Does the halting problem place an actual limit on computation?

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Abstract

Hehner and Stoddart agree that the halting problem has an inconsistent, unsatisfiable specification. Hehner and Macias agree that a key issue with the halting problem is that it requires a: subjective specification(Hehner) / context dependent function(Macias). When a problem has an unsatisfiable specification because this specification is inconsistent then the unsatisfiability of the specification is anchored in its error thus does not actually limit computation.

Analysis of the Halting Problem

> A specification is subjective if the specified behavior varies depending on the agent that performs it. (Hehner:2017)
>
> But there is a class of computer functions whose behavior is dependent on the context in which they are called or used: these may be called Context-Dependent Functions (CDFs). (Macias:2014)

To unify these two views
(a) Subjective specifications derive context dependent functions.
(b) Objective specifications prohibit context dependent functions.

Both Macias and Hehner deal with uncomputable sequences. For Hehner twisted self-reference in a subjective specification makes halts() have an unsatisfiable and inconsistent specification. Macias has a similar view shown below:

> Now run BAD(BAD) and consider what happens: ...
> Note that these are the only two possible cases, and in either case (whether HALT returns 0 or 1), HALT′s behavior is incorrect, i.e., HALT fails to answer the Halting Problem correctly. (Macias:2014)

I paraphrase the above: When HALT(BAD,BAD) is a context dependent function and called by BAD then HALT(BAD,BAD) cannot possibly get the correct answer.

So both Hehner and Macias seem to agree that (subjective specifications / context dependent functions) cause a halt decider to get the wrong answer on some inputs.

> The proof of the halting problem assumes a universal halt test exists and then provides S as an example of a program that the test cannot handle. But S is not a program at all. It is not even a conceptual object, and this is due to inconsistencies in the specification of the halting function. (Stoddart: 2017)

Stoddart supports Hehner's view using the similar terminology above. Hehner's view is that it is logically impossible to solve the halting problem because it has an inconsistent, unsatisfiable and subjective specification. The view of Macias was shown to be very similar.

When a problem has an unsatisfiable specification because its specification is inconsistent this is saying that the problem itself is defined to be unsatisfiable. A problem definition that is defined to be unsatisfiable is isomorphic to a question that is defined to have no correct answer.

Self Referential Undecidability Construed as Incorrect Questions
Professor Hehner PhD came up with an English question that is perfectly analogous to the halting problem proofs.

His thought experiment must stipulate that any answer besides [yes, no] is a wrong answer. This preserves its correspondence to a Turing machine halt decider:
Can Carol correctly answer "no" to this [yes/no] question?

> ...is a consistent, satisfiable specification for some agent (anyone other than Carol), and an inconsistent, unsatisfiable specification for some agent (Carol). (Hehner:2017)

If Carol answers "no" to this question she is saying that "no" is the wrong answer, if she is correct then "no" is the right answer making her necessarily incorrect.

If Carol answers "yes" to this question she is saying that "no" is the correct answer thus making "yes" necessarily the wrong answer.

Thus both [yes, no] are the wrong answer from Carol, thus "no" is the correct answer from anyone else.

Since the question posed to Carol has no correct answer from Carol and the same word-for-word question does have a correct answer from anyone else linguistics understands that these are two different questions because they have different meanings depending on the linguistic context of who is asked.

A concrete example of how the meaning of the same word-for-word question has an entirely different answer depending on who is asked: **Are you a little girl?**

We can see that Carol's question posed to Carol is self-contradictory for Carol because the question contradicts both yes/no answers from Carol.

Upon careful examination we can see that Carol's question posed to Carol is isomorphic to input D to decider H where D has been defined to do the opposite of whatever Boolean value that H returns.

```
// The following is written in C
//
typedef int (*ptr)(); // pointer to int function  see (x86utm)
int H(ptr x, ptr y)   // uses x86 emulator to simulate its input

int D(ptr x)
{
  int Halt_Status = H(x, x);
  if (Halt_Status)
    HERE: goto HERE;
  return Halt_Status;
}

void main()
{
  H(D,D);
}
```

The question: Does directly executed D(D) halt? is contradicted by D for every H
From the above source-code we can see that there cannot possibly exist any H that can return a value corresponding to the direct execution of D(D).

The reason for this is that D has been intentionally defined to do the opposite of whatever value that H returns. This makes the question: Does D(D) halt? A self-contradictory (thus incorrect) question for H.

Conclusion
Linguistics understands that the same word-for-word question sometimes has a different answer depending on who is asked. When H1 is asked the same question: **Does D(D) halt?** It can correctly answer, thus proving that the same word-for-word question has a different meaning depending on who is asked. (x86utm)

The halting problem proofs merely show that self-contradictory questions have no correct answer thus place no actual limit on computation

[1] E C R Hehner. **Objective and Subjective Specifications**
WST Workshop on Termination, Oxford. 2018 July 18.
See https://www.cs.toronto.edu/~hehner/OSS.pdf

[2] Nicholas J. Macias. **Context-Dependent Functions: Narrowing the Realm of Turing's Halting Problem**
13 Nov 2014
https://arxiv.org/abs/1501.03018
arXiv:1501.03018 [cs.LO]

[3] Bill Stoddart. **The Halting Paradox**
20 December 2017
https://arxiv.org/abs/1906.05340
arXiv:1906.05340 [cs.LO]

Addendum

This is apparently the original version of Carol's question that was directed at me twenty years ago. I repeated it as Bill's question hundreds of times forgetting the original authorship. I finally tracked down the original author and Daryl confirmed original authorship.

On 6/25/2004 6:30 PM, Daryl McCullough wrote: sci.logic
> You ask someone (we'll call him "Jack") to give a truthful
> yes/no answer to the following question:
>
> Will Jack's answer to this question be no?
>
> Jack can't possibly give a correct yes/no answer to the question.
USENET Message-ID: **<cbiciv02k04@drn.newsguy.com>**

This is my 2004 work that proposes that the halting problem has an unsatisfiable specification thus asks an ill-formed question.

Alan Turing's Halting Problem is incorrectly formed (PART-TWO) sci.logic
On 6/20/2004 11:31 AM, Peter Olcott wrote:
> PREMISES:
> (1) The Halting Problem was specified in such a way that a solution
> was defined to be impossible.
>
> (2) The set of questions that are defined to not have any possible
> correct answer(s) forms a proper subset of all possible questions.
> …
> CONCLUSION:
> Therefore the Halting Problem is an ill-formed question.
>
USENET Message-ID:
<kZiBc.103407$Gx4.18142@bgtnsc04-news.ops.worldnet.att.net>