Infinity in Pascal's Wager

The standard formulation of Pascal's Wager casts it in terms of absolute utilities: we use '∞' to represent the absolute utility of wagering for God if God exists. The standard formulation supposes that the decision set-up is represented by the following table:

	God Exists **Pr(God Exists) = p, 0<p<1**	**God does not Exist** **Pr(God does not Exist) = 1-p**
Wager for God	∞	A, where A is finite
Fail to Wager for God	B, where B is finite	C, where C is finite

There are two available courses of action: wager for God, or fail to wager for God. There are two relevant conceivable states of the world: God exists, or God does not exist. The credence that is assigned to the claim that God exists is p, whence the credence assigned to the claim that God does not exist is 1-p. The utility of wagering for God, given that God exists, is infinite; all of the other utilities are finite. The expected utility of wagering for God is $p.\infty + (1-p).A = \infty$. The expected utility of failing to wager for God is $p.B + (1-p).C$ = a finite value. Given that one ought always act so as to maximize expected utility, one ought to wager for God.

One might worry that the mathematics is suspect: '∞' is not a standard number. For now, let's suppose that there is no difficulty involved in revising the standard numbers N_S to the extended numbers N_S' via the addition of '∞' (and '$-\infty$'), subject to the following rules:

(1) $\forall x \in N_S'$: $\infty + x = \infty$
(2) $\forall x \neq 0 \in N_S'$: $x.\infty = \infty$
(3) $\forall x \in N_S$: $\infty - x = \infty$
(4) $\forall x \in N_S$: $x - \infty = -\infty$
(5) $\infty - \infty$ is undefined
(6) $0.\infty = 0$

If you think that there are some rules missing from this list, don't worry: we'll have more to say about the rules governing '∞' later on. So far, I have just considered rules that might be required for the calculation of expected utilities in variations of Pascal's Wager, on the assumption that probabilities only take standard values. Also, note that I have not yet specified *which* standard number system is designated by 'N_S'.

There are many concerns that have been raised about Pascal's Wager in this standard formulation.

First, there are worries about the assignment of a credence to the claim that God exists. Does it make sense to suppose that we do or can assign credences to that claim? And, if it does make sense to suppose that we do or can assign credences to that claim, are we justified in supposing that the assigned credence can or must be strictly greater than zero and strictly less than 1?

Second, there are worries about the assignment of infinite utility to wagering on God's existence given that God exists. Even if we can construct a satisfying mathematical formalism, we might think that it makes no sense to suppose that we can have infinite utilities, perhaps because infinite utilities violate the axiom of continuity. And, even if we suppose that it does make sense to suppose that we can have infinite utilities, we might think that it is a mistake to suppose that the utility of wagering on God's existence if God exists is infinite.

Third, there are worries about exactly what 'wagering for God' amounts to. It cannot be 'believing in God', because believing in God is not an action that you can just choose to perform. But, on Pascal's theology, it is not clear that anything less than believing in God will secure 'an infinity of infinite happiness'. It is simply not obvious that there *is* an action that you can choose to perform that has infinite utility if God exists.

Fourth, there are worries about the suggestion that it can be proper to adjust one's credences in the light of one's utilities. However, exactly, the Wager argument is supposed to work, the overarching idea is that your utilities alone give you a reason to raise the credence that you give to the claim that God exists. But it seems questionable whether your utilities alone ever can give you an adequate reason to revise your credences.

Fifth, there are worries about the soteriological import of Pascal's wager. The construction of the wager seems to suggest that you may obtain 'an infinity of infinite happiness' merely by acting on a dominant desire to acquire, for yourself, an infinity of infinite happiness. Experience teaches that, in this life, merely acting on a dominant desire to acquire happiness for oneself typically does not lead to happiness. Is it really credible to suppose that things stand differently with the next life?

Beyond these worries, there are three objections to the standard formulation of Pascal's wager that seem particularly formidable.

First, there are many different actions that one might take that are bundled together under 'fail to wager for God'. For example, I might toss a coin in order to decide whether to wager. The expected utility of doing this is ∞. More generally, there is a vast range of 'mixed strategies' that I could pursue, all of which have infinite expected utility. The assumption, in the standard formulation of the wager argument, that there are just two available courses of action that need to be considered is false; and, when we correct this assumption, we no longer get out the conclusion that, in order to maximize expected utility, we must wager for God. (For more detailed development of this objection, see Duff (1986) and Hájek (2003a). For worries about the objection, see Hájek's contribution to the present volume.)

Second, there are many different conceivable ways that the world might be that are bundled together under 'God does not exist'. In particular, there are conceivable sources of infinite utility that we might obtain by performing actions other than wagering for God. Consider, for example:

(a) Very Nice Gods who reward everyone regardless of whether and how they wager;
(b) Nice Gods, each of whom rewards those who wager for it, and all of whom reward wagers on some conceivable Gods while also not rewarding wagers on all of the other conceivable Gods;
(c) Very Perverse Gods who reward everyone except those people who wager for it;
(d) Perverse Gods, each of whom does not reward those who wager for it, and all of whom reward wagers on some conceivable Gods while also not rewarding wagers on all of the other conceivable Gods.

All of these conceivable Gods reward some kinds of wagering on Gods, but many reward different kinds of wagering on Gods. So, there are many different wagers all of which have infinite expected utility. The assumption, in the standard formulation of the wager argument, that there are just two relevant conceivable ways that the world might be is false; and, when we correct this assumption, we no longer get out the conclusion that, in order to maximise expected utility, we must wager for God.

Third, given that we are countenancing infinite utilities—i.e. utilities with value ∞—it seems that we should also countenance infinitesimal credences—i.e., credences with value ε. If ∞ is so large that it cannot be increased, then ε is so small that it cannot be properly decreased: if you remove a proper part of something of infinitesimal measure, you are left with something that is also of infinitesimal measure.

It is worth noting that all three of these 'formidable' objections to Pascal's wager are keyed to the presence of ∞ in the calculations. In this chapter, I shall consider the best contemporary effort to deal with these 'formidable' objections. But, before we get to that, we need to have some preliminary discussion of the mathematics of ∞ and ε.

1. Infinity and Infinitesimal

If we suppose that both ∞ and ε are added to N_S to form N_S', then it is, at least initially, plausible to suppose that we have the following rules (in addition to those given earlier):

(7) ∞/∞, n/∞, and ε/∞ are all undefined
(8) $\infty.\varepsilon$ is undefined
(9) $\varepsilon+\varepsilon = \varepsilon$
(10) $\forall n\neq 0 \in N_S$: $n.\varepsilon = \varepsilon$
(11) ε-ε is undefined

If, in our standard formulation of Pascal's wager, we set $p=\varepsilon$, then it turns out that the expected utility of wagering for God is undefined. And, in that case, the advice that we ought to maximize expected utility will not give us the conclusion that we ought to wager for God.

Perhaps it might be objected that we cannot make sense of infinitesimal credences. But it is hard to see a good reason for supposing that, while we can make sense of infinite utilities, we cannot make sense of infinitesimal probabilities. If we can make sense of the idea that a utility can be so large that it cannot be increased by adding to it, why can't we make sense of the idea that a credence can be so small that it cannot be properly decreased by subtracting from it? 'Infinitesimal' is a natural dual to 'infinite'.

Even if we waive other objections, the system that we have to this point is not satisfactory. There is just too much that is undefined: for example, if we ask what is the sum of ($n+\varepsilon$) and ($m-\varepsilon$), we get back the answer that this sum is undefined. The best way to avoid this result that I have found so far is to suppose that the introduction of ε 'perturbs' all of the standard numbers: instead of n, we have

$n\pm\varepsilon$. We then have the following rules for the extended rationals (where n and m are standard rationals):

(12) $(n\pm\varepsilon)+(0\pm\varepsilon)=n\pm\varepsilon$
(13) $(n\pm\varepsilon)+(m\pm\varepsilon)=(n+m)\pm\varepsilon$
(14) $\infty+(0\pm\varepsilon)=\infty$
(15) $\infty+(n\pm\varepsilon)=\infty$
(16) $\infty+\infty=\infty$
(17) $(n\pm\varepsilon)-(0\pm\varepsilon)=(n\pm\varepsilon)$
(18) $(n\pm\varepsilon)-(m\pm\varepsilon)=(n-m)\pm\varepsilon$
(19) $\infty-(0\pm\varepsilon)=\infty$
(20) $(0\pm\varepsilon)-\infty=-\infty$
(21) $\infty-(n\pm\varepsilon)=\infty$
(22) $(n\pm\varepsilon)-\infty=-\infty$
(23) $(0\pm\varepsilon)-(0\pm\varepsilon)=(0\pm\varepsilon)$
(24) $\infty-\infty$ is undefined
(25) $(n\pm\varepsilon).(1\pm\varepsilon)=(n\pm\varepsilon)$
(26) $(n\pm\varepsilon).(0\pm\varepsilon)=(0\pm\varepsilon)$
(27) $(n\pm\varepsilon).(m\pm\varepsilon)=(n.m)\pm\varepsilon$
(28) $\infty.(1\pm\varepsilon)=\infty$
(29) $\infty.(0\pm\varepsilon)$ is undefined
(30) $\infty.(n\pm\varepsilon)=\infty$
(31) $\infty.\infty=\infty$
(32) $(n\pm\varepsilon)/(1\pm\varepsilon)=(n\pm\varepsilon)$
(33) $(n\pm\varepsilon)/(0\pm\varepsilon)$ is undefined
(34) $(0\pm\varepsilon)/(n\pm\varepsilon)=(0\pm\varepsilon)$
(35) $(n\pm\varepsilon)/(m\pm\varepsilon)=(n/m)\pm\varepsilon$
(36) $\infty/(1\pm\varepsilon)=\infty$
(37) $\infty/(0\pm\varepsilon)$ is undefined
(38) $(n\pm\varepsilon)/\infty=(0\pm\varepsilon)$
(39) $\infty/(n\pm\varepsilon)=\infty$
(40) ∞/∞ is undefined
(41) $(0\pm\varepsilon)/(0\pm\varepsilon)$ is undefined
(42) $(0\pm\varepsilon)/\infty=(0\pm\varepsilon)$.

The interesting rule, given that we take this route, is that, despite gains in definition elsewhere, $\infty.(0\pm\varepsilon)$ is undefined. The intuitive justification for this is that, even if we think that the product of the infinite and the infinitesimal should be finite, there is no satisfactory way of choosing any particular finite number to be that product. Speaking *very* loosely, it is quite intuitive to suppose that $n.(0\pm\varepsilon) = (n.0)\pm(n.\varepsilon) = 0\pm\varepsilon$. But, when we consider $\infty.(0\pm\varepsilon) = (\infty.0)\pm(\infty.\varepsilon) = 0\pm(\infty.\varepsilon)$, there is no way to sensibly assign a value to $\infty.\varepsilon$. It must be that ∞/∞ is not well-defined: if you divide an infinite set into infinitely many equinumerous subsets, the cardinality of those subsets could be any of 1, 2, 3, … ∞. But, if ∞/∞ is not well defined, then neither is $\infty.\varepsilon$, if ε is the multiplicative inverse of ∞. And $\infty.\varepsilon$ is also not well-defined if we suppose that, while ∞ is so big that addition makes no difference to it, ε is so small that subtraction of a proper part makes no difference to it.

What is our number system N_S? Strictly, I think, N_S is the rational numbers. What motivates the addition of '∞' to the rational numbers is the countable denumerability of the rationals: if we do not add '∞' to the rationals, there is a sense in which there are rational quantities that are not explicitly represented in the rational number system. Given that our concern is expressive completeness, if we want to extend the real numbers by including symbols for infinite numbers, then we cannot make do with just the pair of symbols: '∞' and 'ε'. On the assumption that the only infinite cardinal below the infinite cardinal that measures the real numbers is the infinite cardinal that measures the rational numbers, we should extend the reals by adding two infinite numbers—∞_{little} and ∞_{big}—and two infinitesimals—ε_{little} and ε_{big}. (We should want to be able to say that there are ∞_{little} rational numbers and ∞_{big} real numbers, where $\infty_{little}<\infty_{big}$: there are fewer rational numbers than there are real numbers, even though there are infinitely many rational numbers.) I leave it as an exercise for the reader to think about the rules that should govern N_S', when N_S is the standard real numbers.

Of course, you might think that it is just a mistake to couch discussion of Pascal's wager in terms of these extended number systems. You might well prefer to couch your discussion in terms of some other kind of non-standard number theory—Robinson's non-standard arithmetic, or Conway's surreal number system, or vectorial representations, or the like—or you might prefer to couch discussion of Pascal's wager in terms of standard number systems and insist that the utility of wagering for God if God exists is very large but finite. But on all of these other approaches, the result of the calculation of the expected value of wagering for God turns out to be sensitive to the precise values that are attributed to the probability that God exists and the utility that is attributed to wagering for God if God exists, and there is nothing that privileges any particular assignment of those values over other assignments of those values. In order for it to have any bite, it seems that Pascal's wager requires '∞'-valued utilities; and yet there seem to be very formidable obstacles that confront Pascal's wager if it is formulated in terms of '∞'. (For a more careful and detailed argument for this conclusion—paying consideration to all of the just-mentioned non-standard number theories—see Hájek (2003a). Perhaps some will find Hájek's argument not quite as persuasive as I take them to be.)

2. Relative Utility and Stable Equilibrium

In some very interesting publications, Paul Bartha (2007) (2012) has explored ways of rehabilitating Pascal's wager that are intended to avoid the bad consequences that follow from the formulation of the wager in terms of '∞'-valued utilities, but without having recourse to some other kind of non-standard number theory. By working with 'relative utilities', Bartha hopes to be able to show that Pascal's wager can be formulated in a way that entirely avoids both the Duff/Hájek objection from 'mixed strategies' and the many gods objection. Bartha makes the following bold conjecture:

> If (1) we can assign positive probability to the existence of deities, (2) we can make sense of infinite utility, (3) we can justifiably revise our beliefs on pragmatic grounds, and (4) we can provide a valid formulation of Pascal's original argument, then the many gods objection poses no additional threat. (Bartha (2012:205).)

Since my list of worries about Pascal's wager is slightly more extensive, I shall interpret this conjecture in the following way: *if we meet all of the other objections to Pascal's wager, then the many Gods objection is thereby already met*.

Bartha argues in detail that, if you think that the field of conceivable Gods is made up of Very Nice Gods, Nice Gods, Very Perverse Gods, and Perverse Gods (described above), along with

(5) Jealous Gods, each of whom rewards all and only those who wager for it (of course, Pascal's God is a Jealous God); and
(6) Indifferent Gods who reward no one, no matter how they wager

then you should apportion all of your credence to the Jealous Gods. If he's right about that, then Very Nice Gods, Nice Gods, Very Perverse Gods, Perverse Gods and Indifferent Gods simply do not make any difficulty for Pascal's wager.

The *first stage* of Bartha's rehabilitation of Pascal's wager is to recast it in terms of *relative utilities*. Let *A«B* mean that B is preferred to A under the weak preference ordering «. If Z«A and Z«B, then U(A,B;Z) is the utility of A relative to B with base point Z. Let [pA, (1-p)B] be a gamble that offers A with probability p, and B with probability 1-p. We have the following three special cases:

1. $U(A,B;Z) = \infty \leftrightarrow B \ll [pA, (1-p)Z]$ for $0<p\leq 1$
2. $U(A,B;Z) = 0 \leftrightarrow A \ll [pB, (1-p)Z]$ for $0<p\leq 1$
3. $U(A,B;Z) = 1 \leftrightarrow [pA, (1-p)Z] \ll B$ and $[pB, (1-p)Z] \ll A$ for $0\leq p<1$

A relative decision matrix is a table of relative utilities. We compute the matrix entries relative to the optimal outcome, i.e., in the case of Pascal's Wager, we compute U(A, salvation; Z). According to Bartha, this means that, when we are dealing with pure outcomes and simple cases, all values in our table are either 0 or 1. '1' indicates the best outcome: salvation; '0' represents all of the other outcomes. When we move to the framework of relative utilities, our initial decision matrix is transformed to look like this:

	God Exists **Pr(God Exists) = p, $0<p<1$**	**God does not Exist** **Pr(God does not Exist) = 1-p**
Wager for God	1	0
Fail to Wager for God	0	0

This recasting in terms of relative utilities immediately disposes of the standard 'mixed strategies' objection. If we represent the expanded decision matrix as follows:

	God Exists and Coin Falls Heads **Pr(God Exists and Coin Falls Heads) = p/2, $0<p<1$**	**God Exists and Coin Falls Tails** **Pr(God Exists and Coin Falls Tails) =p/2**	**God does not Exist** **Pr(God does not Exist) = 1-p**
Outright Wager for God	1	1	0

Toss Fair Coin to Decide Whether to Wager for God	1	0	0
Neither of the Above	0	0	0

then we get:

EU (Outright Wager for God) = p
EU (Toss Fair Coin to Decide Whether to Wager for God) = p/2
EU (Neither of the Above) = 0.

In order to maximize expected utility, one must wager for God.

It may be tempting to suggest that, if we countenance infinitesimal credences, and if we accept that, for infinitesimal p, p+p=p, then the 'mixed strategies' objection survives in the case in which p is infinitesimal. Given that our representation in terms of relative utilities only does away with the infinite values, in the case in which credence for the existence of God is infinitesimal, we have:

Pr(God exists and coin falls heads) = Pr (God exists and coin falls tails) = p

EU (Outright Wager for God) = p+p+0 = p
EU (Toss fair coin to Decide Whether to Wager for God) = p+0+0 = p
EU (Neither of the above) =0.

The advice to maximise expected utility does not tell us what to do.

However, since the relative utility approach depends upon the assumption that we work only with standard real-valued probabilities, this criticism does not quite hit the mark. (We shall subsequently return to considerations about infinitesimal credences.)

The *second stage* in Bartha's rehabilitation of Pascal's wager is to introduce some constraints on acceptable credences with respect to the many Gods objection. Given that the thrust of Pascal's Wager is that, in certain cases, you ought to update your credences in the light of your utilities, it may seem plausible that, for those cases, your credences ought to be stable equilibrium points under the updating in question. (Compare Arntzenius (2008) and Skyrms (1988).)

The specific updating rule that Bartha proposes is as follows:

> Suppose that there are finitely many possibilities $S_1, \ldots, S_n$ with corresponding wagers $W_1, \ldots W_n$, a relative decision matrix A, and an initial subjective probability vector $\mathbf{p} = (p_1, \ldots, p_n)$, where p_i is the initial subjective probability for S_i. Let $U(W_i) = A(\mathbf{p})$ represent the expected relative utility of wager W_i. Let $\hat{U} = p_1U(W_1) + \ldots + p_nU(W_n)$ represent the average (relative) expected utility. Then the updated subjective probability for S_i is $p_i' = p[U(W_i)/\hat{U}]$.

The consequent constraint that Bartha imposes on acceptable credences with respect to the many Gods objection is that a viable probability distribution for a Pascalian decision problem should be a stable equilibrium under this updating rule. (A probability distribution $[p_i]$ over possibilities $S_1 \dots S_n$ is an *equilibrium distribution* if $p_i' = p_i$ for all i. An equilibrium distribution $[p_i]$ over possibilities $S_1 \dots S_n$ is *stable* if for any small (and mathematically admissible) set of changes Δp_i, application of the updating rule to the distribution $[p_i + \Delta p_i]$ leads to convergence to $[p_i]$.)

Bartha observes that the precise details of the rule are not important; the rule is but one of a large family of evolutionarily robust updating rules that would deliver similar results across the kinds of cases in which we are interested.

What kinds of scenarios meet the requirement of being a stable equilibrium? As Bartha notes, one obvious scenario of this kind is one in which one gives full credence to a single Jealous God. However, as Bartha also notes, not all cases of stable equilibria consist of a single jealous God.

Consider, first, a decision scenario in which you are deciding between wagering on a Jealous God and wagering on a Very Nice God. For simplicity, we ignore the wager on neither.

	Jealous God Exists	**Very Nice God Exists**
Wager on Jealous God	1	1
Wager on Very Nice God	0	1

Suppose that the probability assigned to the Jealous God is $p>0$, so that the probability assigned to the Very Nice God is $(1-p)<1$.

EU (Wager on Jealous God) = $p.1 + (1-p).1 = 1$.
EU (Wager on Very Nice God) = $p.0 + (1-p).1 = 1-p$
$\hat{U} = p.1 + (1-p).(1-p) = p + (1-p)^2 = 1 + p^2 - p$
$P_J' = p.\ 1/1+p^2-p = P_J.[1/1-p+p^2]$
$P_N' = (1-p).(1-p) / p + (1-p+ p^2) = P_N.[(1-p)/(1-p) +p^2)]$

Clearly, $P_J' > P_J$ and $P_N' < P_N$. (Remember that $0<p<1$, so $0<p^2<p$, whence $1-p+p^2 <1$.) If we iterate the process of redistribution credence, P_J' goes to 1 and P_N' goes to 0. Moreover, as a special case, if we suppose that P_J' is $1-\varepsilon$, and P_N' is ε, under redistribution, P_J' goes to 1 and P_N' goes to 0. So the only equilibrium point is $P_J=1$ and $P_N=0$, and it is a stable equilibrium point. Given a choice between wagering on a Jealous God and wagering on a Very Nice God, Bartha's constraint on credences requires you to be giving all of your credence to the Jealous God.

Consider, second, a decision scenario in which you are deciding between a Perverse God and a Nice God, but where there is also the 'atheist' option of wagering on neither of these Gods,

	Perverse God Exists	**Nice God Exists**	**No God Exists**
Wager on Perverse God	0	0	0
Wager on Nice God	0	1	0
Wager on No God	1	1	0

Suppose, again, that the probability assigned to the Jealous God is $p>0$, the probability assigned to the Nice God is $q>0$, and the probability assigned to there being no God is $(1-[p+q])<1$.

EU (Wager on Perverse God) = p.0 + q.0 + (1-[p+q]).0 = 0
EU (Wager on Nice God) = p.0 + q.1 + (1-[p+q]).0 = q
EU (Wager on No God) = p.1 + q.1 + (1-[p+q]).0 = p+q
$\hat{U}$ = p.0 + q.q + (1-[p+q])(p+q) = (p+q-[p^2+2pq])
P_G' = p.0 / (p+q-2pq) = 0 / (p+q-[p^2+2pq]) = 0
P_N' = q.q / (p+q-2pq) = q^2 / (p+q-[p^2+2pq])
P_O' = p+q (p+q-[p^2+2pq])

Since $q^2 < q < p+q$, $P_G' < P_O'$ and $P_N' < P_O'$. So, in this case, the only stable equilibrium point is wagering on No God.

In order to get the result that the Pascalian wants, we need a further condition. Bartha opts for the following:

> A viable probability distribution for a Pascalian decision problem must be a strongly stable equilibrium. (An equilibrium distribution [p_i], $1 \le i \le n$, over possibilities S_1, …, S_n is *strongly stable* if for any new possibility S_{n+1}, and any (mathematically admissible) set of changes Δp_i, $1 \le i \le n+1$, application of the updating rule to the distribution [$p_i + \Delta p_i$] leads to convergence to [p_i], and, in particular to $p_{n+1}=0$.

The motivation for this proposal is the observation that, if we expand our decision scenario by adding in some additional Nice Gods and Perverse Gods who reward one another but do not reward those who wager on No God, then it will cease to be the case that wagering on No God is a stable equilibrium.

Even if the condition of strong stability gives Bartha the result that he wants, I think that a slightly different condition may be mandated. When deciding what to do, you really should take all of the relevant possibilities into account. Given our account of the various Gods that are up for consideration, a *serious* decision scenario is one in which there are no asymmetries introduced in connection with the Nice Gods and the Perverse Gods. Consider, for example, the following, somewhat more complex, decision scenario (where wagering actions are specified in the left column, states of the world are specified in the top row, J is a Jealous, I is indifferent, VN is Very Nice, VP is Very Perverse, the Ni are Nice, the Pi are Perverse, and 0 is the state in which there are no Gods):

	J	**I**	**VN**	**VP**	**N1**	**N2**	**N3**	**N4**	**P1**	**P2**	**P3**	**P4**	**0**
J	1	0	1	1	1	0	0	0	1	0	0	0	0
I	0	0	1	1	0	1	0	0	0	1	0	0	0
VN	0	0	1	1	0	0	1	0	0	0	1	0	0
VP	0	0	1	0	0	0	0	1	0	0	0	1	0
N1	0	0	1	1	1	0	0	0	1	0	0	0	0
N2	0	0	1	1	0	1	0	0	0	1	0	0	0
N3	0	0	1	1	0	0	1	0	0	0	1	0	0
N4	0	0	1	1	0	0	0	1	0	0	0	1	0
P1	0	0	1	1	1	0	0	0	0	0	0	0	0
P2	0	0	1	1	0	1	0	0	0	0	0	0	0
P3	0	0	1	1	0	0	1	0	0	0	0	0	0

P4	0	0	1	1	0	0	0	1	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	1	0	0	0

In this scenario, the only stable equilibrium position is to give all of your credence to the Jealous God. Since this scenario has the kind of symmetry that is plausibly the target of the strong stability condition, it is plausible to conclude that the assessment of this scenario establishes that, if there is a choice between a Jealous God, an Indifferent God, a Very Nice God, a Very Perverse God, the full range of Nice Gods, the full range of Perverse Gods, and no God, you should wager on the Jealous God.

Does this mean that Bartha's conjecture is vindicated? Bartha himself urges caution:

> At the moment, I'm unsure whether or not other types of deity can participate in a strongly stable equilibrium. That leaves room for a remnant of the many-gods objection, and for doubts about the sufficiency of the requirement of strong stability.

I am undecided about whether Bartha's caution is justified. Perhaps, if you accept the requirement of strong stability, you can conclude that the only stable position is to assign all of your credence to a jealous God, or to a perverse cartel (i.e. a bunch of perverse Gods who only reward each other's followers). But, even in view of the considerations currently in play, it is not clear to me that we should accept the requirements of stability and strong stability. Consider, for example, the possibility of a jealous cartel: a group of Gods, each of whom rewards all and only those people who wager on one among that group of Gods. (How should we think about a jealous cartel? Where a Jealous God says 'You must believe in me (in order to obtain salvation)!' a God in a jealous cartel says 'You must distribute all of your credence over Gods who are enough like me (in order to obtain salvation): it needn't *be* me; near enough is good enough!')

Let J be a regular Jealous God, let J1 and J2 form a jealous cartel, let P be a very perverse God, and let N be a very nice God. If we are deciding just between these five, then the table that represents our decision problem is as follows:

	J	**J1**	**J2**	**P**	**N**
J	1	0	0	1	1
J1	0	1	1	1	1
J2	0	1	1	1	1
P	0	0	0	0	1
N	0	0	0	1	1

In this decision problem, the only stable equilibrium distribution is to give all your credence to the jealous God. However, while there is no stable equilibrium distribution that gives all of your credence to the jealous cartel, any distribution that gives all of your credence to the jealous cartel is an equilibrium distribution that is also a *quasi-stable-equilibrium* distribution: any small perturbation of an [equilibrium] distribution of your credence over a jealous cartel will take you to a very nearby [equilibrium] distribution of your credence over the very same jealous cartel. Rather than insist that you should ensure that your distribution of credence is a stable equilibrium, you could insist that your distribution of credence be a quasi-stable-equilibrium. Why should you prefer a stable equlilibrium distribution to a quasi-stable-equilibrium distribution?

There may be considerations that will lead you to prefer to wager on a jealous cartel rather than on a lone jealous God. In the example above, so long as you give more initial credence to the jealous cartel than to the jealous God, iterated application of the updating rule will lead you to an equilibrium in which you give all your credence to the jealous cartel. (In particular, if you distribute your initial credence *equally* over all of the Gods, iterated application of the updating rule will lead you to an equilibrium in which you give all your credence to the jealous cartel.) Moreover, as the membership of the jealous cartel increases—i.e. as you allow more and more Gods to belong to the jealous cartel—the proportion of initial distributions of credence that will converge to an equilibrium in which you give all your credence to the jealous God decreases. Indeed, in the limit, as the membership of the jealous cartel goes to infinity, the range of initial distributions of credence that converge to an equilibrium in which you give all of your credence to the jealous God goes to zero. At the very least, anyone who is not inclined to load their initial probability distribution heavily in favour of the jealous God might think that these considerations speak in favour of settling for a quasi-stable-equilibrium distribution of your credence across a jealous cartel.

This result looks like bad news for Pascal and Bartha. Their God is a Jealous God; but if you can reasonably think that it is just as good as, or better than, wagering on a single jealous God to wager on a jealous cartel, then—even if everything is otherwise in order—Pascal's wager does not give the result that Pascal wants. But that's not to say that Pascal's wager does tell you to wager on a God who belongs to a jealous cartel. There are several problems here.

First, if you are to wager on a jealous cartel, there is a question about how big it should be. It looks as though the best answer to this question is: infinitely large! But, if that's right, then we are now looking at a decision problem involving at least infinitely many Gods that belong to jealous cartels, infinitely many Perverse Gods and infinitely many Nice Gods. That means that we'll have an infinite number of occurrences of '1' in many of the rows in our table. But, if this is right, then our attempt to extricate ourselves from entanglement with infinities by moving to the framework of relative utilities appears to have foundered.

Second, I see no good reason to suppose that the list of kinds of conceivable Gods that we have been considering is complete. In particular, the Gods that we have considered so far distribute their rewards according to the Gods that are believed in by those who would like to have the rewards. But there are lots of conceivable Gods who, while represented as Indifferent on the table—because they do not distribute rewards according to the God-beliefs of those who would like to have the rewards—nonetheless do differentially bestow rewards. Consider, for example, a God who rewards only those who do not allow their credences to be affected by their utilities. More generally, consider the—plausibly infinite—class of conceivable Gods who will not reward acts ultimately founded on the aim of maximising expected utility in the light of Pascalian calculation.

Perhaps it might be objected that, while it was fine to countenance a Very Nice God, a Very Perverse God, an Indifferent God, and a range of Nice Gods and Perverse Gods, it is not fine to countenance the greatly expanded range of Gods that have crept into my discussion. Bartha says the following, in the context of justifying the requirement of strong stability:

> It is important that [a] new theological possibility is 'in the neighbourhood'. From the bare fact that a deity appears to be logically possible, one need not—indeed, cannot always—infer positive probability. The thought here is that the many-gods objection rests on the view that it is not reasonable to assign positive probability only to one deity. I am generalising this point to

> the ‘pantheon of possibilities’ [mentioned above]: anyone who assigns one of these gods a positive probability should be willing to entertain a tiny positive probability for the other types … These are relevant possibilities for anyone who takes Pascal’s argument and the many gods objection seriously. (202)

This might seem to open up the prospect of admitting into consideration Jealous, Very Nice, Very Perverse, Nice, Perverse, and Indifferent Gods, while not admitting into consideration jealous cartels and Gods who do not reward acts ultimately founded on the aim of maximising expected utility in the light of Pascalian calculation. However, it seems to me that it would be very odd to admit Jealous, Very Nice, Very Perverse, Nice, Perverse, and Indifferent Gods for consideration while not admitting jealous cartels for consideration; and it seems to me that, if anything, it would be even odder to admit Jealous, Very Nice, Very Perverse, Nice, Perverse, and Indifferent Gods for consideration while not also admitting for consideration Gods who do not reward acts ultimately founded on the aim of maximising expected utility in the light of Pascalian calculation. If we are prepared to countenance the rather hard to motivate behaviour of the Very Perverse God and at least some of the Nice Gods and Perverse Gods, surely we ought to be prepared to countenance members of a jealous cartel whose behaviour is plausibly motivated in much the same way that the behaviour of Jealous Gods is motivated. And surely, too, we ought to be prepared to countenance Gods who do not reward acts ultimately founded on the aim of maximising expected utility in the light of Pascalian calculation, since it seems readily intelligible that one might suppose that the behaviour of such Gods is motivated by their respect for rationality and integrity.

Bartha justifies the claim that one cannot always ‘infer positive probability from logical possibility’ by appeal to ‘Gale’s denumerable infinity of sidewalk crack deities’. Here is what Gale (1991: 350) says:

> From the fact that it is logically possible that God exists, it does not follow that the product of the probability of his existence and an infinite number is infinite. In a fair lottery with a denumerable infinity of tickets, for each ticket it is true that it is logically possible that it will win, but the probability of its doing so is infinitesimal, and the product of an infinite number and an infinitesimal is itself infinitesimal. … There is at least a denumerable infinity of logically possible deities who reward and punish believers … For instance, there is the logically possible deity who rewards with infinite felicity all and only those who believe in him and step on only one sidewalk crack in the course of their life, as well as the two-crack deity, the three-crack deity, and so on, *ad infinitum*.

I do not think that we should be persuaded by Gale’s *argument*. In N’, it is not true that ‘the product of an infinite number and an infinitesimal is itself infinitesimal’; more generally, I do not think that there is any coherent theory of infinities and infinitesimals on which the product of an infinite number and an infinitesimal is always an infinitesimal. If N’ is our background mathematical theory, then it is not true that it is logically possible for there to be a fair lottery with a denumerable infinity of tickets. The requirement that the lottery is fair means that each ticket has an equal chance of winning. But, in N’, there is no number x which satisfies $x.\infty=1$. Moreover—though I admit that this is controversial—I do not think that there is any coherent theory of infinities and infinitesimals against which we can establish that it is logically possible that there is a fair lottery with a denumerable infinity of tickets. (See Oppy (2006: 188); compare Wenmackers and Horsten (2013) and Pruss (2014). Pruss’s arguments leave it open that one might prefer to say that things can happen that have no chance of happening. I think that we should strongly prefer not to say that.)

Earlier, I gave a *prima facie* case for admitting infinitesimal credences: but that *prima facie* case involved uncountably many Gods. While, in a case involving denumerably many Gods, it is possible to give positive credence to all of them—for example, if, for all n, one gives probability $1/2^n$ to the n-crack deity, then one gives positive credence to all of Gale's sidewalk crack deities—there is no way that one can give positive credence to all of uncountably many Gods. If one is prepared to allow that there are uncountably many possible Gods, one can rightly insist that there is no legitimate inference of positive probability from logical possibility.

But, as I have already insisted, whether or not one is prepared to countenance uncountably many possible Gods, there is another option: one can allow that some of the Gods admitted for consideration in Pascal's wager are given only infinitesimal credence. If one takes this option, then one will say that the many-gods objection rests on the view that it is not reasonable to assign *non-zero* probability only to one deity. When we 'generalise to our pantheon of possibilities', what we say is that anyone who assigns one of these gods a positive probability should be willing to entertain a tiny—i.e. non-zero—probability for the other types.

3. Concluding Remarks

Time to take stock.

Bartha conjectured that, if we meet all of the other objections to Pascal's wager, then the many-Gods objection is already met. Moreover, he showed that, if all other objections to Pascal's wager are already met, then, in a choice between a Jealous God, an Indifferent God, a Very Nice God, a Very Perverse God, the full range of Nice Gods, the full range of Perverse Gods, and no God, you should wager on the Jealous God. However, he worried that there might be other types of Gods that can participate in strongly stable equilibria—and, if that were so, then it would remain the case that, even if all other objections to Pascal's wager were met, the many-Gods objection would still be a significant objection to Pascal's wager.

I have argued that the requirement of strongly stable equilibrium—and, indeed, the requirement of stable equilibrium—is not well-motivated. There are other types of Gods, no less worthy of consideration than those that figure in Bartha's deliberations, which are intuitively no worse wagers than the Jealous God. In particular, I have suggested that one does no worse to wager on a jealous cartel than one does to wager on a Jealous God.

I have also argued that there are other types of Gods, no less worthy of consideration than those that figure in Bartha's deliberations, that make trouble for Pascal's wager, but not because one would do better to wager on them rather than on a Jealous God. In particular, I have suggested that a God who does not reward acts ultimately founded on the aim of maximising expected utility in the light of Pascalian calculation plausibly makes trouble for Pascalian wagering. Consider the following decision scenario (in which the 'Virtuous' God does not reward acts ultimately founded on the aim of maximising expected utility in the light of Pascalian calculation, but does reward properly motivated virtue):

	Jealous God Exists	**Virtuous God Exists**	**No God Exists**
Wager on Jealous God	1	0	0
Wager on Virtuous God	0	0	0
Virtuously Refuse to Wager	0	1	0

In this case, Bartha's updating rule does not apply, and there are no relevant notions of stable equilibrium and strongly stable equilibrium. Once we introduce Gods who reward non-wager actions, we introduce a barrier to the updating of credences in the light of utilities. And, once we've done that, we definitely do not get out the conclusion that one ought to wager on the Jealous God in preference to merely acting virtuously. (Or so it seems. But perhaps there is something paradoxical about considering gods who reward those who virtuously refuse to wager in a Pascalian context in which one is trying to decide what to do.)

Finally, I have argued that there is an objection to Pascal's wager that Bartha does not consider, but that interacts in interesting ways with Bartha's treatment of the many-Gods objection. If we are prepared to countenance infinitesimal credences, then we should baulk at the move that recasts Pascal's wager in terms of relative utilities. In the original formulation of Pascal's wager, when infinite utility meets infinitesimal credence, we do not get well-defined results (and quite properly so). But, if we suppose that infinitesimal credences are in no worse standing the infinite utilities, then we cannot accept the assumption—built into the relative utilities framework—that there cannot be infinitesimal credences.

If I take seriously the idea that there are uncountably many possible Gods, and I understand the requirement on strongly stable equilibrium to require that all possible Gods are taken into account, then it will certainly be the case that there are undefined expected utilities in my wagering calculations. Even if Bartha's treatment of some cases involving finitely many Gods is quite compelling, it seems that the uncountably-many-Gods objection remain a serious, independent objection to Pascal's wager.

I think that there is a more general lesson here. I have focused on Bartha's approach because I think that it is the best extant treatment of infinity available to those who support Pascal's wager. As I noted above, I agree with Hájek that, on all other coherent extant approaches, the result of the calculation of the expected value of wagering for God turns out to be sensitive to the precise values that are attributed to the probability that God exists and the utility that is attributed to wagering for God if God exists, and there is nothing that privileges any particular assignment of those values over other assignments of those values. To date, there is no treatment of infinity in Pascal's wager that gives full satisfaction to its aficionados.

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