

Philosophy in Einstein's Science

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1. Introduction

Albert Einstein read philosophy. It was not an affectation of a celebrity-physicist trying to show his adoring public that he was no mere technician, but a cultured thinker. It was an interest in evidence from the start.

In 1902, Einstein was a poorly paid patent examiner in Bern seeking to make a few extra Francs by offering tutorials in physics. Maurice Solovine answered the advertisement. The tutorials quickly vanished when they discovered their common fascinations in reading and talking. They were soon joined in their raucous meetings by Conrad Habicht, completing what they dubbed their "Olympia Academy." Their explorations were wide-ranging, devouring texts and sausages with gusto. They read the philosophers and philosophically-minded scientists of the day, including Pearson, Mach, Hill, Hume, Spinoza, Avenarius, Clifford and Poincaré.¹

The philosophical interest endured. In the late 1920s, there were three portraits on the walls of Einstein's study in Berlin. Two were unsurprising: the great English physicists Michael Faraday and James Clerk Maxwell. And the third? It was not one most people would predict. It was Arthur Schopenhauer. However, as Don Howard² has shown in detail, once one is alerted to

look, the imprint of Einstein's reading of Schopenhauer is clearly visible in his writing and thought.

Einstein's own philosophical writings have in turn attracted considerable attention.³ In the years following his discovery of the general theory of relativity, Einstein was read and consulted by many philosophers, including Rudolf Carnap, Hans Reichenbach and Moritz Schlick. As a result, Einstein's writings in physics and philosophy enjoyed a commanding presence in the new movements in modern philosophy that gained strength in the 1920s and 1930s.⁴

My purpose in this note is not to attempt a synthetic portrait of Einstein's philosophy. For reasons I will indicate later, I am not sure how useful that would be. Rather I want to draw attention to what I believe is the most important aspect of Einstein's interest in philosophy. Einstein quite consciously integrated philosophical analysis into his physical theorizing. Its explicit use was part of how Einstein found his way to new theories and defended them. Here I will sketch a few of episodes in Einstein's physics in which philosophical analysis played an important role. I will try to explain at a level relatively free of the technicalities of physical theories just what role the philosophical analysis played.

The first episode, recounted in Section 2, is Einstein's adopting an empiricist theory of concepts in order to legitimate an extraordinary new physical proposal concerning time in special relativity. Section 3 will recount what Einstein described as the "epistemological defect" in earlier theories that motivated him to seek his general theory of relativity. Section 4 will describe how Einstein twice grounded his theorizing in principles that distinguish the real from the unreal. One pertains to the completion of the general relativity and the grounds his co-authored efforts to prove the incompleteness of quantum theory. Finally, in Section 5, I will describe how Einstein came to adopt a form of mathematical Platonism as the way to find to new theories, such as his unified field theory. In the conclusion in Section 6, I will explain why I believe Einstein was correct and appropriately unapologetic in portraying himself as an "unscrupulous opportunist" in the view of a systematic epistemologist.

2. An Empiricist Theory of Concepts

In June 1905, Einstein⁵ sent the journal *Annalen der Physik* the manuscript of his paper, “On the Electrodynamics of Moving Bodies.” In it, he laid out his special theory of relativity. Its first “Kinematical Part” is both a brilliant departure from earlier thinking and a deceptively easy analysis to read. In order to solve certain problems in electrodynamics, Einstein tells us, he will posit two principles: the principle of relativity and the light postulate. The first asserts the equivalence of all inertial motion. The second assigns a unique speed to light propagating in vacuo.

The two principles are “apparently irreconcilable,” Einstein mentions in passing on the paper’s first page. The reader is left to imagine why. It is not hard to do. When the two principles are combined, they entail that all inertially moving observers will find the same speed for the one beam of light. Imagine that you measure the speed of a light beam and find some value, “ c .” If I am chasing rapidly after that same beam at the great speed of $c/2$, should I not find it to propagate at $c - c/2 = c/2$? No, the two principles say. I must find the same value, c ; and that just does not seem right.

Einstein turned immediately to a simple explanation of why this irreconcilability is only apparent. Implicit in our judgments of the speed of light are further assumptions about space and time. In concrete terms, an observer measures the speed of light by timing how long light takes to traverse a known distance; and that time difference is measured by clocks placed at either end of the distance. The procedure requires that the two clocks be properly synchronized. Each must read “12 o’clock” at precisely the same instant. It is easy to assume that, if one observer judges the clocks to be properly synchronized, then so also will another observer in relative motion. That, Einstein proceeded to demonstrate, is incorrect. His famous demonstration involved an ingenious thought experiment with clocks and light signals and drew on his theory’s two principles. The essential outcome is that the two observers will not agree on which spatially separated events are simultaneous; and thus they will not agree on whether the two clocks are properly synchronized. In their attempts to measure the speed of light, the two observers will use clocks synchronized differently. The resulting differences turn out to be exactly sufficient to ensure that both recover the same value c for the speed of light.

This effect, “the relativity of simultaneity,” was the first of the novel results of the new special theory of relativity. It led Einstein immediately to argue that observers in relative motion

will, in general, not agree on the lengths of objects and the time durations measured for processes. The analysis is so crisp and simple that it is hard to suppress the image of impish Einstein casually tossing off the analysis from the comfort of an armchair one sunny afternoon.

The reality of the discovery was quite different. As I have recounted in some detail elsewhere⁶, Einstein had become convinced years before that the principle of relativity must hold in Maxwell's theory of electrodynamics, even though that theory was based on an ether in which there was a preferred state of rest. Worse, Maxwell's theory asserted that light always travels at just one speed, $c=186,000$ miles per second in vacuo in relation to this ether. It seemed to Einstein that the principle of relativity would force him to give up this constancy. He struggled to find a modification of Maxwell's theory in which the speed of light would vary according to speed of the emitter. After many fruitless attempts, Einstein finally realized that he could find no sustainable emission theory of light. Maxwell's theory and the constancy of the speed of light must stand. It was a point of desperation for him. How could he keep both the principle of relativity and the constancy of the speed of light?

Einstein later recalled in his *Autobiographical Notes* how he finally solved the problem:⁷ Today everyone knows, of course, that all attempts to clarify this paradox satisfactorily were condemned to failure as long as the axiom of the absolute character of time, or of simultaneity, was rooted unrecognized in the unconscious. To recognize clearly this axiom and its arbitrary character already implies the essentials of the solution of the problem.

To solve his problem, Einstein had to see what everyone before him had missed: that the absoluteness of simultaneity is an assumption that can be challenged. And further he needed something to give him the courage to mount that challenge and abandon the assumption. Einstein continued the above remarks by noting that this essential support came from his reading in philosophy:

The type of critical reasoning required for the discovery of this central point was decisively furthered, in my case, especially by the reading of David Hume's and Ernst Mach's philosophical writings.

Einstein affirms here *that* reading Hume and Mach's philosophical writings were decisive. However he does not tell us *how* they were decisive or even *which* writings were at issue. It has

been natural to assume that it was Hume and Mach's writings specifically in philosophy of space and time.

When we begin to explore Einstein's other writings and remarks, another possibility emerges. In 1924, Einstein remarked⁸

After seven years of reflection in vain (1898–1905), the solution came to me suddenly with the thought that our concepts and laws of space and time can only claim validity insofar as they stand in a clear relation to experiences; and that experience could very well lead to the alteration of the concepts and laws. By a revision of the concept of simultaneity into a more malleable form, I thus arrived at the special theory of relativity.

The same idea is given more succinctly in a remark from Einstein's 1917 popular account of relativity theory⁹

The concept [of simultaneity] does not exist for the physicist until he has the possibility of discovering whether or not it is fulfilled in an actual case.

The breakthrough was not grounded in some novel philosophical insight into space and time specifically. Rather it was a general view about how concepts are properly employed in physical theories.

The concepts of physical theories must, Einstein here asserts, be properly grounded in experience; else they are fictions. Once one has this clue, one recalls immediately that just this sort of empiricist approach to concepts is fundamental to the thought of Hume and Mach and one can see that is this aspect of their writing to which Einstein referred. The analysis of David Hume's *Treatise* depends on just this simple grounding of concepts ("ideas") in experience ("impressions"). The introductory section concludes with the synoptic assertion:¹⁰

...all our simple ideas proceed either mediately or immediately, from their correspondent impressions.

This then is the first principle I establish in the science of human nature... Later he makes clear that concepts cannot extend beyond this grounding in experience without introducing a fiction. For example, he writes:¹¹

Ideas always represent the objects or impressions from which they are deriv'd, and can never without a fiction represent or be appl'd to any other...

One finds a similar empiricist approach to concepts in the writings of Ernst Mach. More relevantly, we know that Einstein found it in Mach, for Einstein tells us just this in his obituary for Mach¹²

Science is, according to Mach, nothing but the comparison and orderly arrangement of factually given contents of consciousness, in accord with certain gradually acquired points of view and methods....

...concepts have meaning only in so far as they can be found in things, just as they are the points of view according to which these things are organized. (Analysis of concepts)

What is important is that this empiricist approach to concepts is quite general. It is not limited to the analysis of space and time, but applies to all concepts. Most famously, Hume applied it to causation.

All this is only the beginning of a fascinating tale. Einstein elsewhere averred that it was Hume “still much more” than Mach who guided him; and we find some differences in the way Einstein was willing to accept fictional concepts not properly grounded in experience in his theorizing as conventions.¹³

3. “An (Inherent) Epistemological Defect”

In his analysis of 1905, Einstein eliminated the ether state of rest from physics and reinstated the relativity of motion only as far as inertial motion, that is, uniform motion in a straight line. Over the ensuing decade, Einstein sought a new theory that would extend the principle of relativity to all motion, including accelerated motion. Einstein believed that he had achieved this in 1915 with the completion of his general theory of relativity.

What is important for our purposes are the motivations Einstein reported for seeking this extension of the principle of relativity. In 1916, Einstein published a definitive review article of the completed theory. In an early section, “§2 The Need for an Extension of the Postulate of Relativity,” Einstein gives what is surely a type of reason that is rarely found stated explicitly in the physics literature:¹⁴

In classical mechanics, and no less in the special theory of relativity, there is an inherent epistemological defect with was, perhaps for the first time, clearly pointed out by Ernst Mach.

Einstein's German—*ein erkenntnistheoretischer Mangel*--was a little weaker than the standard Perrett and Jeffrey translation given here and is captured more literally merely as “epistemological defect.” However I have reported the stronger Perrett and Jeffrey translation since that it has been in the standard edition of Einstein's paper since its 1923 translation; and I like to pretend that it captures the passionate energy of its author, a barely 37 year old Einstein at the moment of his greatest scientific creativity.

Either way, it is an extraordinary idea. Our best theory of gravity and Einstein's greatest contribution to modern physics is motivated in part by the need to remedy an epistemological defect of earlier theories!

Einstein proceeded to explain the problem. Both classical physics and special relativity posit certain preferred inertial motions. These were the uniform straight-line motions followed by free bodies, unaffected by perturbing forces. These motions in turn define inertial spaces of reference; they are, loosely speaking, the spaces carried with each set of bodies moving together inertially. So-called “inertial forces” arise if a body is constrained to accelerate, that is, to deviate from inertial motions. Newton¹⁵ imagined water swirling in a bucket and the resulting acceleration led the water to be hurled outward and climb up the wall of the bucket, producing a concave water surface. Analogously, fluid spheres in rotation, such as stars and planets, bulge at their equators.

What causes this bulge, Einstein asked. We are, he noted, inclined to answer that the cause is rotation with respect to inertial spaces. This answer is rejected thunderously:

No answer can be admitted as epistemologically satisfactory, unless the reason given is an *observable fact of experience*. The law of causality has not the significance of a statement as to the world of experience, except when *observable facts* ultimately appear as causes and effects. [Einstein's emphasis]

He continued a few sentences later:

...the privileged [inertial] space of ... Galileo thus introduced, is merely a *factitious* [“bloss fingierte” = ad hoc, JDN] cause, and not a thing that can be observed. (Einstein's emphasis)

Then Einstein turned to the cause that he would accept, distant masses and their motions. He thereby foreshadowed the form that he hoped his final theory would take. In it, nothing intrinsic to a space distinguishes one space from another. The discrimination of spaces into inertial and accelerating comes only by virtue of the masses distributed in them. If the masses of the universe are at rest in a space, it is an inertial space. If those masses swirl around, it is a space with inertial forces that pull water up the sides of Newton's bucket and lead fluid bodies to bulge at their equators.

The analysis is driven by Einstein's conception of an epistemological defect. In his popular account of relativity, written at the end of 1916, Einstein gave a more prosaic and visceral illustration of it:¹⁶

I am standing in front of a gas range. Standing alongside of each other on the range are two pans so much alike that one may be mistaken for the other. Both are half full of water. I notice that steam is being emitted continuously from the one pan, but not from the other. I am surprised at this, even if I have never seen either a gas range or a pan before. But if I now notice a luminous something of bluish colour under the first pan but not under the other, I cease to be astonished, even if I have never before seen a gas flame. For I can only say that this bluish something will cause the emission of the steam, or at least possibly it may do so. If, however, I notice the bluish something in neither case, and if I observe that the one continuously emits steam whilst the other does not, then I shall remain astonished and dissatisfied until I have discovered some circumstance to which I can attribute the different behaviour of the two pans.

It is hard for a philosopher to read this and not see an account here of the violation of a venerable principle, Leibniz's principle of sufficient reason.¹⁷ This, however, was not Einstein's reading. He proceeded to assert that it was "E. Mach [who] recognised [the epistemological defect of prior theories] most clearly of all..."

Einstein clearly had in mind Mach's celebrated analysis of Newton's notions of absolute space and time in his *Science of Mechanics*, including his famous remark on Newton's bucket:¹⁸

Newton's experiment with the rotating vessel of water simply informs us, that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces are produced by its relative

rotation with respect to the mass of the earth and the other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick. The one experiment lies before us, and our business is, to bring it into accord with the other facts known to us, and not with the arbitrary fictions of our imagination.

Einstein has left us in little doubt as to how he read Mach's critique. His first published statement of what he later dubbed "Mach's Principle" came in 1912 when Einstein had developed only a rudimentary forerunner to his general theory of relativity. The statement asserts:¹⁹

...the entire inertia of a point mass is the effect of the presence of all other masses, deriving from a kind of interaction with the latter.

Lest there be any doubt as to the origin of the idea, Einstein—notorious for his meager citation habits--appended a footnote to the section of *Science of Mechanics* in which the above bucket quote appears:

This is exactly the point of view which E. Mach urged in his acute investigations on the subject. (E. Mach, *The Development of the Principles of Dynamics*. Second Chapter. *Newton's Views of Time, Space and Motion*.)

Einstein's reading of Mach's remark is curious. Mach asserts (my emphasis) "*No one is competent to say* how the experiment would turn out if the sides of the vessel increased in thickness..." Yet Einstein took this as a license to say just what would happen. Were the walls of bucket so enlarged and set into rotation, they would drag the water in the bucket slightly. This dragging would be a massively weakened version of what Einstein believed happens when all the masses of the universe rotated around the bucket.

Einstein sought to derive these "Machian" effects in his developing theories of gravity of Einstein prior to the completion of general relativity.²⁰ They are recovered in various forms in the final theory as well, as Einstein explains in his *Meaning of Relativity*, the closest Einstein came to writing a textbook for his theory.²¹

Matters did not continue as one might expect. Einstein later came to renounce his fascination with Mach's critique. Writing in 1946 in his "Autobiographical Notes," he reflected:²²

...in my younger years, however, Mach's epistemological position also influenced me very greatly, a position that today appears to me to be essentially untenable. For

he did not place in the correct light the essentially constructive and speculative nature of all thinking and more especially of scientific thinking; in consequence, he condemned theory precisely at those points where its constructive-speculative character comes to light unmistakably, such as in the kinetic theory of atoms.

It was also never clear that the general theory of relativity did meet the Machian-inspired demands concerning the origin of inertia. Eventually Einstein withdrew his support for these demands, as he noted again later in his “Autobiographical Notes”:²³

Mach conjectures that in a truly reasonable theory inertia would have to depend upon the interaction of the masses, precisely as was true for Newton's other forces, a conception that for a long time I considered in principle the correct one. It presupposes implicitly, however, that the basic theory should be of the general type of Newton's mechanics: masses and their interaction as the original concepts. Such an attempt at a resolution does not fit into a consistent field theory, as will be immediately recognized.

Finally it remains unclear that the critique of absolute space Einstein read in Mach's writings is the one Mach intended. Einstein found the critique as authorizing the search for a new theory of inertia, whereas Mach may merely have intended it to support an austere formulation of an otherwise unaltered classical physics, everywhere purged of the mention of metaphysical notions, such as Newton's absolute space.²⁴

4. The Real

A perennial theme in philosophy is the separation of reality from appearance. Present day physics is replete with techniques that effect this separation. They are associated with the notions of invariance, symmetry and gauge transformations, whose lineage in physics traces back to Einstein's work. A century ago, his theories of relativity demonstrated the same reality can have very different appearances in different frames of reference. However the idea of using group theory and distinguishing the real as the invariants of the transformations of groups is a nineteenth century notion. It was a commonplace of geometry before it was brought into twentieth century physics, in large measure through the stimulus of Einstein's theories of relativity.

These are broad themes. My concern in this section, however, is two narrow episodes concerning the real. In them, Einstein sought to resolve a pressing problem in physics by positing what we might call a reality principle; that is, a principle that separates reality from appearance.

4.1 The Point-Coincidence Argument

The first of these episodes arose with the completion of the general theory of relativity. We saw that Einstein's initial concern was to implement a generalized principle of relativity that extended to accelerated motion. By 1916, that demand had evolved into a requirement of general covariance. To see what it amounts to, we should recall that spacetime theories label events with four numbers. They are usually three spatial coordinate and one time coordinate. However one can make new numerical labels for any event by adding, subtracting or taking any combination of the more traditional choices of the event's coordinates, and any rescaling of them. These manipulations create arbitrarily many more spacetime coordinate systems. If one has a physical theory that can employ any of these coordinate systems, no matter how jumbled and rescaled, then the theory is generally covariant.

The central conception of Einstein's general theory is a connection between gravitation and the curvature of the spacetime geometry. His decision to seek a generally covariant theory was pivotal. It enabled Einstein to draw on the elaborate body of mathematical techniques emerging from the nineteenth century for understanding curvature. As long as he kept his equations generally covariant, this body of mathematics admitted remarkably few possibilities for the implementation of his theory. That fact is routinely used today in motivating Einstein's theory.

Hence it can come as a surprise to modern readers to learn that Einstein considered and rejected general covariance in 1913. Then he and his mathematician friend, Marcel Grossmann, published a sketch of what was the general theory of relativity in all its parts, excepting its most essential part.²⁵ That was its gravitational field equations, the theory's analog of Newton's inverse square law of gravitation. They announced that they had been unable to find physically admissible generally covariant gravitational field equations. In place of these equations, they published gravitational field equations of limited covariance.²⁶

In a little over two years, Einstein would recognize this rejection of general covariance for the catastrophe it was. However, before then, Einstein turned his powers towards making a

bad situation worse. If he could not find admissible generally covariant gravitational field equations, then he would demonstrate that omission no failing, for he would prove that such equations are physically uninteresting. What resulted was his “hole argument.”²⁷ It was published in four versions in 1913 and 1914. One was in an addendum to his joint paper with Grossmann²⁸. The most complete version was in a 1914 review article.²⁹

General covariance gives the theorist the power to represent the one physical reality in many different coordinate systems. What Einstein found, however, was that it also permitted a reverse capacity. One could fix the coordinate system and induce many apparently distinct physical realities in it and this could be done in a way that seemed to compromise determinism.

The essential idea can be conveyed in an analogy to different map projections. One sheet of paper can host many projections of the world. One of the oldest and best known is the Mercator projection of 1569 of Figure 1.

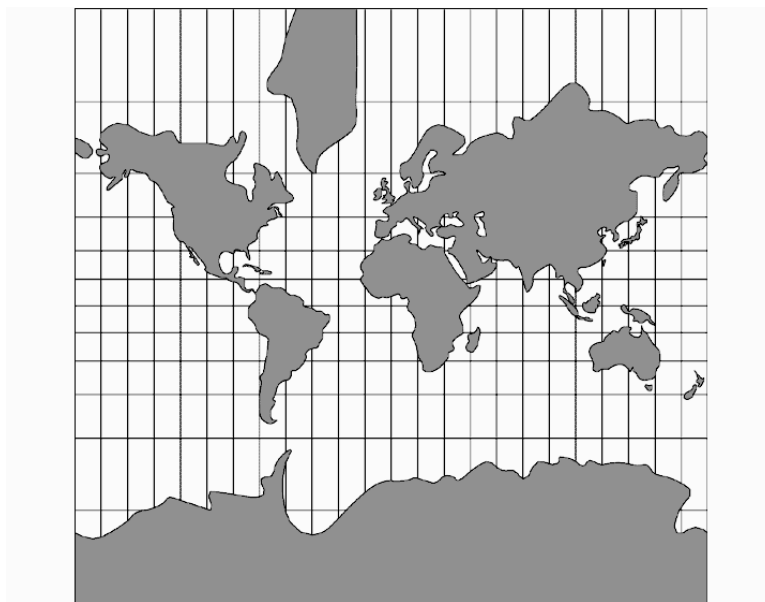


Figure 1. Mercator Projection³⁰

Many more projections are possible. Another is the Lambert projection of 1772 of Figure 2.

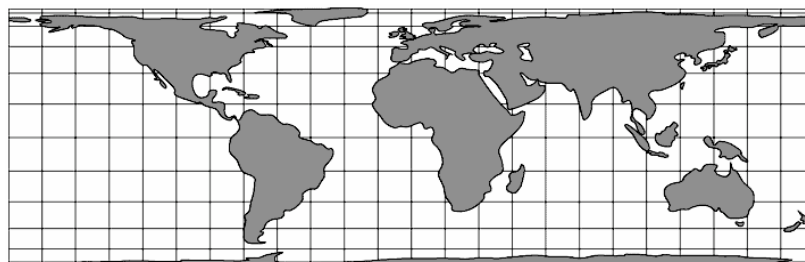


Figure 2. Lambert Projection

The continents in the two projections look rather different. Antarctica in the Mercator projection looks enormous in comparison to Antarctica in the Lambert projection. We know, of course, that this difference is purely an artifact of the different projections and represents nothing real.

Imagine, however, that one did not realize that the differences were artifactual. One would then imagine that these are maps of two different worlds: the Mercator world, with its enormous Antarctica, and the factually distinct Lambert world, in which Antarctica is a mere sliver. Worse, we can construct further factually distinct hybrid worlds. The Southern hemisphere may be extracted from the Mercator projection and the Northern from the Lambert projection. That would be yet another world depicted in Figure 3.

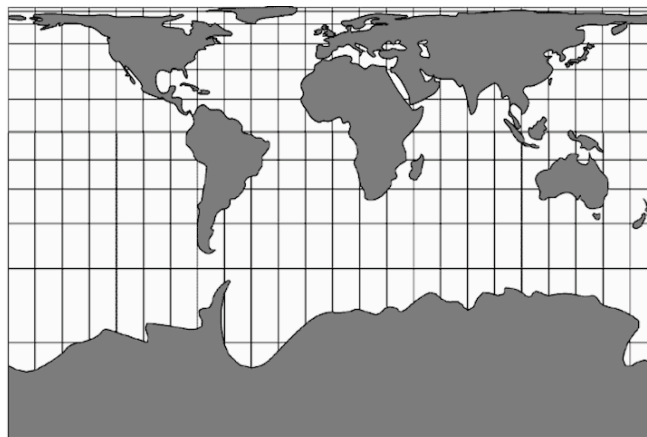


Figure 3. Hybrid Mercator-Lambert Projection

One would then be faced with an odd problem if one is trying to determine which of the Mercator or Lambert worlds is our world. We might check the world everywhere in the Southern hemisphere and find that everywhere in the Southern hemisphere our world conforms precisely with the Mercator world. We still could not know whether our world *in toto* is the Mercator world, for it could extend into the hybrid world of Figure 3 in which the Northern hemisphere is Lambertian. Perhaps that is our world. Even though we are dealing with a limited set of atlases, the geography of the Southern hemisphere does not fix the atlas that applies to the Northern hemisphere.

This structure is essentially the one Einstein implemented in his hole argument. The sheet of paper that carries the different map projections corresponds to Einstein's spacetime coordinate system. The figures printed on the paper correspond to the fields of Einstein's theory. General covariance was the license that permitted him to spread his fields over the one spacetime

coordinate system with the same freedom as we have in moving between map projections. In 1913, Einstein believed that the different spreadings of the fields corresponded to factually distinct realities. If we imagine that the Southern hemisphere of our map corresponds to the past of Einstein's spacetime and the Northern hemisphere to its future, the problem becomes a failure of determinism. Fixing the past of the spacetime does not fix its future.

What of the "hole"? Einstein realized that there was a sharpened version that was even more troubling. In the map analogy, instead of grafting hemispheres of different projections together, one could perform the grafting in just a small portion of the page. We might have a projection that is everywhere Mercator, except in the region holding Australia, into which we graft some other projection, being sure to smooth the join neatly, as shown in Figure 4.

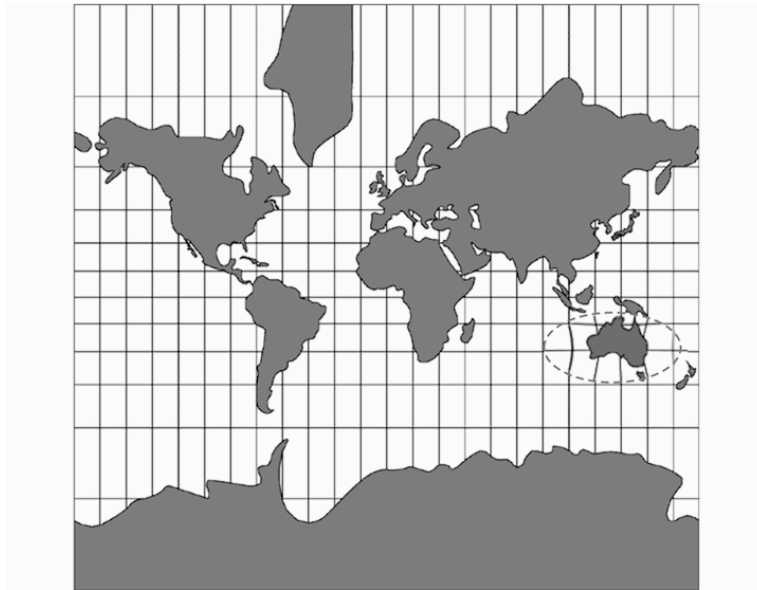


Figure 4. Mercator Projection with a Distorted Hole

Allowing this possibility means that knowing the geography everywhere except Australia fails to fix the geography of Australia.

In Einstein's theory, the corresponding construction realizes a more severe failure of determinism. The hole corresponds to a small region of space that persists for a short time. It might just be a volume of space the size of a basketball that lasts for a second. Fixing everything in spacetime in its past, in its future and in all other parts of space throughout the universe still fails to fix what happens inside the hole. In 1913 and 1914, this result satisfied Einstein. It assured him that the generally covariant equations he could not find would not be worth finding. They would visit a radical form of indeterminism on his theory.

Matters had changed by the end of 1915. Then Einstein had returned to general covariance and had proposed the generally covariant gravitation field equations for which he became famous, the “Einstein field equations.” Clearly some repair work was needed. Einstein now saw that, if he paid careful attention to what is real and what is not, the hole argument establish neither indeterminism and nor the physical inadmissibility of a generally covariant theory.

The map analogy lets us see how this comes about. While we have the freedom to spread our pictures of the continents in many ways over the one sheet of paper, there are no factual differences in geography to be found in the different spreadings.

The town of Alice Springs lies in the heart of Australia. Its exact placement on the sheet of paper will differ in the different map projections of the hole. However none of these differences will translate into verifiable geographical facts. Alice Springs lies roughly North of Adelaide on the South coast of Australia. Someone who leaves Adelaide and drives about 1500 km Northward along the highways, mostly the Stuart Highway, will arrive there. The drive will intersect that of someone who leaves Darwin on the North coast of Australia and drives roughly 1500 km Southward, again much of it on the Stuart Highway. The two projections will agree that the two travelers will meet at Alice Springs, as they will agree upon any other matter of geography that one can check. What is outside the hole fails to determine some aspects of what is within. But any aspects that remain undetermined are purely artifacts of the different projections; no geographic fact that can be checked is left uncertain.

Einstein’s resolution was essentially identical to this analysis in geography, but with the claims carried over into spacetime theory. He wrote in his 1916 review article:³¹

All our space-time verifications invariably amount to a determination of space-time coincidences. If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meetings of two or more of these points. Moreover, the results of our measurements are nothing but verifications of such meetings of the material points of our measuring instruments with other material points, coincidences between the hands of a clock and points on the clock-dial, and observed point-events happening at the same place at the same time.

This is the “point-coincidence” argument.³² Its immediate purpose is to establish the conclusion that the factual content of a physical theory is exhausted by the catalog of spacetime coincidences that it licenses. The different spreadings of the fields over the one coordinate system in the hole argument agreed in all spacetime coincidences. It now followed that any differences between the spreadings were purely artifactual; they correspond to no factual differences. The threat of the hole argument is averted.

The possibility of the multiple spreadings of fields had been inferred by Einstein from the requirement of general covariance, that is, the requirement that a physical theory can use any spacetime coordinate system. The hole argument had been deployed to demonstrate the untenability of general covariance. Einstein’s position was now reversed. He was advocating general covariance strongly. The point-coincidence argument was called to his aid and it was extended into an argument for general covariance. The extension depended upon the fact that the two spreadings of the fields differ only in the spacetime coordinates at which the various spacetime coincidences may be found. Hence the assigning of spacetime coordinates is purely artifactual and has no physical content.³³ “The introduction of a system of reference [coordinate system] serves no other purpose than to facilitate the description of the totality of such coincidences,” Einstein wrote immediately after the quote above. It follows that we should be free to use any coordinate system we like.

In his review article, Einstein did not make clear that this point-coincidence argument was his answer to the hole argument. It introduced as an argument for general covariance. We can understand that Einstein would be reluctant to call any further attention to an argument that he had published four times but now deemed erroneous. That the point-coincidence argument was explicitly intended to resolve the hole argument was made clear when Einstein explained to correspondents why he felt authorized to restore the demand of general covariance. For our purposes, these explanations are interesting because they contain strong statements about the division between what is real and what is not.

On January 3, 1916, he wrote to his friend, collaborator and confidant, Michele Besso:³⁴ Nothing is physically *real* but the totality of space-time point coincidences. If, for example, all physical happenings were to be built up from the motions of material points alone, then the meetings of these points, i.e. the intersection of their world

lines, would be the only real things, i.e. observable in principle. (Einstein's emphasis)

A letter from Einstein to Ehrenfest of December 26, 1915, contains similarly strong assertions concerning what is real:³⁵

The following considerations should replace [the 1914 presentation of the hole argument]. The physically real in what happens in the world (as opposed to what depends on the choice of the reference system) consists of *spatio-temporal coincidences*. [Einstein's footnote here: "and in nothing else!"] For example, the points of intersection of two world lines, or the assertion that they *do not* intersect, are real. Such assertions referring to the physically real are thus not lost because of any (single-valued) coordinate transformations.

This correspondence is also helpful in that it affirms that the map analogy captures Einstein's understanding. The analogy is close to one Einstein used in explaining the point-coincidence argument to Ehrenfest in another letter to him from late December 1915. Einstein took Ehrenfest's example of light from a star passing through an aperture to strike a photographic plate and then considered how the fields representing the system might be spread differently over a spacetime coordinate system. Einstein used a homely construction to illustrate how the resulting solutions of the field equations—Ehrenfest's A and B—are related. Ehrenfest was to trace a drawing of the system onto deformable paper. The different spreadings were then produced merely by deforming the paper. Is there any factual difference between the two resulting figures? Einstein continued:³⁶

What is essential is this: As long as the drawing paper, i.e. "space," has no reality, the two figures do not differ at all. It is only a matter of "coincidences," e.g., whether or not the point on the plate is struck by light. Thus, the difference between your solutions A and B becomes a mere difference of representation, with physical agreement.

In all this, what is intriguing to a philosopher is to see that strong pronouncements concerning the real figure prominently in a major scientific discovery. Taken in isolation they are strong, even programmatic announcements of a fundamental principle: "Nothing is physically *real* but the totality of space-time point coincidences." and "The physically real consists of *spatio-temporal coincidences*. (and in nothing else!)" Even more striking is the strongly

verificationist tone that underpins them. The real and the observable appear to be identified (“the only real things, i.e. observable in principle.”) As a result, it has proven tempting to philosophers to regard these claims by Einstein as the anticipation of a grander verificationist view of science and one that, perhaps, should be regarded with suspicion for its extremism.³⁷

We should approach that portrait of Einstein with caution. The Einstein who wrote these words was not the armchair philosopher grappling with the problem of reality and appearance at the most abstract level. The Einstein who wrote these words was jubilant at his great success with a scientific theory, general relativity, but physically and emotionally exhausted. He had struggled for nearly three years with his extraordinary new theory of gravity, in imperfect and misshapen form. Its defects were now finally identified and eliminated. The struggle was over. All that remained was for Einstein to correct the errors of these three past years. One senses his weariness when he introduces his explanation of the error of the hole argument to Ehrenfest in his letter of December 26 1915.³⁸

It is comfortable for Einstein. Each year he retracts what he wrote the previous year; now my duty in the extremely sad business of justifying my most recent retraction. Einstein’s proclamations about reality are not forward-looking, the anticipation of a new tradition of verificationism in philosophy. They are backward-looking, a convenient device that brings closure to an episode that Einstein now finds painful. In these circumstances, it is not surprising that Einstein would fail to give his pronouncements the pedantic precision that characterizes careful analysis by a professional philosopher.

Don Howard³⁹ has identified a significant and pertinent conflation. Are Einstein’s coincidences “point-coincidences” or “pointer-coincidences”? The former are mathematical abstractions akin to idealized Euclidean points in geometry. They may be designated as the real in the sense that they are invariant, which means that they remain the same in all spacetime coordinate system. The latter are the coincidences of macroscopic objects, such as pointer needles and scale marks. They are observable and hence real. Einstein’s writing runs the two together. However the first, point-coincidences, is what Einstein needed to deflect his hole argument and restore general covariance. The second, pointer-coincidences, is what the ensuing tradition in verificationism needed to read in Einstein’s writing if it was to claim him as their patron.

4.2 The Incompleteness of Quantum Theory

There is another, better known criterion of reality associated with Einstein. It too arose in the context of a significant problem in physical theory: Einstein's critique of quantum theory.

The Einstein of 1916 was the discoverer of general relativity. He was the theorist who stood in the vanguard of new work in physics. This was the same Einstein who had, in 1905, proposed the revolutionary concept of the light quantum. Now, in 1916, he continued his contributions to quantum theory with his "A and B coefficient" quantum analysis of heat radiation, laying the grounding for the modern theory of lasers.

The Einstein of 1926, a decade later, was drifting into a different role. The energy of physics had been drawn into the emergence of the so-called "new quantum theory." In Schrodinger's famous formulation, each particle of the new theory was associated with a wave. Since a wave is spread out in space, one could associate no definite position with the particle, even though, on position measurement, the particle would always manifest in a definite position. Correspondingly, in general, a particle has no definite momentum, but it will always manifest a definite momentum upon momentum measurement. The best the new theory could provide was the probability that particle would manifest in this position or with that momentum on the corresponding measurement.

Einstein joined his colleagues in recognizing that this new quantum theory was a worthy achievement that resolved accumulating difficulties of the "old quantum theory." However Einstein resisted one aspect of it resolutely. Does the quantum wave associated with a particle provide a complete description of the particle? Or are there further facts about a particular particle that is not expressed in the wave? The mainstream adopted the first view. Einstein urged the second view, incompleteness. A full accounting of why Einstein found himself a critic of the mainstream view of completeness would require a discussion of his reluctance to admit the arcane possibilities of non-locality and non-separability.⁴⁰

However, in seeking the grounding of Einstein's discomfort, one cannot overlook his much repeated quip that God does not play dice.⁴¹ If the quantum wave provides a complete description of the state of a particle, then quantum theory is indeterministic. Fixing the full state of the present does not fix the future. If we fix the quantum state of particle now, the best we can recover for the future are merely probabilities for the particle being measured in this or that position. We have seen that Einstein recoiled from indeterminism when it was threatened in the

hole argument. Einstein then described this failure of determinism as a failure of the “law of causality.”⁴² That reveals a decidedly nineteenth century aspect to Einstein’s thinking, for in the nineteenth century, causality was purged of all its embellishments and reduced to the simple notion of determinism. Hence the failure of determinism would be viewed as a failure of causality itself. This orientation is reflected in the general description in the earlier part of the twentieth century of the indeterminism of quantum theory as a violation of causality, a failure that many find inherently troubling. Einstein, it would appear, was sufficiently nineteenth century in his thinking to find the indeterminism of quantum theory an unacceptable violation of causality.⁴³

While the origins of Einstein’s discomfort with quantum theory may be diffuse, there was a single argument that Einstein favored as the way to establish the theory’s incompleteness. The best-known presentation of the argument was given in his co-authored paper with Boris Podolsky and Nathan Rosen, the celebrated “EPR” paper.⁴⁴

Establishing that the quantum wave provides an incomplete description of a particle is not straightforward. If one measures the position or momentum of a particle, under the standard account, its wave collapses to a new state with a definite position or momentum, according to the measurement undertaken. The old state is destroyed and one cannot preclude the possibility that the definite measurement outcome was created by the measurement process itself. Direct measurement no longer necessarily reveals the properties of particles possessed prior to measurement.

What the EPR paper recognized was that indirect measurement should succeed in revealing the properties really possessed by a particle, where direct measurement may fail. In classical physics, if two qualitatively identical particles are flung symmetrically from some central explosion, they will carry duplicate properties. When the first particle has moved some distance—say 100 m to the left—the other particle will have moved the same distance—100 m—to the right. Therefore measuring the position of one particle will reveal the position of the other. Similarly, since momentum is conserved, the momentum of one particle is simply the negation of the momentum of the other. So measuring the momentum of one particle will tell us the momentum of the other. It turns out that quantum theory allows similarly correlated particles in what is known as an entangled state. So the same indirect measurements are possible for quantum particles.

Once that fact is recognized, the remainder of the analysis is straightforward. We prepare two entangled particles as above. We know that were we to perform a position measurement on one particle, we could recover a definite position for it. Hence we would know that the other remote particle would reveal the corresponding, definite position on measurement. Since our measurement here cannot affect the remote particle some great distance away, we know that position property is possessed by the remote particle and not created by the measurement operation. That is already enough to establish the sought after incompleteness, for, in general, quantum theory does not allow a single definite position for the particle. The argument can then also be repeated for the momentum of the particle.

This simple and beautiful argument depends upon some apparently innocuous assumptions. One must assume that a measurement here cannot instantly affect a remote particle there; this is an assumption of locality. One must also assume that the two particles, once separated spatially, have independent existences with their own definite properties; this is an assumption of separability. The two assumptions must be made if we are to infer from measurements on local particles to the real properties possessed by distant particles entangled with them. The EPR authors recognized that assumptions along these lines were being made. While harboring no evident doubts, they asserted their version of the assumptions clearly in italic text at the start of the EPR paper:

If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

This is the famous criterion of reality from the EPR paper of 1935. It is the foundation of the argument mounted by EPR and Einstein writing as a single author for the incompleteness of quantum theory.

All this is just the beginning of a long saga. The EPR paper failed to move the mainstream of physics. Einstein's antagonist, Niels Bohr, wrote a rebuttal that, sadly, was as obscure as it was influential. The EPR analysis did not receive the response it deserved until the work of John S. Bell. He demonstrated that no theory that agrees empirically with quantum theory could preserve locality and separability. Since these last two notions were, one way or another, necessary for the EPR reality criterion, Bell's work forced a choice between abandoning

the empirical adequacy of quantum theory or abandoning the assumptions needed for the EPR criterion of reality. The mainstream of physics has chosen to abandon the latter.⁴⁵

5. The Power of Platonism

Most of the examples of the philosophy we have seen so far in Einstein's work have empiricist, positivist or verificationist underpinnings. Hence we might conclude that Einstein's commitments are to empiricist and verificationist principles. To some extent, that was true. Yet, however strong these commitments may have been, they were subordinate to a deeper commitment. It was simple commitment to whatever ideology best led him to new theories. Hence, we should expect Einstein's commitments to empiricism and related approaches to be negotiable and even dispensable. And they were. Platonism is anathema to an empiricist. Yet, in the episode to be recounted, Einstein quite gladly adopted a mathematical Platonism when he sensed that it might be a more fertile aid to him in generating new theories.

During the research that led up to his discovery of the general theory of relativity, Einstein reflected explicitly on the methods he was using.⁴⁶ On the one side, he identified a physical approach. It took as the guide to new theories physical principles that were usually closely grounded in experience. They included the principle of relativity of his special theory and the principles of conservation of energy and momentum. The physical approach also gave special weight to limiting cases whose content is assured by physical reasoning. For weak gravitational fields, for example, his new theory had to replicate Newtonian theory. The purest embodiment of the physical approach came in the thought experiments for which Einstein was famous. In them, our physical sensibilities would direct us inexorably towards a particular outcome.

This approach was contrasted with a formal or mathematical approach. According to it, we are guided to new theories by the mathematical properties of the structures involved. That transformations must form a group in the mathematical sense can be a powerful restriction. So also is the requirement of covariance, that the equations of the theory preserve their form in transformations among some stipulated range of spacetime coordinate systems. In this approach, mathematical theorems can reduce the viable theories to a very small selection. The purest embodiment of the approach is the use of formal naturalness and mathematical simplicity as a guide in theory selection.

Einstein's early inclinations had been strongly towards the physical approach, along with a discomfort and even distaste for the formal approach. This distaste is still evident in 1912 when he was turning to devote his attentions more fully to the developing general theory of relativity. One episode reveals it clearly. A gravitation theory that competed with Einstein's was formulated by Max Abraham by the simple expedient of transporting Newton's theory of gravity in special relativity in the mathematically simplest way. Einstein immediately proclaimed his displeasure with the theory to his correspondents, denouncing it as "totally untenable," "incorrect in every respect," and more. Most revealing, however, was that he was prepared to condemn the theory precisely because it had followed the formal approach. "...the thing is probably wrong." he wrote to Heinrich Zangger on 27 January 1912, "This what happens when one operates formally, without thinking physically."⁴⁷

After his move to Zurich in August of 1912, Einstein became embroiled in the new mathematical methods needed by his developing theory. With the assistance of a friend, the mathematician Marcel Grossmann, he began to learn the absolute differential calculus of Ricci and Levi-Civita (now called "tensor calculus"). As he developed his theory, Einstein continued to use his well-worked physical approach. However he could not ignore how the mathematical tools supplied by Grossmann offered certain natural structures from which to build his theory. Most importantly, it provided the Riemann curvature tensor as the structure that described the curvature of spacetime and from which the gravitational field equations should be built.

We are fortunate to have an intimate window onto Einstein's deliberations at this crucial time in the development of general relativity. His research notes have survived in the form of the "Zurich Notebook."⁴⁸ In it we can trace his earliest efforts to relate gravity and spacetime curvature and their ultimate fate. Einstein's expectation was that he could apply both approaches, physical and formal, and that they would agree. However, as his investigations proceeded, Einstein failed again and again to secure the agreement. Eventually he was forced to a choice: should he accept the results of the physical approach or those of the formal approach? Fatefully, Einstein chose in favor of the physical approach, abandoning general covariance. He struggled for nearly three more years with the resulting misshapen theory.⁴⁹

Einstein's return to general covariance became a matter of public record in November 1915. Then he published four papers, one each week, with a series of proposals for gravitational field equations, based on the Riemann curvature tensor. It was a difficult month, made all the

more tense by the knowledge that David Hilbert in Goettingen, perhaps the greatest mathematician of the age, was in that same month working and publishing on the gravitational field equations of Einstein's theory. The month closed with the greatest achievement of Einstein's career. He completed the general theory of relativity and, in the process, received a welcome affirmation of its correctness. A jubilant Einstein found that his perfected theory could now account precisely for the anomalous motion of Mercury.

As that fateful month began, Einstein made no secret that his advances derived from a reversion to the mathematical or formal approach. In the first of the four papers, he reported:⁵⁰

I completely lost trust in the field equations I had chosen and looked for a way to restrict the possibilities in a natural manner. Thus I went back to the requirement of a more general covariance of the field equations, which I had left only with a heavy heart when I worked together with my friend Grossmann. In fact we had then already come quite close to the solution of the problem given in the following.

Einstein makes a similar report to Sommerfeld in correspondence at the end that month:⁵¹

Once all trust in the results and methods of the earlier theory had gone, I saw clearly that a satisfactory solution could only be found through a connection to the general theory of covariants, i.e. to Riemann's covariant.

For our purposes, what is striking is just how willing a formerly scornful Einstein was to heap praise upon the fertility of the mathematical approach. He wrote in the first of the papers of November 1915:⁵²

Hardly anyone who has truly understood it can resist the charm of this theory; it signifies a real triumph of the method of the general differential calculus, founded by Gauss, Riemann, Christoffel, Ricci and Levi-Civita.

Einstein drew an important moral from this experience. Had he only taken the formal approach more seriously at the start, he would have spared himself much suffering. This moral entered into Einstein's methods. His subsequent search for a unified field theory depended essentially on seeking the mathematically simplest equations.

By the time of Einstein's 1933 Herbert Spenser lecture "On the Methods of Theoretical Physics," Einstein's (1933) advocacy of mathematical Platonism is explicit and powerful. There he wrote the following.⁵³

Our experience hitherto justifies us in believing that nature is the realisation of the simplest conceivable mathematical ideas. I am convinced that we can discover by means of purely mathematical constructions the concepts and the laws connecting them with each other, which furnish the key to the understanding of natural phenomena. Experience may suggest the appropriate mathematical concepts, but they most certainly cannot be deduced from it. Experience remains, of course, the sole criterion of the physical utility of a mathematical construction. But the creative principle resides in mathematics. In a certain sense, therefore, I hold it true that pure thought can grasp reality, as the ancients dreamed.

One should not mistake these words for the abstract musings of an armchair philosopher. They are the mature reflections of a philosophically sophisticated Einstein, reporting the methods that had worked in his earlier researches and that he hoped would lead him to his unified field theory.

6. Conclusion

Albert Einstein took philosophy seriously. He read it; he wrote it; and he engaged in exchanges with the leading philosophers of his time. Most importantly, philosophical analysis was incorporated directly into his theorizing in physics. He was a physicist who could use the word “epistemological” in a physics paper. However Einstein was not a philosopher. His concern was physics and his allegiance was to whatever instrument would advance his theorizing. Hence, taken in isolation, Einstein’s philosophical commitments may appear capricious, changing at whim. But that is a short-sighted appraisal. It merely reflects that the Einstein who wrote philosophy was not a dogmatic philosopher who would defend his system come what may. Rather, it reflects an Einstein who used philosophy pragmatically for other purposes.

One might find this assessment of Einstein’s philosophical commitments slighting. However Einstein was quite self-aware and it is, I believe, his own assessment. Responding to critics later in life in early 1949, he wrote:⁵⁴

... no sooner has the epistemologist, who is seeking a clear system, fought his way through to such a system, than he is inclined to interpret the thought-content of science in the sense of his system and to reject whatever does not fit into his system. The scientist, however, cannot afford to carry his striving for epistemological

systematic that far. He accepts gratefully the epistemological conceptual analysis; but the external conditions, which are set for him by the facts of experience, do not permit him to let himself be too much restricted in the construction of his conceptual world by the adherence to an epistemological system. He therefore must appear to the systematic epistemologist as a type of unscrupulous opportunist: he appears as *realist* insofar as he seeks to describe a world independent of the acts of perception; as *idealist* insofar as he looks upon the concepts and theories as free inventions of the human spirit (not logically derivable from what is empirically given); as *positivist* insofar as he considers his concepts and theories justified *only* to the extent to which they furnish a logical representation of relations among sensory experiences. He may even appear as *Platonist* or *Pythagorean* insofar as he considers the viewpoint of logical simplicity as an indispensable and effective tool of his research.

¹ As collected and reported by the editors the Einstein Papers. John Stachel, et al. (1989) *The Collected Papers of Albert Einstein Volume 2, The Swiss Years: Writings, 1900-1909*, (Princeton: Princeton University Press, 1989), xxiv-xxv.

² Don Howard, "A Peek behind the Veil of Maya: Einstein, Schopenhauer, and the Historical Background of the Conception of Space as a Ground for the Individuation of Physical Systems." In *The Cosmos of Science: Essays of Exploration*. John Earman and John D. Norton, eds. Pittsburgh-Konstanz Series in the Philosophy and History of Science, vol. 6. (Pittsburgh: University of Pittsburgh Press; Konstanz: Universitätsverlag, 1997), 87–150.

³ For a good introduction to his philosophical writing and thought, see Don Howard "Albert Einstein as a Philosopher of Science," *Physics Today*, 58, no 12 (Dec. 2005), pp. 34-40; Don Howard, "Einstein's Philosophy of Science", *The Stanford Encyclopedia of Philosophy* (Summer 2010 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/sum2010/entries/einstein-philsience/>.

⁴ See, for example, Thomas Ryckman, *Reign of Relativity: Philosophy in Physics 1915-1925*. (Oxford: Oxford University Press, 2007).

⁵ Albert Einstein, “Zur Elektrodynamik bewegter Körper,” *Annalen der Physik*, 17 (1905), 891-921; translated as “On the Electrodynamics of Moving Bodies” in Albert Einstein et al., *The Principle of Relativity*. Trans. W. Perrett and G. B Jeffrey. (New York: Dover.Einstein et al., 1952), 37-65.

⁶ John D. Norton, “Einstein’s Investigations of Galilean Covariant Electrodynamics prior to 1905.” *Archive for History of Exact Sciences* 59 (2004): 45–105.

⁷ Albert Einstein, “Autobiographical Notes,” in P. A. Schilpp, , ed., *Albert Einstein: Philosopher-Scientist*. Evanston, (IL: Library of Living Philosophers, 1949) 2-95 on p. 53.

⁸ The remark is in a voice recording, transcribed and presented in the German in F. Herneck, “Zwei Tondokumente Einsteins zur Relativitätstheorie.” *Forschungen und Fortschritte* 40(1966), 133–35; and translated in John Stachel, et al. *The Collected Papers of Albert Einstein Volume 2, The Swiss Years: Writings, 1900-1909*, 264.

⁹ Albert Einstein, *Ueber die spezielle and die allgemeine Relativitätstheorie (Gemeinverstaendlich)*. Braunschweig: Friedr. Vieweg & Sohn, 1917; 15th expanded edition translated by R.W. Lawson as *Relativity: the Special and the General Theory*. London: Methuen, 1954. section VIII.

¹⁰ David Hume, *A Treatise of Human Nature*. 2nd ed. Oxford: Clarendon, 1978, Book 1, Part 1, Section 1.

¹¹ David Hume, *A Treatise of Human Nature*. Book 1, Part II, Section III.

¹² Albert Einstein, “Ernst Mach,” *Physikalische Zeitschrift*, 17 (1916), 101-104.

¹³ For elaboration of these issues, see John D. Norton, “How Hume and Mach Helped Einstein Find Special Relativity,” in M. Dickson and M. Domski, eds., *Discourse on a New Method: Reinvigorating the Marriage of History and Philosophy of Science*. (Chicago: Open Court, 2010), 359-386.

¹⁴ Albert Einstein, “Die Grundlage der allgemeinen Relativitätstheorie,” *Annalen der Physik*, 49 (1916), pp. 769-822; translated as “The Foundation of the General Theory of Relativity” in Albert Einstein et al., *The Principle of Relativity*, 111-164. From §2.

¹⁵ Isaac Newton, *Mathematical Principles of Natural Philosophy and his System of the World*, 1729, tr. Andrew Motte, revised Florian Cajori, Berkeley: University of California Press, 1934. Vol. 1, pp. 10-11.

¹⁶ Albert Einstein, *Ueber die spezielle and die allgemeine Relativitaetstheorie (Gemeinverstaendlich)*, Ch. XXI.

¹⁷ Perhaps I need not add that philosophers now a century removed from Mach's positivism will find the insistence on the direct observability of causes excessive.

¹⁸ Ernst Mach, *The Science of Mechanics: A Critical and Historical Account of Its Development*, 6th ed. T. J. McCormach, trans. (LaSalle, Illinois: Open Court, 1960), 284.

¹⁹ Albert Einstein, "Gibt es eine Gravitationswirkung, die der elektrodynamischen Induktionswirkung analog ist?" *Vierteljahrsschrift für gerichtliche Medizin und öffentliches Sanitätswesen*. 44(1912): 37-40; on 39.

²⁰ Albert Einstein, "Gibt es eine Gravitationswirkung, die der elektrodynamischen Induktionswirkung analog ist?"; Albert Einstein, "Zum gegenwärtigen Stande des Gravitationsproblems," *Physikalische Zeitschrift*. 14 (1913): 1249-1262.

²¹ Albert Einstein, *The Meaning of Relativity*. (1922) 5th Expanded Edition. (Princeton: Princeton University Press, 1956) 101-103.

²² Albert Einstein, "Autobiographical Notes," 21.

²³ Albert Einstein, "Autobiographical Notes," 29.

²⁴ For investigation of this issue, see John D. Norton, "Mach's Principle before Einstein." in J. Barbour and H. Pfister, eds., *Mach's Principle: From Newton's Bucket to Quantum Gravity: Einstein Studies*, Vol. 6. (Boston: Birkhäuser, 1995), 9-57.

²⁵ Albert Einstein and Marcel Grossmann, *Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation*. (Leipzig: B.G.Teubner, 1913) (separatum); with addendum by Einstein in *Zeitschrift für Mathematik und Physik*, 63(1914), pp.225-61.

²⁶ What precisely went wrong? We have a rather complete record of the computations Einstein undertook during his preparation of the 1913 paper in the "Zurich Notebook." For an analysis of what it reveals, see John D. Norton, "How Einstein found his Field Equations: 1912-1915," *Historical Studies in the Physical Sciences*, 14(1984): 253-316; reprinted in Don Howard and

John Stachel (eds.), *Einstein and the History of General Relativity: Einstein Studies*, Volume 1, (Boston: Birkhäuser, 1989), 101-159; and Juergen Renn, et al. *The Genesis of General Relativity. Vol. 1 Einstein's Zurich Notebook, Introduction and Source. Vol. 2 Einstein's Zurich Notebook: Commentary and Essays*. (Springer, 2007).

²⁷ The hole argument and its resolution have been revived in more recent philosophy of space and time as presenting an insurmountable dilemma for certain versions of spacetime substantivalism. For a review, see John Earman and John D. Norton, "What Price Spacetime Substantivalism? The Hole Story." *British Journal for the Philosophy of Science*, 38(1987), 515-525; and John D. Norton, "The Hole Argument", *The Stanford Encyclopedia of Philosophy* (Winter 2008 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/win2008/entries/spacetime-holearg/>..

²⁸ Albert Einstein and Marcel Grossmann, *Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation*.

²⁹ Albert Einstein, "Die formale Grundlage der allgemeinen Relativitätstheorie," *Königlich Preussische Akademie der Wissenschaften (Berlin). Sitzungsberichte* (1914), 1030-1085. For an account of these four versions and related issues, see John D. Norton, "Einstein, the Hole Argument and the Reality of Space," in J. Forge (ed.), *Measurement, Realism and Objectivity*, (Dordrecht: Reidel, 1987) 153-188.

³⁰ The two map projections are adapted from the US Geological Survey websites, <http://mcmcweb.er.usgs.gov/DSS/ImgHTML/Mercator.html> and <http://mcmcweb.er.usgs.gov/DSS/ImgHTML/Lambert0deg.html>

³¹ Albert Einstein, "Die Grundlage der allgemeinen Relativitätstheorie."

³² Einstein's resolution is more severe than the one just sketched concerning map projections. For Einstein urges that all factual content reduces to space-time coincidences. In the map projection analogy, we took as a geographical fact that the driving distance from Adelaide to Alice Springs is 1500km. We can adhere to the strictures of Einstein's argument by reducing facts about driving distances to facts about coincidences. If a car has tires with 2m circumference, then the fact of driving 1 km = 1000m along the road reduces to the fact of 501 coincidences of a spot on the tire surface with different spots on the road.

³³ In the map analogy, different map projections would place Alice Springs at different locations on the paper. Hence the locations on the paper by themselves have no geographical meaning.

³⁴ Translation from John Stachel, "Einstein's Search for General Covariance, 1912-1915," pp. 63-100 in Don Howard and John Stachel, eds., *Einstein and the History of General Relativity*. (Boston: Birkhaeuser.Stachel 1989), 86.

³⁵ Translation from John Stachel, "Einstein's Search for General Covariance, 1912-1915," pp. 63-100 in Don Howard and John Stachel, eds., *Einstein and the History of General Relativity*. (Boston: Birkhaeuser, 1989), 86.

³⁶ Translation from Don Howard and John D. Norton, "Out of the Labyrinth: Einstein, Hertz and Göttingen Answer to the Hole Argument," pp. 30-62 in J. Earman, M. Janssen and J. Norton, eds., *The Attraction of Gravitation: New Studies in History of General Relativity*. (Boston: Birkhäuser, 1993), 48.

³⁷ For discussion of how Einstein's pronouncements have been received and interpreted philosophically, see John D. Norton, "Einstein, the Hole Argument and the Reality of Space," 154-155; and Don Howard, "Point Coincidences and Pointer Coincidences: Einstein on Invariant Structure in Spacetime Theories." In *History of General Relativity IV: The Expanding Worlds of General Relativity*. H. Goenner, J. Renn, J. Ritter, and T. Sauer, eds. (Boston: Birkhaeuser, 1999), 463-500. From the modern perspective, Einstein did not need such strong pronouncements for his purpose of defeating the hole argument. A weaker assertion, such as I have used in the exposition, would suffice. He need only assert that the factual content of the solution of his field equations is fixed by its catalog of spacetime coincidences, not that the coincidences themselves are all that is factual.

³⁸ Translation from John D. Norton, "Einstein, the Hole Argument and the Reality of Space," 169.

³⁹ Don Howard, "Point Coincidences and Pointer Coincidences: Einstein on Invariant Structure in Spacetime Theories."

⁴⁰ See, for example, Don Howard, "Einstein on Locality and Separability," *Studies in History and Philosophy of Science*, 16(1985), 171-201.

⁴¹ For example, writing to Max Born on December 4, 1926, he remarked “Quantum mechanics is very worthy of regard. But an inner voice tells me that this not yet the right track. The theory yields much, but it hardly brings us closer to the Old One's secrets. I, in any case, am convinced that He does not play dice.” Max Born, *The Born-Einstein Letters*. (New York: Walker & Co. 1971), 91.

⁴² Albert Einstein, "Die formale Grundlage der allgemeinen Relativitätstheorie," 1066.

⁴³ We have now modified our notions of causality probabilistically so that quantum theory is no longer regarded as a mortal threat to causality. That, in my view, is no victory for causal metaphysics. Rather it provides one of many illustrations of the elasticity of causal notions that in turn reveals that a requirement of causality has no independent factual content. See John D. Norton, "Causation as Folk Science," *Philosophers' Imprint* Vol. 3(2003), No. 4; reprinted in H. Price and R. Corry, eds., *Causation, Physics, and the Constitution of Reality*. (Oxford: Clarendon, 2007), 11-44.

⁴⁴ See Albert Einstein, Boris Podolsky, Nathan Rosen, (1935) "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" *Physical Review*, 47, pp. 777–780; the same argument is developed in Albert Einstein, “Physics and Reality,” *Journal of the Franklin Institute*, 221(1936), pp. 349-382; Albert Einstein, "Quanten-Mechanik und Wirklichkeit," *Dialectica*, 2(1948), 320-24; Albert Einstein, “Autobiographical Notes,” 82-87.

⁴⁵ For a recent account of the incompatibility that requires only the most elementary notions from quantum theory, see John D. Norton, “Little Boxes: A Simple Implementation of the GHZ Result in Spatial Degrees of Freedom,” *American Journal of Physics*, forthcoming.

⁴⁶ This conception of these two approaches was elaborated by a research group working in Berlin whose conclusions are given in Renn, et al. *The Genesis of General Relativity*. See also John D. Norton, “ ‘Nature in the Realization of the Simplest Conceivable Mathematical Ideas’: Einstein and the Canon of Mathematical Simplicity,” *Studies in the History and Philosophy of Modern Physics*, 31(2000), 135-170.

⁴⁷ For the sources of these quotes and more similar ones, see John D. Norton, “ ‘Nature in the Realization of the Simplest Conceivable Mathematical Ideas’: Einstein and the Canon of Mathematical Simplicity,” §3.

⁴⁸ For an early account see John D. Norton, "How Einstein found his Field Equations: 1912-1915"; and for a fuller account see Juergen Renn, et al. *The Genesis of General Relativity. Vol. 1 Einstein's Zurich Notebook, Introduction and Source. Vol. 2 Einstein's Zurich Notebook: Commentary and Essays.*

⁴⁹ For an introductory synopsis of my view of what led Einstein astray in 1913, see John D. Norton, "A Conjecture on Einstein, the Independent Reality of Spacetime Coordinate Systems and the Disaster of 1913," in A. J. Kox and J. Einsenstaedt, eds., *The Universe of General Relativity. Einstein Studies Volume 11.* (Boston: Birkhaeuser, 2005) 67-102.

⁵⁰ Albert Einstein, "Zur allgemeinen Relativitaetstheorie," *Koeniglich Preussische Akademie der Wissenschaften* (Berlin). *Sitzungsberichte* (1915), 778-786 on 778.

⁵¹ Translation from John D. Norton, " 'Nature in the Realization of the Simplest Conceivable Mathematical Ideas': Einstein and the Canon of Mathematical Simplicity," 151.

⁵² Albert Einstein, "Zur allgemeinen Relativitaetstheorie," 779.

⁵³ Albert Einstein, "On the Methods of Theoretical Physics," (1933) in *Ideas and Opinions.* (New York: Bonanza, 1954), 270-276.

⁵⁴ Albert Einstein, "Remarks Concerning the Essays Brought Together in this Co-operative Volume," (1949) in P. A. Schilpp, , ed., *Albert Einstein: Philosopher-Scientist*, 665-688.