perhaps a qualification, which he provides for his prediction. It consists of his attempting to rule out specifications not satisfiable by set theory which are "essentially *metaphysical*", to use his expression.

Hazen realises that it is quite straightforward to produce some sorts of specifications of objects which entail that there will not be a set of them - his example is a specification that there be concrete objects which are isomorphic to the sets under a relation which is isomorphic to set-membership (p. 37). He thinks that it is no serious cost to rule out such specifications as being even logically possible, and argues that ruling out such specifications "is not.. likely to conflict with any genuine modal intuitions", presumably because definitions in terms of the entire set-theoretic hierarchy are definitions which are constructed from material that outruns normal intuition, or because they cannot be formulated as intuitions just about concrete objects rather than concrete objects conceived of in relation to sets. (Grim's definition of a world which specifies what experts there are in terms of the totality of ordinal numbers presumably runs afoul of this qualification).

I am unsure exactly why these definitions are suspect, apart from the question-begging reason that they are inconsistent with the view of possible worlds Hazen is concerned to defend in his article. In any case, he states what he takes to be a consequence of this restriction, and since I think this consequence can be shown to be implausible, doubt can be cast on the premise from which it is derived (however exactly that is to be fleshed out). He says "Thus no intuitive judgment of the form 'Things could have been like *so*,' with the *so* a description cobbled together out of physical and ordinary mathematical language, can imply the possibility of a world containing proper-class many non-abstract objects." (p. 37). Hazen seems to wish to respect judgments of the form he specifies - I imagine therefore it is only descriptions which employ other than "physical and ordinary mathematical language" which acquire the epithet "metaphysical". Even then, he need not frown on all such descriptions for his purposes - for it is only those descriptions which are plausibly contingent that need worry him. Necessary falsehoods, even if they imply a world of more than a set of non-abstract objects, and even if they are cast in "physical and ordinary mathematical" language, need not bother him, since these do not need to be modeled by possible worlds.[16]

Many would not accept Hazen's claim that every structure of mathematical interest can be represented in set theory. Various defenders of class theories and category theories would reject the claim that their entire theory can be represented in the sets, since they take the power of category theory or class theory to outstrip the power of set theory.[17] Some versions of set theory are inconsistent with other versions - Zermelo-Frankel and New Foundations, for example. In addition, there are some theoretical motivations outside mathematics to endorse possibilities that cannot be modeled by sets. For example, I have argued elsewhere that at least one principle formulated with reference to

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[16] There is an interesting puzzle about how Hazen's approach could be extended to handle impossible worlds, if there are impossibilities that cannot be adequately modeled by set theory, but this need not concern us here.

[17] Of course there might be attenuated senses of "represent" according to which set theory does represent or model some of the behaviour of proper classes or categories – but that set theory can provide a fully satisfactory model of the intended interpretation of their theories is controversial, to say the least.

cardinality (and hence perhaps "metaphysical") has a good deal of intuitive force, and which has as a consequence that some world contains a proper class of objects: the unrestricted principle of recombination for worlds (see Nolan 1996 and Nolan 2002 ch 6, though this consequence is not drawn explicitly).

Let me, however, grant Hazen his restriction, and take him to be claiming that no non-metaphysical description, in the sense of "metaphysical" defined above) could imply that there were a proper-class of "concrete" or "physical" objects.[18] Hypergunk still meets this restriction. The definition of hypergunk relies on some standard physical vocabulary like "part of", and very ordinary mathematical notions: perhaps it relies on talk of sets (or groups, or collections) in the most ordinary and commonsense way, and apart from that the only mathematical vocabulary is the use of the phrases "strictly greater" or "more than", hardly highly technical notions. Even these can be spelled out a little further, if it is objected that the notions of "more than" or "strictly greater size" take on special 'metaphysical' meanings in transfinite contexts. For these concepts in turn can be spelled out in terms of equinumerosity - in terms of there being exactly one of each of one sort of thing for exactly one of each sort of another sort of thing. This is surely one of the most ordinary of mathematical (or pre-mathematical) concepts we possess. The way of employing this concept to spell out "strictly greater" or "more" is reasonably obvious, but just to make it explicit:

Something is Hypergunk iff it is atomless (i.e. every (proper) part of it has (proper) parts), and for every set *S* containing only parts of it there is another set *S'* containing only parts of it which has a subset whose members are equinumerous with the members of *S*, but *S'* is not itself equinumerous with *S*.

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[18] His claim can in fact be weaker - non-metaphysical descriptions that implied that there were a proper-class of objects which were not *prima facie* consistent would not be a problem for Hazen. His claim should of course be read with this restriction by those who believe that inconsistencies imply everything - for it is easy to frame a run-of-the-mill contradiction without the suspicion of 'metaphysical' devices.

(The variables *S* and *S'* can be dispensed with too to provide a sentence of ordinary language using anaphora, but such sentences are difficult to disentangle without careful reading).

or, to employ the plural form (again assuming plural quantification does not deliver any more expressive power than set theory):

Something is Hypergunk iff it is atomless, and whenever there are some of its parts there are some others of its parts such that the first are not equinumerous with the second, but are equinumerous with some of the second.

These sentences can be made even more ordinary by substituting a more ordinary sounding definition for the technical word "equinumerosity", but even as the claims stand they obey Hazen's stricture against 'metaphysical' definition, as far as I understand it.

Since there will be a one-one correspondence between the sort of propositions expressed by sentences of the form "*X* exists" and those pieces of hypergunk, it will follow that there cannot be a set of all the contingent truths in a world where there is hypergunk, even if we ignore those contingent truths cooked up by conjoining contingent propositions to necessary ones. Trouble seems to be looming for theories that suppose every possible world is to be identified with a set of propositions: worlds with hypergunk have too many basic contingent things true according to them.

All is not yet lost for a Hazen-style conception of worlds, according to which the set must provide a "supervenience base" for all the other contingent truths of that world. For it is a plausible (though not uncontroversial) view that a proper class of objects can be represented with a mere set of propositions. Indeed, one might think that a description of a world of this form is by itself complete:

There is a piece of Hypergunk, and all of its parts have property *F*, and that's the only fundamental property any of them have, and that's all there is.

Remember that in defending the notion of possible worlds as complete descriptions (or as sets of propositions which together provide a complete description) Hazen is not defending the traditional logical atomist conception of such descriptions - he (or other defenders) need not be defending a modified version of Carnapian state descriptions. So there need be no problem with having quantified propositions in the subvenient base, even quantified propositions besides some sort of totality clause (a "and that's all there is" clause).

However, not everyone will be happy with this. One might think that this account leaves something out, something which is not implied by the quantified statements, but requires propositions of the form "*X* exists". This is because one might think that the question of which object is which is an issue over and above the general account of how many objects there are, their qualitative natures, and the relations holding between them. Heaccaetists may think that there could be two worlds satisfying the above description, but that there be some possible object that is a part of one of the pieces of hypergunk but not of the other - or even that there be two possible objects both of which exist in each world, but have their positions swapped. Such people should reject the claim that quantified propositions can do the work I suggest. If, on the other hand, one were to believe that facts about the identities of particular objects supervened on the qualitative facts (including relational facts) of a world - facts which could in principle be captured using quantifiers - then one could maintain the claim that possible worlds were sets of propositions in the face of the possibility of Hypergunk.

Hypergunk may still cause trouble for those who reject the heccaeistic theory discussed in the previous paragraph. For when (and if) the metaphysical possibility of Hypergunk is admitted, it becomes plausible that the parts of Hypergunk could possess different properties - perhaps so many differences that they could not be captured in a set of propositions, even of propositions which corresponded to sentences containing quantifiers. And it seems easy enough to say something to this effect, without employing 'metaphysical' terminology (in Hazen's sense). To say, for example, that there could be

some Hypergunk such that each piece of it had a property that none of the others did, employs no language out of the ordinary. Perhaps something needs to be added to the effect that the properties must not supervene on a terribly small supervenience base, but this again can be specified easily enough (adding a clause like "and the fact of the possession of these properties does not follow from the formal characteristics of each piece or the possession of any properties by anything else" should probably do the trick, with perhaps an extra bell or whistle or two). Other ways of ensuring that there will be sufficient complication are of course also possible.[19]

Of course, one could think that this problem has a solution analogous to the one available in the case above. One could hope that propositions quantifying over properties would imply the other truths about the qualitative character of the objects, just as quantification over individuals arguably obviated the need to include a separate claim about the existence of each individual. Indeed, some will find this an adequate solution - some structuralists about properties will think that once it has been determined how many properties there are, what the pattern of their instantiation is, and what relations hold between them (which may in turn only need to be described structurally) then all there is to be said has been said (or at least all there is to be said has is entailed by that determination). I do not find this as convincing - I think that there is more to a property (or relation) than its pattern of instantiation and the other relations in which it might stand, and I find myself wanting to ask "which part has which property?" when the case described employing quantification over properties is raised. For those that believe that there is more to properties than their qualitative (including relational) character[20], a world constructed with only such quantified statements about properties will not suffice – and I take it there are many who do think there is more to a property than its structural profile in a single world. Without the rejection of haecceitism and the acceptance of this sort of structuralism about properties, variegated hypergunk defeats even Hazen's set-theoretic worlds.

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[19] This plugs the loophole noted in footnote 14.

[20] This view is sometimes described as the belief that properties have "suchnesses" (analogously to haecceities or "thisnesses") - I owe the term to John Hawthorne.

4. Other Challenges

The possibility of hypergunk causes trouble for some other views in metaphysics and logic besides theories of possible worlds. One variety consists of the views which take logical consequence to be defined model-theoretically, and takes the relevant models to be set-theoretic or class-theoretic objects (see the "traditional realism" of Shapiro 1993 p 461). The problem here is that set-theoretic models are prima-facie unsatisfactory for modeling entailments when there are propositions about a proper-class of objects – any proposition true in every set-sized model will be counted as a theorem of logic (because it holds in all models), even though some of those propositions are false in a possible world where hypergunk exists. The risk is that these theories will count sentences as logically true, and perhaps also logically necessary, which are possibly false – and in a world where there is hypergunk, these theories may claim that something which is false in that world is logically true there as well, which is not a happy result. (An obvious example of the sort of proposition that might cause trouble is "no object has more than set-many parts".)

Another problem arises for those who take logical entailment to be a relation that holds only between *sets* of propositions. As an example, take the proposition expressed by a sentence of the form "*a* has more parts than are members of any set".[21] *Prima facie*, enough claims of the form "*x* is a part of *a*", "*y* is a part of *a*", along with enough claims of the form "*x* is distinct from *y*", will jointly entail "*a* has more parts than are members of any set", but no set of such claims will do so.

The debate about what is going on here gets into deep water very quickly, and many of the relevant issues have already arisen in the literature with respect to entailments between propositions about sets. So I do not want to come to any conclusions here about

[21] Perhaps it is not propositions that logical entailments hold between, but sentences that express propositions, or propositions-plus-logical-forms. Whatever your preferred theory of the relata, a modification of the example should make the point.

expressibility or the use of model-theory to explain logical consequence – I only intend to point to this as an area of interest where the possibility of hypergunk may raise a challenge. Another place where it may make a difference is the debate about whether the use of a quantifier is always to be associated with a set-sized domain of quantification which supplies the potential values of the relevant variable. (That is, when I say "all cows eat grass", for example, there is some *set* associated with the quantifier such that the sentence is true just in case all the members *of that set* are either non-cows or grass-eaters). Interpreting a sentence of the form "all the parts of *a* are *F*", for example, will be difficult to do if *a* has more parts than can form a set, *and* we are to suppose that there is a set-sized domain associated with the sentence, *and* that the domain at least contains all of *a*'s parts. Insistence that the parts of *a* must all be part of a set-sized domain would be inconsistent with *a*'s being a piece of hypergunk. (For another example that would cause trouble, consider the sentences on p 21, interpreted so that all the quantifiers are associated with only set-sized domains). Again, there is much that could be said – but hypergunk can give us a distinctive example that may cause distinctive trouble.

5. Conclusion

These musings about hypergunk cannot hope to be conclusive – we could in the end decide that the virtues of some theory that forbids it are worth the cost. What I have tried to do is argue that denying the possibility of hypergunk would indeed be a cost – it answers to one plausible conception of unlimited division, it is formally consistent, it violates no obvious analytic truths, and it can be characterised in relatively simple terms that seem to pass even Hazen's demand that if we are to take cases like this seriously they must be able to be stated in "non-metaphysical" language.

Hypergunk causes several sorts of problems for theories of possible worlds. It causes trouble for those that take there to be a set of all possible objects. It causes trouble for those who take there to be a set of atomic facts in each world, or atomic propositions describing each possible world. Hypergunk, or more exotic varieties like variegated hypergunk, causes trouble for worlds represented by sets of non-atomic propositions, if other assumptions are made. And it causes trouble not just for realists about possible

worlds and possible objects, but some, such as Rosen's (1990) fictionalist or Sider's (2002) pluriverse ersatzer, who seek the benefits of the corresponding realist theories. Often the modifications needed do little or no harm, and even for general approaches that would seem to be in the most trouble – the approaches which take worlds to be sets of propositions, for example – modifications can be made to accommodate hypergunk, for example along some of the lines suggested in the previous section. So while taking the possibility of hypergunk seriously can be significant, it need not be seen as destructive.

If we are to rule out the possibility of the kind of unlimited divisibility that hypergunk represents, it would be good to at least explicitly specify what modal principle is to exclude it, and we should hope that such a principle can be plausibly motivated. *Prima facie* possibilities do not furnish knock-down counter-examples, but they can point to directions in which to develop alternative theories. In the case of hypergunk, it suggests we should not tie logical or metaphysical possibilities for objects to the limitations of set theory. Those who do think this tie is sufficiently important are invited to articulate what it is about set theory that they think makes it a guide to what is necessary about the size of the non-mathematical realm.

The focus of this paper has been on hypergunk's consequences for modality, with a mention of its effect on formal semantics and model theory. But the issues raised about what division is possible may be of interest elsewhere as well. One area of interest is in trying to tease out strands in the concept of unlimited divisibility, both in a pre-theoretic notion (if there is one), and the notions that have appeared in the history of ideas. Another is in debates about the relation in size between the mathematical universe and the non-mathematical universe (is there a set of all ur-elements, for example? If the mathematical realm can be said to be "indefinite" in size, should we say the same about the non-mathematical realm? etc.) These are investigations for another occasion.[22]

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