Erratum to "The Ricean Objection: An Analogue of Rice's Theorem for First-Order Theories" *Logic Journal of the IGPL, 16(6): 585–590(2008)*

IGOR CARBONI OLIVEIRA and WALTER CARNIELLI, *Centre for Logic, Epistemology and the History of Science (CLE) and Department of Philosophy (IFCH), State University of Campinas - UNICAMP, C.P. 6133 - 13083-970 Campinas-SP, Brazil.*

There is an error in the proof of the main theorem on page 6 ("Rice's Theorem for Logic"). If ϕ is undecidable in Q, the claim that *Ax1* and *Ax2* are tautologies is not valid, and this fact compromises the rest of the proof. Furthermore, the following decidable property is a counter-example to our result:

Definition 1. *Let T be a theory. A theory T^+ is an extension of T if $Th(T) \subseteq Th(T^+)$.*

Definition 2. *$Sub(T)$ denotes the set $\{Th(T^-) : T$ is an extension of $T^-\}$.*

Proposition 1. *If T is a consistent decidable theory, then $Sub(T)$ is a non-trivial decidable property.*

Proof. By the consistency of T, $Sub(T)$ is non-trivial. For each finite $A \subseteq L_\Sigma$, the following holds:

$$Th(A) \in Sub(T) \Longleftrightarrow T \text{ is an extension of} A \Longleftrightarrow T \vdash \bigwedge_{\phi \in A} \phi.$$

Since T is decidable, there exists an algorithm which decides for given finite $A \subseteq L_\Sigma$ whether $T \vdash \bigwedge_{\phi \in A} \phi$ or not. That is, *Sub(T)* is decidable. ■

For instance, $Sub(\{\forall_x \forall_y (x = y)\})$ is a counter-example to our result. This negative result, which is contrary to our initial intuition, led us to consider the existence of these "Ricean" undecidability results in a more general sense.

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Definition 3. *Let T be a theory and Γ be a set of sentences. Γ is a property on T if the following holds for any sentences ϕ and ψ:*

$$T \vdash \phi \leftrightarrow \psi \Rightarrow [\phi \in \Gamma \Longleftrightarrow \psi \in \Gamma].$$

A property Γ is trivial if it is the empty set or the set of all sentences.

In particular, note that a property P in the sense of our paper corresponds to the property $\{\bigwedge_{\phi \in A} \phi : Th(A) \in P$ and A is finite$\}$ on $\emptyset$. If we consider sufficiently expressive theories such as Q (Robinson arithmetic), it is indeed possible to prove that they are undecidable in the sense of Rice's theorem.

Theorem 1. *Every non-trivial property on Q is undecidable.*

For instance, since $Th(Q)$ is a property on Q, we derive as a particular case of this general result that Q is an undecidable theory. Theorem 1 is a consequence of the diagonal lemma, and the reader is referred to [1] for a general treatment of this elegant result and its consequences. A new interpretation of this result will appear in a forthcoming paper.

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References

[1] C. Bernardi. On the Relation Provable Equivalence and on Partitions in Effectively inseparable sets. *Studia Logica* **40** (1981), 29–37.