Husserl's Theory of Manifolds and Ontology: From the Viewpoint of Intentional Objects

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Introduction

The concept of the manifold (*Mannigfaltigkeit*) is discussed by Husserl in two contexts. One is that of mathematics, the foundations of mathematics or mathematical logic in particular, while the other, which is not specific to mathematics, is that of ontology and the theory of science in general. Husserl's theory of manifolds is a unifying framework that links these two contexts. As Husserl states in the Prolegomena of the *Logical Investigations* (hereafter the first edition is referred to), mathematicians call a manifold "the objective correlate of the concept of the possible theory, definite only in respect of form", and the "*most general idea*" of the theory of manifolds is "to be a science which definitely forms the essential types of possible theories [(or domains)], and investigates their lawful relations with one another" (Hua XVIII, 250f.; Husserl's emphasis, an insertion in the second edition is marked by []).

It is natural that Husserl develops his theory of manifolds, which originated in mathematics, in more detail in the context of mathematical logic, as seen in *Formal Logic and Transcendental Logic*. Contemporary mathematical interpretations of Husserl's theory of manifolds have centered around comparisons with modern model theory; for example, examinations of the relationship between Husserl's theory and the problem of semantic completeness.

On the other hand, Husserl consistently discusses manifolds not only in the context of pure mathematics but also in the broader context of ontology and the theory of science: in the *Logical Investigations, Ideas I,* and in *Formal Logic and Transcendental Logic*, manifolds are discussed in relation to formal ontology (cf. Hua XVIII, §69; Hua III/1, §72). However, these discussions are exclusively developed at a fairly high level of generality and few examples are discussed. We need clarification on how the concept of the manifold relates to ontology and the theory of science, and how it plays a role in the specific issues associated with them, in order to better understand the theoretical scope and the significance of Husserl's theory of manifolds.

In order to develop the discussion on the theory of manifolds from the perspective of ontology in general, we focus on the concept of the manifold in an early essay of Husserl's, dated 1894 and entitled "Intentional Objects" (Schuhmann 1991, 142–176; referred to as IG hereafter).[1] Husserl's interest in the theory of manifolds can be traced back to at least the early 1890s, around the same period as this essay was written; his 1891 review article includes a discussion on Schröder's concept of the manifold in the algebra of logic (Hua

XXII, 3–43). In IG, Husserl refers to mathematical manifolds with respect to mathematical objects, and at the same time places fictional objects, such as those found in myths and stories, with mathematical objects in the category of intentional objects.

In this paper, we aim to show how the theory of intentional objects described in IG can be reconstructed in connection with the theory of manifolds. In Section 1, we first review the textual background of the 1894 essay "Intentional Objects". In Sections 2–3, we provide a detailed discussion of the concept of the manifold and its relation to the intentional object as laid out in the essay, and then argue that the formal/real (formal/material) distinction in mathematics and fiction determines the theoretical status of manifolds in a broad sense. Finally, in Section 4, we discuss the concept of the manifold described in the essay in relation to formal ontology and compare it with recent interpretations of Husserl's theory of manifolds. We conclude that Husserl's discussion in IG suggests the conceptual priority of the manifolds over the real world in the context of the ontology of fiction as well as that of mathematics. In Sections 2–3, we deal with the theory of manifolds only in terms of Husserl's discussion of it in IG, before reconsidering it in a broader context in Section 4.

1. The 1894 essay "Intentional Objects"

The unpublished essay "Intentional Objects" is the surviving second part of a longer manuscript written in 1894 and entitled "Presentation and Object" (cf. Schuhmann 1991, 138). It presents Husserl's own view on intentional objects and is critically based on the discussion in Twardowski's *On the Theory of the Content and Object of Presentations*, which was published in the same year.

The essay had been important to Husserl in relation to Meinong's theory of intentional objects for over a decade, including in the years before and after the publication of the *Logical Investigations*, and Husserl even attached the manuscript of the essay to a letter to Meinong in 1992 (cf. Rollinger 1999, 187f.). Because of their priority dispute, Meinong declined to read the manuscript in his reply. However, in a private note dated 1906, Husserl considers publishing the essay in a revised form as a "confrontation" (*Auseinandersetzung*) with Meinong (Hua XXIV, 447).

The essay is distinctive in that it draws a parallel between fictional objects, such as mythological characters, and mathematical objects, such as numbers and geometric figures, within the category of intentional objects. This parallel between fiction and mathematics contributes to the discussion on manifolds as well. In terms of mathematics, Husserl mentions manifolds as geometric spaces, that is, mathematical manifolds of a sort (IG, 159). On the other hand, Husserl also discusses "a comprehensive multiplicity [manifold] of assertions" (*eine umfassende Mannigfaltigkeit von Aussagen*) standing under some assumption (*Assumption*); and the fictional assumption (*fiktive Assumption*) such as that of poetry and of mythology is juxtaposed with the scientific assumption (*wissenschaftliche Assumption*) (IG, 160). As we shall see, a mathematical manifold is *the correlate of* a comprehensive multiplicity of assertions standing under a mathematical assumption. Our concern is whether the mathematical manifold and the correlate of the comprehensive multiplicity of assertions *in general*, which we will refer to as the *manifold posited by assertions*, can be equated.

However, putting aside the discussion on the theory of manifolds, the parallel between fiction and mathematics raises a problem: if we want to distinguish mathematical truths from merely fictional truths, we need to account for the scientific nature and privileged status of mathematics. As Rollinger points out, Husserl did not further develop the view he expresses in IG, which implies that mathematical truth is a kind of fictional truth, because it was incongruous with his demand for a distinction between mathematics and fiction (Rollinger 1999, 151f.).

Nevertheless, in spite of Husserl's choice, the basic direction taken in IG is advantageous in terms of ontological simplicity and intuitive persuasion, and is comparable to Meinong's theory of intentional objects and its contemporary interpretations (Rollinger 1999, 151).

2. Theory of "Intentional Objects"

In IG, Husserl focuses on the problem of objectless presentations: how to explain the presentation of something merely possible like a "golden mountain" or something impossible like a "round square". If we characterize presentations by their relation or directedness to some object, it should follow that "every presentation has an object"; but objectless presentations seem to contradict that position. Husserls also discusses the problem of general presentations, such as presentations of "a lion" or "a triangle", which are not presentations of a particular individual object.

As a first step in discussing this issue, Husserl distinguishes between the proper (*eigentlich*) and the improper sense of the statement "every presentation has an object". Husserl denies that every presentation has an object in the proper sense of the statement, namely, that some existent object "corresponds" (*entsprechen*) to every presentation. Husserl nevertheless asserts that in the improper sense, every presentation "represents" (*vorstellen*) an object, in terms of "intentional" objects (*"intentionale" Gegenstände*). In this way, Husserl introduces the notion of the "intentional object" into his discussion (IG, 151).

Although Husserl does not explicitly discuss the ontological status of the intentional object, he uses the expression "intentional object" in a theoretically substantial way, even if it is an improper expression. Husserl sees, for example, "a round square", "a square", and "Cerberus" as intentional objects (IG, 151). However, Husserl does not recognize them as intentional objects in Twardowski and Meinong's sense, namely as nonexistent objects. In Husserl's view, intentional objects are admitted under some "hypothesis" (*Hypothese*) or "postulate".

This view is illustrated most concretely by the example of geometry and its objects (IG, 159). In Husserl's view, the existence of a geometric figure as an intentional object actually means that its existence follows from some system of geometry or, equivalently, that it exists in some geometric space or manifold posited by some system of geometry.

Based on his exposition, Husserl's view can be elaborated as follows. A complete and pure system of geometry, or the foundations (*Grundlagen*) of such a system, is nothing but a definition that is divided into two kinds of statements, namely, existential statements and general, nomological (*nomologisch*) statements, about the objects posited by the system. This amounts to the hypothetical positing (*Setzung*) of a manifold, a geometric space in the context of geometry (IG, 159). In other words, there are a "system of the pure consequences" (*System der reinen Konsequenzen*) and a manifold, and the former posits the latter. To amplify Husserl's example, we may think of the axiomatic system of Euclidean geometry and the Euclidean space posited by it as a specific example. What the existence of a geometric object means is that "'[i]n the (definite) space' there exists a square, but not a round square, and a triangle, but not an equilateral right triangle, and so forth" (IG, 159). We can further broaden this argument to see that what exists will also differ between Euclidean and non-Euclidean spaces, for example.

Husserl finds that intentional objects in general, and in particular fictional objects, have precisely this structure: as he states, "[s]imilarly we say, after all, that 'in Greek mythology' there are nymphs, 'in the German fairy tale' a Little Red Riding Hood, and so forth: only here we are not concerned with scientific hypotheses and pure deductions" (IG, 159). In other words, a work of fiction can be understood as a system of existential and nomological statements. Although—unlike in mathematics—the entire content of a work of fiction is not determined solely by purely logical consequences, the existence or nonexistence of a fictional object is still determined to some extent by what is deduced from the statements. The assumption "in the work of fiction X, …" is often implicit in fiction as well as in mathematics (IG, 160).

As a result, in IG Husserl can be seen as devising a framework for intentional objects that consists of the deductive system in a broad sense and the correlate posited by them. In this framework, as is the case with the system of geometry and the geometric manifold (geometric space), it holds for intentional objects in general that the existential statements in some deductive system determine the domain of intentional objects in the correlate of the system, and the nomological statements in the system determine the nomological relations among objects in the correlate. Thus, in IG the mathematical theory of manifolds, as in the case of Euclidean geometry, and the theory of "manifolds posited by assertions" in a broader sense covering the ontology of fiction as the theory of fictional objects, share a common framework.

3. Theory of manifolds and the formal-material distinction

In IG, Husserl offers a unifying framework for the ontology of intentional objects in general, including both mathematical and fictional objects, on the basis of (1) a deductive system of some sort as the assumption positing objects and (2) a correlate posited by the assumption. In mathematics, especially in geometry, mathematical manifolds are considered to be the correlates. It is thus possible to contrast the concept of the manifold in the narrow mathematical sense with the concept of a manifold posited by assertions, which is not limited to the mathematical context. In this way, the concept of the manifold can be understood as applicable to the ontologies of both mathematics and fiction.

Is it possible, then, to consider the manifolds of intentional (and particularly fictional) objects posited by assertions in general to be exactly the same thing as mathematical manifolds? As already indicated in IG and consistently emphasized in Husserl's subsequent developments of the theory of manifolds, the important difference between the two lies in the distinction between the material and the formal (or, equivalently, between the real and the formal). In other words, Husserl assumes that mathematical objects are formal, while fictional objects are usually "materially filled".

Let us take a closer look at this point. Husserl contrasts "formal mathematics" (*formale Mathematik*) and "real mathematics" (*reale Mathematik*)[2], and considers his theory to concern mathematical objects in the former sense (IG, 157). While real mathematics is supported by intuitions (like the mental image of a geometric figure), formal mathematics is detached from all intuitions and deals with "the pure forms of mathematical connections and systems in the most general generality" (IG, 157). Hence, the correlate of a system in "formal mathematics" is not only formal in terms of its nomological relations but also purely formal in terms of the objects belonging to its domain of objects. In a 1901 text related to his lecture at the Göttingen Mathematical Society, Husserl simply refers to such objects as "formal objects" (*formale Objekte*), and in the *Logical Investigations*, he refers to the theory of manifolds in mathematics as "*formal mathematics* in the most general sense" (Hua XII, 452; Hua XVIII, 250; Husserl's emphasis). Unlike formal objects that lack material determinations, intentional objects in general are often materially filled: for example, the fictional object Cerberus.

We then need to clarify how the manifold posited by assertions, objects belonging to which can be materially filled, relates to the manifold in formal mathematics, the manifold as the formal. In this regard, it is important to note that the concept of the manifold as such does not necessarily preclude the possibility of objects being materially filled. Da Silva argues that Husserl himself could have considered the concept of a manifold which encompasses materially filled objects (Da Silva 2000, 425). The concept of the manifold can thus be extended based on this freedom, consistent with the concept of the manifold posited by assertions described in IG. In this interpretation, the manifolds posited by assertions that involve intentional objects in general are manifolds in the extended sense, and mathematical manifolds are special cases. As such, the application of the theory of manifolds to the ontology of fiction allows us to determine the truth of statements about fictional objects as intentional objects in relation to the manifold posited by a work of fiction.[3]

4. Manifolds as the formal and the possible worlds

Given the parallel between the correlate of the system of fiction and that of the system of mathematics as the correlates of the multiplicities of assertions, it is relatively faithful to the text to generalize the concept of the manifold so that it can be materially filled. However, in light of the subsequent developments of Husserl's theory of manifolds, this interpretation conflicts with the fact that the theory of manifolds is considered to be the science of the formal in the context of ontology and the theory of science beyond mathematics. In fact, Husserl calls the theory of manifolds the theory of "theory-forms" (*Theorieformen*) in the *Logical Investigations* (Hua XVIII, 248; cf. Hua XVII, 110). It is the theory of manifolds as the science of the formal that what relates Husserl's theory of manifolds to formal ontology, and given that later in 1906 Husserl refers to "general-formal ontology" (*allgemein-formale Ontologie*) as one of the matters that should have been discussed in the 1894 essay, the relation of Husserl's view in IG to formal ontology deserves substantial consideration (Hua XXIV, 447).

If we confine the concept of the manifold to a formal one, considering these external conditions, then the role of the manifold will also be confined accordingly. Such a manifold in the narrow sense, the formal manifold, is what remains after eliminating real (material) constituents from the "correlate". While mathematics is a purely formal theory of manifolds, the "correlate" of a system of fiction will contain not only a formal manifold concerning formal ontology but also material constituents due to the materially filled objects, which go beyond the concerns of the theory of manifolds.

This interpretation of the correlate as a formal manifold with material constituents is similar to that of Smith (2002). Smith observes that Husserl in the *Crisis* describes the formal manifold as "the formal-logical idea of a 'world-in-general'", and considers the correlate as a whole as the world, or the (sub-worldly) totality of states of affairs, and the manifold as its form ("world-form") (Smith 2002, 106f., 110).

Smith develops his interpretation by drawing mainly on the passages on manifolds in the *Logical Investigations*, but Husserl employs the very expressions "world of myth", "world of poetry", and "world of geometry" in IG (emphasizing that there is only one real world that can be truly called a "world"; IG, 159f.). Thus, it seems that the theory of manifolds in IG is compatible with Smith's interpretation of the correlate as a kind of possible world.[4] Adding to this point, Husserl seems to allow states of affairs to constitute the correlate, given that he explicitly applies his argument not only to objects of presentation but also to states of affairs as objects of judgment (IG, 143). The theory of manifolds in IG is better understood by considering the correlate, as a whole with material constituents, to be equivalent to a world or a totality of states of affairs, and taking a manifold as its "world-form".

An important aspect of Husserl's view in IG is, however, that it fundamentally calls attention to the notion of "world", which occupies a central position in Smith's interpretation of manifolds. In IG, Husserl does not actually endorse the view of the world as a correlate, but uses it to distinguish the real world from the "world of myth", the "world of poetry", and the "world of geometry"; and the "worlds" distinguished from the real world are immediately described in terms of "a comprehensive multiplicity of assertions" (IG, 159f.). It is certainly acceptable to employ the notion of "world" as a concept derived from the real world when the correlate is assumed to be analogous to the real world, as is often the case in the "world of mythology", for example. However, how is it possible to consider every "world of geometry", such as that of Euclidean or non-Euclidean geometry, to be analogous to the real world in a proper sense? The same applies to other kinds of mathematics and sciences. In contrast, neither the ontology of intentional objects in mathematics and fiction nor the theory of science generalizes the "world" as a concept derived from the real world. Rather, from a starting point in the mathematical theory of manifolds, the ontology of intentional objects in general and the theory of science must be described by generalizing and extending the concept of the manifold. From the standpoint Husserl takes in IG, it is therefore the manifold that is the most fundamental concept prior to the material constituents of the correlate. A correlate of a system of assertions is legitimately taken as a "world" only when the correlate is analogous to the real world, as it is in many works of fiction.

Conclusion

Husserl's theory of manifolds poses the question of the manifold's position and role as an ontological concept that is not limited to a mathematical context. According to our argument, the ontologies of mathematics and fiction in IG can be reconstructed in relation to the theory of manifolds, in terms of the deductive systems based on statements and their correlates. Mathematical and fictional objects differ in terms of the formal/material distinction, but we may generalize the concept of the manifold to one that can be materially filled so that both can be viewed as manifolds in a sense. If we confine the concept of the manifold to a formal one instead, in light of the subsequent developments of Husserl's theory of manifolds, and, particularly, if we interpret the manifold as a form of the world, it follows from Husserl's discussion in IG that the concept of the manifold is more fundamental than the concept of "world".

Rollinger compares Husserl's view in IG to Meinong's theory of intentional objects and its contemporary interpretation (Rollinger 1999, 151f.). The concept of the manifold in IG provides a clue to understanding how the view put forth by Husserl in IG can be related to formal logic, given the connection between the theory of manifolds and formal logic. In this respect, the view taken by Husserl in IG makes it possible to anticipate a theory that could be contrasted with contemporary Meinongianism, especially with Meinongian semantics as a formal-logical theory of intentional objects.

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Notes

1. Cf. Hua XXII, 303–348. In what follows, we refer to Schuhmann's edition, which is translated in Appendix 1 of Rollinger (1999).
2. The term "real" is used here not in the sense of reality as contrasted with non-existence or ideality, but in the sense contrasted with formality.
3. Ryba makes a connection between the ontology of fiction and the theory of manifolds in a similar way, implicitly introducing this broader concept of manifolds based on *Ideas I* and *Crisis* (Ryba 1990, 234ff.).
4. Rang mentions a similar possible-world interpretation in the Editor's Introduction to Hua XXII (Hua XXII, XL–XLI). However, Rang's discussion on it does not involve the manifold as the formal.

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