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Unremarkable Contextualism: Dispositions in the Bohm Theory

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One way to characterize dispositions is to take them to be reducible to categorical properties plus experimental arrangements. We argue that this view applied to Bohm's ontological interpretation of quantum theory provides a good picture of the unremarkable nature of spin in that interpretation, and so explains how a simple realism of possessed values may be retained in the face of Kochen and Specker's theorem. With this in mind we discuss Redhead's influential analysis of Kochen and Specker's theorem which does not appear to allow for the above view.

1. INTRODUCTION

Our motivation for writing about contextualism in the Bohm theory is twofold. Firstly, in most books on the philosophy of quantum mechanics the treatment of possible responses to Kochen and Specker's famous "no hidden variables" theorem[1] is seriously handicapped by an omission of any discussion of the Bohm theory. We believe that despite Bell's efforts,[2] a presentation of the position of the Bohm theory vis-à-vis this "no hidden variables" theorem is needed, not least because two of the most recent and important books in the field, Redhead's *Incompleteness, Nonlocality and Realism*[3] and van Fraassen's *Quantum Mechanics*,[4] do not even refer to the Bohm theory, let alone discuss it. Although we shall concentrate on Kochen and Specker's proof, what we say will also be applicable to the more recent "no hidden variables" proofs of Peres[5] and Mermin.[6]

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Secondly, since it is well known that most responses to Kochen and Specker's proof need to introduce some sort of contextualism, it is important to be clear about exactly what sort of contextualism one finds in the Bohm theory. Of particular relevance is the contextualism of spin quantities which are the properties used in all the above "no hidden variables" theorems. That the contextualism of these quantities in the Bohm theory is of a particularly intuitive and unremarkable sort (and very different in that respect from other proposals), we believe can be brought home most effectively by drawing the familiar philosophical distinction between categorical and dispositional properties within the Bohm theory.

In the next section dispositions are briefly discussed and the Bohm theory is introduced (only qualitatively) in light of this discussion. In Sec. 3 the position of the Bohm theory with respect to the interpretations of quantum mechanics discussed by Redhead[3] is identified. In Sec. 4, Redhead's influential analysis of the possible responses to Kochen and Specker's proof is discussed with the Bohm theory in mind.

The issue of how the Bohm theory escapes the Kochen–Specker contradiction is pursued with more technical detail in Sec. 5 where an analysis is given of the measurement of the squared spin components of a spin-1 particle according to the Bohm theory. (Such an account of this example that Kochen and Specker presented as paradigmatic of the implications of their theorem is, as far as we know, missing from the literature.)

2. PROPERTIES AND CONTEXTUALISM IN THE BOHM THEORY

We shall adopt a minimal form of realism whereby measurement results are taken to be grounded in some real property of the object measured. That there are properties of objects which are properly called dispositional is generally agreed. Whether all properties are of this kind, or whether there are categorical bases for dispositions which are of a non-dispositional kind and which alone qualify as real occurrent states of the object ("possessed values") is a matter of debate (see, for example, Mellor[7] and Mackie[8] for expositions of these and alternative views on dispositions).

The debate has significant bearing on the philosophy of quantum mechanics. The view that all properties are dispositional, which under the above minimal realist assumption makes dispositions real occurrent states of objects (and which we shall call the antireductionist view since dispositions are taken not be reducible to nondispositional properties), provides the philosophical framework for an alternative realist interpretation of quantum mechanics (alternative to "hidden variable" theories). In this interpretation an object not in an eigenstate of some observable possesses a probabilistic disposition or propensity to produce various results upon measurement of that observable (see, for example, Margenau,[9] Heisenberg,[10] Mellor,[11] and Popper[12]; there are, of course, differences between their accounts).

That said, our concern here will be with a particular hidden variable theory, the Bohm theory, with sure-fire dispositions rather than the probabilistic ones considered above, and with the view that dispositions are reducible to categorical properties plus experimental arrangements (which we shall call the reductionist view). We shall find that this view of dispositions is a natural, indeed, arguably the only way of understanding certain properties in the Bohm theory.

Before embarking on a description of the Bohm theory, a word about the connection between dispositions of the reductionist view and contextualism. In an almost trivial sense, dispositions of this view are contextual properties. This is just because they are not real properties, but, roughly speaking, a façon de parler, a linguistic shorthand if you like, for what happens when an object with a certain categorical property is subjected to a certain experimental context.[3] Their ascription, therefore, necessarily requires the specification of a context.[4] That is all that is meant by "contextualism" here. Since dispositions are not real properties their "contextual" nature clearly has no ontological significance. We shall call this unremarkable "contextualism" of dispositions *contextualism*$_1$, distinguishing it from the *causal* dependence of *possessed* values on the context of measurement, which we shall call *contextualism*$_2$.

Bohm's theory[13] is a (nonrelativistic) deterministic theory which captures all the predictions of quantum mechanics. Roughly speaking, the picture is that of particles whose trajectories are determined by their initial positions and a new type of physical field, the ψ-field. The particle positions at any time together with the ψ-field, which is determined by the wavefunction of the system interpreted as a field defined over configuration space (the manifold of possible instantaneous configurations of the particles in the system), determine the values a measurement would yield at any time for all dynamical quantities associated with the particles. The fact that most quantum predictions are probabilistic reflects our ignorance of the

[3] It has to be said that efforts to make this statement rigorous have faced difficulties, notably concerning the mutability of dispositions (see Mellor,[7] pp. 106–107).

[4] In common parlance the full context is more often than not left unmentioned, not because it is not required, but because it is assumed as obvious. For example, in "salt is soluble," the context "in water at room temperature and pressure" is usually what is implied.

(uncontrollable) initial positions of the particles, not any irreducibly stochastic behavior of the system. It will help to be a little more precise in our explanation of how the Bohm theory works, first with regard to the ψ-field, then with regard to measurement.

The evolution of the ψ-field in configuration space from some initial time depends on the evolution of the wavefunction (via the Schrödinger equation), which in turn depends on the initial wavefunction and on whether and how the system interacts with measurement devices or other quantum systems. However, the field at a given time does not depend on the actual position in configuration space that the system previously occupied. In this sense, unlike a classical field, the ψ-field has no sources.

The ψ-field exerts a force in real space on the particles in the system. For any one particle this is calculated by taking the component in the subspace of that particle of the field vector at the point in configuration space that the system of particles actually occupies. Therefore, for those cases where the ψ-field in configuration space is not decomposable into independent components corresponding to each of the subspaces of the particles in the system (which will occur if the wavefunction is not factorizable), the force exerted on any one particle will depend on the positions of all the other particles in the system, as well as on the ψ-field (and so on whether and how the system has interacted with measurement devices, whether distant to the particle in question or not).

This nonlocal dependence of the force exerted on any one particle on the positions of all the particles is not our main concern here. Our concern is rather with the dependence of this force on the ψ-field (and so on any measurement interactions), i.e., with contextualism. This, as we shall see below, breaks down into contextualism$_1$ and contextualism$_2$.[5] We saw above that for entangled (i.e., nonfactorizable) multi-particle systems the context for any one particle includes measurements on distant particles, so that there is also a nonlocal aspect to contextualism. But, for the most part, our discussion here will be confined to single-particle systems where neither nonlocal effect arises. (In passing, however, it is worth reiterating that neither of these nonlocal effects arise through any "back-reaction" or influence of the particles on the ψ-field.[6])

We shall raise the issue of the ontological status of the ψ-field in configuration space only to set it aside for another occasion. Whether or not ontological significance is invested in the ψ-field in configuration space (rather than to nonlocal and nonlinear waves in three-dimensional

[5] When without a subscript, "contextualism" should be taken to refer to both contextualism$_1$ and contextualism$_2$.

[6] See Butterflied[14] for a philosopher's introduction to the Bohm theory account of the nonlocality of a two-particle Bell experiment.

space) marks one of the main differences between de Broglie's and Bohm's interpretations (although the two are often conflated).[7] It is also true, of course, that this is a general problem: all realist interpretations of quantum mechanics have to make some ontological sense of the wavefunction.

We now turn to measurements. In the Bohm theory the measurement of a dynamical quantity is reducible to measuring the position of the particle after the measurement interaction has correlated its position to the value of this dynamical quantity, the correlation being brought about through the ψ-field's evolution during the interaction. Since the values of dynamical quantities depend partly on the ψ-field in this way, it follows that measurement results for all dynamical quantities depend partly on the context of measurement (which, as we said earlier, plays a role in determining the evolution of the ψ-field). So measurement results are contextual; a realist needs to ask further what this fact entails about the properties of the object measured that "ground" these results.

There are two accounts of the dependence in the Bohm theory of measurement results on the context of measurement, corresponding to whether the object possesses a value for the property measured or not.

The measurement of a categorical property other than position will not, in general, reveal the possessed value for this property immediately prior to measurement. This is not just because the possessed value will in general change with the introduction (or change) of the measurement context (which would simply be a disturbing measurement from which the pre-measurement possessed value might still be calculated), but because this change depends on both the pre-measurement possessed value of the quantity measured and the (also unknown) pre-measurement position of the particle. Thus, from a particular measurement result one cannot, in general, "trace back" to a unique possessed value (the exception is the measurement of an eigenstate which will reveal the possessed value because in this case all possible initial positions must lead to the same final position).

There are two things, therefore, to say about categorical properties in the Bohm theory. The first is that they are not *faithfully measured*: measurement does not in general simply reveal the pre-existing (i.e., pre-measurement) possessed value[8] (so, although measurement results are

[7] For further details see Dewdney *et al.*[15]

[8] This does seem like an abuse of terminology: an interaction which fails to reveal the pre-existing value of some property can hardly be called a measurement of that property! All such an interaction does according to the Bohm theory is, *for given initial state of the system*, reveal the set of possible initial positions which give rise to the measurement result obtained. However, we shall persist with the abuse in order to make the point that what is regarded as a measurement in orthodox quantum mechanics cannot, in general, be regarded as such in the Bohm theory.

always eigenvalues, possessed values need not be). The second is that categorical properties are contextual in the sense of contextualism$_2$, since the possessed value causally depends on the measurement context (which, of course, may or may not pertain to the measurement of that value, since it may be another property or a property of another (entangled) object which is being measured).

If it is a dispositional property that is "measured," the dependence of the measurement result on the context is just contextualism$_1$, the "contextualism" of dispositions. There is no issue of faithful measurement, of course, since there is no possessed value to reveal. In this instance the property that "grounds" the measurement result is not the one "measured" but the categorical property to which (in addition to the context) the dispositional property is reducible to.

On a natural reading, although all quantities in the Bohm theory with continuous spectra (e.g., position and linear momentum) are categorical properties, spin quantities are dispositions of the reductionist view; dispositions for the particle under consideration to behave in certain ways in certain contexts. They have a categorical basis, the position of the particle, which together with the (context-dependent) ψ-field uniquely define them.

A word or two is in order to explain what we mean above by "a natural reading." For a realist theory like the Bohm theory there are two ways to deal with quantities like spin, which according to the formalism of quantum mechanics can only have discrete spectra.

On the one hand, such quantities may be taken to be categorical properties with a continuous spectrum of possessed values (clearly not in general eigenvalues), the discreteness dictated by the formalism applying only to measurement results (and, in the Bohm theory, ensured by the ψ-field's evolution).

On the other hand, these quantities may be interpreted as dispositions; in the Bohm theory, reducible to the position of the particle and the context. In this case, the discreteness is seen to arise simply because certain questions which must be answered experimentally are reducible to questions with a discrete spectrum of possible answers. For example, the question "What is the value of spin in a certain direction of a certain spin-1/2 atom?" is reducible to asking whether the position of the atom as it entered the magnetic field of the Stern–Gerlach apparatus was above or below a certain plane; a question with only two possible answers. This is the "natural reading" referred to above.

In their account of spin in the Bohm theory, Bohm, Schiller, and Tiomno[16] and, more recently, Dewdney *et al.*[17] can be seen as essentially taking the former approach, for they postulate that particles possess a spin vector which defines their state of rotation. We shall have occasion to mention the BST approach again briefly in Sec. 4. The latter (and present) approach is essentially that taken by Bell[18] in his account of spin (albeit not couched in terms of dispositions).[9]

Dispositions in the Bohm theory differ from their classical counterparts only in that for multi-particle systems the contexts needed for their definition are not always local and so the contextualism$_1$ for such systems is of a nonclassical sort. This is just a reflection of the fact that quantum mechanics is a nonlocal theory. Indeed, as was noted above, the Bohm theory makes clear the connection between this sort of nonlocality and contextualism.

It should now be clear what we mean by contextualism in the Bohm theory. There is contextualism$_1$ which pertains to spin quantities and there is contextualism$_2$ which pertains to categorical properties like linear momentum and kinetic energy. We shall contrast this contextualism with the standard accounts within the philosophy of quantum mechanics in Sec. 4.

3. REDHEAD'S INTERPRETATIVE FRAMEWORK

We would like to set the scene for discussing Redhead's analysis of Kochen and Specker's proof[3] by first describing the framework within which Redhead operates and attempting to locate the Bohm theory within this framework.

Redhead distinguishes three interpretations of the quantum mechanical formalism which he presents as answers to the question: "What can one say about the value of an observable, call it Q, in QM when the state of the system is not in an eigenstate of Q?" (Redhead,[3] p. 45). Roughly, view A (hidden variables) says that Q has a sharp but unknown value; view B (propensities) says that Q possesses a propensity or disposition to produce various possible values upon measurement; and view C (Copenhagen) says that Q has no value until an appropriate measurement context is specified.

At first blush the Bohm theory seems to fall squarely in the category of view A, since, in a loose sense, all observables have precise values at all times. However, as we have seen, some values (those of dispositional properties) cannot be said to be possessed by the particle. Also, faithful measurement (which Redhead builds into view A) is only true for position in the Bohm theory (making position the only true "observable").

Perhaps then the Bohm theory has elements of both views A and B? In the present paper we have certainly emphasized the dispositional

[9] See also Albert and Loewer[19] and Albert.[20]

character of certain quantities in the Bohm theory. However, the view of dispositions in operation in the Bohm theory is very different from that of view B (reductionist in the former, antireductionist in the latter), the latter not allowing for categorical properties.

Of course, under the antireductionist view of dispositions, *all* properties in the Bohm theory are dispositional. This, however, is an unfortunate combination of views. Firstly, the main attraction of the antireductionist view vis-à-vis interpreting quantum mechanics is supposed to be that it provides for a realist interpretation of quantum mechanics not saddled with the incompleteness of that theory. This attraction is lost on the above combination of views. Secondly, and more importantly, it is arguable that this combination of views is inconsistent with our minimal realist assumption. To see this, consider that on an antireductionist view of dispositions an arguably necessary condition for real properties is that they display themselves in more ways than one.[10] However, such multiple display for spin observables is precisely what Kochen and Specker's proof shows is not always possible—there is no guarantee that a spin observable measured in two different contexts will be found to have the same value. This leaves one with no real occurrent properties at all grounding spin observables, contra our minimal realist assumption. (This, of course, is true for the antireductionist irrespective of whether (s)he takes the BST or the present approach to spin.)

Authors on the Bohm theory (see, for example, Bell[(2)]) have not left unmentioned the similarity between the holism and complementarity of view C with the built-in contextualism of the Bohm theory. It is important to stress, however, that this similarity is entirely superficial. Complementarity is an epistemological constraint in the Bohm theory, whereas it is a fundamental principle of ontological significance in the Copenhagen interpretation. The Bohm theory is a contextual theory while still being thoroughly realist, unlike the Copenhagen interpretation where dynamical quantities become real occurrent states of an object within certain contexts but remain undefined otherwise.

Our conclusion then has to be that the Bohm theory, although closest in spirit to the view A interpretation, is not really captured by that view. There are two reasons for this: view A does not allow for dispositional

[10] On all accounts, dispositions should explain their displays. On the reductionist view such explanations are "grounded" on the categorical properties to which they are reducible. But on an antireductionist view, such "explanations" seem to beg the question if the only criterion for a disposition's ascription is its own display. So, to maintain that dispositions are real properties and that they are explanatory in the above sense, it is arguably necessary that there be grounds for their ascription that are independent of just a single display (see Mellor,[(7)] pp. 117-118).

properties of the reductionist view, and it stipulates faithful measurement of possessed values. The Bohm theory, then, is basically a view A interpretation of quantum mechanics without faithful measurement and with the important amendment that spin quantities are dispositions whereas quantities with continuous spectra are categorical properties (the former contextual_1, the latter contextual_2).

4. REDHEAD'S ANALYSIS OF KOCHEN AND SPECKER'S THEOREM

We shall take the demonstration of the Kochen–Specker contradiction as given and concentrate on Redhead's examination of the justification of the assumption which led to the contradiction: FUNC (the functional composition principle).

Formally, FUNC constrains the functional relations obtaining between the values of observables to be the same as those obtaining between the corresponding operators. For Hilbert spaces of dimension greater than two this constraint leads to Kochen and Specker's coloring contradiction.

Redhead shows how FUNC can in fact be derived from three more intuitive principles; so, in view of the Kochen–Specker contradiction, one of these three has to be rejected (Redhead,[3] p. 133–136). They are: View A; the correspondence rule (that there is a 1-1 correspondence between self-adjoint operators and observables); and the reality principle (roughly, that if a measurement results in numbers distributed according to the statistical algorithm of quantum mechanics for some operator, then there exists an observable associated with that operator which is measured by those numbers).

Redhead's approach is to "see how far we can get in interpreting QM with a simple realism of possessed values" (*op. cit*, p. 136). With his interpretative framework in mind, it is clear that (unless the concept of possessed values is jettisoned altogether), a rejection of view A would entail embracing a contextualism of possessed values. This does not qualify as a "simple realism of possessed values" for Redhead, probably because the contextualism he has in mind is a *logical* dependence of possessed values on the context, making them possessed but relational attributes. A rejection of the reality principle, he argues, would leave one in the awkward position of having to decide which are genuine and which are nongenuine measurements of an observable. This leaves no option but the denial of the correspondence rule, an option proposed (but not defended) by van Fraassen and called *ontological contextuality* by Redhead.

According to ontological contextuality, there are as many different observables corresponding to each nonmaximal operator as there are different maximal operators of which the nonmaximal operator is a function. This is supposed to explain why the measurement of a given nonmaximal operator under different contexts, assumed to be picked out by the maximal operator through which it is measured, will in general produce different results. (In Kochen and Specker's proof FUNC is applied to commuting nonmaximal operators (since different measurement contexts of the same observable are needed for the proof and these are, of course, only available for nonmaximal operators), thus this ontological contextuality infects only nonmaximal operators.)

The ontological extravagance of this approach, with each quantum mechanical object possessing uncountably many values for each of an uncountable number of observables beyond those envisioned by orthodox quantum mechanics, is difficult to stomach. Indeed, it hardly qualifies as a *simple* realism of possessed values! The unphysicality of this approach is also disquieting—we are left none the wiser about the physical basis of contextualism. If this is the best we can do with view A, then it seems untenable. But what of the Bohm theory? Perhaps a simple realism of possessed values can be kept after all?

Kochen and Specker's theorem makes use of spin quantities. These are dispositions in the Bohm theory, so the theorem has to be formulated in terms of counterfactually definite measurement results rather than possessed values. More importantly, however, since dispositions are contextual$_1$, FUNC cannot be assumed and no contradiction ensues. This is because FUNC assumes that the values, or counterfactually definite measurement results, assigned to nonmaximal operators are independent of which context they are measured in, which is (by definition!) not the case for dispositions of the reductionist view. Thus, the conclusion of Kochen and Specker's theorem can be avoided just with contextualism$_1$; hardly the sort of contextualism to write home about.

It is also true, of course, that the Bohm theory is contextual$_2$. So, it seems that in the long run Redhead is right. Contextualism of possessed values is unavoidable if view A is rejected, although, as we have seen, this is certainly not a direct consequence of Kochen and Specker's theorem. Whether or not the Bohm theory, by underpinning it with physics, manages to make contextualism$_2$ (the *causal* dependence of possessed values on the context) respectable and worthy of the title "simple realism of possessed values" is only a matter of words. At the very least it seems more plausible than ontological contextuality.

5. THE PHYSICS OF CONTEXTUALISM IN THE BOHM THEORY

Our purpose in this section is to give the physics behind the Bohm theory's instantiation of contextualism$_1$ and consequent circumvention of Kochen and Specker's theorem. For that purpose, it is most appropriate to use Kochen and Specker's own example of measuring the squared spin components of a spin-1 particle via spin Hamiltonian measurements (for further background quantum mechanical details, see Kochen and Specker[1] (pp. 72–73) and Redhead[3] (pp. 38–39 and Chap. 5).

As mentioned in the previous section, Kochen and Specker's theorem has to be formulated in terms of counterfactual measurement results (rather than possessed values) to be applicable to the Bohm theory under a reductionist view of dispositions. Let $[S_x^2]_{H_{xyz}}$ represent the counterfactual measurement result that would register if the squared spin component in the x-direction S_x^2 ($=1$ or 0) were measured via measuring the spin Hamiltonian $H_{xyz} = aS_x^2 + bS_y^2 + cS_z^2$ (with a, b, and c distinct real numbers), let $[S_y^2]_{H_{x'yz'}}$ be the counterfactual measurement result if S_y^2 were measured via a measurement of $H_{x'yz'} = aS_{x'}^2 + bS_y^2 + cS_{z'}^2$ for a distinct orthogonal triad $\{x', y, z'\}$, etc.

Recall that Kochen and Specker's central assumption is FUNC, which in this case requires that the above square-bracket values be "noncontextual": that the measurement result that would register were any (squared) spin component measured does not depend upon which orthogonal triad of components it is measured with, i.e., on which spin Hamiltonian it is measured through. Using the above notation, FUNC therefore entails the equations

$$[S_x^2]_{H_{xyz}} = [S_x^2]_{H_{xy'z'}}, \qquad [S_z^2]_{H_{x'y'z}} = [S_z^2]_{H_{xyz}}, \text{ etc.} \tag{1}$$

To keep things simple, let $a=1$, $b=-1$, and $c=0$ in our spin Hamiltonians above, and focus on measuring S_z^2 in two different (incompatible) ways—through measuring $H = S_x^2 - S_y^2$ and through measuring $H' = S_{x'}^2 - S_{y'}^2$, each of which has eigenvalues -1 $(=b+c)$, 0 $(=a+b)$, and $+1$ $(=a+c)$. Since $(S_x^2 - S_y^2)^2 = S_z^2$ for any orthogonal triad $\{x, y, z\}$, in both cases the measured value of S_z^2 is obtained by squaring the result of measuring the appropriate spin Hamiltonian, H or H'. The rest of this section will be devoted to spelling out the physical reasons why a given initial particle position and ψ-field can, contrary to (1), lead to different measurement results for S_z^2 depending on whether it is measured through H or H'.

To apply the Bohm theory, we must first start with an orthodox analysis of a spin Hamiltonian measurement to determine the statevector for the composite apparatus + object system during the measurement interaction. The statevector will then be reinterpreted, in the usual way, as a field guiding the particle into one or another region of a detecting screen placed in the path of the particle as it leaves the interaction.

To measure H (a similar prescription will apply for H'), it suffices to consider the application of an interaction Hamiltonian to the particle of the form (see, for example, Bell[2] p. 161):

$$H_T = g i^{-1}(\partial/\partial q)\, H \qquad (\text{setting } \hbar = 1) \tag{2}$$

where g is a (positive) coupling constant, nonzero only during the time T of the measurement interaction, and q is the component of the particle's position which is correlated to the value of H so that it can be measured. Such a correlation could, at least in principle, be arranged by passing the particle through a suitable inhomogeneous *electromagnetic* field which functions much like a Stern–Gerlach apparatus—see Swift and Wright.[21] [11] If one makes the usual assumption that the measurement is "impulsive," the Schrödinger equation reduces to $\partial\Psi/\partial t = -i^{-1}H\Psi$ during T, and [using (2)] has the integrated solution

$$\Psi(q, t) = \exp[-g(\partial/\partial q)\, Ht]\; \Psi(q, 0), \qquad \text{for} \quad t \in [0, T] \tag{3}$$

Taking the initial state to be of the general form

$$\Psi(q, 0) = \phi(q) \sum_j c_j\, |H = j\rangle \tag{4}$$

where $\phi(q)$ is a narrow wave packet symmetric about $q = 0$ and $|H = j\rangle$ is the eigenstate of H with eigenvalue j ($= -1$, 0 or $+1$), one finds [inserting (4) into (3)]:

$$\Psi(q, t) = \sum_j c_j \phi(q - gjt)\; |H = j\rangle \tag{5}$$

Note that to make negligible the overlap between adjacent wavepackets at time T so that the particle's H-value is almost always discernible from its post-measurement deflection in the q-direction, we must make the usual assumption that gT is significantly larger than the (already narrow) width of $\phi(q)$ (cf. Bohm,[22] p. 597). The measurement result "$H = j$" then corresponds to finding the particle displaced from $q = 0$ by the amount gjT as it leaves the electromagnetic field.

With statevector (5) in hand, we can apply the Bohm theory's prescription for determining the possible trajectories of the particle through the field. Following Bell[23] (pp. 10, 131, 162), the easiest way to do this is

[11] We can ignore any deflection of the particle in the plane orthogonal to q during T since that motion will not be correlated to the value of H and is therefore irrelevant to determining its measurement results.

not to introduce "hidden" ("classical") variables to describe the particle's spin (as in the BST approach), but simply to define the probability density, P, and current, J, by summing over the spin indices. (This is the "natural reading" of spin in the Bohm theory that we adopted in Sec. 2.) Since, during T, the Schrödinger equation $\partial\Psi/\partial t = -iH_T\Psi$ [with H_T as given in (2)] is easily seen to imply $\partial\,|\Psi(q,t)|^2/\partial t + \partial[\Psi^*(q,t)\; gH\Psi(q,t)]/\partial q = 0$, this gives a continuity equation for $P \equiv |\Psi(q,t)|^2$ and $J \equiv \Psi^*(q,t)\,gH\Psi(q,t)$. One can then adopt a classical interpretation for the particle's velocity dq/dt, taking it to be equal to just J/P. (This interpretation could be motivated by appeal to Hamilton–Jacobi theory, but one can also just stipulate that the particle obeys the guidance formula $dq/dt = J/P$ since it suffices to reproduce the usual predictions.) Using (5)'s $\Psi(q,t)$, this makes the quantum state, now defining a causally efficacious field, impart to the particle a velocity

$$dq/dt = g\sum_j j\,|c_j|^2\,|\phi(q-gjt)|^2 \Big/ \sum_j |c_j|^2\,|\phi(q-gjt)|^2, \qquad \text{for} \quad t\in[0,T] \tag{6}$$

With the possible initial positions confined to the set $\{q\colon |\phi(q)|^2 > 0\}$, (6) can in principle be solved to determine the measurement results (gT, 0, or $-gT$) corresponding to various initial positions. Furthermore, by the continuity equation, these results will have to be probabilistically distributed (as in the orthodox interpretation) according to $|c_j|^2$ if our ignorance about which initial position obtains in a given measurement trial is represented by the initial distribution $|\phi(q)|^2$. (So we see probabilities arise here just as in classical statistical mechanics.)

Obviously, the above analysis can be applied in exactly the same way to describe the measurement of H'. Analogously to (6), the particle's velocity will be determined by

$$dq'/dt = g\sum_j j\,|c_j'|^2\,|\phi(q'-gjt)|^2 \Big/ \sum_j |c_j'|^2\,|\phi(q'-gjt)|^2, \qquad \text{for} \quad t\in[0,T] \tag{7}$$

where the c_j''s are coefficients in the expansion of the same general initial state (4) over H' eigenstates instead, and q' is the position component of the particle coupled to H' during the measurement. We must distinguish between q and q' because the spatial orientation of the field required to measure any spin Hamiltonian H_{xyz} will in general depend upon which orthogonal triad of (squared) spin components $\{x, y, z\}$ one wants to measure, and so the particular position component correlated during the measurement will share this dependence.

Equation (7) presupposes that the position part of the wavefunction has the same profile in the q'-direction as in the q-direction, viz. ϕ, and assumes that the H' measurement is of the same strength, g, and duration, T, as the H measurement. How is it, then, that a different measurement result could be produced in each of the two cases for a given initial position and spin state? The answer is, of course, that the c_j's (the expansion coefficients of the initial spin state over H eigenstates) will necessarily differ from the c_j''s (the expansion coefficients over H' eigenstates), since $[H, H'] \neq 0$. This reflects the fact that the ψ-field will be affected differently (more precisely, bifurcate differently) in each measurement context due to the differing measurement interaction Hamiltonians needed to measure H and H', and so will, in general, affect a given initial position differently.

We can see why this is so without having to integrate (6) or (7). From (6), the q-velocity of the particle at any time is determined solely by its position in the q-direction (for a given initial spin state). Therefore, it is not possible for distinct possible trajectories of the particle to cross along the q-axis. For if they did, then at their intersection they would have matching position (and, therefore, velocity) in the q-direction forcing the rest of their trajectories in that direction, both before and after the intersection point, to match as well (by determinism). What this means is that those possible trajectories which are measurement deflected by gT must have started with initial positions more positive (in the q-direction) than those which get 0 deflection; and (similarly) those trajectories deflected by $-gT$ must have arisen from initial positions more negative (in the q-direction) than those with 0 deflection. The same applies for the H' measurements and the q'-axis [by (7)]. Therefore, in each case, the possible initial positions the particle can take up within the initial wavepacket will be divided into three distinct, nonoverlapping intervals, or "bins," each of which corresponds to obtaining one of the three possible measurement results.

That a given initial position will in general be affected differently by different interaction Hamiltonians now just follows by noting that the size of these "bins" will be determined by the particular spin Hamiltonian, H or H', that is measured. For we know that a fraction $|c_j|^2$ of the initial positions (in an ensemble distributed according to $|\phi(q)|^2$) produce the result j when H is measured, whereas $|c_j'|^2$ is the appropriate fraction for that result when H' is measured. The change of the fraction of initial positions in each bin because of this change of the size of the bins when an H' rather than an H measurement is performed is what delivers contextualism$_1$: a measurement of S_z^2 through H' may, for the same initial position and spin state of the particle, yield a different result than if it had been measured through H. (Exactly what fraction of initial positions yield different results under a change of context will also, of course, depend on the angle between the q and q' axes.) Thus, at least one of the equations in (1) fails and Kochen and Specker's argument is blocked—in a physically natural way.

6. CONCLUSION

What we've argued here about Kochen and Specker's proof is equally applicable to subsequent "no hidden variables" proofs in the same vein, for they also deal with spin quantities—see, for example, Peres[5] and Mermin[6] (who deal with two- and three-particle systems respectively, and so nonlocal contexts). Dewdney[24] discusses the Peres and Mermin proofs with the Bohm theory in mind, but adopts the BST approach to spin. This means that on his approach it is because of contextualism$_2$ that the Bohm theory avoids the conclusion of these theorems. Apart from the general advertisement of the Bohm theory, the aim of this paper has been to show that, on a natural reading, spin quantities in that theory are dispositions of the reductionist view, so that these proofs and Kochen and Specker's fail for a simpler reason: the unremarkable contextualism of dispositions.

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