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A DEMOCRITEAN PHENOMENOLOGY FOR QUANTUM SCATTERING THEORY*

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ABSTRACT

Granted that the basic logical requirement of a particle theory is to be able to assert that a particle is "here" "now" rather than "there" and/or "then", the implied operational device is a system of particle detectors which can make these basic discriminations. Given such devices, their successful operation requires the existence of a limiting velocity. Given devices that can change particle velocities in both magnitude and direction in a reproducible way, a Lorentz-invariant mass can be operationally defined. Accepting the wave-particle duality (operationally definable either in terms of a grating or in terms of the uncertainty principle in energy-time) implies (under the Democritean assumption of a smallest mass) changes in particle number (the Wick-Yukawa mechanism). Granted this mechanism, the conventional quantum scattering formalism for hadron scattering at finite energy can be recovered without postulating either "interactions" or "analyticity". By identifying the velocity-changing devices as electromagnetic fields whose sources can be calculated from particle wave functions, and by calculating the systemic properties of particles which move in the fields generated by other particles, the devices needed for the operational definition of particles can be described in terms of the particles so defined, to an accuracy of order $e^2/\hbar c$. Thus the operational definition of particle can be made self-consistent and self-generating to that accuracy.

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The atoms of Leucippus and Democritus had no "natural" or "original" motion; their random collisions were strikingly similar to the nineteenth century model for the kinetic theory of gases. Epicurus assumed that the atoms were falling in straight lines and that it was necessary to postulate that some of them "swerve" in order to initiate the processes which lead to the generation (and decay) of worlds. His random element in atomic theory has been criticized as foreign to the basically materialistic and deterministic focus of this natural philosophy. In recent years we have learned from the success of quantum mechanics that determinism does not inhere in the individual atomic events. The approximate validity of determinism stems from the calculable flow of probability amplitudes from the past up to some event in which the massive particles involved in that event manifest (potentially) observable aspects of their particulate behavior. The random character of these individual events supplies a quantum mechanical resolution of the puzzle posed by ancient atomism. We present below a complete descriptive phenomenology for the scattering of massive particles as the starting point of a coherent development of this modern atomic point of view.

The "derivation" of the quantum scattering formalism in Section III originated in response to a query from John Bell[1] about a paper entitled "Fixed Past and Uncertain Future" by this author[2]. That paper made use of the interpretation of quantum mechanics developed by Thomas E. Phipps, Jr.[3,4,5] which perfects the correspondence between classical and quantum mechanics by retaining degrees of freedom represented classically by constants of the motion as a phase factor of the Schrœdinger wave function. Phipps' theory is dynamical in that he starts from the Hamilton-Jacobi equations, including the "interaction terms"; the scattering theory developed below is "kinematical" in the sense that it makes use only of the (non-interacting) "free particle solutions" of those equations. Thanks to discussions with Ted Bastin[6,7] revolving around the idea that it is mass rather than "action" that is quantized, the author eventually reached the "obvious" conclusion that quantum scattering theory need make no reference to interactions; all that is needed is an understanding of the phenomenology of particle wave functions and the existence of some mechanism which generates scattering events. The philosophy lying behind this approach is already implicit in Wick's[8] derivation of the connection between the mass of Yukawa's[9] heavy quanta (now called mesons or more specifically pions) and the range of nuclear forces. Wick's argument shows that <u>any</u> theory which couples the uncertainty principle to the mass-energy relation necessarily implies the possibility of fluctuations in particle number within small enough space-time volumes, and <u>hence</u> scatterings of massive particles. Whether or not these fluctuations arise from "interactions" is a separate question, which we claim can be severed from the phenomenology by an incisive use of Occam's Razor. We perform this operation below by first providing a precisely limited definition of the concept of "particle" as it enters into scattering theory and then proceeding from that definition to the construction of a phenomenological transition matrix which is connected to cross sections ("observables") in the conventional way. We then justify the measuring devices used <u>a posteriori</u> by showing that the theory implies that they can be constructed.

II. WHAT IS A PARTICLE?

In order to distinguish particles from the void we must be able to distinguish something from nothing. The basic operational device for a particle theory is a "particle detector" that can tell us whether a particle is "there," or conversely that only void is "there." If this statement cannot change, we cannot decide whether the detector is functioning or not. A detector must be able to tell us whether or not the particle is there "now."

We make this vague requirement quantitative by means of auxilliary devices which — to start with — are simply the "rigid rods" and "uniform rate clocks" of special relativity. From the start we assume that the measurements made by rods and clocks are macroscopic, i.e. that the human observer can uniquely define distance intervals and time intervals relative to those rods and clocks by a sequence of "operations" (in Bridgman's sense[10,11]. For the moment all we need assume is that to some finite accuracy a particle detector can say that a particle was "there" within a distance Δx "during" a time interval Δt, that the distance between two particle detectors can be measured by a rigid rod to be $\sigma = | x_1 - x_2 |$, that we can state unambiguously whether a particle was in detector 1 before or after it was in detector 2, that the time interval between these two happenings $\tau = t_2 - t_1$ can be measured by a uniform rate clock, and that the uncertainties in Δx_1, Δx_2, Δt_1, Δt_2, σ, and τ can all be as small, both individually and collectively, as the smallest of any of these quantitative measures.

If the average velocities between detectors, i.e. σ/τ, can be arbitrarily large, then the particles with these very large velocities could be used to measure the dimensions of the detectors themselves to arbitrarily high precisions. If we could refine their size measurement down to arbitrarily small dimensions, this would imply that the particles can be arbitrarily small, and our basic separation of particles from the void would dissolve into a continuum. We claim, therefore, that there is a logical (and not just an experiential) difficulty in assuming that velocities can be arbitrarily large in any Democritean theory. Our way of meeting this difficulty is simply to postulate that there is a limiting velocity defined relative to the rods and clocks we have introduced. Then non-overlapping particle detectors specify a unique labeling x_i, t_i for each event in each detector i, up to the resolutions of the detector Δx_i and Δt_i.

The problem would be simple if, experientially, we could reduce the basis set of detectors to four, and relate all other events to the distances between these detectors, and time scales calibrated by the passage of particles through this reference set. That we cannot do so reflects genuine inhomogenieties in the space-time description of events, such as rotation relative to the "receeding galaxis" and the "rod shift." These spacial and temporal inhomogeneities, though observationally "real" enough, could very well be due to a process of development in time, and need not necessarily indicate basic inhomogeneities in the space-time framework itself.

Although we invoked "galactic coordinates" to indicate the experiential limitations of our construction, all we have said that does not depend on them could refer to motion of particles along a single line. We postulated a unique sense in which time increases by taking $\tau = t_2 - t_1$, but cannot yet distinguish left from right because for distances we only assumed that $\sigma = |x_2 - x_1|$. This implies either a symmetry in nature not exhibited by the events themselves, or an inadequacy of description, which cannot be removed by postulating a <u>single</u> direction perpendicular to the relative velocity between two detection systems referred to in the Lorentz transformations. Two directions orthogonal to each other as well as to this axis suffice to allow us to set up four labeled (non-coplanar) detectors which allow us to define "left" as distinct from its non-superposable "right" alternative. In other words, we insist that both the direction of time sense and the macroscopic distinction between "left" and "right" are primitive experiential facts which our operational definitions must be capable of describing. Granted this, our assumption that (to some accuracy) inertial systems cannot be used to determine galactic coordinates, we can complete the construction of the Lorentz transformations in Minkowski space-time in the usual way.

Using this kinematical (geometrical) definition of particle motion, we can go on to a kinematical (mechanical) definition if we have available auxilliary devices that can change either the magnitude or the direction of the velocity of the particles. For charged particles these can be, respectively, electric and magnetic fields. We invoke these (external, static) "fields" in order to define the mass of a charged particle m as a positive scalar invariant under Lorentz transformations $m = +\sqrt{(\epsilon/c^2)^2 - (p/c)^2}$. The quantities ϵ and p are called the energy and momentum respectively. They are defined mathematically in terms of the velocity of the particle $\underline{v}$ by $\epsilon(v) = mc^2(1 - v^2/c^2)^{-1/2}$ and $\underline{p}(\underline{v}) = m\underline{v}(1 - v^2/c^2)^{-1/2}$. Operational definitions assume that the change in the energy of a particle of electric charge e that crosses a gap across which a potential difference V (as measured by a voltmeter) is sustained is eV (the average electric field $\mathscr{E}$, if the distance across the gap in the direction in which the velocity of the particle changes is D, is simply V/D); similarly ρ, the radius of curvature of the trajectory of a particle moving through a uniform magnetic field of strength $\mathscr{B}$ is related to its charge e and momentum p by $\rho = cp/e\mathscr{B}$. Since this radius of curvature can be defined geometrically, i.e., using particle detectors, all we need to complete the operational definition is the fact that the "magnetic field" at the center of a circular loop of wire of radius r in which a current I as measured by an ammeter is flowing is given by $\mathscr{B} = 2\pi I/r$, if the correlated definition given above refers to the radius of curvature projected onto the plane of the loop.

These operational definitions, in addition to geometrical concepts and the unspecified devices "voltmeter", "gap", "ammeter", "wire loop", introduce a new constant e which is also assumed to be a Lorentz scalar, along with two new types of measurement. The whole system of particle detectors, voltage, current, velocity change and radius of curvature (velocity direction change) enable us to determine <u>both</u> the electric charge e and the mass m of a particle. If the initial velocity is measured to be $v_0 = \sigma_0/\tau_0$, the energy change in crossing a voltage gap V is related to the final velocity $v = \sigma/\tau$ by $\epsilon(v) - \epsilon(v_0) = eV$; if the direction rather than the magnitude of the velocity is changed by a magnetic field of strength $\mathscr{B}$ producing a radius of curvature ρ (in which case $v = v_0$) then $p(v) = p(v_0) = e\mathscr{B}\rho/c$. These relations can be solved to determine e and m in terms of the measurable quantities $\sigma_0, \sigma, \tau_0, \tau, V, I, \rho$ in various ways depending on the experimental setup. Similarly, once we have measured the charge and mass of identifiable particles, we can use the above relations to calibrate the readings given by additional voltmeters and ammeters, and hence <u>indirectly</u> to measure $\vec{\mathscr{E}}$ and $\vec{\mathscr{B}}$ via particulate motion. Historically, the measurement of elementary charges was carried out by Millikan using microscopic observations of the behavior of oil drops in air, but now that detectors which measure the passage of charged particles as individual events are standard equipment, we believe the operational definition just given is much simpler.

Experientially, of course, the above definitions would not be of much use were there not a large number of very different experimental contexts in which we could both measure charges and masses of particles and demonstrate these charges and masses to be Lorentz scalars. Once we have convinced ourselves that this is so, in any single coordinate system describing a system of particles, each moving with constant velocity, it is a trivial consequence of our formalism that (measured in a single inertial frame) the sum of their energies and the vector sum of their momenta are individually conserved. That this is also true when, in an isolated system, the individual velocities change from their initial values to some other set, was the basic reason for introducing the concept of mass into mechanics in the first place. To the extent that these conservation laws hold, the operational definitions given above for charged particles can be extended to define what we mean by electrically neutral particles and measure their masses, as has been discussed elsewhere.[12]

Up to this point the operational definitions we have set up suffice to establish the classical relativistic kinematical description of a system of particles, but this is not enough to establish a system that we call "Democritean". The first reason has already been hinted at above. Unless there is some way to put a limit on the (smallest) size of detector, our basic distinction between atoms and the void dissolves back into the continuum. One way to set such a limit would be to assume that there is a smallest charge e and lightest mass m_e, which is obviously a Democritean postulate, in agreement with experiment, and defines experientially a unit of length $e^2/m_e c^2 = 2.8 \times 10^{-13}$ cm. But Eherenfest already showed in the nineteenth century that no system containing only charged particles is stable. When Lorentz tried to force the Maxwell theory down to the limiting distance $e^2/m_e c^2$ (at which the electrostatic energy associated with a charge of still smaller average dimensions exceeds its rest mass-energy), he ran into inconsistencies and infinities that were never resolved. This length is a good estimate of where classical relativity <u>must</u> break down, but does not indicate where to look for a different "fundamental length" or some other novel concept that might resolve the classical paradoxes.

The second reason why we cannot accept a classical relativistic particle theory as Democritean, even if some consistent way were found to introduce a fundamental length into it, is more subtle. Such a theory is <u>static</u> in the (four-dimensional) Minkowski space, and incapable, internally, of describing change. In practice, theories of this type are used to make causal predictions <u>after</u> the system has (at least implicitly) been set up by the experimenter (or some other "diety"). Such an implicit introduction of an entity outside the system in order to give it operational significance (a logical requirement of deterministic theories ?) seems even more foreign to an atomic and materialistic philosophy than the "swerve" of Epicurus, or its modern counterpart, the random events of quantum mechanics. But these random aspects of particles are by now well known, operationally definable as we demonstrate below, and lead to the additional constant (beside m and c) needed to set the (local) scale of our Democritean particle theory.

The measurement of the particulate unit of electric charge via the oil drop experiment is not easy to describe. Our proposed operational definition of "charged particle" using particle detectors is, hopefully, more direct. One way to approach a simple operational definition of the wave-particle duality aspects of quantum mechanics, is to exhibit the measurement of deBroglie waves by means of a "reflection grating". This need not be a new device, since we can invoke the measurement system already defined to justify the idea of a rectilinear array of particle detectors with equal spacings d between them. Then if a "beam" of particles falls on this array, and we have a system of detectors at a distant region close to the direction of specular reflection θ_s from the array, we find a system of maximum and minimum counts (as we accumulate data) as a function of angle θ with maxima at $\theta_n = \theta_s \pm n\Delta\theta$. The measured constant $\Delta\theta$ is related to the spacing between detectors in the "grating" and the momentum (assumed well defined) p of the particles in the beam by $(h/p) = (d/2)\left[\cos(\theta_s + \Delta\theta) - \cos\theta_s\right]$, where h is Planck's constant. It would be a useful exercise for a quantum mechanics class to show that if the setup is used to record both the single reflection event from single detectors in the "grating" and the correlated count in a single detector in the angular array, these correlated counts would give only the "diffraction" patterns determined by the individual detectors in the "grating", while the uncorrelated distribution in θ exhibits the "interference pattern" used above to define h. Historically, of course, the measurement of these "deBroglie waves" by Davisson and Germer was a confirmation of a quantum mechanical prediction, and not a defining characteristic.

The interference phenomena illustrated in the last paragraph require a definition of what we mean by a particle which not only allows us to define its velocity via detections in two or more particle detectors, and its energy, momentum, and mass via electromagnetic measurements and geometrically defined trajectories (which we claim to have accomplished), but also to predict the statistical accumulation of interference patterns exhibiting the appropriate deBroglie wave length if we send enough similar particles through appropriate geometrical setups to provide the requisite counting statistics.. It remains unite those two aspects of particles in a single formalism. This problem can be met by Born's interpretation of the "wave function", as follows. If we "prepare the system" in such a way that at time t=0 the probability of the particle being in volume element dxdydz surrounding the point $\underline{x}$ is $|f(\underline{x})|^2$ dxdydz and if at some subsequent time t we place a particle detector of resolution $\Delta x \Delta y \Delta z \Delta t$ (and 100% efficiency) at a position enclosing $\underline{x}$, the probability of obtaining a count is $|\Psi(\underline{x},t)|^2 \; \Delta x \Delta y \Delta z \Delta t$ where

$$\Psi(\underline{x},t) = \int \frac{F(\underline{p})\, e^{i[\underline{p}\cdot\underline{x} - \epsilon(p)t]}}{\sqrt{p^2+m^2}}\, dp_x\, dp_y\, dp_z \tag{1a}$$

$$F(\underline{p})/\sqrt{p^2+m^2} = \frac{1}{(2\pi)^3} \int dx\, dy\, dz\; e^{-i\underline{p}\cdot\underline{x}}\, f(\underline{x}) \tag{1b}$$

and

$$\int dx\, dy\, dz\ |f(\underline{x})|^2 = 1 \ ; \qquad \hbar = 1 = c \tag{1c}$$

If this prediction is to be Lorentz invariant, $F(\underline{p})$ must be a Lorentz scalar.

That this definition has the appropriate wave character to exhibit the type of interference phenomena for beams of a single type of particle of well defined momenta needed to describe deBroglie waves is obvious from wave theory; we will return to the problem of dealing with more than one type of particle in Section III. But Born's interpretation puts definite limitations on how far we can push the definitions of energy, momentum, and mass used without critical examination up to now. To begin with we cannot associate the "phase velocity" that goes with the deBroglie wave length $v_p = \epsilon(p)/p$ with the particle velocity, since it is unbounded as the momenta go to zero, and limited from below by the limiting velocity. If we make up a "wave packet" centered on some momentum $\underline{p}_0$ this center will move with the "group velocity" $v_g = d\epsilon/dp = p/\epsilon$ evaluated at $p = p_0$ and in the direction of $\underline{p}_0$, which is the same as the particle velocity. Different momenta in the packet will move with different velocities; the uncritical use of particle trajectories employed above implies that the actual sizes and geometries of particle detectors used in scattering experiments do not hit these limitations. This problem is discussed in detail by Goldberger and Watson.[13] Here we illustrate it for the case of most interest to our own development — namely the uncertainty relation between the energy of a particle and the time over which that energy is measured.

Because of the connection between energy and momentum already contained in our operational definition of particle mass, and the Born interpretation of the wave function as a probability wave amplitude, the spacial uncertainty associated with wave packets implies a temporal uncertainty in energy; otherwise we would have to abandon Lorentz invariance or introduce some new dimensional parameter. This limitation, immediately derivable from Eq. (1) using the Born statistical interpretation, is that we can only determine the energy of the particle during a time measured to accuracy Δt with an uncertainty $\Delta\epsilon \geq \hbar/\Delta t$; if we try to exceed this accuracy, individual measurements will exhibit statistical fluctuations around the central valve; these deviations can be measured by this limiting uncertainty. These fluctuations have not actually been observed for high energy particles, or we would use them for our operational definition and derive the deBroglie wave formula from them and Lorentz invariance rather than going through the operational definition of a "grating" used above.

Current techniques can define the position of a particle using wire spark chambers whose wires are 0.1 cm across, with time resolutions that are being pushed down toward the pico-second range (10^{-12} sec); $\Delta t = 10^{-12}$ sec converted to spacial resolution for a fast particle is even less than the wire size ($c\Delta t \sim 0.03$ cm). To find the requirements for observing the uncertainty principle fluctuations in energy, we take these resolutions as representative. What distance, σ, between two detectors is needed to measure the time of passage of a particle between them (τ) to an accuracy which exceeds the limit? $v = \beta c = \sigma/\tau$ is the measured quantity. If the distance between detectors is measured to the same accuracy as the individual resolutions, and similarly for the time interval, and we add all six uncertainties incoherently, the uncertainty in the dimensionless parameter β is $\Delta\beta = (\Delta x/\beta\sigma)(3+3\beta^2(c\Delta t/\Delta x))^{1/2}$, or less than $2\,\Delta x/\beta\sigma$ for particles with velocities close to c (we make that restriction so that the energy needed to activate the detector can be small compared to the kinetic energy of the particle). Since $\epsilon = mc^2/(1-\beta^2)^{1/2} \equiv \gamma mc^2$, we find that $\Delta\epsilon\,\Delta t/\hbar = 2\gamma^3(c\,\Delta t/\hbar/mc)\,(\Delta x/\sigma)$ as the quantity which must be less than unity if we are to test the uncertainty principle in energy directly by observing the statistical fluctuations it generates. For electrons ($\hbar/m_e c = 3.86 \times 10^{-11}$ cm) with $\overleftarrow{\gamma} \sim 1$, this works out to a distance σ of about 10,000 miles between the two detectors. Hence, improving the spacial and temporal resolutions of the detectors each by a factor of only 100 would bring the requirement in length down to the dimensions of vacuum flight paths currently available in high energy accelerator laboratories.

This calculation shows why we use deBroglie waves for our operational definition of the fundamental constant $\hbar$, rather than the energy fluctuations. The Wick-Yukawa mechanism depends on energy fluctuations that are not yet demonstrable, but may become detectable in the future. Conversely, the fact that these fluctuations are not now directly observable allowed us to start our discussion and operational definition with classical relativistic particulate concepts; we only had to introduce the wave-particle duality at a later stage. A similar analysis would show that if we calculate the particle trajectories through the macroscopic electromagnetic fields needed to measure energy and momentum using classical electromagnetic theory we will, within the accuracy of current measurements, get the same result as if we calculated trajectories as the perpendiculars to the wave fronts of Eq. (1) using the prescription $\underline{p} \rightarrow \underline{p} - e\underline{A}(\underline{x}, t)$, $\epsilon \rightarrow \epsilon - e\,\Phi(\underline{x}, t)$ in the exponential; here $\underline{A}$ and Φ are the vector and scalar "potentials" corresponding to the fields $\vec{\mathscr{E}}$ and $\vec{\mathscr{B}}$ through which the particle is moving. Thus our operational definitions are justified in the realm where they are actually used to determine particle masses and charges, but have built in limitations of which we must be wary when we try to extrapolate the theory down to atomic dimensions.

The most fundamental of these limitations, from our point of view is the restriction on the concept of particle number and the necessity for particle scattering entailed by that restriction. The argument was originally due to Wick;[8] we restate it for our purposes as follows. Think of two wave packets initially representing distinguishable particles (e.g., spacially separated or of different mass) that come to occupy (with high probability) the same volume of approximate radius δr for a time of duration approximately δt. Because of the uncertainty principle, the energy within this region is uncertain by an amount $\delta\epsilon \gtrsim \hbar/\delta t$. If we make the Democritean postulate that there is some smallest relevant mass m, we cannot tell whether or not this particle is present if the time δt is so short that $\hbar/\delta t \geq mc^2$. If we further postulate that momentum is still conserved when such a particle appears, the momentum of the two initial particles must change, and when they subsequently separate far enough so that we can be reasonably certain that the mass m can no longer be there, the individual momenta with which they emerge need not be the same as those with which they started, even though the total momentum of the system is conserved. The dimension of the region δr in which this can happen is limited to the distance which the particle m can move during the time δt, and hence must be less than $c\delta t$. Hence $\delta r \leq c\delta t \leq c\hbar/\delta\epsilon \leq \hbar/mc$. We conclude that our operational definition of particles predicts scattering at distances of approach (geometrically defined from external wave-packet trajectories) less than $\hbar/mc$, independent of any assumptions about "interactions". This is Wick's explanation of the connection between the mass of Yukawa's "heavy quanta"[9] and the range of nuclear forces. The same derivation immediately suggests that if the energies of the two initial particles are high enough, the mass m can be materialized in the process and emerge as a third free particle in the final state. Verification of the Yukawa hypothesis occurred when the particle whose mass, spin, parity, and charge states had been predicted[14] from observed properties of "nuclear forces" was materialized in accelerator experiments and shown to possess all of the predicted properties.[15] This particle is now called the pion, and particles which can produce or absorb pions with high probability (i.e., cross sections of order $\pi(\hbar/m_\pi c)^2 \sim 6\times 10^{-26}$ cm^2) are called hadrons.

To summarize our operational definition, a particle is

(1) a system characterized by the Lorentz scalar $m = +\sqrt{\epsilon^2 - p^2}$ where ϵ and p can be measured using electromagnetic fields if the particle is charged or the conservation of momentum and energy if it is neutral.

(2) Until the characteristics of the system change in a (potentially) observable way, the wave function $\Psi_i(\underline{x}_i, t)$ of each particle i in the system which was known at time t=0 to be within dxdydz of the point $\underline{x}$ with a probability $|f(\underline{x}_i)|^2\,dxdydz$ is given by Eq. (1); this definition implies that m and $F(\underline{p})$ are Lorentz scalars.

(3) The probability of producing a count at $(\underline{x}, t)$ in a particle detector of resolution $\Delta x\Delta y\Delta z\Delta t$ surrounding that event converges in the sense of the law of large numbers to $|\Psi(\underline{x}, t)|^2\,\Delta x\Delta y\Delta z\Delta t$.

III. PARTICLE SCATTERING AND PRODUCTION

In many instances the study of elementary particles follows a route which can be easily described using the wave functions developed in the last section. Beams of particles of unique mass and very precisely defined energy and momentum are prepared by electromagnetic devices and allowed to intersect each other or to strike a stationary target containing particles also of known mass. The particles emerging from these regions of overlap are then analyzed as to momentum and energy (and hence mass) again using macroscopic electromagnetic devices, and the passage of individual particles recorded using particle detectors. When possible, enough statistics are accumulated to obtain a reasonable estimate (e.g., to a confidence level of 1-5%) of the probability of that type of particle emerging in that direction with that energy and momentum; the particulate events themselves are individually "random". This probability is quoted in terms of the area of the beam (normalized to one particle in the beam) which has been diverted to produce one event of the type detected, and is called a "cross section". In this section we assume that all fluctuations in particle number which the uncertainty principle allows to occur in the volume of the intersecting beams (or beam and target) which can occur for a specified (finite) number of particle types do occur. We further assume that the total energy and momentum of the emerging particles is equal to that of the particles initially present, and that the fluctuations are summed in such a way as to prevent the recovery of any knowledge of where in the volume the events occurred (knowledge which would violate the uncertainty principle and introduce hidden variables). These assumptions are enough to allow us to recover the conventional scattering formalism.

The precise statement of the "scattering boundary conditions" described verbally in the last paragraph is that at some distant time in the past, there were N_A particles of mass m_n and precisely defined momenta $\underline{P}_n$ present in the region of interest. The wave function describing this situation is:

$$\Psi_A = \frac{1}{(2\pi)^{3N_A/2}}\, e^{i \sum_{n=1}^{N_A} \underline{P}_n \cdot (\underline{x}_n - \underline{X}_n)}\, e^{-iE_A t}$$

$$E_A = \sum_{n=1}^{N_A} \epsilon_n(P_n) \qquad (\hbar = 1 = c) \tag{2}$$

where we have introduced the (unmeasurable) parameters $\underline{X}_n$ in order to preserve translational invariance; this function describes the system as t approaches $-\infty$. After the events that (because of the Wick-Yukawa mechanism) are sure to occur have happened, and the resulting system of N_B particles with masses m_n and momenta $\underline{K}_n$ has separated sufficiently so that the individual momenta can be measured to high precision and the individual masses identified, the appropriate wave function describing this situation will be

$$\Psi_B = \frac{1}{(2\pi)^{3N_B/2}}\, e^{i \sum_{n=1}^{N_B} \underline{K}_n \cdot (\underline{y}_n - \underline{Y}_n)}\, e^{-iE_B t}$$

$$E_B = \sum_{n=1}^{N_B} \epsilon_n(\underline{K}_n) \tag{3}$$

If we assume that this "scattered" wave function arose from a single fluctuation at some time t', $-\infty < t' < t$, with a probability amplitude $A_{BA}^{t'} \Psi_A$ (i.e., linearly proportional to the probability amplitude of the initial state), then the wave function describing this single event will be

$$\Psi^{t'}_{BA} = \psi_A(\underline{x}_n, \underline{X}_n, t) + A^{t'}_{BA}\,\psi_A(t)\;\theta(t-t')$$
$$\equiv \psi_A(\underline{x}_n, \underline{X}_n, t) + P^{t'}_{BA}\,\psi_B(\underline{y}_n, \underline{Y}_n, t-t')\;\theta(t-t') \tag{4}$$

Even though we are describing events which lead to only N_B particles each of a specified mass, there can be many different states B, so to obtain a wave function which will describe this situation we must sum over all of these as well as over all possible fluctuations up to the present time t. That is

$$\Psi_{N_B N_A} = \psi_A(\underline{x}_n, \underline{X}_n; t) + \int_{-\infty}^{t} dt'\; \Sigma_B\, P^{t'}_{BA}\, \Psi_B(\underline{y}_n, \underline{Y}_n, t-t') \tag{5}$$

In order that this description reduce to Ψ_A as $t \to -\infty$, we introduce the obvious convergence factor $e^{+\eta(t'-t)}$ (with the implied limit $\eta \to 0^+$ to be taken later) and with

$$\Psi_{N_B N_A} = \Psi_A(\underline{x}_n, \underline{X}_n, t) + \int_{-\infty}^{t} dt'\; e^{+\eta(t'-t)}\, \Sigma_B P^{t'}_{BA}\; \Psi_B(\underline{y}_n, \underline{Y}_n, t') \tag{6}$$

This satisfies our boundary conditions at $t = -\infty$, but there are additional conditions which must be imposed on the (so far arbitrary) amplitude fluctuation probability A_{BA}. Note in particular that interference between the two pieces of the wave function could, in general, give information about the $\underline{X}_n$ and $\underline{Y}_n$ and hence introduce hidden variables. In a <u>particular</u> situation which at time t leads to some unique system B with specified momenta $\underline{K}_B$ we have an <u>irreversible</u> transition in which the (so far virtual) processes we are describing become determinate (though only partially retrodictable) and join the fixed past. This can happen at any time t', whether <u>recorded</u> (observed) or not. But $\underline{P}_n, \underline{X}_n$ and $\underline{K}_n, \underline{Y}_n$ are canonical variables just as subject to the uncertainty principle as the dynamical variables $\underline{p}_n, \underline{x}_n$ or $\underline{k}_n, \underline{y}_n$. Thus to satisfy our boundary condition of precisely known $\underline{P}_n$ and precisely knowable $\underline{K}_n$, we must insure that $|\Psi_{N_A N_B}|^2$ contains no reference to $\underline{X}_n$ or $\underline{Y}_n$. This can be accomplished by postulating that

$$A_{BA}^{t'} = i(2\pi)^{3N_A/2}\, e^{i\underline{K}_n\cdot\underline{Y}_n}\, \mathscr{T}_{BA}(\underline{K}_1\cdots\underline{K}_{N_B};\, \underline{P}_1,\ldots,\underline{P}_{N_A})$$

$$e^{-i\sum_{n=1}^{N_A}\underline{P}_n\cdot\underline{X}_n}\; e^{-i\sum_{n=1}^{N_B}\underline{K}_n\cdot\underline{Y}_n}\; e^{iE_A t'}\; e^{-iE_B(t-t')} \tag{7}$$

where the energy variables

$$E_A = \sum_{n=1}^{N_A}\epsilon_n(\underline{P}_n) \qquad \text{and} \qquad E_B = \sum_{n=1}^{N_B}\epsilon_n(\underline{K}_n) \tag{8}$$

can differ because of the uncertainty principle connecting energy and time. Equivalently, because of Eqs. (4), (5), and (6), we could assume that

$$P_{BA}^{t'} = i(2\pi)^{3N_A/2}\, e^{\eta(t'-t)}\, e^{i\left(\sum_{n=1}^{N_B}\underline{K}_n\cdot\underline{Y}_n - E_A t'\right)}\, \mathscr{T}_{BA}(\underline{K}_1\cdots\underline{K}_{N_B};\underline{P}_1\cdots\underline{P}_{N_A}) \;. \tag{9}$$

In order to conserve momentum we also postulate that

$$\mathscr{T}_{BA}(\underline{K}_1\cdots\underline{K}_{N_B},\underline{P}_1\cdots\underline{P}_{N_A}) = T_{BA}(\underline{K}_1\cdots\underline{K}_{N_B},\underline{P}_1\cdots\underline{P}_{N_A})\,\delta^3\left(\sum_{n=1}^{N_B}\underline{K}_n - \sum_{n=1}^{N_B}\underline{P}_n\right) \tag{10}$$

Substituting into Eq. (7) and letting $\Sigma_B \rightarrow \int d^3K_1\ldots d^3K_{N_B}$ we obtain

$$\psi_{N_B N_A} = e^{-i\left(\sum_{n=1}^{N_A}\underline{P}_n\cdot\underline{X}_n + E_A t\right)}\left[\frac{1}{(2\pi)^{3N_A/2}}\, e^{i\sum_{n=1}^{N_A}\underline{P}_n\cdot\underline{x}_n} + \frac{i}{(2\pi)^{3N_B/2}} \times\right.$$

$$\left. \times \int d^3K_1\ldots d^3K_{N_B}\int_{-\infty}^{t} dt'\, e^{i(E_A + i\eta - E_B)(t-t')}\, T_{BA}\, e^{i\sum_{n=1}^{N_B}\underline{K}_n\cdot\underline{y}_n}\,\delta^3\left(\sum_{n=1}^{N_B}\underline{K}_n - \sum_{n=1}^{N_A}\underline{P}_n\right)\right] \tag{11}$$

Thus, by insuring that there is no way of determining $\underline{X}_n$ or $\underline{Y}_n$ if $\underline{P}_n$ and $\underline{K}_n$ can be precisely known (i.e., by postulating the uncertainty principle), and insuring momentum conservation we show that these boundary conditions predict

$$\Psi_{N_B N_A} = e^{-i \sum_{n=1}^{N_A} \underline{P}_n \cdot \underline{X}_n} \Phi_{N_B N_A}(\underline{x}_n, \underline{y}_n)\, e^{-iE_A t} \tag{12}$$

where $\Phi_{N_B N_A}$ is the usual stationary state wave function as given, for example, in Goldberger and Watson[13]

$$\Phi_{N_B N_A} = \frac{1}{(2\pi)^{3N_A/2}} e^{i \sum_{n=1}^{N_A} \underline{P}_n \cdot \underline{x}_n} + \frac{1}{(2\pi)^{3N_B/2}} \int d^3K_1 \ldots d^3K_{N_B}$$

$$\frac{T_{BA}\, e^{i \sum_{n=1}^{N_B} \underline{K}_n \cdot \underline{y}_n}\, \delta^3\left(\sum_{n=1}^{N_B} \underline{K}_n - \sum_{n=1}^{N_A} \underline{P}_n\right)}{\sum_{n=1}^{N_A} \epsilon_n(\underline{P}_n) + i\eta - \sum_{n=1}^{N_B} \epsilon_n(\underline{K}_n)} \tag{13}$$

with the conceptually significant difference that T_{BA} is now an <u>arbitrary</u> function referring to fluctuations and not to interactions.

We hope that this derivation of the conventional scattering wave function (from which the restrictions on T_{BA} required for Lorentz invariance, and the connection to observable cross sections now follow in the usual way) makes it clear that, from a particulate point of view, there is no necessity to postulate any specific type of "interaction mechanism" (such as a "potential" or "interaction Lagrangian") in order to derive a Lorentz-covariant description of any conceivable hadronic scattering process connecting a finite number of particles in and out at a finite (asymptotic) energy. One way of viewing what we have just done is to say that we have presented a complete kinematics for quantum scattering processes, and have cleanly separated that descriptive problem from the dynamical problem of actually computing T_{BA}.

One fact worth noting in this derivation is that it was originally[2, 16] arrived at in quite a different way. Phipps' interpretation of quantum mechanics[3, 4, 5] starting from the Hamilton-Jacobi (operator) equations makes it clear that in the quantum limit in which the action goes to the unique value $\hbar/i$, the only difference from conventional Schrœdinger theory is that the Schrœdinger wave function $\phi(x_k, t)$ is multiplied by the phase factor $\exp\left(-i\Sigma_k P_k X_k\right)$ where P_k and X_k are (in the classical limit) both canonical variables and constants of the motion, and the sum over k runs over the degrees of freedom. Phipps notes that, because of their classical interpretation, it is natural to assume that they are also constants in the quantum limit, but change, both randomly and irreversibly, when any virtual process is completed and joins the fixed past. This was the starting point of the author's paper[2] entitled "Fixed Past and Uncertain Future", which took shape once he had insured covariance by using only free-particle wave functions and fluctuations in particle number as the basic concepts, and had noted that the Phipps phase factor already occurs in conventional scattering theory (e.g., Goldberger and Watson[13]) where it is used to construct wave packets. To satisfy questions about that approach, the derivation given above was constructed, using the Lippmann-Schwinger Ansatz,[17] to exhibit explicitly how this phase factor still occurs in a theory that does not mention "interactions". The operational definition of particle provided in the last section now allows us to drop also any reference to the Hamilton-Jacobi equations, and to construct a hadronic scattering theory from first principles.

The kinematics just presented has some pecularities, in that the T_{BA} introduced above, which obviously need not conserve particle number, also need not even conserve probability (i.e., is not guaranteed to be unitary). Thus, if we are not to get into trouble with macroscopic facts about thermal equilibrium (detailed balance), we must make sure that any dynamics used to compute T_{BA} provides explicitly for unitarity. One such dynamics, if the S-matricists are right, could be provided by postulating unitarity and analyticity (i.e., crossing symmetry) which then, according to Stapp,[18] leads to CPT and the usual connection between spin and statistics. But our approach, if we take care to insure only unitarity, need not contain the second postulate, and hence might be of use in constructing theories which violate these two conditions in a Lorentz-covariant way; such models could be used to increase the precision of experimental tests of these fundamental regularities.

It is not the purpose of this article to develop a dynamics for T_{BA}, but we note that for three-body problems, an appropriate dynamics exists in the non-relativistic limit: the on-shell Faddeev equations. We have shown[19] that these equations allow non-trivial calculations of three-body scattering problems using only measured two-body phase shifts and binding energies as input, if restrictions derived from the known mass spectrum of hadrons are taken seriously.

IV. PARTICLE DETECTORS CONSTRUCTED OF PARTICLES

Given particle detectors and the geometry they can be used to articulate, the Lorentz invariants m and e can be used to provide operational definitions of the "fields" through which a particle has passed, and hence to calibrate the whole scheme of charged particles in a Minkowski universe, flipping back and forth between (small) detectors and (large) regions in which there are fields. But to deal withsituations in which the fields used to measure particulate properties can influence the detectors, and to understand the detectors themselves, we need to extend the theory. The missing additional postulate is that the densities of the charge and current sources in Maxwell's equations due to a single particle of charge e whose wave function is $\Psi(\underline{x}, t)$ are, in a volume surrounding $\underline{x}$, t of dimension dxdydzdt, given by

$$\rho(\underline{x}, t) = e\,|\Psi(\underline{x}, t)|^2$$
$$\underline{j}(\underline{x}, t) = \frac{e\hbar}{2mi}\left[\Psi^* \underline{\nabla}\Psi - \Psi\underline{\nabla}\Psi^*\right] \tag{14}$$

Note that because our basic definition (Eq. (1)) is covariant, a constant amplitude in momentum space does not correspond (as it would in a non-relativistic theory) to a δ-function in coordinate space but to a source of finite size whose charge distribution falls off exponentially with a scale length $\hbar/mc$. This is a warning that if we try to discuss problems at distances shorter than that we must not use single particle wave functions, but instead the multiparticle scattering wave functions developed in the last section; this is just another example of the basic importance of the Wick-Yukawa mechanism in any quantum particle theory.

The most important single case we can discuss is a system of two particles of opposite charge described accurately only for distances large compared to

$\hbar/mc$ (or equivalently frequencies small compared to $mc^2/\hbar$), and to first order in $e^2/\hbar c$. Then, if we treat one particle as the source of the field in which the other moves, the external field which we introduce into the wave function according to the prescription given in Section II is just the field due to the Coulomb potential $-e^2/|\underline{x}_1 - \underline{x}_2|$. By expanding the exponential to first order in $e^2/\hbar c$, factoring out the time dependence $\exp(-i\, mc^2 t/\hbar)$, and keeping only the first term in an expansion in powers of $(p/mc)^2$, the resulting wave function for the relative motion of the two particles satisfies the usual non-relativistic atomic Schrœdinger equation, e.g., that for the "hydrogen atom" if $m = m_e/(1+m_e/M_p)$. Thus, in the appropriate limit our particle theory can generate to first order all of non-relativistic atomic theory, once we add dicotomic spin variables and the Pauli principle (see below). The whole system of particle accelerators, analyzing magnets, particle detectors, counters, electronic computers, etc., introduced in rather abstract form in the operational definition of particles given in Section II and articulated for scattering experiments in Section III can then be explained (to order $e^2/\hbar c$) using conventional theories.

This construction, which we have no space to develop here, implies that both "photons" and "fields" are simply convenient ways of describing how the motions of some charged particles in the past are correlated with the motions of other charged particles in the future — a point of view developed in detail by Feynman and Wheeler in the forties.(20) Further, if we concentrate on the restrictions of measurability of electromagnetic fields using particulate measuring apparatus which satisfies the uncertainty principle, the analysis of Bohr and Rosenfeld(21) demonstrates that the "commutation relations" of the Maxwell fields more conventionally derived via "second quantization" are completely and precisely reproduced. That their analysis (like ours) cannot be extended down to dimensions comparable to $\hbar/mc$ is explicitly recognized in their paper.

Although, so far, we have derived equations only for spinless particles, the extension of the theory (to order $e^2/\hbar c$) to charged particles of spin 1/2 is straightforward. We simply use the appropriate positive energy two-state (e.g., helicity) spinors for particles with one sign of charge, and corresponding "charge-conjugate" (but still positive energy) spinors for the other sign. Details are given in Stapp[18] together with the additional assumptions needed to insure CPT and spin-statistics connections originally derived from "field theory". Following the same route we used above to obtain the Schrœdinger equation, we can then derive the one-particle Dirac equation, and the properties of electron spin needed for most atomic theory. Note that we know, a priori, that this theory is inadequate for energies greater than $2m_ec^2$, or distances shorter than $\hbar/2m_ec$.

Whether this phenomenological theory can now be extended to a deeper level, and actually become the starting point for a particle theory that might replace the conventional quantum field theories remains uncertain. For hadronic physics, the situation looks promising, as had been discussed elsewhere.[16, 22] For quantum electrodynamics, the first question that must be resolved is whether the calculation of the motion of two charged particles can be pushed beyond $e^2/\hbar c$ to describe to the next order the reaction back of the motion of one particle moving in the "field" generated by the other on the motion of the first particle. To the extent that this motion depends on frequencies greater than $2m_e c^2/\hbar$, we know that this is partly a four-body problem involving an electron-positron pair as well as the two particles with which we start. In some ways the problem is simpler than in conventional quantum electrodynamics, as we have no "self-energies" or "vacuum fluctuations", and the particles themselves in some sense have finite size. The success of French and Weisskopf[23] in computing the Lamb shift using conventional perturbation techniques and positrons in positive energy states makes correct results likely, but since the French-Weiskopf calculation still required "renormalization" to remove various infinities, success is by no means guaranteed.

V. CONCLUSION

The basic concept of a particle exhibiting a Lorentz-invariant mass and charge, and wave-particle duality, can be established by operational definitions using particle detectors and external electromagnetic fields in a manner analogous to actual practice in high energy particle physics. Given these definitions, and the recognition that they entail fluctuations in particle number, and hence scattering and particle production, it is then possible to set up the conventional relativistic stationary-state scattering formalism, with the conceptual difference that the transition matrix is an arbitrary function that may be descriptive (kinematical) and need not be derived from a dynamical model (in particular need not require a postulate that there are "interactions"). By invoking the usual prescription which attributes electromagnetic fields to the charge and current densities calculated from particle wave functions, all properties of conventional quantum electrodynamics can be recovered to order $e^2/\hbar c$. This in turn allows a description to the same order of the particle detectors needed to start off the analysis, and also allows the electromagnetic field to be viewed as a mathematical convenience introduced to facilitate the calculation of dynamical correlations between particulate motions. Whether this self-consistent phenomenology can be extended to the calculation of effects of order $(e^2/\hbar c)^2$ and higher in agreement (to current experimental accuracy) with the results of quantum electrodynamics remains a problem for future research.

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