Abstract Objects

According to Naturalism, Causal Reality is entirely natural: there are none but entirely natural causal entities possessing none but entirely natural causal properties. Moreover, according to Naturalism, there are none but entirely natural causal relations that hold between none but entirely natural causal states and/or none but entirely natural causal events and/or none but entirely natural causal processes.

According to Theism, Causal Reality is a mix of natural and supernatural causal entities possessing a mix of natural and supernatural causal properties. On this view, there are natural and supernatural causal relations that hold between natural and supernatural states and/or natural and supernatural events and/or natural and supernatural processes. Furthermore, on this view, there is an initial part of Causal Reality that is entirely supernatural: in the beginning, Causal Reality involves no causal beings other than God and no instantiated causal properties other than the supernatural causal properties instantiated by God. However, according to this view, at some non-initial point in Causal Reality, God makes or creates Natural Reality, which—at the very least—is populated by causal entities that are not entirely supernatural and that possess causal properties that are not entirely supernatural.

When we compare the relative merits of Naturalism and Theism, we often focus primarily on comparison of their respective accounts of Causal Reality. Given an agreed conception of theoretical virtues—simplicity, goodness of fit with data, explanatory breadth, predictive fruitfulness, and so forth—we can compare the theoretical virtues of Naturalism and Theism against a wide range of 'causal' data: the global shape of Causal Reality; cosmic fine-tuning; the history of our universe; the history of the earth; the history of humanity; the suffering and flourishing of human beings and other living creatures; the nature and distribution of consciousness and reason; reports of miracles and religious experiences; the nature and distribution of religious scriptures, religious authorities, religious organisations, and religious traditions; and so forth.

However, it is not immediately obvious that a comparison of the relative merits of Naturalism and Theism should focus exclusively on their comparative theoretical virtues in the light of 'causal' data. True enough, if Reality is exhausted by Causal Reality, then Naturalism and Theism are properly construed as no more than competing theories about Causal Reality. But, if there is more to Reality than Causal Reality, then, as comprehensive theories, Naturalism and Theism will be properly construed as competing theories about that more extensive Reality. And then it is at least appears conceivable that it could turn out that, while one of these theories leads when only Causal Reality is taken into account, the other theory wins when all of Reality is taken into account.

The wider question that serves as background to this paper is whether there is any serious prospect that, while one of Naturalism and Theism leads when only Causal Reality is taken into account, the other theory wins when all of Reality is taken into account. However, our principal focus will be somewhat narrower. Suppose that we call the difference between

Reality and Causal Reality 'Abstract Reality'. The question that will be our principal focus is this: Does either of our theories—Naturalism and Theism—afford a better account of Abstract Reality? Of course, it is plainly conceivable that there is no difference between Reality and Causal Reality; and, in that case, the answer to the question just framed is trivial. But there are well-known considerations that have prompted some philosophers to suppose that there is an Abstract Reality; and we shall take it that all responses to those well-known considerations are 'accounts of Abstract Reality'. If we conclude that there is no significant difference between the accounts that Naturalism and Theism can give of Abstract Reality, then we shall have a negative answer to our wider background question, and we shall be able to insist that any further comparison of the relative merits of Naturalism and Theism need only consider their respective accounts of Causal Reality.

In what follows, I shall (1) make some further remarks about the distinction between Causal Reality and Abstract Reality; (2) provide a taxonomy of accounts of Abstract Reality; (3) say something about the merits of competing accounts of Abstract Reality; (4) give a detailed argument for the conclusion that considerations about Abstract Reality do not differentially support either Theism or Naturalism; (5) explain why tokenism and conceptualism are not genuinely competitive accounts of Abstract Reality; (6) offer some critical remarks on Plantinga's sketches of arguments for the existence of God from numbers, sets and propositions; and (7) draw some appropriate final conclusions.

1.

There are many potential denizens of Abstract Reality: numbers, sets, classes, functions, mappings, structures, groups, rings, algebras, states, patterns, propositions, contents, intentional objects, properties, universals, attributes, characteristics, types, normative principles, values, utilities, generic objects, arbitrary objects, intensional objects, mere possibilia, impossibilia, incomplete objects, and so on. Of course, friends of Abstract Reality debate amongst themselves which of these—and other—potential denizens of Abstract Reality *are* denizens of Abstract Reality. Various questions arise.

One question concerns the distinction between Causal Reality and Abstract Reality, or the allied distinction between concrete objects and abstract objects. There are various candidates for distinguishing between concrete objects and abstract objects—e.g. that the latter are 'non-mental and non-sensible', or 'non-mental and non-physical', or 'non-spatial and causally inefficacious', or 'generated by an abstraction function', etc. In my introductory remarks, I have taken it for granted that the right way to draw the distinction is in terms of causation: concrete objects are denizens only of Causal Reality; abstract objects are denizens only of Abstract Reality. But not everyone is persuaded that the distinction can be drawn in these terms. Consider, for example, Rosen (2012):

It is widely maintained that causation strictly speaking is a relation among events or states of affairs. If we say that the rock—an object—caused the window to break, what we mean is that some event or state (or fact or condition) *involving* the rock caused the breaking. If the rock itself is a cause, it is a cause in some derivative sense. But this derivative sense has proved elusive. The rock's hitting the window is an event in which the rock 'participates' in a certain way, and it is because the rock participates in events in this way that we credit the rock itself with causal efficacy. But what is it for an object to *participate* in an event? Suppose John is thinking about the Pythagorean Theorem and you ask him to say what's on his mind. His response is an event—the utterance of a sentence; and one of its causes is the event of John's thinking about the theorem. Does the Pythagorean Theorem 'participate' in this event? There is surely *some* sense in which it does. The event consists in John's coming to stand in a certain relation to the Theorem, just as the rock's hitting the window consists in the rock's coming to stand in a certain relation to the glass. But we do not credit the Pythagorean Theorem with causal efficacy simply because it participates in this sense in an event which is a cause. The challenge is therefore to characterize the distinctive manner of 'participation in the causal order' that distinguishes the concrete entities. This problem has received relatively little attention. There is no reason to believe that it cannot be solved. But in the absence of a solution, this [characterisation] must be reckoned a work in progress.

It seems to me to be entirely plausible to suppose that Rosen's challenge can be met. Suppose, for example, that we think that causation is marked by transfer of conserved quantities: wherever there is causation, there is transfer of conserved quantities between entities. When the rock hits the window, there is transfer of conserved quantities—energy and momentum— between the rock and the window. When John tells you what is on his mind, there is transfer of conserved quantities between his neural states and yours, via an intermediate chain of such transfers. John's thinking about the Pythagorean theorem just is his being in a certain kind of neural state—that is appropriately characterised using the expression 'thinking about the Pythagorean theorem'—but there is no transfer of conserved quantities anywhere in the relevant causal chain between the Pythagorean theorem and other entities. Whereas the rock and window 'participate in the causal order in the manner that is characteristic of causal entities', the Pythagorean theorem does not.

Now, of course, not everyone accepts that causation is marked by transfer of conserved quantities, and not everyone accepts that mental states just are neural states—but these assumptions are not essential to the meeting of Rosen's challenge. What matters is that there be some story told about causation that, on the one hand, brings the rock and the window into the causal domain *as causal entities*, in the way that the invocation of transfer of conserved quantities between the rock and the window does; and, on the other hand, invokes the Pythagorean Theorem only in roles—e.g. of characterisation of the content of mental states— that do not bring *it* into the causal domain *as a causal entity*. (Perhaps it might be added here that we should be a bit cautious in accepting that there is some sense in which the Pythagorean Theorem 'participates' in the event of John's thinking about the theorem. For

suppose, instead, that John had been thinking about Santa Claus. Is there really a good sense in which Santa Claus 'participates' in the event of John's thinking about Santa Claus? Non-existent entities simply cannot be causal entities in the causal domain; it is *hopelessly* wrong to suppose that Santa Claus is involved in transfers of conserved quantities with other causal entities that belong to Causal Reality.)

Another question concerns allegedly distinctive properties of the denizens of Abstract Reality. It is often claimed that the denizens of Abstract Reality are distinguished by facts concerning the *necessity* of their existence and nature, the extent to which it is possible to have *a priori* knowledge of their existence and nature, and the *absolute* truth or falsity of claims made concerning their existence and nature. However, it is important not to go beyond what is properly defensible in making claims of these kinds. In particular, it is important to note that the denizens of Abstract Reality appear to divide into *two* classes. On the one hand, there are the *pure* abstracta, which: (a) exist of *necessity*; (b) have only *essential* intrinsic properties and *essential* relations to other pure abstracta; (c) can be known *a priori* to exist and to have the intrinsic properties and relations to other pure abstracta that they do have; and (d) can be described or referred to in sentences that are true or false *absolutely*, and not merely true or false relative to a certain type of theory or model. On the other hand, there are the *impure* abstracta, which (a) exist *contingently*, but whose existence is necessary given the existence of appropriate denizens of Causal Reality; (b) have *accidental* intrinsic properties and *accidental* relations to other abstracta, but only in cases where those intrinsic properties and relations are necessary given the existence of appropriate denizens of Causal Reality; (c) can only be known *a posteriori* to exist and to have the intrinsic properties and relations to other abstracta that they do have, despite the fact that it can be known *a priori* that these abstracta exists and have the properties and relations that they do given the existence of appropriate denizens of Causal Reality; and (d) may be described or referred to in sentences that are merely true or false relative to a certain type of theory or model, depending upon whether or not that theory or model adverts to the existence of appropriate denizens of Causal Reality. Putative examples of pure abstracta include numbers and pure sets (the iterative hierarchy generated from the null set); putative examples of impure abstracta include impure sets (e.g. unit sets of denizens of Causal Reality). (Cf. Yablo (2002).)

2.

The most obvious distinction to draw, in connection with views about Abstract Reality, is the distinction between (a) views which claim that there are some abstract objects (and hence which affirm that there is a domain of Abstract Reality), and (b) views which claim that there are no abstract objects (and hence which deny that there is any such domain as Abstract Reality). I shall refer to the former class of views as *realism* about Abstract Reality, and to the latter class of views as *anti-realism* about Abstract Reality.

Within realist views about Abstract Reality, we can distinguish between views that are full-bloodedly committed to abstract objects, and views that have a more deflationary commitment to abstract objects. Examples of what I take to be full-blooded commitments to abstract objects include the commitment of Quine (1960) to sets, the commitment of Armstrong (1978) to universals, and the commitment of Gödel (1964) to numbers. An example of what I take to be more deflationary commitment to abstract objects is the commitment of Hale and Wright (2009) to numbers.

Within anti-realist views about Abstract Reality, we can distinguish between *struthioism*—which barefacedly denies that straightforward talk ostensibly about abstract objects brings with it commitment to the existence of any such objects—and *fictionalism*—which aims to 'explain away' the apparent commitment to abstract objects in different kinds of things that we say. Struthioism is, as far as I know, a mere theoretical possibility: in particular, I do not know of any contemporary defenders of it. On the other hand, a good example of fictionalism is the general program of Yablo (2000) (2002) (2005). There are, of course, many recent instances of fictionalism about particular abstracta—e.g. Field (1980) and Melia (1995)—but we are here interested in versions of fictionalism that treat the entire domain of Abstract Reality as fiction. (Perhaps Rosen (1990) might also have been offered as an example of fictionalism about particular abstracta; but it is debatable whether he offers a fiction about *abstracta*.)

The above taxonomy may appear to omit generalisations of traditional versions of nominalism. In particular, some may say that I have overlooked *tokenism*—which eschews commitment to abstracta in favour of commitment to 'extra-mental' denizens of Causal Reality, such as linguistic tokens, or tokenings—and *conceptualism*—which eschews commitment to abstracta in favour of commitment to 'mental' denizens of Causal Reality, such as 'concepts', or 'ideas in the mind', or the like. However, I deny that there are viable views that fall under either of these labels. (More about that anon.)

3.

It is no part of my present project to take a stance on which is the correct view to hold about Abstract Reality. However, it will be useful for me to say something about the comparative plausibility of realism and fictionalism. (I take it that struthioism is not a view that deserves serious consideration.)

Yablo (2000) (2002) (2005) claims that certain pieces of language that appear to commit us to abstract objects function as representational aids that boost expressive power. In particular, in Yablo (2005), he sets out a meta-myth which shows how the "myth of mathematics" might have arisen as the result of the adoption of a series of representational aids aimed at boosting the expressive powers of language. According to the meta-myth, we start out with a first-

order language quantifying over concreta, and then add further resources—involving various kinds of 'pretence' or 'supposition' or 'making as if'—in order to facilitate expression of useful claims about concreta. First comes "finite numbers of finite numbers", then "operations on finite numbers", then "finite sets of concreta", then "infinite sets of concreta", then "infinite numbers of concreta", and finally "infinite sets (and numbers) of abstracta".

The kind of idea that Yablo expresses here in connection with numbers finds application in other domains. It is commonplace in discussions of truth that the truth-predicate serves an evident need: without it, we would need to cast around for some other means of expressing the thought that *everything that the Pope says is true*, and the like. While there are theoretical alternatives—e.g. infinite disjunction or insistence on infinitely many instances of a sentential schema—there is no alternative that admits of finite expression. (This same example demonstrates the value of propositional quantification, a value that we can exhibit using examples that have nothing to do with truth. Consider, for example: *He had nothing new to say*. This claim could also be expressed as an infinite disjunction, or via insistence on infinitely many instances of a sentential schema.) Similarly, in the discussion of universals, while it is has been argued by some that sentences such as *Napoleon has all of the attributes of a great general* demonstrate that we are committed to universals, it is clear that we can understand explicit talk of attributes in this kind of case as a representational aid that serves to boost the expressive power of our language. (In this case, too, there are similar theoretical alternatives; e.g. infinite conjunction or insistence on all the instances of a sentential schema.) Of course, there are other examples of sentences alleged to demonstrate commitment to universals—consider, e.g., *Red resembles orange more than it resembles blue*, discussed in Jackson (1977) and Lewis (1983)—that raise different considerations. I'm inclined to think that this sentence is just false—but that's really a story for another occasion.

While it seems to me to be plausible to suppose that often—perhaps even always—where we feel pressure to postulate abstracta, we find representational aids that boost the expressive power of language, it is not obvious that this gives us a decisive argument against realism about abstracta. After all, it is clearly conceivable that, in finding representational aids that boost the expressive power of our language, we make discoveries about the denizens of Abstract Reality. Nonetheless, I am inclined to think that, at least, wherever we can point to connections between apparent commitments to abstracta and devices that boost the expressive powers of language, we have a powerful motivation to think that fictionalism affords the best account of the abstracta in question.

4.

For each of the candidate views that might be held about Abstract Reality, there is a straightforward argument for the conclusion that that view affords no differential support to just one of Theism and Naturalism.

(a) **Realism**: If we are realists about abstracta, then we suppose that there is an independent domain of abstracta, with the distinctive properties of the denizens of Abstract Reality identified earlier, to which we are committed by claims that we make in connection with Causal Reality. While we noted earlier that there is room for disagreement about the thickness of the conception of object that is invoked in connection with abstracta, we shall see that this disagreement has no implications for the assessment of the comparative merits of Theism and Naturalism.

The key observation to make here is that, according to realism, the only connections between Abstract Reality and Causal Reality are necessary connections. If we suppose that Abstract Reality divides into Pure Abstract Reality and Impure Abstract Reality, then Pure Abstract Reality is absolutely independent of Causal Reality, and Impure Abstract Reality is dependent upon Causal Reality only in the sense that there are certain necessary connections between Causal Reality and Impure Abstract Reality. Since Pure Abstract Reality is absolutely independent of Causal Reality, and since—*ex hypothesi*—Theism and Naturalism differ *fundamentally* only in their accounts of Causal Reality, it is impossible for considerations about Pure Abstract Reality to favour one view over the other. Furthermore, since Impure Abstract Reality is dependent upon Causal Reality only in the sense that there are certain necessary connections between Causal Reality and Impure Abstract Reality, it is impossible for it to be the case that there are considerations about Impure Abstract Reality that favour one view over the other that do not simply mirror considerations about Causal Reality that favour that same view over the other. (It could not be, for example, that one view does better, on account of its commitments to impure unit sets unless that view also does better on account of its commitments to the causal entities that figure in those impure unit sets.)

Perhaps some might be tempted to object that it is a mistake to suppose that Abstract Reality is independent of Causal Reality, given that God belongs to Causal Reality. In particular, some might be tempted to claim that, since God is the creator, or source, or ground of all else, God is the creator, or source, or ground of Abstract Reality. But this cannot be right.

First, if God is the creator, or source, or ground of Abstract Reality, then Abstract Reality has a cause, or source, or ground—whence, Abstract Reality is part of Causal Reality, in contradiction with our initial assumption about the nature of Abstract Reality. Moreover, we cannot repair this problem by supposing that God belongs to Abstract Reality: for then God would not be the cause, or source, or ground of anything, and, in particular, would not be the cause, or source, or ground of Natural Reality.

Second, if realism requires that the denizens of Abstract Reality are either necessary, or else necessary given the mere existence of denizens of Causal Reality, then it is impossible that anything—even God—be their creator, or ground, or source. Anything that is created, or that has a ground or a source, is *dependent* upon the thing that is its creator, or ground, or source for its existence, and so is something whose existence is *contingent*, in contradiction with our assumption that it either exists of necessity, or else exists of necessity given the mere

existence of other things. Dependence and contingency are asymmetric modal relationships: if A depends upon or is contingent upon B, then either it is possible not to have B, or else it is possible to have B without A—and, either way, it follows that it is possible to not have A. (True, in the *Third Way*, Aquinas writes of "necessary beings that owe their necessity to something else". But, in this context, the "necessary beings that owe their necessity to something else" are merely eternal beings whose existence is metaphysically contingent upon God's creative activities. So there is no counterexample to be found in that part of that *Summa*.)

Third, even setting the preceding two considerations aside, there seems to be a further difficulty in the idea that God might be the cause, or ground, or source of Abstract Reality that arises from the role that the denizens of Abstract Reality have in the *characterisation* of the denizens of Causal Reality. If, for example, there are universals in Abstract Reality, then entities in Causal Reality *participate* in at least some of those universals; if, to take another example, there are propositions in Abstract Reality, then entities in Causal Reality *have* at least some of those propositions as *contents*; and so forth. But, on the assumption that God creates Abstract Reality, it follows that there is a part of Causal Reality—the part that is (causally) prior to the creation of Abstract Reality—that is not related to Abstract Reality by the appropriate kinds of characterisation relations. On the one hand, we are to suppose that we are required to believe in Abstract Reality because of the *essential* role that it plays in 'characterising' elements of Causal Reality; and yet, on the other hand, we are to suppose that there is an entity in Causal Reality that lies beyond all of these allegedly essential 'characterising' elements: at least prior to the creation of Abstract Reality, God exists in some way and yet participates in no universals, has thoughts even though there are no contents of thoughts to be had, and so forth. I think that I will not be alone in suspecting that this overall picture is just incoherent.

(b) **Struthioism**: If we are struthioists about abstracta, then we suppose that, while some of our talk brings with it *prima facie* commitment to abstracta, there is actually no Abstract Reality, and no need for any explanation in connection with our *prima facie* commitments to abstracta. Needless to say, in the nature of the case, there is nothing in this assumption that favours either Theism or Naturalism. So we conclude that, on struthioism, there is no significant difference between the accounts that Naturalism and Theism give of Abstract Reality.

I noted earlier that struthioism appears to be a merely theoretical possibility; as far as I know, it has no actual defenders. This should not be surprising: it seems hard to resist the pull of the thought that there is *apparent* commitment to abstracta in some of the ways that we talk, and hence of the further thought that, at the very least, there is something here that requires explanation if, in fact, the commitment in question is *merely* apparent. However, these points do not affect the argument given in the previous paragraph. It is, after all, quite plain that, if struthioism were correct, it would yield no differential support to either Theism or Naturalism.

(c) **Fictionalism**: If we are fictionalists about abstracta, then we suppose that we can *explain* apparent commitments to abstracta by (a) appealing to the utility of adopting linguistic devices that generate the apparent commitments, and (b) adding some further story about how we take on the linguistic devices without taking on the apparently generated commitments. The further story may involve claims about the attitudes that we actually have towards the linguistic devices in question—perhaps claiming that, in fact, we view them in such a way that we take the apparent commitments that they generate to be merely apparent—or it may only involve claims about the attitudes that we could justifiably take towards the linguistic devices in question—perhaps claiming that we could justifiably view the apparent commitments that they generate as merely apparent. (Roughly, the distinction to which I am adverting here is the distinction between *hermeneutic* and *revisionary* fictionalisms. There are many other philosophical domains in which similar distinctions are drawn: consider, for example, the different ways in which contractarian political and ethical theories can be formulated.) And, of course, the further story might also involve claims to the effect that different parts of Abstract Reality are treated in different ways: 'hermeneutic' fictionalism is appropriate for some abstracta (perhaps, e.g., 'sakes'), while 'revisionary' fictionalism is appropriate for other abstracta (perhaps, e.g., natural numbers).

The key point to note about fictionalism is that it appeals only to facts about human beings: the languages that we speak, the linguistic devices that we have invented, and the interpretations that we ourselves place upon the languages that we speak and the linguistic devices that we employ. Given that there is nothing in the data about human beings, the languages that they speak, and the linguistic devices that they employ, that favours one of Theism and Naturalism over the other, there is nothing in fictionalist accounts of Abstract Reality that favours one of Theism and Naturalism over the other.

Perhaps some may be tempted to object that facts about the evolution of human beings, or the evolution of human intellectual capacities, or the evolution of human languages actually favours one of Theism and Naturalism over the other. However, even if it were true that facts about the evolution of human beings, or the evolution of human intellectual capacities, or the evolution of human languages do favour one of the Theism and Naturalism over the other, these considerations would all belong to the 'causal' data that is the standard focus of traditional arguments about the existence of God. Unless there is something, about representational devices that boost the expressive power of languages at the cost of merely apparent commitments to abstracta, that *adds* to considerations about the evolution of human beings, or the evolution of human intellectual capacities, and the evolution of human languages, there is no special place for considerations about Abstract Reality in the decision between Theism and Naturalism.

Given that realism, struthioism and fictionalism are the candidate views about Abstract Reality, and given that none of these views provides differential support to one of Theism and Naturalism, we are justified in concluding that considerations about Abstract Reality do not provide differential support to one of Theism and Naturalism.

5

As I noted above, some may think that the conclusion drawn at the end of the preceding section is premature, because there are other candidate views about Abstract Reality that have not yet been considered. I turn now to the task of arguing that these other proposed candidate views—tokenism and conceptualism—do not offer viable accounts of Abstract Reality.

(d) **Tokenism**: According to tokenism, apparent commitments to abstracta turn out to be genuine commitments to extra-mental denizens of Causal Reality. So, on this view, there really are abstracta—such things as numbers, sets, classes, functions, mappings, structures, groups, rings, algebras, states, patterns, propositions, contents, intentional objects, properties, universals, attributes, characteristics, types, normative principles, values, utilities, generic objects, arbitrary objects, intensional objects, mere possibilia, impossibilia, incomplete objects, and so on—but these things are extra-mental denizens of Casual Reality: linguistic tokens, or linguistic tokenings, or appropriately-shaped regions of space-time, or the like.

This view seems to be to be entirely misconceived. When we produce linguistic tokens, or make linguistic tokenings, those linguistic tokens or linguistic tokenings are, themselves, causal entities that belong to Causal Reality. But, when we ask what we *commit ourselves to* in producing these linguistic tokens or making these linguistic tokenings, it simply isn't part of the correct answer that we *commit ourselves* to the very linguistic tokens that we have produced, or the linguistic tokenings that we have made, and nor is it part of the correct answer—at least, in general—that we *commit ourselves* to some other linguistic tokens or linguistic tokenings (that are appropriately related to the linguistic tokens that we have produced or the linguistic tokenings that we have made). To suppose that abstracta—such things as numbers, sets, classes, functions, mappings, structures, groups, rings, algebras, states, patterns, propositions, contents, intentional objects, properties, universals, attributes, characteristics, types, normative principles, values, utilities, generic objects, arbitrary objects, intensional objects, mere possibilia, impossibilia, incomplete objects, and so on—are linguistic tokens, or linguistic tokenings, or appropriately-shaped regions of space-time is just to make a kind of category error.

(e) **Conceptualism**: According to conceptualism, apparent commitments to abstracta turn out to be genuine commitments to mental denizens of Causal Reality. So, on this view, there really are abstracta—such things as numbers, sets, classes, functions, mappings, structures, groups, rings, algebras, states, patterns, propositions, contents, intentional objects, properties, universals, attributes, characteristics, types, normative principles, values, utilities, generic objects, arbitrary objects, intensional objects, mere possibilia, impossibilia, incomplete objects, and so on—but these things are mental denizens of Casual Reality: concepts, or ideas, or the like.

This view seems to me to be vitiated by an ambiguity in talk of concepts, or ideas, or the like. When we talk about concepts, or ideas, there are two different things that we might be meaning to discuss. On the one hand, we might be talking about mental tokens: causal entities that are denizens of Causal Reality. On the other hand, we might be talking about the contents of mental tokens: putative abstracta that would be denizens of Causal Reality if realism is the appropriate attitude to take towards them. And there is no third thing that we might be talking about: either we are talking about mental tokens, or we are talking about contents of mental tokens.

However, if we are talking about mental tokens, then conceptualism is just a variant of tokenism, in which the tokens in question are mental rather than extra-mental. But whether the tokens are mental or extra-mental makes no difference to the viability of tokenism: either way, it is just a category mistake to suppose that putative abstracta are causal tokens.

On the other hand, if we are talking about the contents of mental tokens, then we have not been offered any account of putative abstracta: for what our theory of Abstract Reality is supposed to do is to give us an account of such things as the contents of mental tokens. If conceptualism claims only that putative abstracta are contents of mental tokens, then it simply fails to be a theory of Abstract Reality.

Either way, then, conceptualism is not a viable theory of Abstract Reality. Whatever account we might give of abstracta, we cannot say that abstracta are concepts, or ideas, or the like.

What, then, of the intuition that many putative abstracta are 'mind-dependent'? Well, if we are realists about Abstract Reality, we shall say that, at least for the range of cases for which there really are denizens of Abstract Reality, the intuition is simply mistaken. And if we are fictionalists about Abstract Reality, then we shall say that our fictionalist theory gives us all of the mind-dependence that we could require: for, of course, the representational aids that boost the expressive powers of our language, thereby generating apparent commitments to abstracta, are products of human minds. If Yablo's 'myth of the seven' captures something important about our apparent commitment to numbers, then Yablo's 'myth of the seven' establishes a significant sense in which 'numbers are mind-dependent'. What more could you want?

6.

Plantinga (2007) claims that there are various good arguments for the existence of God concerning denizens of Abstract Reality; in particular, he sketches arguments concerning natural numbers, sets, and intentional objects:

Argument from Numbers:

It ... seems plausible to think of numbers as dependent upon or even constituted by intellectual activity. … So, if there were no minds, there would be no numbers. … But … there are too many of them to arise as a result of human intellectual activity. We should therefore think of them as among God's ideas. (213)

Argument from Sets:

Many think of sets as displaying the following characteristics …: (1) No set is a member of itself; (2) Sets … have their extensions essentially; hence sets are contingent beings and no set could have existed if one of its members had not; (3) sets from an iterated structure: at the first level, sets whose members are non-sets, at the second, sets whose members are non-sets or first-level sets, etc. Many [are] also inclined to think of sets as collections—i.e., things whose existence depends upon a certain sort of intellectual activity—a collecting or 'thinking together'. If sets were collections, that would explain their having the first three features. But of course there are far too many sets for them to be a product of human thinking together; there are far too many sets such that no human being has ever thought their members together. That requires an infinite mind—one like God's. (211f.)

Argument from Intentionality:

Consider propositions: the things that are true or false, that are capable of being believed, and that stand in logical relations to one another. They also have another property: aboutness or intentionality … [they] represent reality or some part of it as being thus and so. … Many have thought it incredible that propositions should exist apart from the activity of minds. … But if we are thinking of human thinkers, then there are far too many propositions: at least, for example, one for every real number that is distinct from the Taj Mahal. On the other hand, if they were divine thoughts, no problem here. So perhaps we should think of propositions as divine thoughts. (210f.)

I do not think that we should be quick to agree with Plantinga that these are promising routes to arguments for the existence of God. There are several reasons for this.

First, of course, there is the ambiguity in talk about 'God's ideas', 'divine thoughts', and the like. When Plantinga says that 'we should think of numbers as among God's ideas', or that 'we should think of propositions as divine thoughts', what he says is ambiguous. He could mean: we should think that numbers and propositions are God's mental state tokens—causal things that belong to Causal Reality. Or he could mean: we should think that numbers and propositions are contents of God's mental state tokens—abstracta that belong to Abstract Reality. However, as we have already noted, numbers and propositions cannot be things that belong to Causal Reality; and the observation that numbers and propositions are contents of God's mental state tokens simply fails to be an account of putative abstracta. Since there is no third construal that can be placed upon talk about 'God's idea', 'divine thoughts', and the like, we can conclude that there is no way that the arguments that Plantinga sketches here can be carried through.

Second, if we grant that there is some *prima facie* plausibility to the thought that numbers are 'dependent upon or even constituted by' intellectual activity—and that sets are 'things whose existence depends upon a certain sort of intellectual activity', and that it is 'incredible that propositions should exist apart from the activity of minds'—we can explain away this *prima facie* plausibility by appealing to fictionalist accounts of numbers (and sets and propositions). We might say, for example, that our commitments to numbers—and sets and propositions— arise from our adoption of certain representational aids that boost the expressive power of our language (and advert to, say, Yablo's 'myth of the seven). Without the activity of our minds that went into developing—and goes into supporting—the use of those representational aids, we would not have any inclination or reason to suppose that we have even *prima facie* commitment to the existence of numbers (or sets or propositions), and questions about the existence of numbers (and sets and propositions) would not so much as arise.

Third, even if we were to follow Plantinga in supposing that it is literally true that the existence of, say, numbers is constituted by intellectual activity, it is not clear that the kind of argument that he tries to develop will go through. Suppose, for example, that the existence of numbers is constituted by there being people whose minds contain tokens of sentences like this one: 'there are infinitely many natural numbers'. If the occurrence of a token of this kind in someone's mind is enough to make it the case that there are infinitely many natural numbers, then it seems that Plantinga is simply mistaken when he claims that there are too many natural numbers for them to arise as the result of human intellectual activity. If I can have a bunch of tokens in my head that *entail* infinitely many further claims, even though it is impossible for me to have infinitely many tokens—separately representing each of those infinitely many further claims—in my head, then it is unclear why the infinite nature of mathematical domains should be thought problematic.

For these—and other—reasons, I conclude that there is no prospect of developing successful arguments for the existence of God from considerations about natural numbers, sets, propositions, or any other abstract objects, along the lines that Plantinga proposes. Of course, this conclusion dovetails nicely with the more general conclusion, argued for in the earlier parts of my essay, that considerations about abstract objects do not favour either Theism or Naturalism. (Perhaps it is worth noting here that there are other arguments for the existence of God—concerning our *knowledge* of matters involving putative abstracta—that would require further discussion. All that has been canvassed here is the possibility of arguments for the existence of God based upon the *metaphysics* of abstract objects. Arguments for the existence of God based upon the *epistemology* of abstract objects will have to wait for some other occasion.)

7.

I took as the principal question for this essay whether Theism or Naturalism affords a better account of Abstract Reality. The conclusion for which I have argued is that considerations about abstracta favour neither theory. In particular, I have argued that realism and fictionalism give no differential support to either Theism or Naturalism, and that there are no other plausible accounts of Abstract Reality. Given this answer to my principal question, I also have an answer to the wider question whether there is any serious prospect that, while one of Naturalism and Theism leads when only Causal Reality is taken into account, the other theory wins when all of Reality is taken into account. My answer to that question is negative: comparison of the relative merits of Naturalism and Theism need only consider their respective accounts of Causal Reality.

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