Can Diagrams Have Epistemic Value? The Case of Euclid

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1 Title of the Tutorial

"Can Diagrams Have Epistemic Value? The Case of Euclid"

2 Names and Affiliations of the Instructor

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3 Benefits

The tutorial will give attendees:

- A firm grasp of an important debate in the history of ideas.
- A logical framework within which to assess questions of diagrammatic justification.
- Knowledge of some key philosophical arguments for and against the epistemic value of diagrams.
- A deeper understanding of the nature of diagrammatic reasoning.

4 Content

The Issue. Can diagrams have epistemic value? Can reasoning with diagrams confer knowledge or justify belief? These fundamental questions have been little studied outside the area of logical diagrams, in which the formal syntax of an adequately specified given system rules out the possibility that correctly interpreted

diagrams can mislead the user.¹ In Euclid's geometry, however the paradigm case of a body of exact knowledge until the mid-19th century precisely this possibility seems to exist. And there is the further problem of generality: how reasoning with a single diagram can justify knowledge of a general mathematical claim.

Traditional Responses. Accordingly, the dominant view among philosophers and logicians, following Bertrand Russell, has long been that the diagrams have no epistemic value but are "merely heuristic"; on an older view, which dates back to Kant, they do have epistemic value, but only via a dubious appeal to a postulated faculty of "intuition".

Challenging the Tradition. The tutorial challenges these claims, and the background assumption that they exhaust the available alternatives. Against the "heuristic" view, it shows that diagrams can be of genuine epistemic value, using Euclid's geometry as a case study. Against the "intuitive" view, it shows that this epistemology need make no appeal to a faculty of intuition. Thus the traditional debate mistakenly ignores a third, crucial, possibility.

Structure. The discussion breaks down into three parts. The first part sets up the problem and the logical space of alternative solutions; the second explores the candidate solutions themselves; the third selects, elaborates and defends the preferred solution. To make the analysis and subsequent discussion as specific as possible, the discussion is focused on a single argument: Prop. I.32 of the Elements, to the effect that all triangles have internal angles that sum to two right angles: the so-called "angle sum" property.

- The Problem and the Solution Space. The tutorial introduces a logically exhaustive Framework of Alternatives, covering different theories that can be advanced to account for the apparent justification offered by this reasoning.
- Exploring Candidate Solutions. The tutorial appraises four candidate theories that might be advanced in each of the categories already identified.
 These theories can be plausibly attributed to an interpretation of Plato by W.D. Ross, to J.S. Mill, to Leibniz, and to Kant. Each theory holds that Euclid's argument confers justification, but they differ as to how it does so.
- Developing the Preferred Solution. Three of the candidate theories can be shown to fail. The tutorial then defends a version of the fourth view: it highlights some of its distinctive features and commitments; it shows how it meets three main lines of criticism, and it meets further logical and epistemological tests.

¹ Overall, see e.g. the collections Glasgow et al. [1], and Blackwell [2]. Greaves [3] gives a broad philosophical survey of diagrams in geometry and logic, but does not devote detailed consideration to the epistemology of reasoning with diagrams as such. For diagrams in computing/AI, see e.g., Sowa [4] and Jamnik [5]; in logic, see e.g. the works of Barwise and his collaborators Etchemendy and Allwein, and Barwise's students Shin, Shimojima and Hammer; and, for a case study comparing inference using diagrams and sentences in propositional logic, see Norman [6].

Significance. Questions of epistemic value are fundamental to current philosophical and logical research on diagrams. Historically, virtually every major philosopher of the period 1600-1850 discusses Euclid's geometry including Descartes, Gassendi, Leibniz, Hobbes, Hume, Locke, Berkeley, and Kant and most of them discuss Prop. I.32 in particular. Thus the tutorial does not merely address an important philosophical problem, it also situates and resolves a well-known debate in the history of ideas.

5 Audience

The argument of Prop. I.32 is well-known and easy to understand, making the case study approach used here accessible to researchers in all disciplines. The tutorial, which is based on and extends previously published work, uses no special logical formalism or technical apparatus. It should be of interest to a wide range of researchers into diagrams, including in such areas as cognitive science, artificial intelligence, education and design.

6 Instructor Background

Jesse Norman is currently Departmental Fellow, Department of Philosophy, University College London. He has an MA from Merton College, Oxford University in Classics, and an MPhil and PhD in Philosophy from University College London, and has been the recipient of numerous academic awards and prizes.

Relevant publications include:

"Peirce Provability and the Alpha Graphs", Transactions of the C.S. Peirce Society, Winter 2003.

Visual Thinking in Euclid's Geometry: An Epistemology of Diagrams (University College London: PhD Thesis).

"Iconicity and 'Direct Interpretation'", Multidisciplinary Studies of Visual Representations and Interpretations (Elsevier Science 2002).

"Differentiating Diagrams: A New Approach", in Anderson, M., Cheng, P., and Haarslev, V. (eds.), *Theory and Application of Diagrams* (Berlin, Springer 2000).

The Achievement of Michael Oakeshott (Duckworth, 1992).

7 Requirements List

The tutorial will require an OHP. There are no other requirements.

References

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- [6] Norman, A.J.: Diagrammatic reasoning and propositional logic (1999) MPhil Thesis, University College London. 15
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- [8] Shin, S.J.: The Iconic Logic of Peirce's Graphs. MIT Press, Cambridge, MA (2002)
- [9] Shimojima, A.: On the Efficacy of Representation. PhD thesis, Indiana University (1996) PhD Thesis.
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- [11] Barwise, J., Etchemendy, J.: Computers, visualisation, and the nature of reasoning. In Bynum, T., Moor, J., eds.: The Digital Phoenix: How Computers are Changing Philosophy. Blackwell, Oxford (1998)
- [12] Allwein, G., Barwise, J., eds.: Logical Reasoning with Diagrams. OUP, Oxford (1996)