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ARTICLE

What can we learn about physical laws from the fact that we have memories only of the past?

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Abstract *Not much. I demonstrate this by constructing a model of a memory system governed by deterministic, time reversible laws only, thereby showing that the mere fact of our having memories solely of the past does not necessitate an indeterministic, time asymmetric or stochastic physics, essentially thermodynamic processes or a primitive notion of time asymmetric causation.*

1. Introduction

A great chasm separates our common experience of time from virtually all our physical theories of time. Our experience is dominated by the present, the now, which advances inexorably in one direction. It consumes a future whose content is undecided and leaves behind a past that is immutable. We do not know who if anyone will first scale the highest mountain of our moon; the matter is indefinite. But our highest earthly mountain, Everest, was scaled first by Edmund Hillary and Tenzing Norgay on 29 May 1953, and no exploit within our powers can change it; the fact is immutable. Our physical theories of time make no mention of a present or a now and give no account of its inexorable advance. Virtually all our fundamental physics and all classical and relativistic physics see no difference between future and past. If they license some change over time, then they license its time reverse as well. Quantum physics offers our only fundamental exceptions.[1] Thermodynamics delivers the rise of entropy as a means of distinguishing the past from the future direction. But this rise has been understood near universally since the time of Boltzmann as recoverable from a fully time reversible microphysics as the result of a highly improbable state in the past.

Most aspects of our experience of asymmetry in time are sufficiently vaguely defined for us to have lesser hope of now providing a precise account of them in terms of standard physical theories. How might these theories capture our experience of the relentless advance of the now or our unsettling sense that the future is indefinite? The most promising candidate for physical analysis is the asymmetry of memory, the fact that we remember the past but not the future. This asymmetry distinguishes past from future

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as surely as any indicator. Our knowledge of the hurricane just passed is definite and immediate in a way that far outstrips any expectation of a coming hurricane from fragile weather forecasts. This asymmetry promises to be amenable to physical analysis since it seems closely analogous to ordinary physical processes. Can my memorizing of the name of my second cousin's new nephew be so much different from my writing it in an address book? Can my failure to recall the names of unborn and as yet unnamed children be any different from the failure of public institutions to have records of these names?

So we might well ask how our physical theories explain the time asymmetry of memory. Such efforts have become a staple of philosophy of time with many candidate proposals. (For an entry into this literature, see Earman (1974); Horwich (1987, Chap. 5); Sklar (1993, Chap. 10).) The best known candidate for this explanation is that memories are low entropy traces in branch systems that have become isolated, so that the time direction picked out by memory is fixed by that of thermodynamics (see Reichenbach, 1991, Chap. IV; Grünbaum, 1974, Chap. 9). This last proposal has been subject to serious and telling criticism, since the processes of memory trace formation do not seem to be essentially or necessarily thermodynamic in character (see Earman, 1974; Horwich, 1987, Chap. 5).

The proliferation of proposals and their mixed success raise the question of whether we can infer something about fundamental physical laws from the mere fact of the existence of the time asymmetry of memory. Might the difficulty of the problem arise because the asymmetry can occur only in universes governed by very particular sorts of laws, so that our efforts to understand the asymmetry are doomed to failure until we base them on the presumption of the right sorts of laws? Or is the asymmetry viable in just about any class of physical law? My question is not what actually explains the memory asymmetry of our world, but whether such an explanation is possible at all without severe restrictions on the character of our physical laws. Might the asymmetry force us to one or other class of physical law? For example:

- Might physical laws be essentially stochastic? In a universe governed by deterministic laws, the present state of the world fixes both its future and past states. So in principle it is possible for that present state to contain traces that are just as reliable memories of the future as they are of the past. If the laws are stochastic, however, the future state of the world can at best be given probabilistically for some fixed present state of the world. Might this indefiniteness of the future explain why we have no memories of the future; there is no definite future of which we can have memory traces?
- Might physical laws be time irreversible? In a universe governed by time reversible laws, any admissible process can also occur in the time reversed direction. So if there is an admissible process that leaves us a reliable memory trace of some past state then its time reverse is also admissible. But that reversed process delivers the present a reliable memory trace of a future state. If the laws are time irreversible, however, they might only license processes that afford memory traces of the past but not of the future.
- Might memory processes be essentially thermodynamic even if the fundamental laws are deterministic and time reversible? While a universe with deterministic, time reversible laws would admit the possibility of memories of the future, thermodynamic processes in it are, with high probability,strongly time asymmetric. So, in spite of the objections to the notion mentioned above, if memory processes are somehow entan-

gled with thermodynamic processes, might we have to use that asymmetry in some subtle way to sustain the asymmetry of memory?

- Might we need to resort to a time asymmetric notion of causation, with the asymmetry either taken as a primitive or recovered from another account such as Reichenbach's celebrated common cause principle? Whatever may be its origin, the causal processes of the ordinary world are governed by such an asymmetric notion. The loosened shoelace both happens before and is the cause of the fall and not *vice versa*. Causes do not come after their effects, except in the imagination of H.G. Wells. So, if memories are effects of past causes, then we cannot have memories of the future.

My goal in this note is to show that we can infer to none of these possibilities from the mere fact of the time asymmetric of memory. I will do so by showing that this asymmetry is quite possible in a universe governed by laws that one would think least hospitable to it, laws that are time reversible and deterministic. To show this I will describe a contrived and simplified world governed by time reversible, deterministic laws that will turn out to admit memory systems capable of discriminating past from future in ways analogous to our human memory. My purpose is not to show that our world is governed by time reversible, deterministic laws. Rather it is to show that any inference to the contrary cannot proceed merely from the fact of a time asymmetric memory. It must invoke some particular properties of this memory if significant restrictions on physical law are to be sustained. The mere fact of a time asymmetric memory cannot force us to admit a stochastic or time irreversible physics or any of the other options above.

The presumption throughout is that the processes of memory lie fully within the compass of physical law, so that memory systems are just ordinary physical systems that happen to have special importance to us.

2. An Idealized World

Imagine an idealized world governed by deterministic, time reversible laws. For later reference, these notions are defined as follows:

> A law is *time reversible* if, for every process P that it licenses, it also licenses the time reversed process TP.

The most familiar example is the Newtonian mechanics of elastic particle collisions. If some sequence of collisions is admitted by Newtonian mechanics, then the time reversed sequence is also admitted.

> A law is *deterministic* if it fixes the future and past state of the world once the state of the present is fixed.[2]

A simple example is a field theory governed by hyperbolic differential equations in which waves propagate in the field at a fixed, finite speed, such as the theory of the source free electromagnetic field. Once we fix the state of the field throughout space at one time, the future development of the field is uniquely determined. If we only fix the fields in a portion of space, then the future of that portion will be fixed only insofar as that future lies beyond the reach of waves that can propagate in from outside the portion.

Within the idealized world we suppose there is a physical memory device also governed by deterministic, time reversible laws. The device records memory traces of

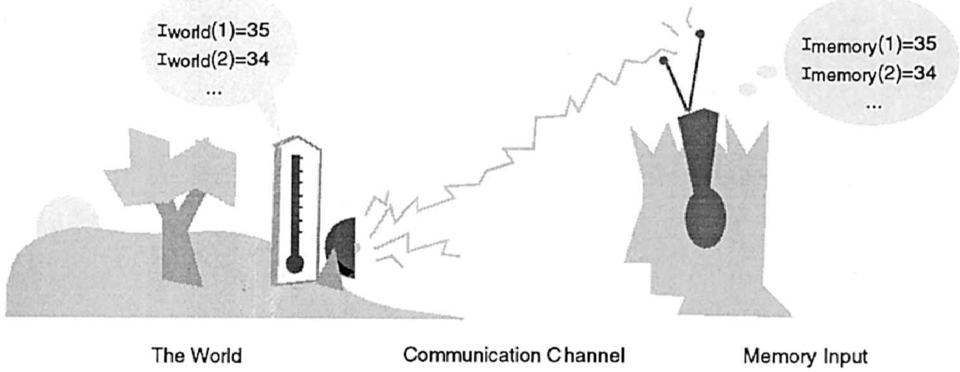

Figure 1. *The world.*

some aspect of the world. For simplicity assume that the aspect of the world can be represented by a real valued variable whose time dependence is continuous and differentiable to high order with time. In our world such variables might be temperature or wind speed. For simplicity we assume that the communication of this variable to the memory system, as shown in Figure 1, is infinitely fast so that we have the input $I(t)$ as a function of time t satisfying

$$I(t) = I_{\text{world}}(t) = I_{\text{memory}}(t) \tag{1}$$

where $I_{\text{world}}(t)$ is the value of the input variable in the world at time t and $I_{\text{memory}}(t)$ the value communicated to the memory device at time t.

The speed of communication is chosen to be infinitely fast since that is the simplest time reversible law. The law (1) is also obviously compatible with the determinism of the whole set of laws governing the world.

The paradigm of a simple memory device is a chart recorder. The input is delivered as the time varying positions of an inked pen tip which is recorded as the trace drawn on a slowly advancing sheet of paper (see Figure 2).

We would like the memory device in the idealized world to function just like this chart recorder. At first glance this may not seem possible since a chart recorder is usually

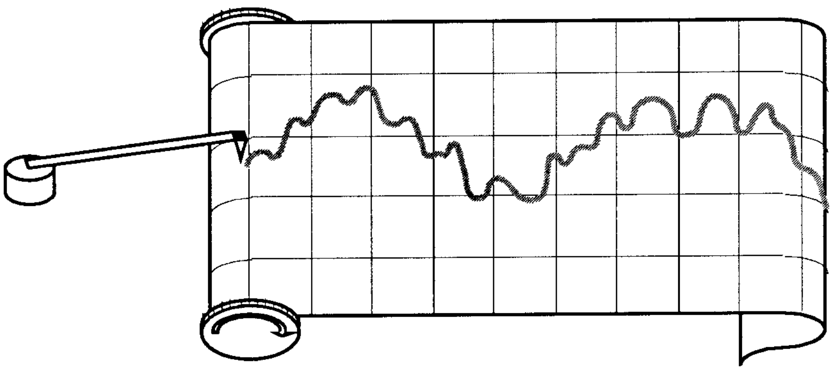

Figure 2. *A chart recorder.*

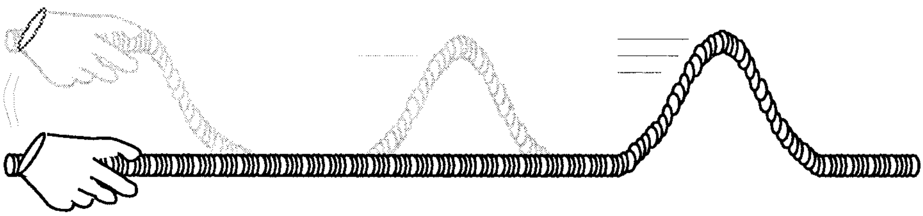

Figure 3. *Wave propagates along a string.*

thought of as operating in an essentially time irreversible fashion. To see why, imagine it running in the reversed direction. The paper chart now slowly winds back towards the inked pen. As it does so, the dried ink takes up moisture from the air and liquefies. Each fully liquefied ink mark arrives at the edge of the chart just at the precise moment that the pen tip passes over it and draws the ink into the pen leaving the paper completely unmarked. So the operation forward in time is admissible by normal physical laws but not the time reversed motion.

3. Building a time reversible memory

The irreversibility of the chart recorder is inessential to its operation. It arises because we have chosen to build the device with irreversible processes, such as the drawing of ink by capillary action from the pen to the paper and the evaporation of water from the ink. Essentially the same operation can be recovered in an idealized device that employs only time reversible processes. Wave motion is a familiar example of a time reversible, deterministic process. Its simplest instance is the wave motion that propagates along an elastic string. If one takes such a string and wiggles one end, a wave will propagate along the length of the string. The way the end is wiggled will fix the outline of the propagating wave that moves along the string (see Figure 3).

If one thinks of the wiggling as the input to a recorder, this propagating waveform records the history in time of the wiggles in exactly the same way as the chart recorder logs the motion of the pen in an inked trace.

This simple behavior can be recovered readily from the standard mathematical treatment of wave motion. If $y(x,t)$ represents the displacement of the string at position x along the string at time t, then this displacement will satisfy the one dimensional wave equation[3]

$$\frac{\partial^2 y}{\partial t^2} - \frac{\partial^2 y}{\partial x^2} = 0 \tag{2}$$

It turns out that the most general solution of this wave equation has an especially simple form. It is just

$$y(x,t) = f_r(x - t) + f_a(x + t) \tag{3}$$

where f_r and f_a are any sufficiently differentiable functions of a single argument.

The term in $f_r(x - t)$ represents a fixed waveform traveling in the $+x$ direction with unit speed. To see this consider a wave described by $y(x,t) = f_r(x - t)$. At times $t = 0$, 1, 2, ... it will be represented by $f_r(x - 0)$, $f_r(x - 1)$, $f_r(x - 2)$, ... These just represent a

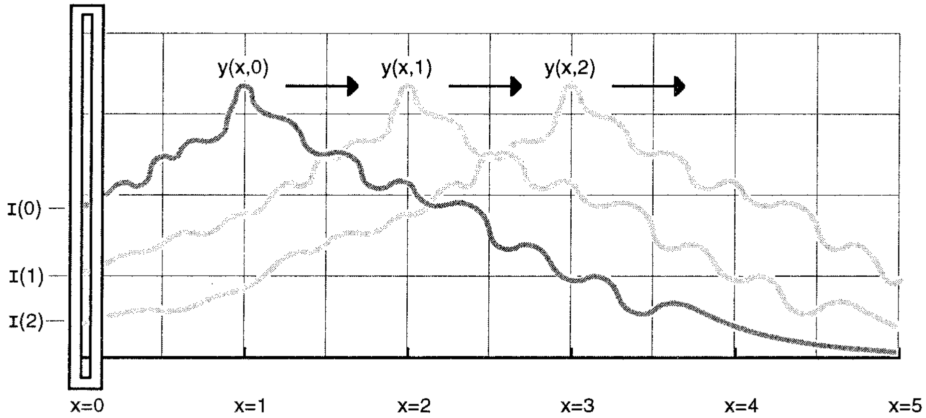

Figure 4. *Retarded solution of one dimensional wave equation.*

waveform translated unit distance in the $+x$ direction for each unit time elapsed. If, for example, f_r has a maximum at $f_r(1)$ as shown in Figure 4, then that maximum will occur at $x=1$ when $t=0$. It will have been relocated to $x=2$ at time $t=1$ (since then $y(2,1)=f_r(2-1)=f_r(1)$) and to $x=3$ at $t=2$ (since then $y(3,2)=f_r(3-2)=f_r(1)$ as well).

In this solution $y(x,t)=f_r(x-t)$, the function f_r remains to be fixed. In the elastic string shown in Figure 3, would f_r represent the form of the propagating wave. That form is fixed in turn by the wiggling of the string at one end. If we consider the string to be a memory device, those wiggles are the data communicated to the device and the resulting waveform in the $x>0$ region records the time dependence of the data. We represent this communication of data in the mathematical treatment by fixing the values that y can adopt at $x=0$ over all times t. That is, we set

$$y(0,t)=I(t) \tag{4}$$

where $I(t)$ is just the input to the memory system. This stipulation of $y(0,t)$ does fix f_r, since we recover from it that $f_r\ (0-\mathrm{t})=I(t)$ so that $f_r\ (t)=I(-\mathrm{t})$. The way that $I(t)$ varies with time is encoded in the waveform f_r which then propagates along the string as an enduring record. Since the waveform propagates away from the input at $x=0$, it is known as the "retarded" solution.[4] Points on the string at $x>0$ do not learn of the value of the input at $t=0$ until a retardation time $t=x$ has elapsed and the corresponding point of the waveform passes.

The analysis of the term $f_a(x+\mathrm{t})$ is virtually identical, excepting the direction of travel of the waveform. The term $f_a(x+\mathrm{t})$ represents a fixed waveform traveling in the "$-x$" direction with unit speed. An example would be the time reverse of the retarded solution shown in Figure 4. In that advanced solution in the $x\geq 0$ region, the same waveform now propagates toward the position $x=0$. Just as the retarded wave recorded the input function $I(t)$ at $x=0$, the advanced wave anticipates the input function $I(t)$ at $x=0$. The magnitude of the wave that arrives at $x=0$ will match exactly the corresponding input $I(t)$, so that, figuratively speaking, the approaching waveform knows in advance the values to be communicated as input at later time (see Figure 5).

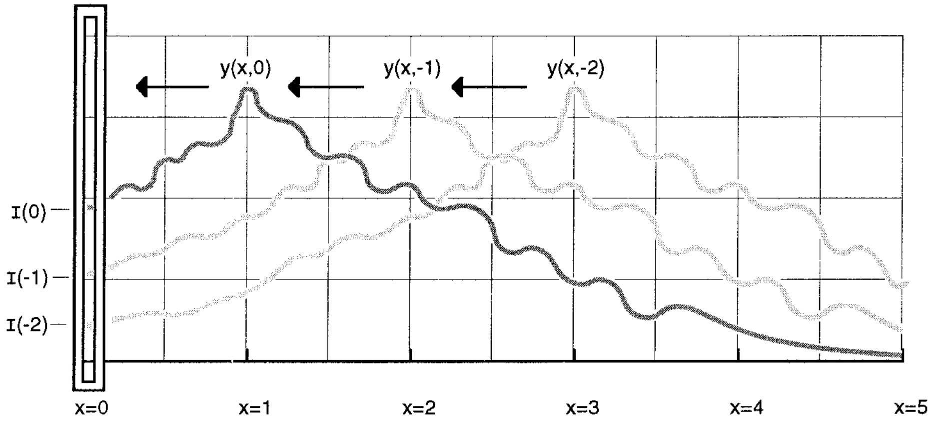

Figure 5. *Advanced solution of one dimensional wave equation.*

The most general solution of form (2) with both terms present is the resultant of two waveforms propagating in opposite directions.

To complete our idealized world, we suppose that the memory device is governed by the wave equation (2) and that the input variable from the world is supplied to it as specified in equation (4); that is, the input fixes the value of the displacement variable y at $x=0$ at all times t. To function as a memory device, we would require the system to employ retarded solutions only and we will need to find some way to preclude advanced solutions. (More on this below!) The fact that f_r can be any suitably differential function is what makes possible the functioning of such a system as a memory device. It means that the system is able to record any suitably differential input function $I(t)$ in the retarded waveform $f_r(t) = I(-\mathrm{t})$.

In principle the memory device could consist of a long, elastic string. The input would be delivered by displacing one end of the string and the memory of the displacements held in the propagating waveform. We might read the displacement of the string by reflecting light pulses off it. In practical terms, however, an elastic string memory device might not be the best choice. Its functioning would be impaired by friction with the structures that support the string. However any device governed by the same equations could be used. The present issue is not the convenience of the storage device but its possibility in principle. In the following we leave open the exact construction of the memory device, assuming only that it obeys (2) and (4). For concreteness in the following, I will talk about "the chart" as the medium that would correspond to the elastic string in whatever device we choose. "The trace" will designate whatever corresponds to the displacement of the string.

4. Model I. No time asymmetry in set up conditions: memories of past and anticipations of future

With our model of the world and idealized memory device now fixed, we can begin to explore their operation. Our goal in the model is to minimize any supposition of time asymmetry and check whether memory of the past alone is still assured. We have components that each obey time symmetric laws. So we can assure ourselves of complete absence of time asymmetry in the operation of the memory device if we introduce no time asymmetry whatever in the contingencies of its set-up. In this case, however, our failure to recover memories of the past alone is a foregone conclusion. If the memory device is governed by time symmetric laws, given a time symmetric set up and communicated data from a world itself governed by time symmetric laws, then we cannot guarantee an essential time asymmetry in the operation of the memory device. The entire system is governed by time symmetric laws with no special conditions to break the symmetry. So if it can manifest some process then it can also manifest that process's time reverse. If the process is one in which a memory trace of the past is recorded, then the time reversed process will involve a recording of a memory trace of the future.[5] While this result is a foregone conclusion, it is helpful to see how it comes about in a concrete case. It will illustrate the interaction of advanced and retarded solutions and aid us designing more models.

To illustrate this operation of the system, we presume that the input variable $I(t)$ is communicated to a memory device without any further supposition about the initial state of the device's chart. We make no supposition, for example, that the chart is set to zero displacement throughout at some initial time. This condition of "no supposition" is a simple time symmetric set-up since it obviously favors neither past nor future. The result is that the operation of the memory device is underdetermined.[6] Many recorder processes are compatible with any given input function $I(t)$. The simplest are the unique retarded and advanced solutions in the $x \geq 0$ region. Each are fixed by the input $I(t)$ if that input is given for all $-\infty < t < \infty$. In each, the waveform is the same; it is essentially the spatial image of the input function $I(t)$. In the retarded case, the waveform allowed by (3)

$$y_r(x,t) = f_r(x - t)$$

propagates away from the input at $x = 0$ and records a memory trace of its past values. It is fixed uniquely by $I(t)$ through the condition (4)

$$y_r(0,t) = f_r(-\mathrm{t}) = I(t)$$

In the advanced case, the waveform allowed by (3)

$$y_a(x,t) = f_a(x + t)$$

propagates towards the input at $x = 0$ and this time anticipates a future value of the input. It is fixed uniquely by $I(t)$ through the condition (4)

$$y_a(0,t) = f_a(t) = I(t)$$

(see Figure 6). Finally, we can find infinitely many more chart processes compatible with the input $I(t)$. These are simply suitably weighted sums of the retarded wave $y_r(x,t)$ and the advanced wave $y_a(x,t)$.[7] In general these sums admit no simple interpretation beyond their being a confusing mix of memories of the past and anticipations of the future.

5. Model II. Memory device with initialized chart: memories of the past alone

Model I illustrates how our memory device will fail to yield the asymmetry of memory unless we inject some time asymmetric presupposition. In the possibilities canvassed in the introduction that time asymmetry was imposed in a quite pervasive, global manner

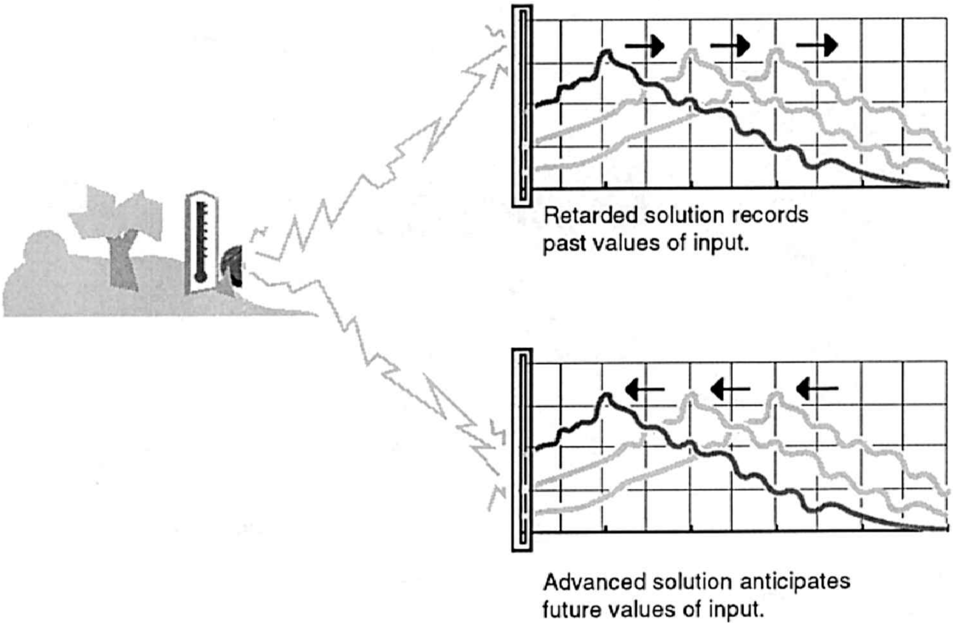

Figure 6. *Fully time symmetric set up of memory device.*

by building it into the very laws that govern all processes at all positions in space and all moments in time. What this second model shows is that a far more modest presumption of time asymmetry is sufficient to enable the asymmetry of memory to be realized. The presumption is:

> *Time Asymmetric Set-up Condition*: At some initial time $t=0$, the chart displacement is set to zero throughout the chart.

This condition is so weak that it is, perhaps, not immediately obvious that it is a time asymmetric condition at all. Setting the chart trace to 0 displacement at time $t=0$ does not seem to favor either future or past. The asymmetry resides in the notion that $t=0$ is an *initial* time. It indicates that we will consider the operation of the chart for an interval of time that begins with $t=0$ and proceeds into a future picked out by $t>0$. To bring this about, we would need to ensure that the state of the chart is decoupled from any earlier states. But we would allow the natural operation of the chart to bring about the chart states at later time $t>0$. This intervention is essentially asymmetric in time. It blocks out interactions with the past ($t<0$) but does not interfere with connections to the future ($t>0$).

With this time asymmetric set-up, the behavior of the memory device is fully determined by the input function $I(t)$ for all times $t \geq 0$.[8] That is, the trace $y(x,t)$ is fully fixed by the initialization condition and coupling condition (4)

$$y(x,0)=0 \text{ for } x \geq 0 \qquad y(0,t)=I(t) \text{ for } t \geq 0$$

These two conditions preclude an advanced term in the general solution (3) excepting

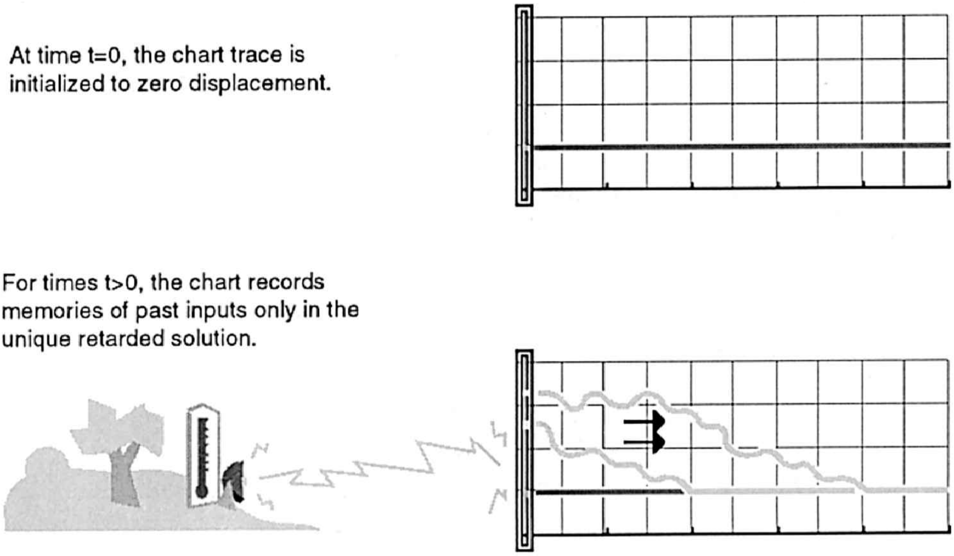

Figure 7. *Operation of memory device with time asymmetric initialization.*

the trivial case in which $I(t) = 0$ for all $t \geq 0$. So for all $t \geq 0$, the unique solution is the retarded solution[9]

$$y(x,t) = f_r(x - t) = I(t - x)$$

This retarded solution represents operation in which the trace records a memory of the past input (see Figure 7). The corresponding advanced solution, which anticipates these inputs, is precluded. Such an advanced solution would need to have its anticipation of the future inputs recorded as the waveform present on the chart at $t = 0$. The initialization of the chart at $t = 0$ precludes the admission of such anticipation.[10]

6. Model III. Is your now my now?

Model II of Section 5 shows just how minimal an injection of asymmetry at one moment is sufficient to ensure the asymmetric operation of memory for all time. How much more can we recover from the model? It turns out that there is also a weak sense of "now" built into the memory device's operation. Memories are records of now. The chart trace of Model II is a record of the input of $I(t)$. So if that chart trace is a memory, the input $I(t)$, the arrival of each value of $I(t)$ at t at the chart, are the "nows" it remembers. It seems futile to ask if the device is aware of these "nows" and senses their inexorable motion with the motion of the chart trace. What test would distinguish a device with this awareness from one without? But there is one aspect of our human experience of the now that the system can readily replicate. Two humans in ordinary, undelayed communication always agree on which instant is now. Two memory devices, operating as in Model II, will likewise agree on which instant is now in a natural sense to be developed below.

To facilitate analysis, I will assume that the input in Model II is not just any suitably

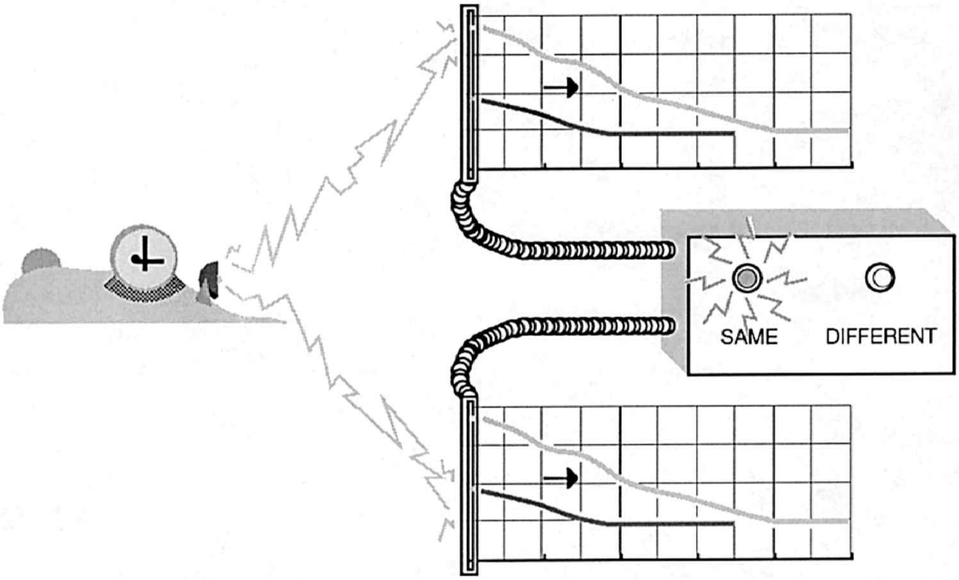

Figure 8. *Two memory devices agree on when is "now".*

differentiable function $I(t)$ but a strictly increasing function of t. This input serves as a kind of clock. It indicates later times by communicating strictly higher values in $I(t)$. We now image two memory devices both run from this same input. The now experienced by each device will be tagged by the corresponding unique value of $I(t)$ delivered to the $x = 0$ point on the chart. So we can ask if each chart sees the same label for its present now at the same time. That is a question that can be asked trivially with electronic circuitry. That is, we sample the input value to each chart and, using circuitry idealized as having zero delay, communicate the values to a device whose function is to tell us instantly whether its two inputs are the same. The agreement in the values is inevitable (see Figure 8). Insofar as it makes any sense at all to say that there is a "now" associated with the devices' operation, the memory devices agree on its occurrence, just as we humans agree on when is now.

This achievement should not be construed as showing that these memory devices come close to the functioning of real human memory. Indeed the devices are capable of mimicking behavior that is solely the province of fiction. In his *The Sword in the Stone*, Terence White described a Merlin who experienced time backwards. He meets Arthur for what is the first time for Arthur but the last time for him. That would cause no end of confusion were it not for the happy fact that both agreed on which moment was now, even though they could not agree on which was the direction of the future. This Merlin and Arthur can be mimicked by a set-up similar to that of Figure 8 but in which one of the memory devices, the Merlin memory, operates with an advanced solution. The Merlin device registers memories of what the other device regards as the future. But the comparison circuitry will show that both agree on which moment is now, in the sense that they will agree that the same time labels are delivered as input to their memory devices at the same moment.

7. Conclusion

The mere fact that we have memories of the past only tells us very little about our physical laws. If this memory asymmetry can be accommodated in the most inhospitable case, that of time reversible, deterministic laws, then it would surely be compatible with the more hospitable cases listed in the introduction. A corollary is that an account of time accrues little support from showing that it can accommodate the asymmetry of memory. That asymmetry would seem to be compatible with just about any physical account of time that admits even the slightest time asymmetry. If we are to learn about physical laws from our experience of memory, then our investigations must depend not just on the fact of the asymmetry of memory. They must draw carefully on detailed facts about particular memories and how they function.

Acknowledgement

I am grateful to John Earman for helpful discussion.

Notes

1. The physics community was shocked to discover that the weak interaction (the force that governs radioactive decay) discriminates slightly between past and future. The collapse of the wave packet of quantum measurement—a process which remains besieged by competing proposals—also points uniquely to the future.
2. This definition allows some variation that will not concern us. We might consider variant definitions that require that we fix the entire past before the future is fixed or that we additionally preclude influences that propagate in infinitely quickly from spatial infinity without prior trace in the present. There is also a vagueness in what counts as the "present" that can become of fundamental importance once one considers the spacetimes of general relativity.
3. To see the time reversibility, imagine that we have a solution $y(x,t)$ of (2). Then its time reverse, $y_{\text{rev}}(x,t) = y(x,-t)$ also satisfies (2) since

$$\frac{\partial^2 y_{\text{rev}}(x,t)}{\partial t^2} = \frac{\partial^2 y(x,-t)}{\partial t^2} = \frac{\partial^2 y(x,-t)}{\partial(-t)^2} = \frac{\partial^2 y(x,-t)}{\partial x^2} = \frac{\partial^2 y_{\text{rev}}(x,t)}{\partial x^2}$$

   An alternate and simpler way to see the time reversibility is to use the general solution (3). The time reverse has the form

$$y_{\text{rev}}(x,t) = y(x,-t) = f_r(x-(-t)) + f_a(x+(-t)) = f_r(x+t) + f_a(x-t)$$

   and this is also an admissible solution, although the f_r term now represents the advanced term and f_a the retarded term of the solution.
4. For this characterization of the solution as a retarded solution, it is essential that we consider the $x \geq 0$ portion of the string. In the $x < 0$ portion, this waveform propagates towards $x = 0$ and is an advanced solution that cannot function as a memory.
5. We might suspect a loophole in this argument. Data from the world can be asymmetric in time. For example, a series of pulses, such as the radio signals for setting clocks broadcast by the National Institute of Standards and Technology from Fort William, Colorado, might count up in the direction of future time. Might we imagine another, smarter memory device built from time reversible elements, but programmed to detect which is the direction of the future from the time asymmetry in the data? With that direction discerned, the memory would be able to record just the past of the data stream. This loophole fails because of the time symmetry of the laws governing the world. Assume there is a data stream whose past alone the memory device will record. Then from the time reversibility of the laws of the world, the time reverse of that data stream is also possible. But now the memory would record that reversed data stream's future only.
6. This indeterminism does not undermine the determinism of the wave equation. As indicated in Section 2 above, that determinism requires that the future be fixed if the present is. The "no supposition"

condition leaves the present state undecided. All that is fixed is the input variable $I(t)$ at one point on the chart.

7. That is, we construct a new solution of (2) and (4) $y(x,t) = Ay_r(x,t) + By_a(x,t)$, for any constants A and B between 0 and 1 for which $A + B = 1$. The new solution is compatible with the input $I(t)$ since $y(0,t) = Ay_r(0,t) + By_a(0,t) = Af_r(-t) + Bf_a(t) = AI(t) + BI(t) = I(t)$.
8. For simplicity I will also assume that $I(0) = 0$.
9. The advanced solution $y(x,t) = f_a(x+t)$, where $f_a(t) = I(t)$ for $t \geq 0$, is precluded, since the state of the chart at $t = 0$ would be given by $y(x,0) = I(x)$ for all $x \geq 0$. Since in general $I(x)$ is not identically the zero function $I(x) \equiv 0$, the requirement of initialization cannot be met by the advanced solution. The retarded solution has no comparable problems. It is $y(x,t) = f_r(x-t)$, where as before the waveform is required to mesh with the input according to $y(0,t) = f_r(-t) = I(t)$ for $t \geq 0$. This last condition only holds for $t \geq 0$, since that is the time during which the chart is coupled to the input. So it will only apply a constraint to $y(x,t) = f_r(x-t)$ for values of x satisfying $x \leq t$. When $t = 0$, the requirement that the solution mesh with the input places no constraint on the solution excepting at $x = 0$ that $y(0,0) = 0$. The remaining values of $y(x,0)$ for $x > 0$ can be arbitrarily set to 0 as the initialized state.
10. That is, it precludes it unless the input will be $I(t) = 0$ for all $t \geq 0$, in which case a zero displacement of the trace correctly predicts the future input.

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