Loop quantum ontology:
spin-networks and spacetime

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Abstract

The ontological issues at stake given the theory of loop quantum gravity (LQG) include the status of spacetime, the nature and reality of spin-networks, the relationship of classical spacetime to issues of causation and the status of the abstract-concrete distinction. I this paper I argue that, while spacetime seems to disappear, the spirit of substantival spacetime lives on under certain interpretations of the theory. Moreover, in order for there to be physical spin-networks, and not merely mathematical artifacts, I argue that we must interpret the theory as including a substantival background manifold. This latter result is important for any project which interprets spacetime as an emergent structure from physical spin-networks.

1 Introduction

In the current literature on loop quantum gravity (LQG), one will find the following claims:

> The spin networks do not live in space; their structure generates space. And they are nothing but a structure of relations...
> (Smolin 2002, p.138)

> ...the quanta of the field cannot live in spacetime: they must build "spacetime" themselves... Physical space is a quantum superposition of spin networks...a spin network is not in space it is space.
> (Rovelli 2004, p.9, 21)

> LQG thus seems to entail that space(time) is not fundamental, but emerges somehow from the discrete Planck-scale structure.
> (Wüthrich 2006, p.169)

> ...the emergence of spacetime continuum and geometry will be the result of the quantum properties of the atoms of spacetime.
> (Oriti 2014, p.15)

> One influential idea based on so-called 'weave states' proposes that the spacetime structure emerges from appropriately benign, i.e. semi-classical, spin-networks. (Huggett and Wüthrich 2013, p.279)

Such claims cause one to wonder: what kind of objects are spin-networks which quite literally ground spacetime? Surely, nothing like anything we have ever known. In this paper, I will address what might be the ontology of LQG in general and of spin-networks in particular.

LQG begins with Dirac's quantization procedure and ends with a Hilbert space of states and a set of physical observables. By analyzing these structures, we will be able to begin to understand what the "atoms of spacetime" are like. Like most physical theories, there is no single metaphysical interpretation forced by LQG. Consequently, I am not under the illusion that this work is complete; rather, I aim to clarify some of the issues at stake and establish a foundation upon which further analysis may be continued.[1] Some of the issues involved in the theory of LQG include: the nature of spacetime, the relationship of geometry to spacetime, the status of abstract objects, the relationship of quantum systems to classical structures, the relationship of (pseudo-)Riemannian structures to physical objects (and to concrete objects), the debate over substantivalism, the problem of time, the notion of locality, and the notion of emergence.

Hagar (2014) addresses issues of geometry, Wüthrich (2014) as well as Smolin and Markopoulou (2007) address issues of locality, Huggett and Wüthrich (2013) as well as Lam and Esfeld (2013) address emergence and the issue of local be-ables, and finally Wüthrich (2014), Isham (1992), and Norton (2015) as well as many others address the "problem of time." However, in order to discuss spacetime emergence or locality, for instance, we must first know what objects there are in LQG and what those objects are like. In this paper, I develop eight interpretations of LQG and highlight the ontology suggested by them. Only under some of these interpretations is spacetime missing from the ontology of the theory and is in "need" of emergence, and only under certain others are there spin-networks.

While eight interpretations seem like a tall order, a few of these interpretations are mere variations of each other and for two interpretations, I provide only the broadest of outlines. Since my interest is to understand the ontology of LQG, most of the interpretations are developed just enough to extract some facts about ontology. Due to the limited space of

[1] The following account assumes the "canonical approach" (as opposed to the "covariant approach") to LQG which takes structures in *space* as being fundamental (spin-networks/s-knots) rather than structures in *spacetime* (spin-foam). For the purposes of this paper, this difference will not matter. For ease of expression, I will use "space" and "spacetime" interchangeably.

this article, I cannot address every interpretation or philosophically important nuance of the interpretations I discuss. Rather than covering every possible interpretation, I have chosen to focus on what I hope will be a diverse collection of intuitive interpretations. Moreover, in the course of discussing the ontology of LQG, I will be forced to make use of a variety of physical and philosophical concepts: quantization, emergence, composition, substantivalism, relationism, and yet I do not provide anything like a complete account or overview of how these concepts have been used in general relativity (GR) and how they might be used in LQG. For instance, I will briefly discuss relationism in the context of explicating Rovelli's interpretation, yet I do not provide a general metaphysical account of what exactly relationism amounts to or an overview of different ways one can be a relationist in LQG (which is of special concern since spacetime itself might be missing).

As a means of coming to understand the ontology of LQG, I will answer the following questions on behalf of different interpretations of the theory:

1. In providing a quantum theory of general relativity, does LQG describe spacetime as having gone missing?

2. Are spin-networks included in the ontology of LQG?

3. Is spacetime emergent from or composed of spin-networks?

I will demonstrate that questions one and two depend rather heavily upon one's interpretation of the mathematics of LQG and upon what we take spacetime to be. Only sometimes is there spacetime, and only sometimes are there spin-networks. Regarding question three, I will argue that spacetime is emergent to the extent that it is an effective structure. Whether or not effective structures are real objects of our ontology or merely useful fictions, is a question which I leave open.

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5 Appendix

5.1 Constraints

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2 The theory of LQG and the naïve interpretation

In this section, I will explicate the theory of LQG for physics-informed non-specialists. Since there is no interpretation free way of expressing the content of a physical theory, in the following I will adopt a common theory-ladened-language. This language is colored with explicit statements to the effect that space and spacetime are background structures assumed by the theory. For instance:[2]

> Nowadays, this approach is mostly pursued in a different form, based on ideas of Ashtekar. The idea of "splitting" spacetime [$\mathcal{M}$] into 3-dimensional slices [Σ], and conceiving dynamics as evolution from one slice to another, remains; but the basic dynamical variable is now, not a 3-geometry, but a 3-connection... (Isham and Butterfield 1999, p.22)

> For each given graph γ, considered embedded in the spatial manifold [Σ] where the canonical analysis takes place.

[2] Furthermore, throughout his text book on LQG (1991), Ashtekar consistently references space and spacetime, as background structures (p.xviii) equipped with spatial structure such as spatial topology (p.27). Though, in at least one instance, Ashtekar signals that he is not fully committed to the 3-dimensional manifold Σ as representing space: "The resulting canonical variables are then complex fields on a ("spatial") 3-manifold Σ." (p.16) Notice that Ashtekar places 'spatial' in scare-quotes. Presumably, this is meant to highlight that without a metrical structure, Σ cannot itself denote space?

> (Oriti 2014, p.5)
>
> ... graphs which are embedded in space [Σ] in such a way that only nodes that are within a few Planck distances of each other...
> (Markopolou and Smolin 2007, p.2)

It is unlikely that any of these authors intended to endorse the metaphysical claim that the mathematical 4-manifold $\mathcal{M}$ *is* physical spacetime or that 3-manifold Σ *is* space. What is less clear is how many of these authors endorse the position that the bare manifold $\mathcal{M}$ is sufficiently rich, on its own, to *represent* spacetime. It is usually assumed that one needs both $\mathcal{M}$ as well as some metric g in order to have a structure rich enough to represent spacetime. And yet, in LQG, $\mathcal{M}$ and Σ have no metric on them; consequently, the physical systems we model using $\mathcal{M}$ and Σ are similarly impoverished. I suspect that when physicists refer to $\mathcal{M}$ as being spacetime or the spacetime manifold, they are simply using what ends up being a convenient language to speak of mathematics only and are not endorsing a position on what physical spacetime is or what structure it has.

In the following exposition of LQG, I will follow this linguistic convention and refer to $\mathcal{M}$ as the spacetime manifold or as representing spacetime, but I will go further and explicitly develop, though not necessarily endorse, an interpretation around the conviction that the bare manifold $\mathcal{M}$ does in fact represent spacetime. I will call this interpretation "naïve" though I do not call it naïve in a disparaging sense.

According to the naïve interpretation, LQG is a theory of quantum geometry and not a theory of spacetime or quantum spacetime. It might not be obvious what the difference is between these options, but it will become clear in the following. According to this interpretation, the world consists of a substantival spacetime manifold which I will often refer to as being "physically substantial," represented by $\mathcal{M}$, replete with "geometrically charged" graphs (s-knots) represented by the s-knot states of the LQG.[3] What s-knot states and geometrically charged graphs are, will be explained shortly. The ontology of naïve-LQG is quantum since the geometry associated with each charged graph has quantum features, which I will also discuss shortly.

Some might find the naïve interpretation unattractive or even obviously false since the bare manifold lacks the geometric structure which we have come to associate with spacetime. However, for three reasons, it will be useful to take the naïve interpretation seriously. First, the language of the naïve interpretation is often used in the physics literature itself. Second, in providing a quantum theory to replace GR, we need to loosen our commitment to old associations. For instance, due to our familiarity with general relativity, we have come to associate physical causes with light cone structures and space and time with $\langle \mathcal{M}, g \rangle$. However, just because spacetime was described by $\langle \mathcal{M}, g \rangle$ in GR, does not mean that it ought to continue to have this description in all future theories. In fact, we know

[3] If there is matter or energy in the world, then the world includes these items as well.

that it won't have this description since there is no metric g in LQG. Of course, I do not mean to suggest that just any interpretation of 'spacetime' should be taken seriously, but rather that we cannot rule out interpretations simply by referring to what is true in GR. Third, I will use the naïve interpretation as the starting point for developing alternative interpretations. For this reason, the naïve interpretation will prove to be a pedagogical aide and a contrast against which to discuss additional interpretations. In §3, I will develop alternative interpretations, all of which are less naïve and some of which are not substantival.

One can find many introductions to the theory of LQG, but few are non-technical and most use heuristics which are detrimental for understanding the theory's ontology. In the following account, I have aimed to explain LQG with less mathematics and have relegated some of the technical details to an appendix and others to citations. Though the following mathematical explication of the theory is, at times, terse, I have aimed to provide an account sufficient for understanding the logic of the relevant arguments if not the details. Throughout this text, I will refer to content in the appendix with its section number "[A(§#)]".

2.1 Constraints

The theory of LQG begins with a Hamiltonian formulation of GR, and proceeds to quantize the theory by quantizing the gravitational field following an approach developed by Dirac. Dirac's procedure is the "canonical" route for quantizing classical theories.[4] In building a canonical theory, one begins by constructing the total Hamiltonian, (Gambini and Pullin 2011, p.50):

$$\mathcal{H}_T \equiv \dot{q}_i p^i - \mathcal{L} + \lambda_m \Phi^m.$$

This expression is constructed by subtracting the Lagrangian of the system from the product of the canonical positions and momenta, and then adding terms representing the constraints of the system. In canonical GR, the Lagrangian exactly cancels the contribution of the $\dot{q}_i p^i$ so that the $\mathcal{H}_T$ is nothing but the second class constraints $\lambda_j \Phi^j$ (DeWitt 1967 p.1118). In general these constraints are equations of the form:[5]

$$\lambda_j \Phi^j = 0.$$

and represent trajectories through phase space which don't affect the Hamiltonian (Gambini and Pullin 2011, p.49,96). Since the total Hamiltonian is identical to the constraints, all the information of the dynamics of the system is captured by solving the constraints (Isham 1992, p.34-35). In the case of LQG, there are three such constraints: the Gauss,

[4] For technical details on canonical quantum gravity, see Isham (1992), Henneaux and Teitelboim (1992), Rovelli (2004), Wüthrich (2006, 2014), Thiemann (2008), as well as the appendix to this paper.

[5] For a more detailed expression of theses constraints see Gambini and Pullin(2011, p.94), or Rovelli (2004 p.146, 225).

vector, and scalar constraint. In the literature, there are other names for these constraints: the Gauss constraint is often referred to as the gauge constraint, the vector constraint as the diffeomorphism constraint, and the scalar as the Hamiltonian constraint. I will always use 'Gauss' and 'Hamiltonian constraint' but will switch between 'vector' and 'diffeomorphism constraint.' The Gauss constraint requires the physical system of LQG to be invariant under an internal gauge transformation, the vector constraint requires the system to be invariant under spatial diffeomorphisms, and the scalar constraint requires the system to be invariant under a reparametrization of the time coordinate (Gambini and Pullin 2011, p.93-94, Rovelli 2004, p.146, 225). There is an industry debating whether or not these constraints require or suggest that variation across space and through time is either frozen or missing. This presumed lack of evolution is called the "problem of time" and is thought to be *the* problem in LQG.[6]

In classical mechanics, a constraint equation on phase space, $C(q,p) = b$, is upgraded in the quantum theory to the operator constraint equation: $\hat{C}\Psi(q) = b\Psi$ (Gambini and Pullin 2011, p.99). In LQG, our Hamiltonian is identical to three constraints of this form where $b = 0$ [A(§5.1)] (Gambini and Pullin 2011, p.93, Rovelli 2004, p.225). The goal of LQG is to look at the space of all functionals Ψ of our phase space variable $\mathcal{A}$ and project down onto the space of states which solve all three constraints. This space represents all the physical states of LQG. The scalar constraint is the only constraint which has not been solved.[7] It is conventional to speak of the Hilbert space of LQG in terms of the states which solve the Gauss and vector constraints, though technically the physical Hilbert space will be some subspace of this which solves all three constraints.

2.2 Spin-networks and s-knots

In the following, I will explicate the theory further by discussing first the Gauss constraint and then the vector constraint. At each stage, I will provide the naïve interpretation of the states which solve the relevant constraint(s) and will thereby unpack, in stages, the naïve-ontology. In developing the theory of LQG (2004), Rovelli implicitly endorses the naïve interpretation up through the Gauss-stage and then jettisons it when considering the vector constraint (p.238). Contrary to Rovelli, I will push the naïve interpretation through the vector-constraint-stage as a means of filling out or completing the naïve interpretation.

In order to solve the Gauss constraint, one first identifies a graph of links (lines) and

[6] It turns out that some version of the problem of time is present in any theory which utilizes the Hamiltonian version of GR. In other words, the problem of time is not a special problem for LQG (Earman 2002). For more on the problem of time see Isham (1991, 1992), Kuchař (1992), Earman (2002), Wüthrich (2014) and Norton (2015).

[7] I will make claims regarding the ontology of LQG using only those states which satisfy the first two constraints. Since the true physical states lie in the intersection of the solutions to all three constraints, solving the final constraint will not take us out of the space of solutions of the first two constraints. Once solved, the true space of physical states may suggest modifications to the ontology of LQG as described here.

nodes (points) embedded in the mathematical manifold $\mathcal{M}$. The manifold $\mathcal{M}$, in which the graphs are embedded, is the manifold of GR stripped of its metrical structure. Recall that in GR a model for spacetime is given by the pair $\langle \mathcal{M}, g \rangle$. $\mathcal{M}$ is a four-dimensional continuum of points endowed with a topology and differential structure, g is the metric field and responsible for the geometric properties of spacetime. In LQG, we explicitly quantize only the gravitational field, represented by g, and do nothing to the manifold $\mathcal{M}$.[8] It is because LQG takes $\mathcal{M}$ for granted that the naïf interprets LQG as being a quantum theory of gravity or geometry but not spacetime.

In order to incorporate the physics of general relativity into what will become LQG, we rewrite the metric g in terms of a vector potential defined by an $\mathfrak{su}(2)$-gauge field $\mathcal{A}$ [A(§5.1)] (Rovelli 2004, p.46). We transform the values of this field at each point into an SU(2)-matrix using holonomies along the links of the graph and by "coloring" each link with a representation of the SU(2) gauge group (*ergo spin-network*).[9] In effect, the colorings pick a group of matrices which act on a certain sized vector space.[10] Every link is assigned a potentially different representation, and each point along the link gets assigned a particular matrix from the representation [A(§5.1)]. The idea is that for each point along a network's link, there is an associated matrix determined by the field $\mathcal{A}$ and the color of the link. A spin-network is a graph whose links and nodes are geometrically "charged" due to the $\mathfrak{su}(2)$-gauge field defined on them. Just as in GR, where collections of spacetime points are associated with a geometry, so in LQG, graphs are associated with a quantum geometry.[11] The quantum geometry of LQG (given by the colorings) plus a graph of lines and points is a spin-network. To be clear, at this point in the discussion, I am only speaking about mathematical objects; thus, in saying that the graphs on $\mathcal{M}$ are charged, I am speaking loosely. However, in just a moment, I will translate, on behalf of the naïve interpretation, this mathematical language into a description of physical objects. We associate to each embedded spin-network, a unique gauge invariant functional of the vector potential called a spin-network state $|\mathcal{S}(\cdot)\rangle$, [A(§5.2)] (Rovelli and Peush 1998, p.233-237).[12] These states form a basis of the Hilbert space of gauge invariant functionals [A(§5.2)]. Thus, each embedded spin-network *defines* a basis vector in the gauge invariant Hilbert space:

$$\text{Spin-network} \Rightarrow |\Gamma(\vec{x}), j_n, i_m\rangle \equiv |\mathcal{S}(\cdot)\rangle. \tag{1}$$

[8] For a discussion on the appropriateness of assuming a continuum manifold in canonical quantum gravity, see Isham and Butterfield (1999). Their article also discusses more radical programs for quantizing gravity which do not assume a classical manifold.

[9] Holonomies are built out of parallel – transport maps. Amongst other things, these maps transform the elements of our algebra $\mathfrak{su}(2)$ into elements of the group SU(2) (Rovelli and Peush 1998, p.2).

[10] Each color is associated with a vector space of a different dimension.

[11] As we shall, see this assumption about GR and LQG does not hold up when we consider the diffeomorphism invariance of the theory.

[12] Varying the vector potential changes how much charge the network has; however, a variation equal to a gauge transformation does not change the functional defined on it. By varying the gauge field we vary which SU(2) group element/ matrix is associated with any point.

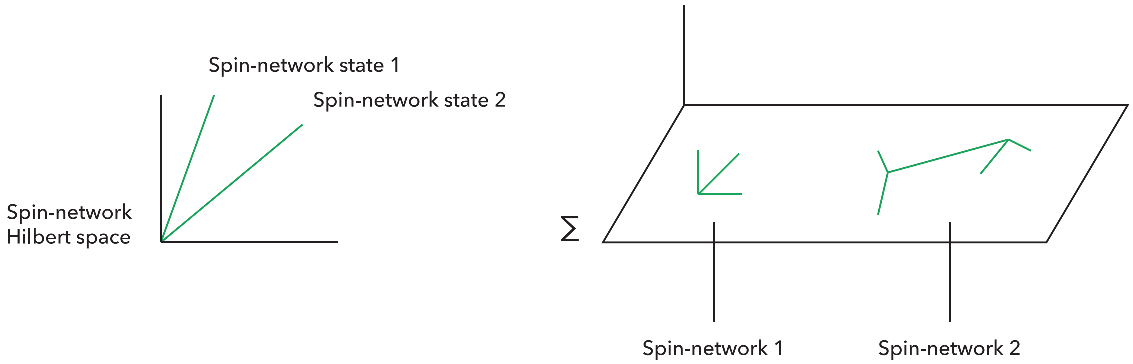


Figure 1: We "color" certain graphs in the sub-manifold Σ of $\mathcal{M}$ with quantum gravitational information. Colored graphs are called spin-networks and are used to construct the Hilbert space of spin-network states. We identify a spin-network state with each embedded network.

The j_n keep track of which links (n) have what algebraic spin information (j) and the i_m keep track of which nodes (m) have what algebraic information (i).[13] The embedded graph $\Gamma(\vec{x})$ is a geometrically contiguous series of links and nodes.

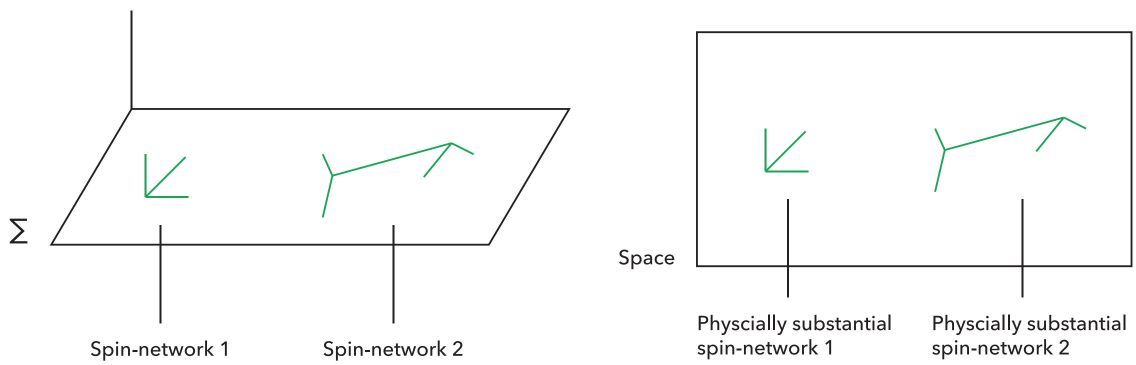


Figure 2: The naïf interprets embedded structures in Σ as literally modeling spatially embedded graphs.

I will stipulate as part of the naïve interpretation that structures on $\mathcal{M}$, which happen to be picked out by the physical states of the theory, are also to be interpreted in a fairly literal way. Consequently, since spin-networks are embedded structures in $\mathcal{M}$ and are picked out by vector states of the gauge invariant Hilbert space, the ontology of LQG, according to the naïf, includes gravitationally "charged" substantival graphs (Figure 2). These graphs are

[13] The nodes of the network are also colored. See [A(§5.2)] for why this coloring is important.

not mere mathematical objects but are composed of spacetime points which are themselves physical objects according to the naïf. These graphs are gravitationally charged since LQG represents them as having suitably quantized gravitational properties encoded by the $\mathfrak{su}(2)$ gauge field. There are times where Rovelli implicitly adopts this ontology (2004 p.147-150), even though, at the end of the day, this is not what he actually thinks the world is like given LQG (see §3.2).

Although I have not yet discussed the observables of LQG (see §2.3), it is consistent with these observables to claim that open sets of spacetime which include highly charged spin-networks, have a large volume or large area. A three-dimensional region of spacetime points which includes a highly charged node is said to have a large volume and a two-dimensional surface of spacetime points which is "cut", by a highly charged link is said to have a large area (Figure 4). The picture that one should have at this stage in the development of the naïve interpretation is a picture of charged networks embedded in the fabric of spacetime. The charge of each network is related to the matter and energy of spacetime and governs the lengths and sizes of spacetime's quantum geometry. The ontology of gravitationally charged, physically substantial graphs, which are responsible for the quantum geometry of physical regions and surfaces, is only possible at the level of the Gauss constraint. In order to solve the diffeomorphism constraint, the vector constraint, we will have to construct a new set of mathematical states as well as a new physical structure for them to represent.

A diffeomorphism can smoothly stretch and shift a network around a manifold, in this case, the three-dimensional manifold Σ. The diffeomorphism constraint requires that our physical states be invariant under this manipulation. This constraint presents a problem if our states are defined with respect to particular embeddings in $\mathcal{M}$ (or more particularly Σ). Networks which are bolted down to locations on $\mathcal{M}$ are not diffeomorphically invariant. Therefore, in implementing the diffeomorphism constraint, our mathematical states are promoted from being tied to particular spin-networks embedded at specific places to equivalence classes, under diffeomorphisms, of such networks (Rovelli and Peush 1998, p.238-242). Formally, this is achieved by mapping each diffeomorphically related spin-network state ($|\mathcal{S}\rangle$) to a specially constructed state ($\langle\mathfrak{s}|$) in its "dual" space [A(§5.2)]. In other words, build an equivalence class of diffeomorphically related states and map each of these equivalence classes to a single state in the dual of the original space:

$$[|\mathcal{S}_k\rangle] \equiv [|\Gamma^{(k)}(\vec{x})j_n, i_m\rangle] \to \langle\mathfrak{s}_{\vec{k}}|. \tag{2}$$

Where, $\langle\mathfrak{s}_{\vec{k}}|$ is a functional on spin-network states $|\mathcal{S}\rangle$ defined by:

$$\langle\mathfrak{s}_{\vec{k}}|\mathcal{S}_1\rangle \equiv \sum_{[|\mathcal{S}_k\rangle]} \langle\mathcal{S}_k|\mathcal{S}_1\rangle \tag{3}$$

$$\langle\mathfrak{s}_{\vec{k}}|(\cdot) \equiv \sum_{[|\mathcal{S}_k\rangle]} \langle\mathcal{S}_k|(\cdot). \tag{4}$$

Here $\langle \mathcal{S}_k|$ is the unique dual vector to $|\mathcal{S}_k\rangle$ such that its inner product (given by the Haar measure) with $|\mathcal{S}_k\rangle$ is one (Rovelli 2004, p.227-228). Moreover, $[|\mathcal{S}_k\rangle]$ is an equivalence class of embedded networks (with identical coloring (j_n, i_m)) such that for any a, b if $|\mathcal{S}_a\rangle$ and $|\mathcal{S}_b\rangle \in [|\mathcal{S}_i\rangle]$, then there exists a diffeomorphism Φ such that:[14]

$$\Gamma^{(a)}(\vec{x}) = \Phi(\Gamma^{(b)}(\vec{x})). \tag{5}$$

A *generic* s-knot is a superposition of s-knot states:

$$\langle \mathfrak{s}| \equiv \sum_{k=1}^{n} \langle \mathfrak{s}_{\vec{i},k}| \equiv \sum_{k=1}^{n} \langle \Gamma_k^{(i)}(\vec{x}) j_n, i_m|. \tag{6}$$

The construction in equations (3) and (4) reads as follows: take the dual "bra-vector" to each of our spin-network "ket-vectors" in the above equivalence class and identify the state $\langle \mathfrak{s}|$, with their sum [A(§5.2)].

The linear span of the $\langle \mathfrak{s}|$-states forms a Hilbert subspace in the dual space.[15] The $\langle \mathfrak{s}|$-states are both gauge and diffeomorphism invariant, and we will refer to them as s-knot *states*. A generic state in this Hilbert space is a superposition of s-knot states; though, I will refer to both kinds of states simply as "s-knot states." When it is important to distinguish these two kinds of states, s-knots and generic superpositions of them, I will do so.

There are different conventions for naming states which are both gauge and diffeomorphism invariant. Some authors use "spin-network state" to refer to any and all states even if they satisfy the diffeomorphism constraint. These authors allow the context to specify which mathematical structures are intended by the slightly ambiguous term. It is important to keep this in mind when reading quotes throughout this paper since, often, claims putatively about spin-networks states or spin-networks are really claims about s-knot states and s-knots.

In the context of the naïve interpretation, I will refer to the physical objects represented by s-knot states ($\langle \mathfrak{s}|$) as *s-knots*. When necessary to distinguish these physical structures from their graphical representation in Σ, I will refer to the physical objects represented by s-knot states as "physically substantial s-knots."[16] I will follow the same convention regarding spin-networks and physically substantial spin-networks (or just physical spin-networks). In most cases, I will allow the context to specify whether I am speaking about mathematical or physical structures.

[14] Of course, any state $|\mathcal{S}\rangle$ which is related to some $|\mathcal{S}_i\rangle$ by a diffeomorphism is, by definition, a member of $[|\mathcal{S}_k\rangle]$. Also, this account is a bit simplistic and extra care is needed since, in general, a diffeomorphism can change more than the graph Γ of a network (Rovelli 2004, p.238).

[15] Technically, we must also take the "closure of the norm" of the vector space formed by the linear span of the $\langle \mathfrak{s}|$ in order to get a Hilbert space. (Rovelli, 2004 p.229)

[16] Though, in §3.2 I will drop this association and will instead refer to the objects represented by s-knots states as simply "quantum spacetime." The reason for this change will become clear.

According to the naïf, s-knots, like spin-networks, are physically substantial networks in the physical manifold. However, the result of making s-knot states diffeomorphism invariant is that they are no longer associated with a single spin-network in $\mathcal{M}$. Since spin-networks in $\mathcal{M}$ are nailed down to locations in the manifold, we have been forced to detach our diffeomorphism invariant states from them. If s-knot states are no longer associated with a single embedded network, what physical thing in spacetime do s-knot states represent?

Here the literature becomes a bit opaque and pushes away from the naïve interpretation. As a consequence of diffeomorphism invariance, Rovelli claims that s-knots are "abstract graphs" and no longer "in space" (Rovelli 2004, p.19-2, p.283). Similarly, Wüthrich claims:

> The (abstract) spin network states result after one has solved the Gauss [gauge] and the spatial diffeomorphism constraints... These spin network states can be represented by abstract graphs. (2006, p.92)

Here, (abstract) spin-network states, according to Wüthrich, are just s-knot states (*ibid*). It is not clear what Rovelli is claiming by calling s-knots abstract, or Wüthrich by describing s-knots states as being abstract (or as being represented by abstract graphs).[17] What these authors mean by calling s-knots abstract and as failing to be in spacetime is complicated, and will take us too far afield if I were to address this issue. For the time being, I will simply note that according to Rovelli and Wüthrich, the states of LQG do not represent networks in a physical manifold, and this has something to do with the diffeomorphism constraint. The vector-stage marks the interpretive split between what will become Rovellian-LQG and the naïve interpretation. In the following, I will argue that the naïf, contrary to Rovelli and Wüthrich, can interpret s-knots as concrete structures in the physically substantial manifold.

In order to argue that it is possible that s-knot states are associated with particular and well-defined structures in $\mathcal{M}$ and thereby with particular substantival networks in the physical manifold, I will first explain how the proof that s-knot states are diffeomorphically invariant works. I will then use this proof to motivate a particular conception of what physically embedded s-knots are.

As a reminder, a generic s-knot state is defined as:

$$\langle \mathfrak{s}| \equiv \sum_{k=1}^{n} \langle \mathfrak{s}_{\vec{i},k}| \equiv \sum_{k=1}^{n} \langle \Gamma_k^{(i)}(\vec{x}) j_n, i_m|. \tag{7}$$

A diffeomorphism U_Φ on a basis vector $\langle \mathfrak{s}_k|$ is mathematically equivalent to $\langle \mathfrak{s}_k| \circ U_{\Phi^{-1}} \equiv$

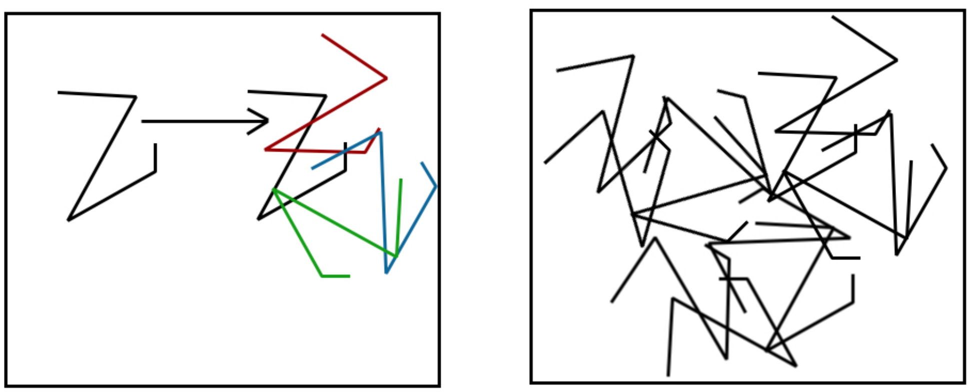
[17] Similarly, in (1994) Baez claims that the states represent a collection of loops which are "not necessarily embedded" in the spacetime manifold. In a private conversation with Baez, he (basically) endorsed the same reason as Rovelli (§3.2), for thinking of the networks as unembedded.

$$\Big(\sum_{[|\mathcal{S}_k\rangle]} \langle \mathcal{S}_k|\Big) \circ U_{\Phi^{-1}} \equiv \tag{8}$$

$$\sum_{[|U_\Phi \mathcal{S}_k\rangle]} \langle \mathcal{S}| \tag{9}$$

Where the summation is over all states $|\mathcal{S}\rangle$ related to $|U_\Phi \mathcal{S}_k\rangle$ by some diffeomorphism. Since the set of states defining the summation in (3) and (9) are the same [A(§5.2)], the states $U_\Phi\langle\mathfrak{s}|$ and $\langle\mathfrak{s}|$ are the same. The preceding proof works by shoving the diffeomorphism into the summation which defines the s-knots states. The proof proceeds by manipulating (U_Φ) each individual spin-network state ($|\mathcal{S}\rangle$) and then noting that, when all is said and done, the set of manipulated states is the same set with which we began.

Following this construction and proof, let me define a geometrically embedded s-knot to be the composite of all diffeomorphically related embedded spin-networks. Just as a single spin-network state corresponds to a single embedded network, so an s-knot state corresponds to the entire composite of diffeomorphically related spin-networks (Figure 3).



Some spin-networks related by a diffeomorphism.

An embedded s-knot, a diffeomorphically smeared spin-network.

Figure 3:

The proof that embedded s-knots, the geometric structures, are invariant under diffeomorphisms follows the proof that s-knot states are invariant under diffeomorphisms. In general, we apply a differomorphism to embedded structures by way of their algebraic descriptions. For example in order to apply a diffeomorphism to a circle, we do not apply the diffeomorphism map to the circular shape directly, but rather to its algebraic representation. In the same way, in order to apply a diffeomorphism to an s-knot, i.e. the knot of networks in Figure 3, we do so by applying the map Φ to the s-knot state. Since s-knot states are invariant under diffeomorphisms, embedded s-knots are too.

We can visualize this algebraic mapping by "cutting out" from $\mathcal{M}$ each network which comprises the s-knot, "shift" and "glue" these networks back onto $\mathcal{M}$. Since the original composite of networks contains all spin-networks related by a diffeomorphism, the result of shifting each network in the same way is to produce no overall change to the collection: each network gets mapped to the location of one of its twin networks and so on. This shifting around of networks produces the exact same configuration of embedded networks with which we began. Since there is no change to the total collection, there is no change to the geometric s-knot.

Previously, I amended the naïve interpretation from merely interpreting $\mathcal{M}$ as representing a substantival manifold to also interpreting structures defined on $\mathcal{M}$ as representing physical objects or structures in spacetime. I used this emendation to include spin-networks as objects in the ontology of naïve-LQG. If we apply the same reasoning to the case of s-knots, our conclusion will be the same. Since geometrically embedded s-knots are picked out by s-knot states, the naïf interprets these structures as representing physically embedded substantival s-knots.

Implicit in the preceding account, the naïf assumes that spacetime points have haecceities, or some other sort of primitive identity and that collections of these physically substantial points are themselves physical. According to the naïf, both spin-networks and s-knots are composed of physically substantial spacetime points where"composed" means that the basal structure of either kind of substantival network is a collection of spacetime points. Just as an aluminum baseball bat is composed of a collection of aluminum atoms, so spin-networks and s-knots are composed of a collection of physically substantial spacetime points. Whatever we mean by "bats are composed of aluminum atoms," is what the naïf means by "s-knots are composed of spacetime points." Similarly, s-knots are a *composite* of all the diffeomorphically smeared spin-networks in so far as s-knots are composed of the substantival points from these networks.

2.3 Observables

Since LQG is a quantum theory aimed to replace GR, it will have observables corresponding to the geometric structure of spacetime. Area and volume observables have been defined in such a way that both spin-network states and s-knot states are eigenvectors of them (Rovelli 2004, p.248, 262 and Rovelli and Pietri 1996, p.15). In just a moment, I will present the area operator though few of the following mathematical particulars will be required for our purposes. I present the area operator merely to highlight its dependence on certain structures in the manifold, and how our embedded spin-networks (not s-knots) are related to the operator through these structures.

$$\hat{\mathcal{A}}(\mathbf{S}) \equiv \lim_{n\to\infty} \sum_{k}^{n} \sqrt{-\left(\int_{\mathbf{S}_k^{(n)}} d\sigma^1 d\sigma^2 \epsilon_{abc} \frac{\partial x^a(\vec{\sigma})}{\partial \sigma^1} \frac{\partial x^b(\vec{\sigma})}{\partial \sigma^2} \frac{\delta}{\delta \mathcal{A}_c^i(\vec{\sigma})}\right)^2} \tag{10}$$

The way to interpret $\hat{\mathcal{A}}(\mathbf{S})$ is that we are measuring the value of some property $\hat{\mathcal{A}}$ of some spatial surface $\mathbf{S}$. The $\hat{\mathcal{A}}(\mathbf{S})$ operator is the concrete "*area*" observable of LQG. The reason for italicizing *area* and *volume* is to distinguish the operators named by them from the classical structures we normally intend. I call the *area* observable "concrete" since it is defined in terms of embedded structures in the manifold $\mathcal{M}$. In fact, the reason for including this equation is to illustrate its dependence on the manifold: the integral is defined in terms of a measure $d\sigma^1 d\sigma^2$ over an embedded surface $\mathbf{S}$. Moreover, the operator $\frac{\delta}{\delta \mathcal{A}_c^i(\vec{\sigma})}$ which acts on the states $|\mathcal{S}\rangle$ is explicitly dependent on the values of the coordinate functions $(\vec{\sigma})$ over $\mathbf{S}$.

The *area* observable acts on spin-network states and has a spectrum of *area* eigenvalues:

$$\hat{\mathcal{A}}(\mathbf{S})|\mathcal{S}\rangle \equiv \sum_{n\in\{\mathbf{S}\cap\Gamma(\vec{x})_{\mathcal{S}}\}} \sqrt{j_n(j_n+1)}|\mathcal{S}\rangle. \tag{11}$$

Embedded spin-networks carry charge (j_n) on their links and so contribute to the value of $\hat{\mathcal{A}}(\mathbf{S})$.[18] An embedded network will affect the value of $\hat{\mathcal{A}}(\mathbf{S})$ for a given surface in two ways: first, the number of its links which cross or *cut* the surface $(\{\mathbf{S}\cap\Gamma(\vec{x})_{\mathcal{S}}\})$ will change the number of things summed over in (11). And second, as we change the charge (j_n) of the links, we affect the size $(\sqrt{j_n(j_n+1)})$ of each term in the sum.[19] Thus, so long as there are no other networks in the vicinity, it is possible to increase the Riemannian area of surface $\mathbf{S}$ and yet not increase $\hat{\mathcal{A}}$. For instance, consider a single embedded network with one link which happens to cross the circular surface $\mathbf{S}$. If we had a metric, we could change the Riemannian size of the circle by doubling its radius though, since we do not change the number of times with which the surface is cut by the link, we will not increase the physical area defined by $\hat{\mathcal{A}}$. This is so, since the physical area described by $\hat{\mathcal{A}}$, is dependent only on the number of links which cross $\mathbf{S}$ and the respective charges of those links. In addition, if we keep $\mathbf{S}$ fixed but increase the charge (j_1), the area associated with $\mathbf{S}$ will increase. These results are similar to the situation in electromagnetism: to increase the electrical charge of a plate we must add more charge, not simply increase the Riemannian size of the plate. In the same way, to increase the *area* of a region, we must change our network, not the bounds of our integration (Rovelli, 2004 p.269-270). A similar situation holds true for our volume observable [A(§5.2)]: integrating over a larger region does not necessarily produce a larger *volume.*

[18] In (11) I indexed the network $\Gamma(\vec{x})$ with "$\mathcal{S}$" in order to highlight that what is summed over depends on the network $|\mathcal{S}\rangle$.

[19] Similarly, there are two ways for a network to affect the volume of a region: the number of nodes of a network in that region and the size of their charge.

The remarkable achievement of LQG and the reason for naming these observables *area* and *volume* is that they produce eigenvalues which approximate their Rimannian namesakes when acting on certain states. For instance, there are special spin-network states $|\mathcal{S}_w\rangle$ such that (Rovelli 2004, p.268):

$$\hat{\mathcal{A}}(\mathbf{S})|\mathcal{S}_w\rangle = (\mathbb{A}(g, \mathbf{S}) + O(l_p/l))|\mathcal{S}_w\rangle \tag{12}$$

$$\hat{\mathcal{V}}(R)|\mathcal{S}_w\rangle = (\mathbb{V}(g, R) + O(l_p/l))|\mathcal{S}_w\rangle. \tag{13}$$

Here $\mathbb{A}(g, \mathbf{S})$ is the Riemannian area of surface $\mathbf{S}$ given by metric g and $\mathbb{V}(g, R)$, the Riemannian volume. As we pull back from the Planck scale ($l \gg l_p$), the values of our observables approach their Riemannian counterparts. However, not all spin-network states satisfy these equations. I have just noted that it is possible to increase the Riemannian area without changing the value produced by $\hat{\mathcal{A}}$. The spin-network states which do satisfy these equations are called "weave states" and are candidates for the coherent states of LQG since they represent structures which most resemble properties of classical geometry. Colloquially speaking, the coherent states of a generic quantum theory are the states which most closely mimic the behavior of the associated classical system.

Throughout the remainder of this paper, I will no longer italicize "area" and "volume" in reference to the observables of LQG. I have made this decision in an effort to signal that if LQG is correct, physical areas and volumes are more accurately described by $\hat{\mathcal{A}}(\mathbf{S})$ and $\hat{\mathcal{V}}(R)$ than by their Riemannian counterparts.

An important prediction of LQG is that the area of surfaces and the volume of regions come in discrete Planck sized packages. This comes about because the graph of a network is modified by adding or subtracting whole numbered links or nodes to it. And since the j_n in equation (11) only takes on integer values, a network can only add discrete units of area to any given surface. Similar reasoning holds for the volume observable. Thus, the geometric observables of LQG do not relate to the manifold as their Riemannian counterparts do since these counterparts can take on a continuum of values. In fact, the important role played by the manifold is in defining which nodes are contained in which regions and which links cross which surfaces (Rovelli 2004, p.262-268). For instance, equation (11) is explicitly dependent on $n \in \{\mathbf{S} \cap \Gamma(\vec{x})_{\mathcal{S}}\}$; where n refers to particular links in the graph $\Gamma(\vec{x})_{\mathcal{S}}$.

Our observables' dependency on structures in the manifold means that they are not "Dirac observables." Since the observables of LQG act on the s-knot Hilbert space, we need them to be both gauge and diffeomorphism invariant. Unfortunately, our observables are explicitly dependent on particular surfaces $\mathbf{S}$ and regions $\mathbf{R}$ (Rovelli 2004, p.266) and thus fail to be invariant under diffeomorphisms. Rovelli has offered some suggestions for how to get around this issue,[20] and claims that once we have gotten around it, the observables will make no reference to particular regions and surfaces in the manifold and be dependent on

[20] Rovelli suggests that we use the gauge freedom of the matter fields to make the observables diffeomorphically invariant, be content with partial observables, or use evolving constants (Rovelli and Peush p.7, Rovelli 2004, p.266).

the algebraic information of the s-knot states alone (Rovelli 2004, p.262-265). This means that the spectrum of the observables, according to Rovelli, will depend only on the coloring of the links and nodes, the number of nodes, and the algebraic-graphical information of the networks (i.e. which nodes connect to which nodes), and not in any way on how the networks are situated in the manifold.

My exposition of LQG from the perspective of the naïve interpretation is almost complete; before moving onto alternative interpretations, I will first address the "non-separability problem." The following account of the problem and its solution will serve to further elaborate the structure of LQG: abstract networks and their relation to spin-networks, notions of physical equivalence in LQG, and an often undiscussed modification of the diffeomorphism constraint. The remaining portions of this section have less to do with the naïve interpretation *per se* and more to do with the structure of the theory. In addition, this discussion will provide further motivation for Rovelli's departure from the naïve interpretation.

2.4 Non-separability

Let us define two weave states to be physically equivalent just in case they yield the same eigenvalue for every observable. Assuming that Rovelli is correct and the observables of LQG rely only on the algebraic information of the states, then two spin-network weave states are physically equivalent just in case they are algebraically identical. In this section, I will argue that, as we have constructed it, LQG is artificially inflated by physically equivalent states.

Let us begin with an "algebraic" graph Γ which tells us which algebraic nodes connect to which algebraic lines. Algebraic lines and nodes are not instantiated as geometric structures in $\mathcal{M}$, they are instantiated purely algebraically. Technically the "algebraic" qualifier is not required as, in and of itself, a graph is not embedded in a manifold. A graph is merely a set of objects with a binary relation. When we embed an algebraic graph, we associate a manifold point to each object in the set and we construct a line connecting any two embedded points whose associated algebraic objects satisfy the binary relation. In common parlance, a graph usually brings to mind an embedded graph, a geometric collection of lines and points. In order to ensure that this geometric graph is not being applied to Γ, I have called it algebraic.

After selecting Γ, we construct the algebraic network $|\Gamma, j_n, i_m\rangle$ by coloring its lines and nodes, which we then embed in two distinct ways. By embedding this network in two distinct ways, we construct two distinct two spin-networks – $|\Gamma^{(1)}(\vec{x}), j_n, i_m\rangle$ and $|\Gamma^{(2)}(\vec{x}), j_n, i_m\rangle$ – from a single algebraic network. Let us assume that our algebraic network contains at least one node of "valence" four or higher (i.e. the node is connected to four or more links).

The problem is, we can embed nodes with four or more links in ways which are not related by a diffeomorphism (Rovelli and Fairbairn 2008, p.5-6). Having four linearly independent links in three-dimensional space means that there are some spatial configurations of the network which cannot be achieved with smooth transformations. This is because four linearly independent lines have one degree of freedom left unconstrained by three-dimensional diffeomorphisms. This limitation, imposed by smoothness, will lead to a non-separable Hilbert space of s-knot states (*ibid*).

Since Γ contains a node of valence four of higher, let us choose $\Gamma^{(1)}(\vec{x})$ and $\Gamma^{(2)}(\vec{x})$ so that they are not related by a diffeomorphism. Now let us impose the diffeomorphism constraint and construct our s-knot states,

$$|\mathcal{S}_1\rangle \equiv |\Gamma^{(1)}(\vec{x})j_n, i_m\rangle \to [|\Gamma^{(1)}(\vec{x})j_n, i_m\rangle] \to \langle \mathfrak{s}_1| \tag{14}$$

$$|\mathcal{S}_2\rangle \equiv |\Gamma^{(2)}(\vec{x})j_n, i_m\rangle \to [|\Gamma^{(2)}(\vec{x})j_n, i_m\rangle] \to \langle \mathfrak{s}_2|. \tag{15}$$

Since $\Gamma^{(1)}(\vec{x})$ and $\Gamma^{(2)}(\vec{x})$ are not related by a diffeomorphism, they belong to distinct equivalence classes and will be mapped to different s-knots: $\langle \mathfrak{s}_1| \neq \langle \mathfrak{s}_2|$.

Generally, it is not problematic to have physical redundancy in our Hilbert space. However, in this case, it is. It turns out that there are infinitely many, non-diffeomorphically related embeddings for any network which contains a node of valence four or higher (Rovelli and Fairbairn 2008, p.5-6). Thus, every spin-network containing a node of valence four or higher will have infinitely many physically redundant copies of itself in the s-knot Hilbert space. This artificial inflating of the s-knot Hilbert space forces the Hilbert space to be non-separable since these s-knots form a basis for our new Hilbert space. Since the new Hilbert space is non-separable, we cannot define an inner product on it which renders the space of little use. In order to solve this problem, let us first review a few things about gauge orbits.

In constraint mechanics, the constraints we generate encode symmetries of our system. If we transform the system in accordance with the constraint, we move along a "constraint-surface" or "gauge orbit" in the phase space and our Hamiltonian does not change. Thus, we can use the constraints to specify regions (i.e. the orbits) of our phase space which represent identical physical situations. Consequently, we have two distinct, though intimately related, notions of physical equivalence: first, two states are physically equivalent just in case our observables cannot distinguish them, and second, two states are physically equivalent just in case they live on a gauge orbit of the theory. In order to ensure that these notions match, we require that our observables be invariant along the gauge orbits of the theory.

Operators which are invariant along the gauge-orbits of the theory are called Dirac observables, and only they are candidates for representing physical properties of our system. Previously, I noted that, as things stand, the geometric observables of LQG are not Dirac observables since they are not invariant under diffeomorphisms. However, if we are able to upgrade our observables and define them in such a way that they are diffeomorphically invariant, then as we move along the gauge orbits associated with s-knot states (i.e. those orbits consisting of all diffeomorphically related spin networks) the observables will not vary. However, according to Rovelli, more than this is the case. Consider the set of equivalence classes of diffeomorphically related embeddings of a single algebraic spin-network: $\{[|\Gamma^{(1)}(\vec{x})j_n, i_m\rangle], ..., [|\Gamma^{(k)}(\vec{x})j_n, i_m\rangle]...\}$. It turns out that we can continuously parameterize this set using variables called moduli. According to Rovelli, if our observables are invariant under diffeomorphisms, then they will be invariant under variations of these moduli as well (Rovelli 2004, p.267). For this reason, Rovelli claims, "these moduli are an artifact of the mathematics: they have nothing to do with the physics" (*ibid*).

If the observables were not moduli-invariant, the moduli would allow the manifold to physically assert itself: some two distinct embeddings $\Gamma^{(1)}(\vec{x})$ and $\Gamma^{(2)}(\vec{x})$ of a single algebraic graph Γ, would be physically distinguished by the Dirac observables of the theory. However since the observables are invariant under variations of the moduli, the remaining remnants of the manifold are erased. It is for this reason that Rovelli claims that the observables of LQG are defined only by the algebraic properties of the states and nothing else. Thus, the true gauge orbit, as seen by the invariance of the geometric observables, is larger than the diffeomorphism-orbit and includes the moduli-orbit as well. Together these orbits cover the entire manifold $\mathcal{M}$: the observables of LQG, according to Rovelli, will not distinguish any two embeddings of an algebraic network. Thus, $\mathcal{M}$ is invisible to the physical observables of LQG.

Since the observables are invariant under diffeomorphisms as well as variations in moduli, Rovelli does not actually impose the diffeomorphism constraint in constructing the s-knot states; instead, Rovelli, and others, impose the "diff*-constraint." I will explain how this constraint solves our non-separable problem and will then explain why we are justified in using it. The diff*-constraint requires that our physical states be invariant under all spatial transformations which are smooth except at, at most, finitely many points (Rovelli 2004, p.232). This constraint is logically stronger than the normal diffeomorphism constraint since it requires that our states be invariant under a much larger class of transformations. We use the diff*-constraint to define our s-knot states using the same recipe as before (equation 2), yet the outcome is different. We begin with the gauge invariant states (the spin-network states) and build equivalence classes of diff*-related networks, and then we map each equivalence class to a single vector in the dual space.

By removing the smoothness requirement at finitely many points, we are able to avoid the trouble posed by nodes of high valence and broaden the number of networks identified in a given equivalence class. The result of imposing the diff*-constraint is that the number of s-knots shrinks to a countable cardinality (Rovelli 2004, p.267). Thus, since the basis vectors are the s-knot states, the s-knot Hilbert space is separable. Besides gaining a usable Hilbert space, this new constraint removes the physical redundancies in our s-knot Hilbert space.

We are justified in using the diff*-constraint rather than the diffeomorphism constraint, since all the states which satisfy the new constraint automatically satisfy the original constraint; as a result, no unphysical states are admitted.[21] In applying the diff*-constraint, we have simply made the requirements for being a physical state more strict. One might be concerned that in tightening the constraint, we will have squeezed out some of the physical states. This need not worry us too much since all the states from the original Hilbert space are found represented in the new Hilbert space. By applying the diff*-constraint, we have identified some old s-knot states by mapping them to a single state in the new Hilbert space. Rather than squeezing out old states, the new constraint merely groups some of them into a new equivalence class. Though the diff*-constraint shrinks the size of the Hilbert space by associating old s-knots states, it does not associate any two states which our observables were able to distinguish. The old s-knots states which end up being bundled together are those states which are physically identical from the perspective of our observables and thereby, we do not remove states which might be required for representing some physical possibility. We bundle up only those states which are representationally redundant.

This completes my exposition of the theory of LQG. At different points in the exposition, I have explicitly endorsed the naïve interpretation. Before considering other interpretations, with competing ontologies, recall what the world is like given the naïve interpretation: spin-networks and s-knots live on a physical manifold and carry gravitational charge along their links and nodes. The more charge a network has, the more volume it produces. The networks of LQG build spacetime geometry one region at a time as geometry "radiates" from them (Figure 4). Since there is a lower bound to how much area and volume a physical network can carry, spacetime can only be geometrically-parsed up to a certain scale – below which, no geometry is defined.

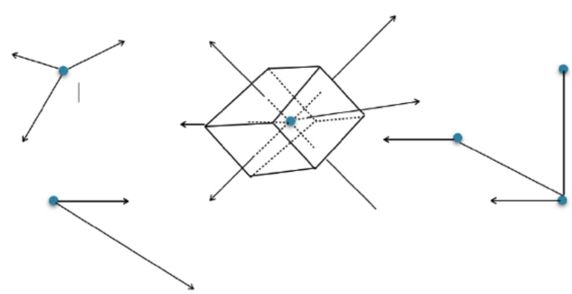


Figure 4: A series of networks: gravitationally charged links and nodes. Each node defines a volume of space and each link, an area.

The ontology of this interpretation is not so different from an equally naïve interpretation of GR. In GR, spacetime is described by $\langle \mathcal{M}, g \rangle$ which we can interpret naïvely as describing a physically substantial manifold bearing a physical geometry. In moving to naïve-LQG, we keep the physically substantial manifold but replace the physical geometry,

[21] Unphysical as determined by our original constraint.

associated with the gravitational field and represented by g, with a quantum geometry, produced by charged networks and represented by $\langle\mathfrak{s}|$.

By identifying s-knot states with diffeomorphically-smeared spin-networks, we can explain how area and volume come to be associated with diverse regions of the physical manifold. The issue with spin-network states is that each state is associated with a single physical network, and this network is nailed down to specific regions of the physical manifold. If our physical network has a few nodes, then the physical manifold will have only certain regions for which there are physical volume and area. It would be a strange world we lived in if only some parts of spacetime had areas and volumes. It is important to remember that each single state of our Hilbert space is supposed to represent the quantum geometry of all of spacetime, and not merely some part of it. By smearing the network over the entire manifold, s-knots are capable of producing areas and volumes across the spacetime manifold.

Enough about the naïve interpretation. Surely, according to the critic, $\mathcal{M}$ cannot represent spacetime on its own? If $\mathcal{M}$ does not represent spacetime, then, as we shall see, spacetime might disappear in LQG.

3 Spacetime disappears

In this section, I will provide seven additional interpretations of LQG, most of which do not include spacetime in the ontology of LQG. Five of the seven differ from one another and from the naïve interpretation merely in what they take spacetime to be. I call four of these interpretations 'naïve*', and the fifth I call 'Rovellian.' The final two interpretations are different insofar as they explicitly or implicitly require that we formally modify LQG. The sixth interpretation I call 'trickle-down' and the seventh 'TaG.' The following analysis will center around whether or not some interpretations have spacetime and physical structures called s-knots in their ontology. These are the two ontological questions which are at the heart of this paper. Only by first understanding how and under which interpretations there are s-knots and not spacetime fundamentally, can we analyze whether or not spacetime is emergent from or composed of s-knots.

3.1 Naïve*-LQG

In the following, I will amend the naïve interpretation to thicken the notion of spacetime from being a structure literally modeled by the bare manifold $\mathcal{M}$ with no physical geometric structure, to being a structure with some kind of geometry or quantum geometry. Unlike the original naïve interpretation, I will show that, according to some of the naïve* interpretations, there is no spacetime in LQG. According to some of these interpretations, there are physically substantial s-knots which are ontologically distinct from spacetime, while in other interpretations there aren't. The motivation for thickening our notion of spacetime to include something like a metrical structure or physical geometry is that, without it, “spacetime” lacks features such as causes, spatial lengths, and durations of time which seem to be constitutive of spacetime. Allow me to explain.

First, were spacetime to be bare and lack the physical structure encoded by g, which I will often refer to as either ‘a physical metrical structure’ or ‘a physical geometry,’ then spacetime would lack segments of spatial length and durations of time. Presumably, that a bare physical manifold lacks spatial lengths and durations of time, is sufficient reason to doubt that a bare physical manifold is spacetime at all. (In the following, I will say ‘bare manifold.’) However, in case one needs more convincing, I will briefly show how certain kinds of causes as well as other physical facts go missing when we treat $\mathcal{M}$, absent g, as representing spacetime. The following considerations are not exactly new and versions of these ideas can be found in Lam and Esfeld (2013) as well as Isham and Butterfield (2011).

According to general relativity, causes are associated with either light-like or time-like trajectories. This requirement forces all causal processes to stay within the light cone of the putative cause. However, since light cones are defined using the metric g, without g, $\mathcal{M}$ does not have a light cone structure. Without a light cone structure on $\mathcal{M}$, we cannot define such causal processes. In the context of LQG, one might not be concerned that a bare manifold lacks GR-causes since, in the context of LQG, GR is no longer considered a fundamental theory. Once out from under the thumb of GR, we may try to formulate a theory of causation which is consistent with a bare manifold.

The trouble in trying to build an account of causation consistent with a bare manifold is that without a physical metrical structure it’s unclear that spacetime contains a very robust sense of change. And without change, in what sense are there causes? For example, consider the broken window: yesterday the window was large, rectangular, and near the book case, today the window lies in a pile of irregularly shaped small glass shards much further from the book case. Standardly, the change in the state of the window would signal some sort of cause, a baseball perhaps, responsible for the change. However, in a bare manifold, since there are no lengths, there are no rectangles, no irregular shapes, no small, no near, no far. Without the physical geometry encoded by g, the sense of change which the world includes is incredibly reduced. Consequently, the kind and number of causes are also reduced. Presumably, that causes are hard to pin down in a bare manifold is actually not so much the issue as it is symptomatic of the fact that so much else has already gone missing: length, duration, shape, size, speed, momentum, force, many kinds of energy, much physical variation and most other physical features which are dependent, in some way, on geometry. Given that so much physical structure is missing in a bare manifold, one begins to doubt that $\mathcal{M}$ is sufficiently rich to model spacetime on its own. I do not claim that the preceding considerations are sufficient reason for rejecting the naïve interpretation but raise these concerns only as motivation for looking beyond it.

Consequently, perhaps spacetime is better modeled by $\langle \mathcal{M}, g \rangle$ or perhaps even by $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$. The latter option should be read as claiming that spacetime is modeled by a topological manifold bearing the quantum geometry of the s-knot named by $\langle \mathfrak{s}|$. In the following, I will modify the naïve interpretation in four ways in order to accommodate these two options. As I mentioned at the start of this paper, these interpretations will be less naïve, though still substantival. Finally, I do not claim that these four interpretations are the only possible elaborations of the naïve position. These interpretations simply provide us with an interesting series of vantage points with which to interpret the mathematics of LQG. As a warning, since I will be cycling through interpretations, I will also be cycling through what 'spacetime' means, or how spacetime is represented by the interpretation. For this reason, it is important to keep in mind which interpretation is being discussed. The following naïve* interpretations are named naïve$_i$ ($i \in \{1, 2, 3, 4\}$).

According to the naïve$_1$ interpretation, spacetime is the composition of a substantival manifold bearing a physical quantum geometry which we represent by the ordered pair $\langle \mathcal{M}, g \rangle$. By defining spacetime as the composition of two things, I am implicitly treating these things as representing physical objects or structures in their own right. However, one need not adopt this position. Rather than treating $\mathcal{M}$ and g as representing physical things which combine to form spacetime, one might instead understand spacetime to be a "simple," non-composite object represented by $\langle \mathcal{M}, g \rangle$. By calling spacetime simple, I do not mean to suggest that spacetime fails to have proper parts in terms of spacetime regions or points, but rather that spacetime fails to have proper parts in terms of structures represented by $\mathcal{M}$ and g. Thus, contrary to naïve$_1$, according to what I will call the naïve$_2$ interpretation, $\langle \mathcal{M}, g \rangle$ represents a simple spacetime in so far as neither $\mathcal{M}$ nor g alone represent anything physical.[22] To be clear then, by definition, according to the naïve$_2$ interpretation, g does not represent the physical geometry *of* spacetime and $\mathcal{M}$ does not represent the physical "basal" structure *of* spacetime. But rather, $\langle \mathcal{M}, g \rangle$, as a unit, represents a geometric-basal structure which we call spacetime. In the following, I will unpack how the naïf, of either variety, might update their belief regarding spacetime in light of LQG.

If spacetime is as according to naïve$_1$, since there is no metrical structure described by LQG, there is no spacetime according to naïve$_1$-LQG. It is likely that the naïf of this variety will interpret the structure $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$ as representing a composite quantum spacetime: $\mathcal{M}$ represents a substantival manifold and $\langle \mathfrak{s}|$ represents a physically substantial network responsible for the quantum geometry of quantum spacetime.

If spacetime is as according to naïve$_2$, since there is no metrical structure describe by LQG, there is no spacetime according to naïve$_2$-LQG. However, since it is likely that the naïf$_2$ will interpret the structure $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$ as representing a simple quantum spacetime, then there is only quantum spacetime and not also physically substantial networks represented by $\langle \mathfrak{s}|$. By definition of what 'simple' means, in this context, it's not the case that $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$ represents a simple structure and $\langle \mathfrak{s}|$ also represents a physical structure on its own.

The general motivation for the following two interpretations is the conviction that if the physical model of LQG includes a four-dimensional basal manifold, then the physical

[22] In different settings, $\mathcal{M}$ and g can represent whatever we want them to. The point here is simply that if we interpret $\langle \mathcal{M}, g \rangle$ as representing a simple structure, then these very same mathematical structures $\mathcal{M}$ and g, in the context of LQG, cannot also represent distinct physical things.

structure being represented is enough like a "container" for the substantivalist to think that a spacetime lives on in LQG. According to substantivalists of this ilk, a spacetime is defined to be that structure in which all physical objects exist, and that which allows objects to both have "geometric" extension and to be "geometrically" related to one another. (Where 'geometry' refers to those physical properties modeled by either GR or LQG.) Substantivalists, of the preceding variety, interpret $\langle \mathcal{M}, g \rangle$ as representing a pseudo-Riemannian spacetime and interpret $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$ as representing a "quantum-Riemannian" spacetime.

According to the naïve$_3$ interpretation, spacetime is the composite of a substantival manifold bearing a quantum geometry which we represent by the ordered pair $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$. According to this interpretation, spacetime, for obvious reasons, does not disappear in LQG. Moreover, since spacetime is a composite, both $\mathcal{M}$ and $\langle \mathfrak{s}|$ represent physical things: there is a substantival manifold with embedded networks which are responsible for the quantum geometry of spacetime. The only significant difference between the naïve$_1$ and naïve$_3$ interpretations is whether or not we require spacetime to be classical. Historically, interpreters of quantum theory have usually opted for versions of the naïve$_{3,4}$-route. For instance, one usually does not interpret the result of quantizing the electromagnetic field as having removed the field from our physical theory. But rather, the usual interpretation is that the physical field has simply gained its proper quantum description. This is the spirt behind the naïve$_{3,4}$ interpretations: spacetime does not disappear in LQG, it just gains its proper quantum description. The difference between the naïve$_3$ and the naïve$_4$ interpretations lies merely in whether or not we take spacetime to be simple or composite in the sense previously discussed.

According to the naïve$_4$ interpretation, spacetime is a simple, non-composite structure represented by $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$. In this case, as previously discussed, since spacetime is simple, neither $\mathcal{M}$ nor $\langle \mathfrak{s}|$ represent anything physical on their own. In particular, the states $\langle \mathfrak{s}|$ do not represent physical things called s-knots but rather these states simply provide some of the requisite mathematical structure for representing the quantum geometric relations between physical structures. As a unit, $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$ represents spacetime according to naïve$_4$-LQG, just as $\langle \mathcal{M}, g \rangle$, as a unit, represents spacetime according to naïve$_2$-LQG. Importantly, according to both the naïf$_2$ and the naïf$_4$, there are no physical structures represented by the states $\langle \mathfrak{s}|$, but only (quantum) spacetime represented by $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$. If we want to think about physical networks which exist somewhat independently of (quantum) spacetime, we need opt for the original naïve interpretation or the naïve$_1$ and naïve$_3$ interpretations. Because the preceding claim is essential for understanding what might be the ontology of LQG, I will repeat the claim in a slightly different manner. If we think that there are physical things called spin-networks (s-knots) and also some other physical thing called spacetime or quantum-spacetime, then we must not interpret $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$ as representing a simple, non-composite structure.

One might object that, despite my claims to the contrary, the states $\langle \mathfrak{s}|$ can play dual roles in the above "simple" interpretations. According to this objection, $\langle \mathfrak{s}|$ partakes in representing the simple structure $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$ and also represents physically substantial s-knots in the physical manifold. However, if $\langle\mathfrak{s}|$ picks out physically substantial networks, then it seems as though the simple structure $\langle\mathcal{M}, \langle\mathfrak{s}|\rangle$ can be conceived to have parts: one of those parts being the physically substantial networks represented by $\langle\mathfrak{s}|$. There might be a subtle way to conceive of $\langle\mathcal{M}, \langle\mathfrak{s}|\rangle$ as being simple eventhough $\langle\mathfrak{s}|$ also represents something on its own; however, since I am unable express this possibility without equivocating or merely insisting that it is the case, I will not attempt to develop this interpretation.

I have introduced the simple interpretations as a means of highlighting the dependence of physically substantial s-knots on a composite interpretation of (quantum) spacetime. In order for there to be physical networks distinct from (quantum) spacetime itself which somehow partake in structuring (quantum) spacetime, we need to conceive of (quantum) spacetime as composite.

All told, there are four less-naïve interpretations of spacetime and these possibilities map the four possible answers to questions 1. and 2. from §1: does spacetime disappear in LQG, and are there s-knots in the ontology of LQG? In summary:

- According to the original naïve interpretation, the manifold $\mathcal{M}$, devoid of any metrical structure, represents substantival spacetime. Spacetime does not disappear in LQG, and there are physically substantial s-knots. Spacetime on this view does not require any particular physical geometry.

- According to the naïve$_1$ interpretation, $\mathcal{M}$, devoid of any metrical structure, represents a substantial pre-spatiotemporal manifold. Since substantival spacetime, on this view, is represented by $\langle\mathcal{M}, g\rangle$, spacetime disappears in LQG. It's reasonable to suppose that a naif of this variety will endorse a composite interpretation of $\langle\mathcal{M}, \langle\mathfrak{s}|\rangle$ as describing quantum spacetime. Spacetime, within this view, requires a classical physical geometry, and quantum spacetime requires a quantum geometry. In any case, since quantum spacetime is composite, there are physically substantial s-knots represented by $\langle\mathfrak{s}|$ which are embedded in the substantival manifold represented by $\mathcal{M}$.

- According to the naïve$_2$ interpretation, substantival spacetime is a non-composite structure represented by $\langle\mathcal{M}, g\rangle$ and fails to be described in LQG. It's reasonable to suppose that a naif of this variety will endorse a non-composite interpretation of $\langle\mathcal{M}, \langle\mathfrak{s}|\rangle$ as describing a substantival quantum spacetime. Since quantum spacetime is simple, in the manner described, quantum spacetime does not have physical parts represented by either $\mathcal{M}$ or $\langle\mathfrak{s}|$.

- According to the naïve$_3$ interpretation, substantival spacetime is a composite structure represented by $\langle\mathcal{M}, \langle\mathfrak{s}|\rangle$. Since spacetime is composite, the states $\langle\mathfrak{s}|$ represent physical networks embedded in the substantival manifold represented by $\mathcal{M}$.

- According to the naïve$_4$ interpretation, substantival spacetime is a non-composite structure represented by $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$ and, as such, spacetime exists in LQG, though substantival s-knots do not.

The theory of LQG neither entails that spacetime disappears, nor that there are physical networks. Whether or not there are such things depends on our interpretation of the theory and, in particular, our interpretation of $\mathcal{M}$ and its relation to spacetime. In order to keep this discussion of ontology from degenerating into a verbal debate, we must specify ahead of time what we take spacetime to be. The philosophical import of this discussion has nothing to do with which mathematical structures get to be labeled 'spacetime,' but it has everything to do with what physical structures we take to be essential for spacetime. If we understand spacetime to be essentially that physical structure described by either the naïve$_1$ or naïve$_2$ interpretations, then, if LQG is true, there is no spacetime fundamentally. However, if we take physical spacetime to be, more generically, that substantival structure in the world partially responsible for some of our "geometrical" experiences then, if LQG is true, we will naturally update our beliefs about that structure and adopt something like the naïve, naïve$_3$, or naïve$_4$ descriptions of spacetime.

Though spacetime might disappear in LQG, it disappears in the same sense that the classical electromagnetic field disappears in quantum field theory. In place of spacetime *qua* $\langle \mathcal{M}, g \rangle$, LQG provides a structure described by $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$. The difference between these two structures is only in the geometric predictions derived from them. In particular, the quantum geometry predicted by LQG and contrary to GR, is discrete and suitably fuzzy. The geometry is fuzzy in two senses: first, as will be explained in §3.2, the geometric shapes associated with weave states will never be sharply Riemannian. Second, since a generic s-knot state is not a weave state, rather than describing areas and volumes which look Riemannian in the classical regime, a generic s-knot state describes a superposition of areas and volumes. How these generic s-knot states come to be associated with the phenomenological world is the big question underlying all instances of the measurement problem. If having a fuzzy quantum-geometric structure entails that spacetime has gone missing in LQG, then so be it; however, one must not think that the disappearance of spacetime, in this sense, is any stranger than the disappearance of any other classical structure when adopting a quantum theory.

One might object that, by proposing the naïve$_3$ and naïve$_4$ interpretations, I am not taking seriously enough the fuzzy physical geometry described by $\langle \mathfrak{s}|$. In essence, this is the same concern which I considered when discussing what goes missing were we to treat spacetime as being modeled by $\mathcal{M}$ alone. There, I argued that since much of what we take spacetime to be goes missing if spacetime were bare, it would not be spacetime after all. For similar reasons, perhaps the structure described by $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$ is not very much like spacetime after all. For example, since a generic s-knot state describes a quantum superposition of geometric structure, spatial lengths are generically described as a superposition of (roughly) classical lengths, Moreover, a similar description applies to durations of time, speed, momentum, electric flux and everything else which we have assumed to rely on the physical geometry of spacetime.[23] In short, the quantum fuzziness described by generic s-knot states leaks into the rest of the world. Can this fuzzy structure really be spacetime? This question is not whether or not spacetime can be recovered from this fuzzy structure, but whether or not spacetime should be identified with it. Is spacetime fuzzy? There comes a point when so much has been lost from what we take spacetime to be, or how we expect physical objects to relate to spacetime, that we must let go of the concept altogether, or so the objection goes. In general, I am sympathetic to this objection, though I will note one important caveat. As discussed above, a substantivalist might regard the fact that $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$ can be interpreted substantivally, as evidence that the spirit of spacetime lives on in LQG. In other words, a substantivalist might require that for spacetime to disappear, we need the substantival "container" to disappear. This concern will be addressed to some extent in §3.3. Before moving beyond the naïve* interpretations, I need to clarify one further point.

According to the original naïve interpretation, the vector states of LQG represent physically substantival s-knots. However, since there are no physically substantial s-knots according the naïve$_2$ and naïve$_4$ interpretations, what do the states of the theory represent? According to these interpretation, (quantum) spacetime is non-composite and modeled by the ordered pair $\langle \mathcal{M}, \langle \mathfrak{s}| \rangle$; consequently, the states of LQG describe different configurations of spacetime itself. In general, for interpretations which do not admit s-knots in their ontology, the states of LQG describe either different configurations of spacetime or of quantum spacetime. In order to streamline the following discussion, I will choose the latter locution.

The three interpretations to which I will now turn, diverge much more radically from the naïve interpretation than do the naïve* interpretations. According to the Rovellian interpretation, the manifold $\mathcal{M}$ is a mathematical artifact and does not encode any physical information; spacetime, according to this interpretation, is only the gravitational field. According to TaG interpretations, $\mathcal{M}$ does encode physical information relevant for spacetime but, as a result, we ought to provide a quantum description for it as well. TaG-LQG diverges from the naïve interpretations in providing, or attempting to provide, a quantization of the spacetime manifold by way of its treatment of $\mathcal{M}$. Similarly, according to trickle-down interpretations, $\mathcal{M}$ encodes physical information relevant for spacetime. However, unlike TaG, according to trickle-down interpretations, the manifold is thought to be automatically quantized under the standard formulation of LQG. Trickle-down interpretations diverge from naïve interpretations insofar as they interpret the effects of quantizing the metric field as trickling down and affecting the base manifold $\mathcal{M}$.[24] Both TaG and trickle-down interpretations are far more programmatic and less well understood than either the

[23] Similarly, Isham and Butterfield note that by replacing g with a suitably quantized alternative, the quantum geometry associated with (quantum) spacetime will not include a stable light cone structure but a superposition of light cones/ causal structures (2001, p. 54, 64).

[24] Technically, the metric g is quantized in canonical quantum gravity and the vector potential $\mathcal{A}$ is quantized in LQG. The difference between these approaches is mostly mathematical since the metric g can be written in terms of "tetrad fields" which are defined by $\mathcal{A}$.

naïve or Rovellian approaches to LQG. As a result, my account of these interpretations will be proportionally less complete.

3.2 Rovellian LQG

The following interpretation is largely inspired by the words and works of Carlo Rovelli; however, I do not claim that the views expressed here are exactly his own. Thus, this interpretation is Rovellian, though perhaps not Rovelli's. According to the Rovellian interpretation, the diffeomorphism freedom found in GR is evidence that $\mathcal{M}$ is a gauge artifact and does not represent a physically substantial manifold (Rovelli 1997, 2004). In fact, this rejection of the substantival manifold is often how diffeomorphisms are employed by those wielding Einstein's infamous hole argument.

Importantly, the diffeomorphism invariance of GR is found recapitulated in the theory of LQG in the form of the scalar and vector constraints. As explained in §2.1, these constraints require that our states be constant in time and across variations of space. However, it turns out, that even more than this is the case: as we saw in §2.4, the observables of the theory are also moduli-invariant. This is important since if the observables were not, they would treat two different embeddings of a single algebraic network differently.[25] If these two embeddings produced physically distinct effects (e.g. if the geometry they produced was distinct), then the manifold would show itself in a physically salient way. However, that is not true; according to Rovelli, two different embeddings of the same algebraic graph produce the same geometry. Thus, since the manifold is invisible to the observables of LQG, Rovelli concludes:

> In fact $\mathcal{M}$ (the spacetime manifold) has no physical interpretation, it is just a mathematical device, a gauge artifact... There are not spacetime points at all. The Newtonian notions of space and time have disappeared... the spacetime coordinates $\vec{x}$ and t have no physical meaning...
> (2004, p.74)
>
> What Newton called "space," and Minkowski called "spacetime," is unmasked: it is nothing but a dynamical object – the gravitational field...
> ...the gravitational field is the same entity as spacetime.
> (2004, p.9, 18)

According to Rovelli, the diffeomorphism invariance of GR and LQG imply that $\mathcal{M}$ plays no role in determining values of our physical observables, and Rovelli concludes that $\mathcal{M}$ is a gauge artifact and ought not be reified.[26] However, just because $\mathcal{M}$ does not play a

[25] Assuming that the network includes a node of sufficiently high valence.

[26] It is unclear how literal we should interpret Rovelli's repudiation of $\mathcal{M}$ as bearing any physical salience. It seems that, at minimum, the global topology of "space" Σ has bearing on what our experiences of the

role in determining what physical values are observed does not require that we treat $\mathcal{M}$ as being a mathematical artifact. How to treat gauge orbits is a thorny philosophical issue, and it is far from settled that all such orbits ought to be taken non-realistically.[27] Rovelli himself recognizes this, in “Halfway Through the Woods” (1997) he acknowledges that though LQG and GR are manifold-invariant, one might still insist that there is a physical background manifold which happens to be unobservable. Though Rovelli acknowledges that the existence of an unobservable manifold is logically possible, it it not the position he endorses. According to the Rovellian interpretation presented here and assumed in Rovelli (2004), there is no physical manifold represented by $\mathcal{M}$.

Since, according to Rovelli, spacetime is just the gravitational field, in quantizing the gravitational field, we quantize spacetime itself (Rovelli 2004, p.17). Since there is no classical gravitational field in LQG, there is no classical spacetime either. However, since the weave states reproduce classical geometry at classical scales, equations (12) and (13), there is a sense in which spacetime arises or is recovered from the quantum phenomena of LQG. When ($l \gg l_p$) and the quantum geometry, or quantum spacetime, is described by a weave state of the theory, the world looks classical. In these cases, we can use GR and g to model some features of the world. In this way, we might say that spacetime is recovered from LQG in the classical regime. However, we must not interpret claims to the effect that spacetime is recovered, in this sense, as necessitating that spacetime, as a new item of ontology, arises in the classical regime. When ‘recovered’ is understood in this way, all that is required is that spacetime, the classical gravitational field, be an effective structure. One might endorse the additional view according to which effective structures are genuine objects of our ontology and distinct in kind from whatever happens to be fundamental. Or, one might view effective structures as merely useful fictions. My argument here is only to note that under Rovellian-LQG there is a sense in which spacetime is recovered from LQG where ‘recovered’ does not necessitate that there actually be such objects.

Much of what I have just said regarding Rovelli’s position can be modified and applied to some of the naïve* interpretations. In particular, according to both naïve$_1$-LQG and naïve$_2$-LQG, classical spacetime can be recovered in exactly the same way as it is for Rovelli: when the states $\langle \mathfrak{s}|$ are weave states, the physical geometry of LQG can be effectively modeled by $\langle \mathcal{M}, g\rangle$ when ($l \gg l_p$). In this way, classical spacetime can be recovered from LQG according to these interpretations. Or more perspicuously, if our naïve interpretation includes physically substantial s-knots, as in naïve$_1$-LQG, classical spacetime is recovered when the physically substantial s-knots come to be described by the weave states of the theory.

Though there is neither a spacetime manifold nor spacetime geometry according to Rovellian-LQG, only the latter is a result of the quantum theory. The spacetime manifold

world is like. For instance, if Σ is compact (i.e. has a dimension which is rolled up like a three-dimensional cylinder), then our theory of (quantum) spacetime should predict that we could travel a finite distance in one direction and come back to where we started.

[27] See, Healey (2009) for a discussion of the issues involved in interpreting gauge variables and orbits.

disappears in Rovellian-LQG for the same reason that it disappears in Rovellian-GR: in one way or another, each theory is diffeomorphically invariant.

Given that the world does not include a physically substantial manifold, according to Rovellian-LQG, the world also does not include physically substantial networks, at least as they have defined throughout this essay. Physically embedded spin-networks and s-knots are physically substantial insofar as they are comprised of points from the physical manifold.[28] This argument: that there are no physically realistic networks since there is no physical manifold, is not an argument which Rovelli makes. However, I will assume that, for Rovelli, there are no physically substantial networks, and I will interpret the following statement, as Rovelli saying as much:

> Such geometrical pictures [of geometric networks] are helps for the intuition, but there is no microscopic geometry at the Planck scale and these pictures should not be taken too literally in my opinion. (Rovelli 2011, p.4)

Additionally Rovelli (2004, p.268-269), describes spacetime like a shirt which, when approached, reveals an underlying weave of threads. However, Rovelli cautions against taking these weaves "as a realistic proposal for the microstates of a given macroscopic geometry [spacetime]" (p.269). Indeed, for Rovelli, there is no shirt since there is no basal structure, there are only "fields on fields" (2004, p.9) or rather fields 'on' fields.[29] I interpret these quotes from Rovelli as cautioning us against naïvely reifying networks in $\mathcal{M}$. The world does not really contain gravitationally charged points and lines which are responsible for the macroscopic geometry of the world. In fact, I take it that this is the reason why Rovelli refers to s-knots as being "abstract" (§2.2).

According to Rovelli, the physically salient aspects of s-knots are algebraic and independent of the manifold altogether. Given their independence of the manifold, we might as well associate s-knots not with $[|\Gamma(\vec{x}), j_n, i_m\rangle]$, but with $[|\Gamma, j_n, i_m\rangle]$ which I have been calling algebraic networks. I take it then that when Rovelli refers to s-knots as being abstract and not in space, that he intends to signal two distinct things: (1) there is no spacetime as represented by $\mathcal{M}$, and (2) the physically salient mathematical structures of LQG are algebraic, not geometric.

A note of caution: I am using 'geometry' in at least two distinct ways. There is the physical geometry given by specifying a metric or an s-knot state, and then there is the geometry of points and lines which we use in constructing the networks in $\mathcal{M}$. In saying that the networks are not to be taken in a geometrically-literal way, the Rovellian is only cautioning against reifying the points and lines of the network. For instance, consider the two levels of geometry contained in Figure 4 (§2.4). This figure contains the geometric

[28] By calling the points, 'spacetime points,' I do not thereby claim that the bare physical manifold is spacetime.

[29] Rovelli does not explain what fields, *qua* physical objects, are. Are fields extended objects, are they composed of field-points and if so, in which ways are physical fields different from a substantival manifold?

graph of the spin-network, as well as the geometry of the cubic volume and square areas. According to the Rovellian, only the volume and areas associated with the network are to be taken seriously. Neither the lines and points of the graph, nor the shapes denoting the volume and areas, are to be interpreted "ontologically." In particular, that the volume is represented as a cube and that the areas are represented as square, are artifacts of the image. I do not have space in this article to discuss these ideas fully, but, in general, the information provided by a network is not enough to fully specify all the angles between adjacent surfaces which define a volume or area. Thus, shapes associated with the networks of LQG have quantum indeterminacy built into them. Since some of the angles are left undetermined, the shape is fuzzy and cannot be represented as being euclidean, as I have done in Figure 4. For more on these issues see Rovelli and Vidotto (2015).

If geometric spin-networks and s-knots are like the manifold $\mathcal{M}$ and fail to refer to substantival links and nodes in the world, what role do they play? According to the Rovellian interpretation, spin-networks and s-knots are mathematical tools useful for encoding the properties of quantized spacetime. According to this view, state-vectors which live in our Hilbert space, networks which live on $\mathcal{M}$, and Γ (networks in some algebraic space), are all merely mathematical structures which encode the geometric properties of quantum spacetime. Both the vector states of LQG and the embedded structures, represent quantum geometric properties of quantum spacetime in terms of the algebraic information they contain. Consequently, no part of a geometric network, not an isolated point or line, is to be taken as physically salient on its own. The network, as a whole, is physically salient insofar as it maps to a vector state in the Hilbert space of LQG.[30]

By repudiating the substantival manifold and by quantizing the gravitational field, Rovellian-LQG is a form of relational quantum spacetime. However, since Rovelli does not say much regarding his relationism (2004, p.77-79), I wilI refrain (mostly) from associating any particular flavor of relationism to Rovellian-LQG.[31] Rather than associating a particular flavor of relationism to Rovellian-LQG, I will leave this particular feature open and will stipulate that however $\langle \mathcal{M}, g\rangle$ is interpreted as describing a relational spacetime in GR, that we import this interpretational stance to LQG and regard $\langle \mathcal{M}, \langle \mathfrak{s}|\rangle$ as describing relational quantum spacetime.[32] By leaving an interpretational variable open, Rovellian-LQG

[30] Caveat: some sub-networks can be treated as being physically meaningful, but only because if we were to extract them from their network, they too would have a copy of themselves in the Hilbert space of LQG. Such networks are not physically meaningful as proper parts of a network, but are physically meaningful as extracted networks in their own right.

[31] See Earman (1989) for different flavors of substantivalism and relationism. Moreover, since it's possible to deny substantivalism and not be a relationalist (Earman, 1979) and since Rovelli has not provided a worked-out metaphysics of (quantum) spatio-temporal relations and how they constitute quantum spacetime, perhaps Rovelli's position is more "non-substantival" than relational.

[32] Some kinds of relationist accounts of GR might also require there to be matter fields before there is spacetime. For instance, if one interprets g as only encoding the spatio-temporal relations between material objects, then, without material objects, all we have is a relation and presumably no spacetime. However, for Rovelli, g encodes more than spatio-temporal relations, g is a physical field in its own right

is not a single relational interpretation of quantum spacetime, but a schema for generating various relational interpretations. Depending on how the relationism is fleshed out, the Rovellian will interpret different s-knot states as representing different relational quantum spacetimes.

One might object that I have omitted an important interpretation of LQG and of s-knot states in particular. Under naïve-LQG and certain version of naïve*-LQG, s-knot states represent physically substantival networks in a physically substantial manifold. One might wonder why I have not considered the analogous relationist interpretation whereby s-knot states represent physically relational networks in a relational spacetime? The reason I have not discussed this interpretive option is that I am not convinced that the suggestion is coherent. In particular, I am not convinced that on a relationist account, there can be a distinction between what the states represent: (a) relational quantum spacetime and (b) some other things called "relational networks." In the case of Rovellian-relationism, what the states represent is relational spacetime and not some conceptually distinct middle man.

The reason that the naïve, naïve$_1$, and naïve$_3$-interpretations coherently distinguish between what s-knot states represent (physically substantial networks) and a physically substantial spacetime, is because they interpret spacetime as a composite and interpret $\mathcal{M}$ substantively. In the case of naïve-LQG, the substantival structure is spacetime and according to naïve$_1$ and naïve$_3$-LQG, $\mathcal{M}$ represents a pre-spatio-temporal substantival manifold. In any case, that there is a physically substantial structure independent of the s-knot states, is what creates the conceptual space for physically substantial s-knots. In the case of relational space-time, what plays the analogous role to the substantival interpretation of $\mathcal{M}$? Or, in other words, if s-knot states do represent physically-relational s-knots, what is relational spacetime and what represents it in our mathematical model?[33] I cannot rule out the possibility that there is a coherent interpretation of LQG whereby there are physically relational networks which live in a relational spacetime, but due to my inability to formulate a coherent instance of such an interpretation, I will not consider it. In the following, I will continue to assume that if there are physical networks distinct from spacetime or quantum spacetime, that these networks are physically substantival.

3.3 Manifold quantization

In this section, I will address both the TaG (topology and geometry) and trickle-down interpretations since they both "quantize" the manifold. In just a moment, I will indicate, as best as possible, what 'quantize' means in this context. However, as I mentioned at the start of this paper, these two interpretations are far more programmatic than serious or well developed versions of LQG; as such, I will not attempt to explain in very great detail

(Rovelli 2004, p.77). That being the case, $\langle \mathcal{M}, g \rangle$ might not require other physical fields in order to be a model of relational spacetime.

[33] One might attempt to interpret $\mathcal{M}$ relationally, though I have serious doubts that this can be done in a convincing way.

how the manifold is quantized, but I will merely indicate what it might mean for spacetime and s-knots if it were "quantized." In short, since TaG and trickle-down interpretations replace the manifold $\mathcal{M}$ with a "quantum" basal structure $\hat{\mathcal{M}}$, the physical structure described by these interpretations is that much more foreign and that much less spatio-temporal. Moreover, in replacing $\mathcal{M}$ with $\hat{\mathcal{M}}$, the naïf cannot interpret the theory as including physically substantial s-knots, at least as s-knots have been constructed thus far.

According to the substantival interpretations considered in this paper, $\mathcal{M}$ plays an essential role in representing a substantival basal structure. Within some interpretations, the manifold $\mathcal{M}$ is interpreted as representing a substantival structure on its own (either spacetime or pre-spatio-temporal); whereas, according to other interpretations, $\mathcal{M}$ is a required component for what ends up representing a substantival structure (either space-time or quantum spacetime). Moreover, what makes Rovellian-LQG non-substantival is precisely the repudiation of $\mathcal{M}$ as being physically significant. For the sake of argument then, let me stipulate more generally that $\mathcal{M}$ is essential for encoding whatever might be substantival about spacetime. If this is correct, then the TaG and trickle-down interpretations are philosophically novel insofar as they describe a theory in which the substantival features of spacetime are "quantized." The point is, if our model replaces $\mathcal{M}$ with a fuzzy background structure "$\hat{\mathcal{M}}$," it is harder to see that there is something like a substantival "container" in which physical things are and dynamical processes occur. This is not to say that one cannot interpret $\hat{\mathcal{M}}$ substantivally and is merely to note that unlike $\mathcal{M}$, $\hat{\mathcal{M}}$, might not model anything like a container. For instance, according to Crowther, spacetime is replaced by a "cloud of lattices" (2014, p.247).

According to TaG versions of LQG, the topology and geometry (TaG) of classical spacetime $\langle \mathcal{M}, g \rangle$ are explicitly replaced by some suitably quantized versions. The impetus behind TaG-LQG is a desire for a more radically background-independent theory of quantum gravity. How one goes about "quantizing" $\mathcal{M}$ however, is far from clear. In general, what quantization means in this context is distinct from what it means according to Dirac's quantization procedure. Moreover, as Isham (1991) notes, since $\mathcal{M}$ is a composite structure consisting of a set of points, topology, and differential structure, one has many options for which structures to quantize in quantizing $\mathcal{M}$. For instance, according to Duston's version of TaG (2012), certain topological features of the manifold are appended to the spin-network states by adding a new internal degree of freedom. Though both Isham and Duston have developed programs to quantize the topological structure of $\mathcal{M}$, one could instead attempt to quantize $\mathcal{M}$ through its differential structures or by *discretizing* the manifold's base set of points. In any case, however one goes about "quantizing" $\mathcal{M}$, in addition to g, the states of TaG-LQG represent physically distinct configurations of $\langle \hat{\mathcal{M}}, \hat{g} \rangle$: a quantum-spatio-temporal structure in all its manifold glories.

Regarding trickle-down interpretations, I intend for this interpretive-scheme to capture any and all interpretations under which the quantization of the gravitational field is thought to automatically affect a discretization of the base manifold.[34] For instance, according to Isham and Butterfield, the discrete spectrum of the area and volume observables (§2.3) suggests that the physical basal structure of spacetime or quantum spacetime is "logically weaker" than a physical continuum (2001, p.78). I do not pretend to understand how trickle down effects work, and my purpose here is not to provide an account of such effects. The point in discussing this interpretive option is merely to highlight its ontology: whatever basal structure there is, the world is structurally impoverished in comparison to that which is described by $\mathcal{M}$. Of course, this is the same result obtained under TaG-LQG. The only significant difference between TaG and trickle-down interpretations is whether or not quantizing the metric field automatically requires that the base manifold be quantized or whether this quantization needs to be imposed on the theory by some other means. In either case, if $\mathcal{M}$ is replaced by $\hat{\mathcal{M}}$ the world being described by LQG is that much less like the spacetime of GR. Consequently, even if we were to adopt a naïve attitude toward these interpretations, we would not conclude that the ontology of the theory includes a physically substantial *manifold*, or perhaps anything which might be interpreted as a substantival container. Moreover, if there is no physically substantial manifold, we will not interpret LQG as including physically substantial s-knots. Allow me to explain.

"Quantizing" the manifold $\mathcal{M}$ will affect the mathematical networks embedded in it and, thereby, will affect what the naïf takes the world to be like. Since the naïf interprets mathematical structures fairly literally, he will interpret the world as including "quantized" substantival s-knots and not the classical substantival s-knots assumed hitherto. What quantum substantival s-knots are, will depend on how exactly $\mathcal{M}$ is quantized. For instance, if the manifold is quantized by "summing over" all possible discretizations of the manifold, then the naïve ontology of this theory would include a fuzzy, discrete, substantival base and yet no s-knots, at least not as they have been defined. Perhaps the substantival networks of this basal structure are superpositions of discretized s-knots? I have no reason to think that the states of either TaG or trickle-down-LQG describe anything like a "⟨fuzzy, discrete, substantival base; superposition of discretized s-knots⟩." The point is simply that in quantizing the manifold, we simultaneously carve away at the substantivalist's container and cut ourselves off from being able to interpret the theory as including physically substantival s-knots.

The TaG and trickle-down interpretations are of special interest, since they push directly up against the substantivalist's intuitions. The container, which the substantivalist takes $\mathcal{M}$ to represent, is explicitly given a quantum description in TaG and trickle-down interpretations. How, or in which ways, $\hat{\mathcal{M}}$ is able to be interpreted substantivally will depend on how, and in which ways, $\mathcal{M}$ is quantized. In any event, it is unlikely that $\langle \hat{M}, \hat{g} \rangle$ or $\langle \hat{M}, \langle \mathfrak{s} | \rangle$ describes a physical structure alike enough to what we mean by 'spacetime' for even the naïf to think that spacetime, or a close kin, exists fundamentally in LQG.

[34] For a general discussion of this idea, see Isham and Butterfield (2001), Isham (1991) as well as Norton (2015).

4 Concluding remarks and challenges

Throughout this account, I have considered eight interpretations of LQG: five naïve interpretations, as well as the Rovellian, TaG, and trickle-down interpretations. Most of these interpretations do not include physically substantial s-knots in their ontology, and some do include spacetime. Thus, claims to the effect that spacetime is composed of or is emergent from spin-networks (s-knots) depend rather acutely on our interpretation. Presumably, if spin-networks compose spacetime, then there must not be a thing called spacetime fundamentally and there must be physical things called spin-networks. I will use this section to provide an analysis of some of the claims which I quoted at the start of this paper. In particular, I will argue that, for many interpretations, spacetime is not composed of or built from physically substantial s-knots and that when spacetime is composed of physically substantial s-knots, they compose spacetime only "weakly," which I will explain.

Following the analysis of whether or not, and in what sense, spacetime might be composed of or built from spin-networks, I will briefly discuss Huggett and Wüthrich's account of spacetime emergence. I will explain how, for these authors, emergence does not require a physical object called spacetime to emerge from some other physical object (a spin-network). I will conclude from these two discussions that the claims quoted at the start of this paper are hard to make true when taken literally and are of limited ontological import when suitably interpreted. This is not to say that there is something wrong or missing from the works which contain such claims. Rather, those works simply have different goals from the ontological focus of this work. I will close this paper by noting how this discussion of ontology might affect related topics in the philosophical foundations of LQG.

4.1 Spacetime is composed of or constructed out of spin-networks

In this section I will analyze claims to the effect that spacetime is composed of or constructed out of spin-networks by placing these claims in the context of three very different interpretations of LQG. The idea is that by considering the claim "spacetime is composed of spin-networks" under three very different interpretations, we might then generalize to some of the other interpretations. In the following I will translate claims of the preceding kind from being about spin-networks to being about s-knots since the proper ontological unit in LQG are s-knots and indeed claims in the literature which are putatively about spin-networks are often really claims about s-knots. Again, this is merely a matter of language and the fact that there is, as yet, no standard vocabulary for LQG.

Case 1:
If we endorse the naïve$_3$ interpretation, then s-knots compose spacetime though only in a technical sense. S-knots "compose" spacetime insofar as spacetime is defined to be that physical structure represented by $\langle M, \langle \mathfrak{s}| \rangle$, and insofar as there are physical things called s-knots. In other words, s-knots compose spacetime since we need $\langle \mathfrak{s}|$ for our model of

spacetime. Moreover, since this interpretation takes for granted a physically substantial manifold, it is not the case that s-knots compose all aspects of spacetime. S-knots, in this situation, merely provide the missing link for spacetime. With the inclusion of s-knots, the physical manifold gains the quantum geometric properties which we have required of spacetime. Since s-knots do not compose all aspects of spacetime, I will say that s-knots *weakly* compose spacetime. This is not a very interesting kind of composition for indeed, the classical gravitational field weakly composes spacetime in GR.

Case 2:
If we endorse the naïve$_1$ interpretation, then s-knots weakly compose an effective spacetime. According to this interpretation, spacetime includes a physical geometry, essentially described by a pseudo-Riemannian metric. Since the world, according to LQG, is never exactly described by a pseudo-Riemannian metric, there is no spacetime fundamentally in LQG. Although, when an s-knot comes to take the form described by some weave state of the theory, we can pretend, in certain regards (equations 12, and 13) and in certain regimes ($l \gg l_p$), that that world is pseudo-Riemannian. In this way, s-knots build an effective spacetime (i.e. quantum geometry looks classical sometimes.) However, as was the case under the naïve$_3$ interpretation, this form of composition is again weak since not all aspects of what we take spacetime to be, are composed of s-knots (e.g. the manifold is not composed of s-knots).

Case 3:
According to the Rovellian interpretation, spacetime is not literally composed of s-knots for two reasons. First, according to the Rovellian, there are no physically substantial networks (for why this is so, see §3.2). Second, since the Rovellian defines spacetime to be the classical gravitational field, and since there is no classical gravitational field in LQG, fundamentally then there is no spacetime. Rather than saying that s-knots compose spacetime, the Rovellian might instead claim that when quantum spacetime comes to take a form described by a weave state, we can approximate LQG with a classical spacetime, e.g. with a classical gravitational field. Thus, spacetime is an effective structure according to the Rovellian. There are no substantival networks from which spacetime is effectively composes but rather, quantum spacetime merely comes to look classical.

Thus, as these cases illustrate, it is not straightforwardly the case that spacetime is composed from or constructed out of s-knots. The closest we get to s-knots, in some literal sense, composing spacetime is in terms of weak or effective composition. In the following, I will briefly discuss Huggett and Wüthrich's account of spacetime emergence. As we shall see, these authors might not be making an ontological claim about new kinds of objects coming to exist.

4.2 Spacetime emerges from spin-networks

According to Huggett and Wüthrich, "the spacetime structure emerges from appropriately benign, i.e. semi-classical, spin-networks" (2013, p.279). Presumably, according to these authors, spacetime is not as described by the naïve, naïve$_3$ or naïve$_4$ interpretations. According to these interpretations, spacetime is fundamental to the theory of LQG and is in no need of emergence.

According to the remaining interpretations, spacetime is essentially related to the physical geometry described by g. Since there is no such geometry in LQG, then there is no spacetime fundamentally. As a reminder, both naïve$_2$ and Rovellian-LQG do not include physically substantial spin-networks (s-knots) as objects in their ontology (§3.1, 3.2). Consequently, if either of these interpretations are what Huggett and Wüthrich have in mind, and they might not, then whatever emergence amounts to, must not require that there actually be physically substantial spin-networks. As it turns out, Huggett and Wüthrich's account of emergence does not require that there actually be physical spin-networks.

According to Huggett and Wüthrich, when two theories satisfy a certain set of pre-specified criteria, the theories (and perhaps some of their substructures) are said to stand in the emergence relation (2013, p.280). Once the criterion is met and the theories stand in an emergence relation, we say either that one theory is emergent from the other or that the physically salient structures of one theory emerge from the physically salient structures of the other. For instance, if GR and LQG satisfy the emergence relation, we might say that GR emerges from LQG, or that spacetime "$\langle M, g\rangle$" or perhaps g, emerges from quantum spacetime "$\langle M, \langle\mathfrak{s}|\rangle$" or perhaps from s-knots (Wüthrich 2006, §9.2). Importantly, though Huggett and Wüthrich's account of emergence requires that spacetime and spin-networks be possible physical structures relevantly related to our experiences and modeled by our theories in order to ground the claim that spacetime is an effective replacement for s-knots, their account does not *require* that there actually be spacetime or spin-networks (2013, p.284).[35] Allow me to explain.

According to Huggett and Wüthrich, spacetime emergence, in the context of LQG, includes two procedures:

> The first procedure, an approximation in the sense of Butterfield and Isham (1999,2001), should show how the dynamics forces the quantum state into semi-classical states with a well-behaved classical counterpart such that, e.g., the quantum superposition is dominated by a single spin network. The second, limiting, procedure then establishes the connection from the semi-classical states to classical relativistic spacetime. (2013, p.280)

The approximating procedure mentioned here is a requirement on the dynamics of LQG to force some generic superposition of s-knot states to take the form of a weave state (§2.3).

[35] Here 'effectivity' is understood in terms of the predictive efficacy of theory given the structure in question, $\langle\mathcal{M}, g\rangle$, for instance. This is the same sense of effective used throughout this essay.

Once done, our authors require that there be some physically salient limiting procedure whereby these weave states might reproduce the empirical content of relativistic spacetime. This limiting procedure is at least partially satisfied by the weave states insofar as they reproduce "the standard [psuedo-Riemannian] area and volume functions" of spatial regions in the limit $l \gg l_p$ (p.279). Notably, neither condition (approximation or limit), require that there actually be a classical relativistic spacetime in the regime $l \gg l_p$; all that these procedures require is that weaves states reproduce the physical geometry of a classical spacetime. For more on this point, see §5.1.

Additionally, these procedures also do not require there to be physically substantial spin-networks (s-knots). For instance, a Rovellian, whose interpretation does not include physical networks, might interpret the aforementioned processes as requiring that relational quantum spacetime, as represented by some s-knot state $\langle \mathfrak{s}|$, come to look like a relational classical spacetime (a described by g) in the regime $l \gg l_p$.[36] According to the Rovellian, using Wüthrich and Huggett's criterion, classical spacetime emerges from quantum spacetime. This example serves to highlight that just because the states of LQG satisfy the conditions as outlined by Huggett and Wüthrich, one cannot infer that there are physical spin-networks (s-knots) from which spacetime emerges. In order for this additional claim to hold, we need to adopt an interpretation which includes physical s-knots. Thus, in claiming that spacetime emerges from the "coherent states of LQG," one might be claiming that there are physical s-knots and from these spacetime emerges. Or, one might merely be claiming that quantum spacetime looks classical in certain regimes.

In sum, the quoted claims with which I began this paper, are of little aid for understanding the ontology of LQG: the nature of spacetime, and its relations the networks of LQG. These claims are of little ontological aid since they are either not true when interpreted literally and, when true, they don't say much about ontology. And then again, it is unlikely that these claims were meant to provide any such aid, but rather these quotes might be better read as providing general heuristics regarding technical results in LQG.

4.3 Conclusion

In the first half of this paper, I provided an exposition of LQG expressed in the language of the naïve interpretation. That interpretation describes the world as including a substantival manifold called spacetime which contains spatially embedded charged graphs. These charged graphs are responsible for the quantum geometric structure of spacetime. The second half of this paper consisted of an analysis of alternative interpretations of LQG. These interpretations differ from one another and the original naïve interpretation in what they take spacetime to be and, consequently, what they take the states of the theory to represent.

As I have argued, whether or not spacetime disappears in LQG depends upon how one interprets $\mathcal{M}$ and how essential a pseudo-Riemannian geometry is for spacetime. What goes

[36] This in fact is exactly what the Rovellian does say in the form of "recovering" classical spacetime (§3.2).

missing in LQG, independent of one's interpretation, is the physical geometry described by g; whether or not spacetime also goes missing is up for debate. Finally, I have provided an analysis of some claims to the effect that spacetime is either composed of or emergent from spin-networks (s-knots) and have argued that, more often than not, these claims cannot be and perhaps are not meant to be interpreted in an ontologically serious sense.

4.4 Afterword: related issues and looking forward

The foregoing analysis will affect other issues in the literature on LQG: the problem of time, the status of locality in LQG, the nature of emergence and of causation in LQG, as well as the distinction between abstract and concrete objects. In this final section, I will only discuss, albeit very briefly, how LQG affects our ability to distinguish between abstract and concrete objects. I will discuss two predominant accounts of the abstract-concrete distinction and will show how LQG makes trouble for them. Included in this discussion will be a brief elaboration (§3.1) of the status of causation in LQG. I here discuss the abstract-concrete distinction as an example of how LQG might force us to reconceive conceptual distinctions or metaphysical doctrines which LQG touches on. In order to streamline the following discussion, I will assume that $\langle M, \langle \mathfrak{s}| \rangle$ represents quantum spacetime (as opposed to spacetime, for instance).

Account one: it is standardly suggested that the difference between abstract and concrete objects, if there be such a distinction, rests in how these objects relate to spacetime. In particular, concrete objects are defined to be just those objects which exist at particular places and times; whereas, abstract objects do not exist at places or times. Tables, chairs, and presumably spacetime itself are concrete objects; whereas, propositions, numbers, and Platonic forms are abstract. As one might expect since according to LQG there is no space, time, or spacetime fundamentally, there is nothing fundamental which exists at spatial places or times. Thus, it seems that so long as the abstract-concrete distinction hinges on there being spacetime, then there is no distinction if LQG is true.[37]

We might try to avoid this conclusion by treating classical spacetime as a genuine object of our ontology in the "emergence" regime. Huggett and Wüthrich's account of emergence utilizes effective structures (see §3.2 and 4.2). If we were so inclined, we could adopt a metaphysics of objects whereby all classical or otherwise effective structures are more than useful fictions and are distinct items of our ontology. These effective structures are distinct insofar as they are not merely fundamental structures which happen to take useful forms. According to this suggestion, there are classical tables and there are also the particles which make up the table. The table is a thing unto itself and is not merely a convenient name for a table-wise arrangement of quantum particles. If we adopt this view, we might try to avoid the collapse of the abstract-concrete distinction since relativistic spacetime exists as an effective structure. While it is true that, according to this metaphysics, there is

[37] Presumably, this result would please any one, e.g. Maddy et al. (1990), for whom some mathematical objects are also concrete.

classical spacetime in the regime $l \gg l_p$, spacetime *qua* an effective structure, cannot play the role which the abstract-concrete distinction requires of it. For instance, while there is spacetime and therefore a distinction between abstract and concrete objects in the regime $l \gg l_p$, what should we say about the non-classical regime $l \not\gg l_p$? Is there no distinction at these energy scales? Do table-particles and numbers, for instance, become metaphysically indistinguishable when $l \not\gg l_p$? Should the fact that there is a distinction between tables or the particles which make up tables and numbers depend on how much energy with which we are probing the table? Presumably not. Thus, even if we were to adopt a split level ontology, we would not thereby save the abstract-concrete distinction *qua* spacetime.

Account two: it is standardly suggested that concrete objects are causal; whereas, abstract objects are not. According to this suggestion, even if there were only quantum spacetime, since tables are causal (let us assume), tables are thereby concrete; since numbers and propositions are non-casual, they are thereby abstract. Of course this suggestion assumes that there are causes in quantum spacetime and, as I argued in §3.1, there might not be causation or at least a very robust account of causation were spacetime defined to be without a classical metrical structure. I will review this argument and strengthen it with a few additional comments.

In section §3.1, I mentioned that if "spacetime" were without any metrical structure, then there would be no lengths, no rectangles, no irregular shapes, no small, no near, no far. I argued that, without these facts, the sense of change and thereby causation, which the world contains, is significantly diminished. We might hope that by modeling the physical basal structure of the world as including a quantum geometry, rather than as being a bare substantival manifold, that these additional quantum geometric features will allow us to model the sorts of changes which we require for causation. However, it turns out that things are actually worse off than I have let on and, in particular, adding the quantum geometry represented by the states $\langle \mathfrak{s}|$ does not help causation. As soon as we include the quantum geometry of LQG in our model of spacetime or quantum spacetime, there is no longer any remaining physical change or variation over time whatsoever. Similar to general relativity, the quantum geometry of LQG is coupled to whatever matter fields there are. Thus, if our matter fields undergo any substantive change, the quantum geometry of LQG will also undergo a change.[38] However, as I have already discussed, the Hamiltonian constraint requires that the quantum geometry of LQG be static. If the quantum geometry is static, so too are the matter fields to which it is coupled. Thus, since the Hamiltonian constraint requires that our matter fields remain static, in what sense are there causes in LQG?

One way to escape this conclusion is to redefine how we model dynamics in LQG, which happens to be an active area of research (Isham 1992, Kuchař 1992). If one of these research projects is able to recapture the missing dynamics of LQG, then presumably we could try to capture causation in LQG using the proposed dynamics. However, according

[38] By substantive change, I mean to exclude cases such as the exchanging of identical particles.

to our current understanding, LQG does not include dynamics and, thereby, does not include enough structure for there to be causation fundamentally. If there is no causation fundamentally in LQG, then what exactly distinguishes concrete and abstract objects?

Thus, if LQG is true, then fundamentally there is neither spacetime nor causation, at least as these concepts have been standardly conceived. Without spacetime or causation, we do not have the conceptual resources for there to be a distinction between abstract and concrete objects, again, at least as this distinction has been standardly conceived. If we think that there is a metaphysical difference, in kind, between mathematical objects and dining room tables, then we will need to upgrade our account of concrete objects so as to distinguish them from abstract objects. I will close this discussion with two final comments.

First, how seriously should we treat the lack of causation and the collapse of the abstract-concrete distinction suggested by LQG when LQG might very well be false? The theory is not completely well-defined, and we don't have a direct way to test any theory of quantum gravity. Given these shortcomings, perhaps it is prudent to set aside the puzzling ontology of LQG until the theory is confirmed. However, such reasoning would be mistaken. Even if LQG turns out to be false, we should take these lessons seriously. LQG has shown us that it is possible to have a physical theory which does not include spacetime or causation. If spacetime and causation are contingent structures, then we should be wary of defining metaphysical doctrines, like the abstract-concrete distinction, in terms of them. Presumably, if there is a distinction between concrete and abstract objects, the distinction is independent of physics.

Second, in light of LQG, we might upgrade our account of the abstract-concrete distinction by making use of quantum spacetime. Under this suggestion, concrete objects are just those objects which are in quantum spacetime and abstract objects are not. Though this is a reasonable and tidy solution, I suggest that we not adopt it for the previously stated reasons. If there is a metaphysical distinction between abstract and concrete objects, we need a metaphysical account of this difference and not another distinction in terms of physical structures. If LQG can erase spacetime, what hope do we have for quantum spacetime?

5 Appendix

In this appendix, I will cover three central mathematical developments in LQG: the constraints derived from Dirac's quantization procedure, the spin-network and s-knot Hilbert spaces, and the area and volume observables. I discuss only these topics because a fuller treatment should be sought for in a textbook, and yet these few topics are sufficient for providing a first level orientation to the mathematics of LQG. This appendix is written as an outline, and many details and caveats are left out. The bulk of this appendix is reproduced from standard textbooks on LQG such as Rovelli (2004), Thiemann (2007), and Gambini and Pullin (2011). When no citation is provided, the corresponding material has been drawn from Rovelli (2004).

5.1 Constraints

In order to use Dirac's quantization procedure, we need to write GR as a Yang-Mills theory. Thus, the task we are first concerned with is how to squeeze GR into a Yang-Mills theory. For more detail or further reading on this material, see Baez (1994). We begin by rewriting Einstein's field equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R+\lambda) = 8\pi T_{\mu\nu}, \tag{16}$$

in terms of the "tetrad fields":

$$e \equiv e^I_\mu \hat{\sigma}_I \otimes dx^\mu. \tag{17}$$

In the case of LQG, with its $\mathfrak{su}(2)$-gauge field, the object $e \equiv e^I \hat{\sigma}_I$ is a vector in Minkowski space and the object $e^I_\mu \hat{\sigma}_I \otimes dx^\mu$ is a "Minkowski-valued" one-form. Generic one-forms are maps from tangent vectors to smooth functions. The above "Minkowski-valued" one-form is a map from tangent vectors to vectors in a Minkowski vector-bundle. Because we are working with vector bundles, every point of the spatial manifold has an associated Minkowski space. The tetrad field associates a vector from each of these vector spaces to every tangent vector at that point in space. Since this mapping is dependent on the spatial manifold, we need both external (spatial) as well as internal (gauge) coordinates in order to fully specify the tetrad (up to coefficients).

For any given internal Minkowski vector $\vec{v} = v^\mu e^I_\mu \hat{\sigma}_I$, we define its length in the usual way:

$$|v| = \sqrt{-\eta_{IJ} v^\mu e^I_\mu(x) v^\nu e^J_\nu(x)}. \tag{18}$$

Unsurprisingly, we can pull this metrical structure back to the base manifold and define the metric on tangent vectors $(v^\lambda \partial_\lambda)$ to be:

$$g_{\mu\nu}\partial^\mu\partial^\nu \equiv \eta_{IJ} e^I_\mu(x) e^J_\nu(x) dx^\mu \otimes dx^\nu. \tag{19}$$

In a similar way, we can rewrite the Ricci tensor and Ricci scalar in terms of tetrad fields which we can then use, in conjunction with (19), to rewrite (16) as:

$$R_\mu^I - \frac{1}{2}Re_\mu^I + \lambda e_\mu^I = 8\pi G T_\mu^I. \tag{20}$$

Here, I have kept only the coefficients of the tensors and have suppressed the basis vectors $\hat{\sigma}_I \otimes dx^\mu$. The practice of keeping only the coefficients is common, though it can lead to confusion if one is not careful to keep track of the indices. Throughout this account, 'I, J, K' range over internal Minkowski coordinates while 'μ, ν, λ' range over external spatial coordinates.

In the same way that we use the Levi-Civita connection to define covariant derivatives in GR (as well as covariant exterior derivatives), we use the $\mathfrak{su}(2)$-gauge field $\mathcal{A} \equiv \mathcal{A}_J^I(x)\hat{\sigma}_I \wedge \hat{\sigma}^J \equiv \mathcal{A}_{\mu J}^I(x)\hat{\sigma}_I \wedge \hat{\sigma}^J \otimes dx^\mu$ to define a similar structure(s) in LQG:

$$D_\mu v^I \equiv \partial_\mu v^I + \mathcal{A}_{\mu J}^I v^J. \tag{21}$$

The gauge field $\mathcal{A}$ is also known as the vector potential in LQG and is one of the new variables which Ashtekar used to reformulate canonical quantum gravity along the lines I am here outlining. The vector potential plays an essential role in writing GR in terms of a Hamiltonian which we need before we can use Dirac's quantization procedure. Using the vector potential, its canonical momenta $\widetilde{E}_I^\mu(x)$, the internal curvature tensor $F_{\mu\nu}^I$, and the Lagrangian multipliers N^μ, N^0, and λ^I, we can write the Lagrangian for GR as:[39]

$$L \approx \int d^3(x)\left(\widetilde{E}_I^\mu \dot{\mathcal{A}}_\mu^I + N^0 \epsilon_{IJK} \widetilde{E}_I^\mu \widetilde{E}_J^\nu F_{\mu\nu}^K + N^\mu \widetilde{E}_I^\nu F_{\mu\nu}^I + \lambda^I (D_\mu \widetilde{E}^\mu)_I\right). \tag{22}$$

According to Dirac's quantization procedure, the three formulas appended by the Lagrangian multipliers are the constraints of LQG:

$$D_\mu \widetilde{E}_I^\mu = 0, \tag{23}$$

$$\widetilde{E}_I^\nu F_{\mu\nu}^I = 0, \tag{24}$$

$$\epsilon_{IJK} \widetilde{E}_I^\mu \widetilde{E}_J^\nu F_{\mu\nu}^K = 0. \tag{25}$$

Importantly, all the physics of GR are encoded in the following constraints (Isham 1992, p.34-35). The first constraint is called the gauge or Gauss constraint, the second constraint is called the vector or the diffeomorphism constraint, and the third is the called the scalar or Hamiltonian constraint. In turning GR into a quantum theory, we begin with a space of functionals on the gauge potential $\Psi[\mathcal{A}]$ and promote the vector potential to play a dual role as a multiplicative operator:[40]

$$\hat{\mathcal{A}}_\mu^I \Psi[\mathcal{A}] = \mathcal{A}_\mu^I \Psi[\mathcal{A}]. \tag{26}$$

[39] Gambini and Pullin (2011, p.93)
[40] *ibid*, p.99

The canonical momenta are likewise promoted to the functional derivatives:

$$\hat{\tilde{E}}^{\nu}_{I}\Psi[\mathcal{A}] = -i\frac{\delta\Psi[\mathcal{A}]}{\delta\mathcal{A}^{I}_{\nu}}. \tag{27}$$

Plugging these operators into the aforementioned constraints, the physical states are defined to be those states which are annihilated by the following three operator-constraints:

$$-iD_{\mu}\frac{\delta\Psi[\mathcal{A}]}{\delta\mathcal{A}^{I}_{\mu}} = 0, \tag{28}$$

$$F^{I}_{\mu\nu}\frac{\delta\Psi[\mathcal{A}]}{\delta\mathcal{A}^{I}_{\mu}} = 0, \tag{29}$$

$$\epsilon_{IJK}F^{K}_{\mu\nu}\frac{\delta}{\delta\mathcal{A}^{I}_{\mu}}\frac{\delta}{\delta\mathcal{A}^{J}_{\nu}}\Psi[\mathcal{A}] = 0. \tag{30}$$

The goal, then, becomes to find a set of states which solve these equations and hope that they form a Hilbert space. In the following I will first construct the spin-network Hilbert space whose states solve the Gauss constraint, (28). I will then construct the s-knot Hilbert space whose states solve both the Gauss and diffeomorphism constraints, (28) and (29). Unfortunately, we do not yet have a Hilbert space of states which solve the Hamiltonian constraint.

5.2 Hilbert spaces

Nota bene: throughout the rest of this account, I utilize a formalism which is slightly different from that used in the main body of this text. For instance, rather than representing spin-network states as $|\Gamma(\vec{x}), j_n, i_m\rangle$ and $|\mathcal{S}\rangle$, I will here use $\Psi_{\mathcal{S}}$ and $|\mathcal{S}\rangle$. The majority of the following construction of the states of LQG follows Rovelli (2014) and Rovelli and Peush (2013).

The generic space of states with which we begin is $\mathcal{S}$, a linear space of cylindrical functionals, on the vector potential, $\mathcal{A}$. A generic state in $\mathcal{S}$ is defined as:

$$\Psi_{\Gamma,f}[\mathcal{A}] \equiv f(U(\mathcal{A},\gamma_1), ...U(\mathcal{A},\gamma_L)). \tag{31}$$

Here the γ_k are generic oriented paths in the "spatial" manifold Σ, and each $U(\mathcal{A},\gamma_k)$ is a holonomy along them. We use these states to define an inner product on $\mathcal{S}$ and, in turn, use this inner product to construct $\mathcal{K}$, the "completion" of $\mathcal{S} \subset \mathcal{K}$. In the following, I will whittle this large space $\mathcal{K}$ into a proper subspace $\mathcal{K}_0$ whose states are gauge invariant functionals.

First, we begin with a particular embedded network, $\Gamma(\vec{x})$, in some surface, Σ, and focus our attention on those state functionals whose γ_k are the curves of the graph $\Gamma(\vec{x})$. We will use the graphs $\Gamma(\vec{x})$ to construct basis vectors for the subspace $\mathcal{K}_0$; however, in order to do so, we need to assign an irreducible representation of the $SU(2)$ group to each link of the graph. By choosing a representation, we are able to associate with each point along the links, γ_k, some particular matrices. As the holonomy drags our gauge field along the link, the exponential map converts the $(\mathfrak{su}(2))$ algebra elements to $(SU(2))$ group elements which we associate with some particular series of matrices (provided by the representation). I will briefly explain how this process works and then how we use these representations to build gauge invariant states.

The irreducible representations of $SU(2)$ are given by the spin-j representations[41] where $j \in (0, \frac{1}{2}, 1, ..., \frac{n}{2}), n \in \mathbb{N}$. We assign some spin-label j_l to each of the links l in $\Gamma(\vec{x})$, and pick some matrix element (α_l, β_l) from the corresponding matrix $M_{j_l}(U(\mathcal{A}, \gamma_l))$. We then construct the following "colored" cylindrical state $\Psi_{\Gamma j_l \alpha_l \beta_l}[\mathcal{A}]$:

$$\equiv M_{j_1}(U(\mathcal{A}, \gamma_1))^{\alpha_1}_{\beta_1} M_{j_2}(U(\mathcal{A}, \gamma_2))^{\alpha_2}_{\beta_2} ... M_{j_L}(U(\mathcal{A}, \gamma_L))^{\alpha_L}_{\beta_L}. \tag{32}$$

The difference between these states and those defined in (31) is that the generic functional in (31) is replaced by a generic multiplication of matrix elements. We call the process of assigning an irreducible representation to each link "coloring" the links, and say that the color of some link l is its associated representation j_l. After coloring the links, we color the nodes of the graph by associating a special vector (an "intertwiner") to each node. Each two nodes can have the same or different intertwiners associated with them.

Since the links of the graph are colored, each link has some Hilbert space H_j associated with it. To each node we associate the tensor product of the Hilbert spaces associated with the the links meeting at that node. This giant tensor product of Hilbert spaces contains a subspace of vectors which are invariant under the action of $SU(2)$-gauge group. We color the node by selecting one of these intertwining vectors. Once each node and link is colored, we define a generic spin-network state $\Psi_{\mathcal{S}}[(A)]$ to have the form:

$$\equiv \vec{\mathbb{V}} M_{j_1}(U(\mathcal{A}, \gamma_1))^{\alpha_1}_{\beta_1} M_{j_2}(U(\mathcal{A}, \gamma_2))^{\alpha_2}_{\beta_2} ... M_{j_L}(U(\mathcal{A}, \gamma_L))^{\alpha_L}_{\beta_L}. \tag{33}$$

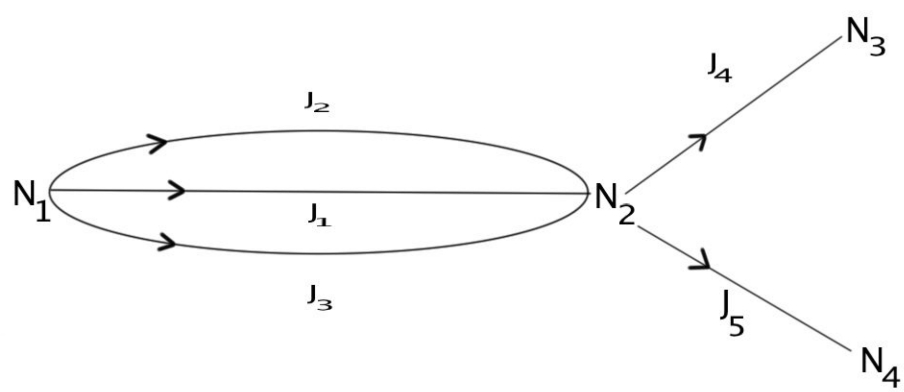
Here, the vector $\vec{\mathbb{V}}$ is tensor product of the intertwiners at the nodes. The difference between these states and those defined by (32) is that these states are defined by contracting all the end-point matrix elements whereas the states in (32) are defined as simply a product of some of these elements. Since the vector $\vec{\mathbb{V}}$ sits in the giant Hilbert space on which these matrices act, it is not hard to pick vectors at the nodes to do the contraction. Generically, $\vec{\mathbb{V}}$ will have the form:

$$\mathbb{V}^m \equiv \mathbb{V}^{\beta^m}_{\alpha^m} \equiv \mathbb{V}^{\beta^m_{l_1} ... \beta^m_{l_{in}}}_{\alpha^m_{l_1} ... \alpha^m_{l_{out}}}. \tag{34}$$

[41] For each j, we construct the Hilbert space H_j out of the polynomials of the form $a_{2j} x^{2j} y^0 + a_{2j-1} x^{2j-1} y^1 + ... a_0 x^0 y^{2j}$ on $\mathbb{C}^2$, where $a_i \in \mathbb{C}^2$. The elements $m \in SU(2)$ are mapped to linear operators $M_j(u)$ on the vectors of H_j. The trivial representation, j=0, is all the functions of the form $a_0 x^0 y^0$ and so is isomorphic to $\mathbb{C}^2$. The fundamental representation of a group is the group itself and only occurs when the group is itself a group of linear transformations on a vector space. The spin-$\frac{1}{2}$ representation is isomorphic to the fundamental representation, and the spin-1 representation is isomorphic to the adjoint representation.

Where m selects the node. The idea is that the α^m index on the node contracts all the α indices of the links which “leave” the node, n, and the β^m contracts the β indices of the links which “enter” the node. In short, spin-network states are defined by contracting all the holonomies around the embedded network in order of how the links enter and exit the nodes of the network. A generic spin-network state $\Psi_{\mathcal{S}}[(A)]$ is picked out by the set of information $(\Gamma(\vec{x}), j_n, i_m)$: an embedded graph in Σ whose links have been colored by selecting representations (j_n) and the nodes have been colored by selecting intertwiners (i_m). It turns out that spin-network states are invariant under gauge transformations and form an orthonormal basis for the (non-separable) Hilbert space $\mathcal{K}_0 \subset \mathcal{K}$.

Figure 5:



For example, the spin-network represented in (figure 5) defines the following spin-network state:

$$\Psi_{\mathcal{S}_5}[(A)] = \mathbb{V}_{\alpha_1,\alpha_2,\alpha_3}[M_{j_1}(U(\mathcal{A},\gamma_1))^{\alpha_1}_{\beta_1} M_{j_2}(U(\mathcal{A},\gamma_2))^{\alpha_2}_{\beta_2} M_{j_3}(U(\mathcal{A},\gamma_3))^{\alpha_3}_{\beta_3}]\mathbb{V}^{\beta_1,\beta_2,\beta_3}_{\alpha_4,\alpha_5}.... \quad (35)$$

Here the ellipsis indicates the contraction along links four and five with the intertwiners at the remaining nodes.

Recall, at this point, our gauge invariant states are in $\mathcal{K}_0$ which is a proper subspace of $\mathcal{K}$. Since both $\mathcal{S}$ and $\mathcal{K}_0$ are subspaces of $\mathcal{K}$ we cannot assume that all the states in $\mathcal{K}_0$ will automatically be one of our cylindrical functions from $\mathcal{S}$. We construct, therefore, $\mathcal{S}_0$, the subspace of states from $\mathcal{K}_0$ which live in $\mathcal{S}$. The space $\mathcal{S}_0$ contains all finite linear combinations of spin-networks states and happens to be dense in $\mathcal{K}_0$. In the following, I will use $\mathcal{S}_0$ and its dual space $\mathcal{S}_0^*$ to construct a space of diffeomorphism invariant states

$\mathcal{S}^*_{\text{Diff}}$. Before turning to this task, however, since I will be switching back and forth between states and their duals, I will use the bra-ket notation: a spin-network state will either be written as $\Psi_{\mathcal{S}}$ or as $|\mathcal{S}\rangle$ and I will refer to the states $\Psi \in \mathcal{S}^*_0$ as either $\Psi^\dagger_{\mathcal{S}}$ or as $\langle\mathcal{S}|$.

In order to construct $\mathcal{S}^*_{\text{Diff}}$, we begin by mapping those spin-network states $\Psi_{\mathcal{S}}$ in $\mathcal{S}_0$ to the state $\langle\mathfrak{s}| \in \mathcal{S}^*_0$; where, $\langle\mathfrak{s}|$ is a functional on states $\Psi_{\mathcal{S}_1} \in \mathcal{S}_0$ defined by:

$$\langle\mathfrak{s}|\mathcal{S}_1\rangle \equiv \sum_{|\mathcal{S}_2\rangle \in \{|\mathcal{S}\rangle\}_\Phi} \langle\mathcal{S}_2|\mathcal{S}_1\rangle. \tag{36}$$

Where the summation is over all states $\Psi_{\mathcal{S}_2}$ related to $\Psi_{\mathcal{S}}$ by a diffeomorphism U_Φ: $\Psi_{\mathcal{S}_2} = U_\Phi\Psi_{\mathcal{S}}$. It is not hard to show that the set of states $\{\Psi_{\mathcal{S}}\}_\Phi$ is the same set of states $\{U_\Phi(\Psi_{\mathcal{S}})\}_\Phi$:

> For any $\Psi' \in \{\Psi_{\mathcal{S}}\}_\Phi$, $\Psi' = U_{\Phi_1}(\Psi_{\mathcal{S}})$, for some Φ_1 diffeomorphism. Since the set of diffeomorphisms form a group, $U_{\Phi_2} = U_{\Phi_1} \circ U_{\Phi^{-1}}$ is also a diffeomorphism for all U_{Φ_1} and $U_{\Phi^{-1}}$. It is easily verified that $\Psi' = U_{\Phi_2}(U_\Phi(\Psi_{\mathcal{S}}))$, and consequently $\Psi' \in \{U_\Phi(\Psi_{\mathcal{S}})\}_\Phi$. The proof for the other direction is similar. Since the two sets contain each other's members, the sets are the same. We will use this result in the following.

The point in mapping the states, $|\mathcal{S}\rangle$, to the dual vectors, $\langle\mathfrak{s}|$, is that we can build diffeomorphism invariance into the mapping. A diffeomorphism U_Φ on $\langle\mathfrak{s}|$ is mathematically equivalent to $\langle\mathfrak{s}| \circ U_{\Phi^{-1}}$:

$$\left(\sum_{\{|\mathcal{S}\rangle\}_{\Phi_i}} \langle\mathcal{S}_2|\right) \circ U_{\Phi^{-1}} = \tag{37}$$

$$\sum_{\{|U_\Phi\mathcal{S}\rangle\}_{\Phi_i}} \langle\mathcal{S}_2|. \tag{38}$$

Where the summation is over all states $\Psi_{\mathcal{S}_2}$ related to $U_\Phi\Psi_{\mathcal{S}}$ by a diffeomorphism. Since the set of states defining the summation in (36) and (38) are the same, the states $U_\Phi\langle\mathfrak{s}|$ and $\langle\mathfrak{s}|$ are the same.

The span of the states $\langle\mathfrak{s}| \in \mathcal{S}^*_0$ form the subspace $\mathcal{S}^*_{\text{Diff}}$ of functionals which are both gauge and diffeomorphism invariant. $\mathcal{S}^*_{\text{Diff}}$ is a Hilbert space whose orthonormal basis vectors are called spin-knot or s-knot vectors. In the main body of this text, I referred to the generic state $\langle\mathfrak{s}|$, as defined in equation (36), as being an s-knot vector; however, this is not quite true. While the generic states $\langle\mathfrak{s}|$ do span $\mathcal{S}^*_{\text{Diff}}$, they are neither linear independent nor orthonormal. Though a generic state of $\mathcal{S}^*_{\text{Diff}}$ does not solve the Hamiltonian constraint, these states are interpreted as representing the physical system of LQG.

5.3 Geometric Observables

In this section, I will focus on the volume ($\hat{\mathcal{V}}(\mathcal{R})$) observable since I already discussed the area observable in §2.3; I will reproduce the area observable below as a point of comparison. This section is based on (Rovelli and Peush, 2013) and (Rovelli and Peitri, 2008).

Both the area and volume observables are defined by regions and surfaces in the "spatial" manifold Σ: there is one observable per region or surface. A generic area observable for some surface $\mathbf{S}$ is defined as:

$$\hat{\mathcal{A}}(\mathbf{S}) \equiv \lim_{n\to\infty} \sum_{k}^{n} \sqrt{-\left(\int_{\mathbf{S}_k^{(n)}} d\sigma^1 d\sigma^2 \epsilon_{abc} \frac{\partial x^a(\vec{\sigma})}{\partial \sigma^1} \frac{\partial x^b(\vec{\sigma})}{\partial \sigma^2} \frac{\delta}{\delta \mathcal{A}_c^i(\vec{\sigma})}\right)^2}. \tag{39}$$

Here $\mathbf{S}_k^{(n)}$ are n-many subdivisions of $\mathbf{S}$. Since a generic spin-network state (§5.2) is identified with a particular embedded, colored graph $(\Gamma(\vec{x}), j_n, i_m)$, in the following I will write $\Psi_{\mathcal{S}}[\mathcal{A}]$ as $\Psi_{\Gamma,\vec{j},\vec{i}}[\mathcal{A}]$. I have changed notation from $\{i_m\}$, $\{j_n\}$ to $\vec{i}$, $\vec{j}$ in order to more easily express the spectrum of the volume observable. But first, the spectrum for the area observable has the rather simple form:

$$\hat{\mathcal{A}}(\mathbf{S})\Psi_{\Gamma,\vec{j},\vec{i}}[\mathcal{A}] \equiv \sum_{n\in(\mathbf{S}\cap\Gamma)} \sqrt{j_n(j_n+1)}\Psi_{\Gamma,\vec{j},\vec{i}}[\mathcal{A}]. \tag{40}$$

As we can see from this expression, the eigenvalues are exclusively controlled by the "charge" or coloring of the links of the graph which cross the surface $\mathbf{S}$.

The classical formula associated with the volume observable is given in terms of the canonical momenta $\widetilde{E}^{\nu J}$:

$$\mathcal{V}(\mathbf{R}) \equiv \int_{\mathbf{R}} d^3x \sqrt{\frac{1}{3!}\epsilon_{\mu\nu\lambda}\epsilon_{IJK}\widetilde{E}^{\mu I}\widetilde{E}^{\nu J}\widetilde{E}^{\lambda K}}. \tag{41}$$

Though one can unpack the *operator* form of the above expression, doing so would take us outside the scope of this short appendix and would add little to our understanding of the geometric structure of the operator. The corresponding spectrum of the volume observable is given by:

$$\hat{\mathcal{V}}\Psi_{\Gamma,\vec{j},\vec{i}} \approx \sum_{\vec{\beta}} \lambda_{\vec{\beta}}\hat{P}_{\vec{\beta}}\Psi_{\Gamma,\vec{j},\vec{i}}. \tag{42}$$

And more specifically by:

$$\hat{\mathcal{V}}\Psi_{\Gamma,\vec{j},\vec{i}} \approx \lambda_{\vec{i}}\Psi_{\Gamma,\vec{j},\vec{i}}. \tag{43}$$

The first thing to note is that this spectrum includes the projector $\hat{P}$ onto the some vector $\vec{\beta}$. This projector projects onto sets of nodes of certain colorings, and is the reason why the volume spectrum does not have as simple an expression as the area spectrum. In particular, the resulting eigenvalues $\lambda_{\vec{i}}$ are determined by which nodes are adjacent in the graph and so cannot be specified generically. There is little else regarding the formal structure of the observables which I feel is important for gaining an orientation to the mathematics of LQG. I will, however, reiterate that the LQG-area and LQG-volume of physical surfaces and regions diverge from the Riemannian-area and Riemannian-volume of surfaces and regions. I mentioned this in §2.3 but is worth saying again: the volume of a region, as defined by (41), does not change as we change the Riemannian size of the region being integrated over, and only changes by changing the set of nodes contained in the region. Similarly, in order to change the physical area of some surface we must change the set of links which "cut" the surface.

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