Hist. Sci., xliii (2005)

GALILEO'S CONSTRUCTION OF IDEALIZED FALL IN THE VOID

Paolo Palmieri
University of Pittsburgh

1. INTRODUCTION

How did Galileo achieve his mathematical natural philosophy? How did he discover the times-squared law, the pendulum isochronism, the parabolic trajectory of projectiles — by experiment or by mathematical reasoning? Questions like these have long divided scholars into two camps: the advocates of experiment and the advocates of geometry. Is there an alternative to the two horns of this dilemma? I am convinced that there is indeed a viable alternative. Before substantiating my claim, however, I need to sketch the essential elements characterizing the division. I will do so through the exemplar case which is central to my study here.

In 1604 Galileo wrote a letter to Paolo Sarpi, in which he put forward an "erroneous" principle from which he claimed that he could derive the times-squared law of fall. The principle is as follows: the ratio of the speeds of fall is the same as the ratio of the spaces fallen through. In referring to that "erroneous" principle, Alexandre Koyré argues that Galileo already knew all the details concerning the phenomenon of fall, such as the *sameness of ratios* between the spaces traversed and the squares of the elapsed times (on *sameness of ratios* more below, in Section 3).[1] What Galileo had long wanted to discover, in Koyré's view, was a general principle from which he could deduce the law of fall geometrically. In other words, Koyré continues, Galileo sought to find the essence, i.e., the definition, or law, of the phenomenon "fall of bodies". Why, asks Koyré, did Galileo adopt the "erroneous" principle? The answer, for him, is clear. The key to classical physics is the geometrization of nature, which implies the application of mathematical laws to the phenomena of motion. But, in Koyré's words, it is much easier to "imagine in space rather than think in time [*imaginer dans l'espace que de penser dans le temps*]"; hence Galileo's "error" in 1604.[2] In the wake of Koyré, William Shea argued that between 1610 and 1632 Galileo worked out "the methodology of his intellectual revolution", especially in connection with his researches on hydrostatics.[3] According to Shea, that methodology basically consisted in mathematically investigating classes of phenomena, such as, for instance, floating bodies, under certain idealized conditions. Further, Shea suggested that the "popular notion of Galileo as the first hard-headed and thoroughgoing experimentalist owes much to his first biographer, Vincenzo Viviani", who, in Shea's view, "seriously misinterpreted Galileo's early interests by claiming that he had discovered the isochronism of the pendulum and the law of uniform acceleration of falling bodies by performing experiments while still a student at Pisa".[4]

Other scholars, notably Stillman Drake and Ronald Naylor, have emphasized

Galileo's aptitude for experimentation. Most of them have pointed to the numerous manuscript sources that expose the empirical basis of Galileo's investigations concerning falling bodies, which may have led to the discovery of the times-squared law.[5] There is now a broad consensus that the crucial period of Galileo's experimental investigations on motion spanned his arrival in Padua, in 1592, to his departure eighteen years later, in 1610.[6] By then Galileo had discovered the times-squared law, the paraboloic trajectory of projectiles, and also the correct sameness of ratios of speeds to times, thus abandoning the 1604 blind alley. Disagreement has by and large been confined to the details of the process by which Galileo is supposed to have corrected his 1604 "error", all of which, however, boil down to the vexed question: experience or mathematics? The basis of all these studies has mostly been the so-called *Manuscript 72*, preserved in the Galileo collection at the National Library in Florence. The manuscript as a whole cannot be dated with certainty, especially because it appears to have been compiled and re-worked by Galileo (and others) at different times. Further, Paolo Galluzzi reviewed the history of the interpretations of Galileo's "error" and concluded that it must have been caused both by the inadequacy of Galileo's diagram representing the quantities involved in the proof and by the inadequacy of his terminology, which would not have allowed him to distinguish clearly between the notions of degree of speed and average speed.[7]

I believe that Koyré's emphasis on the 1604 "error" has led historians astray. Let's see how. The implication of Koyré's argument has always, and rather uncritically, been taken to be that ever since Galileo discovered his "error", allegedly some time before leaving Padua in 1610, he must have automatically embraced the correct version of the sameness of ratios, a turn that would virtually have marked the pinnacle of achievement of his research on motion. Subsequently, the story goes, other interests caught his imagination, especially following his astronomical discoveries with the telescope, thus delaying the publication of *Two new sciences* (1638) until after the notorious condemnation of 1633. In other words, the implication has always been that the correct form of the sameness of ratios was somehow discovered at a precise point in time after the "error" of 1604.

Thus the assumption underlying this reasoning seems to have been that truth came after error. In fact, documentary evidence (based on a short, but revealing writing, *On accelerated motion* [*De motu accelerato*], on which more in a moment) suggests that the regularity of uniform acceleration with respect to time had been considered by Galileo as a strong candidate for a general principle perhaps as early as the 1590s, or not long thereafter. Accordingly, I will argue that the "error" of 1604 was no more than a marginal, very short-lived attempt to mathematize that regularity in terms of Euclidean samenesses of ratios. The so-called *De motu accelerato* is a short writing that Galileo never published. Different views on its dating have arisen. However, I am convinced that the document is in fact to be dated to well before 1604, quite possibly to the late 1590s. First and foremost, the folios are bound with the same early manuscript material known as *De motu antiquiora*, or simply *De motu*, which is by general consensus attributed to the early 1590s. I see no valid reason why it should be detached from this early material. In fact the only substantial reason for this arbitrary re-location seems to have been the very assumption that because in *De motu accelerato* Galileo proposes the "correct" definition of uniformly accelerated motion, and because a portion of it was recast in *Two new sciences* (1638), it must have been conceived well after 1604, perhaps in preparation for *Two new sciences*.[8] My analysis has led me to call into question the very assumption that the rectification of the 1604 "error" came with the discovery of the "correct" sameness of ratios. As we shall see, Galileo soon found a counterargument to the 1604 sameness of ratios between speeds and spaces, which had nothing to do with the "correct" sameness of ratios between speeds and times. In reality, the fact until now unappreciated is that regardless of the form of the sameness of ratios, i.e., regardless of the difficulty in mathematizing the regularity of uniform acceleration with respect to time, Galileo must long have felt caught in a dilemma. For the motivational dynamics driving his researches on the mathematization of motion (and in part causing the delay in publication) ultimately depended on his grappling with a paradox concerning heavy bodies and the continuity of motion in uniformly accelerated fall. It was the resolution of that paradox that was central to Galileo's project of creating an entirely "new science of motion". He had struggled with it ever since the 1590s. As I will suggest, the paradox finally evaporated in 1635–36. In an extraordinary cognitive process of "memory rewriting" Galileo convinced himself that falling bodies somehow become weightless. On that basis he went on to design and carry out experiments with pendula to test the principle that bodies of all types of matter fall at the same speed in the void.

On the other hand, we are now certain that Galileo excogitated and performed ingenious experiments. I believe that Galileo's engagement in a lifelong quest for a general principle, from which he could deduce the times-squared law of falling bodies, was first and foremost a search for an idealization of the phenomena of falling bodies that could escort him safely beyond the quagmire of paradox. In other words, Galileo's mathematical investigations of nature were ultimately based on idealization processes. Their significance for his methodology deserves to be studied in finer detail. Galileo's methodology integrates numerous investigative strategies, such as mental models,[9] analogical thinking, experimentation, cognitive autobiography, and, most important of all, proportional reasoning.[10] Galileo's proportional reasoning is a form of reasoning based on the principled manipulation of ratios and samenesses of ratios, according to the rules set forth in the fifth book of Euclid's *Elements*. As for Galileo's use of proportional reasoning in natural philosophy, a sizable literature is now available, which allows us a better understanding of most of its technical aspects.[11] All of these strategies are flexibly employed by Galileo throughout his career, but essentially they all serve the purpose of what I label "Galilean idealization". In what follows, I will show that Galilean idealization provides an alternative to the two horns of the dilemma presented by Galileo historiography. Before specifying my objectives in further detail, however, I need to clarify my terminology. By "Galilean idealization" I mean the process of abstraction by which phenomena observable in nature, or conceivable in mind, are gradually divested by Galileo of their perceptible properties (type of matter, weight, surface corrugation, for example), in order to construct idealized representations of phenomena, such as, for instance, falling bodies in the void; hence idealized phenomenon of fall in the void.

In this paper, I will study Galileo's life-long construction of the idealized phenomenon of fall in the void. Firstly, I will show that Galileo's quest was deeply rooted in his 1590s writings, especially *De motu*, a set of drafts and notes on motion and mechanics that were deeply influenced by Archimedes. In *De motu*, Galileo set out to investigate the "ratios of motions [*proportiones motuum*]" of bodies in fluid media, initially hoping that Archimedean hydrostatics would afford the strategy needed to disprove Aristotle's views on motion and the impossibility of the void, and eventually overcome the latter's physics of the "plenum". Secondly, I will offer a detailed reading of *De motu accelerato*, in connection with Galileo's ideas on the structure of the continuum. In contrast to received historiographical wisdom, I will argue that Galileo's 1604 hypothesis of the sameness of ratios of speeds and spaces is fully compatible with his previous analysis of accelerated motion in *De motu accelerato*. Thirdly, I will suggest that the paradox of heavy bodies and continuous motion in accelerated fall was the original question motivating Galileo's work with inclined planes, both theoretical and experimental. Thus the times-squared law might even have been a serendipitous by-product of thinking and measuring with inclined planes. Fourthly, I will show that cognitive autobiography, in the context of a culture still influenced by orally-shaped modes of thinking, affected Galileo's development of idealized fall in the void. As revealed by a hitherto little-studied document, the so-called *Postils to Rocco*,[12] cognitive autobiography made Galileo realize that weight cannot be the cause of a body's speed of fall, and that all bodies somehow become "weightless" while falling in the void. This was the breeze that finally allowed Galileo to sail beyond the doldrums of paradox. Most importantly, it emerges from my analysis that Galilean idealization is a progressive strategy, and that its final construct, the idealized phenomenon, is the result of successive transformations at different levels of idealization. Finally, I will suggest that experiments with pendula were carried out by Galileo in order to decide between two competing theories of a body's fall in the void, a restricted one *predicting equality of degrees of speeds for bodies of the same matter*, and a general one *predicting equality of degrees of speeds for bodies of all types of matter*. These experiments, which, in Galileo's view, sounded the death-knell for the restricted theory, sealed his life-long quest for the idealized phenomenon of fall in the void. His pilgrimage was rewarded with a vision of order and uniformity. Any body whatever falls with the same degrees of speed in the void, and the ratio of the successive degrees of speeds is the same as the ratio of the elapsed times. I will conclude the paper with a brief discussion of some responses to Galileo's law of fall by contemporary philosophers, which indirectly shed light on the significance of Galileo's achievement.

2. THE RESTRICTED THEORY: *DE PROPORTIONIBUS MOTUUM*

At one point in *De motu*, Galileo exclaims, "As almost everything he wrote about local motion, even about this question [i.e., the motion of projectiles] Aristotle wrote the opposite of what is true".[13] *De motu* is mostly about ratios of motion, a subject not really prominent in Aristotle's physics. However, Galileo's "reconstruction" of Aristotelian natural philosophy is a reflection of teachings and oral discussions that he certainly took part in when a student and later on a young professor at Pisa university. Thus in Galileo's references to problems raised by Aristotle we must not look for precise textual correspondences with our modern editions of Aristotle's texts. Late sixteenth-century Aristotelianism, as is well known, was the fruit of a long period of elaboration in the Latin Middle Ages and the Renaissance, which brought about a flurry of questions that have long been expunged from the genuine Aristotelian corpus by modern scholarship.[14]

Galileo claims that since natural motion depends on a mobile's gravity or levity, it is necessary first of all to be clear about what it means for a body to be heavier or lighter than another body. Thus he introduces the terminology of *gravitas in specie* [specific gravity]. A body is specifically heavier, or lighter, than another body if, their volumes being equal, the body is heavier or lighter than the other.[15] But all motions appear to occur in media, Galileo notes, such as water, air, or even fire (three of the traditional four elements). In what way? There was a commanding source on the behaviour of bodies in a fluid medium. Archimedes's treatise on floating bodies explains why bodies float or sink in water.[16]

The young Galileo took a profound interest in Archimedes. A few marginal postils to the latter's *On the sphere and the cylinder*, probably written in the late 1580s, strongly suggest that Galileo scrupulously studied this work. At about the same time, Galileo furnished a solution to the problem of Hiero's crown different from that commonly related by the Archimedean tradition.[17] Though published many years later in *Two new sciences*, Galileo's theorems on centres of gravity, whose Archimedean inspiration is all too evident, date from the mid-1580s.[18]

We first of all need to understand Galileo's initial pictorial representations of the idealized phenomenon of floating bodies. They depict solid magnitudes floating in water. These representations have antecedents in Archimedes. In Figure 1 we have diagrams from two Renaissance editions of Archimedes's *On floating bodies*. The diagrams on the left are from Niccolò Tartaglia's edition, those on the right from Federico Commandino's, two works that Galileo would have consulted.[19] The diagrams of the first row represent bodies immersed in water (left portion of the hemisphere) and volumes of water (right portion of the hemisphere). They are located on the surface of a spherical shell of water covering the Earth, whose centre coincides with the Earth's centre. Those of the second represent a body within water and an equal volume of water.

Like Archimedes, Galileo depicts bodies floating on the surface of a mass of water at rest. However, in the first draft of *De motu* (*cf.* the diagrams in Figure 2, left column), the water mass seems confined within a sort of circular sector. Most

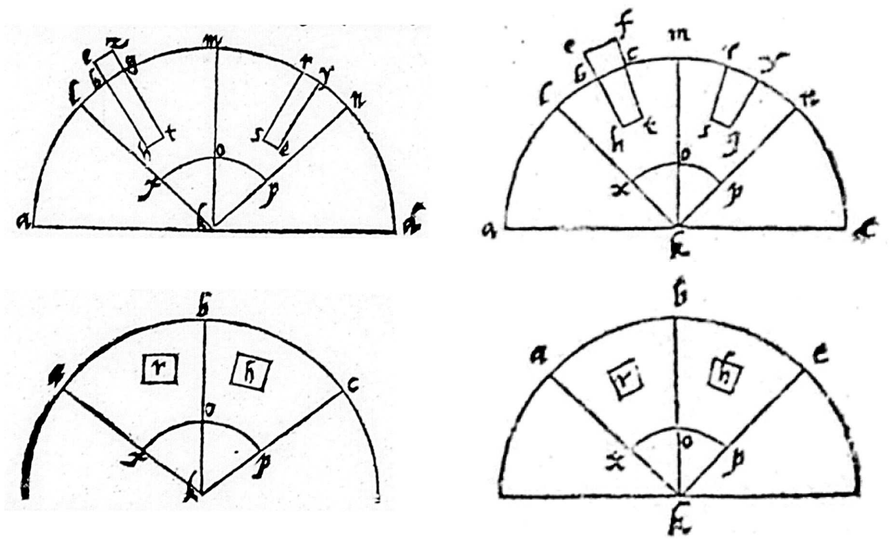

FIG. 1. (*left*) Diagrams from Niccolò Tartaglia's edition of Archimedes's *On floating bodies*: Archimedes, *Opera Archimedis Syracusani philosophi et mathematici ingeniosissimi* (ref. 19), 32 verso – 33 recto; (*right*), diagrams from Federico Commandino's edition of Archimedes: *De iis quae vehuntur in aqua libri duo, a Federico Commandino Urbinate in pristinum nitorem restituti et commentariis illustrati* (ref. 19), 2 verso – 3 verso.

importantly, Galileo always shows the raised level of water, level *g*, for instance, following immersion of the body *ab* (upper left diagram). In contrast to Archimedes's similar figures, the diagrams of *De motu* show bodies against the background of the raised level of water following immersion.

In subsequent drafts of *De motu*, we observe an advance in the construction of the phenomenon of idealized fall. Galileo abandons for good the spherical representation of water in favour of what (deceivingly) appear to be parallelopiped vessels (Figure 2, right column). He never claims that this is the case, in fact. He keeps referring to water as if it had the mass-like nature of one of the four traditional elements, regardless of its possibly being enclosed in a vessel. Although the lettering of the diagrams suggests that Galileo might have regarded them as a representation of bodies immersed in a vessel, he never specifies whether the space enclosed by the vertical and horizontal lines is supposed to represent a vessel. Indeed he does not even mention what those lines are supposed to represent. Galileo's language reflects this mass-like character of elemental water. For example, *abcd* (Figure 2, upper right diagram) is simply called the "state of water".[20]

Galileo abandons the spherical representation of water, which means that he has started thinking in terms of downward tendencies, or directions of motions, along parallel lines, no longer along lines converging to the centre of the Earth. So we might ask ourselves whether Galileo felt that he was justified in his abstracting from

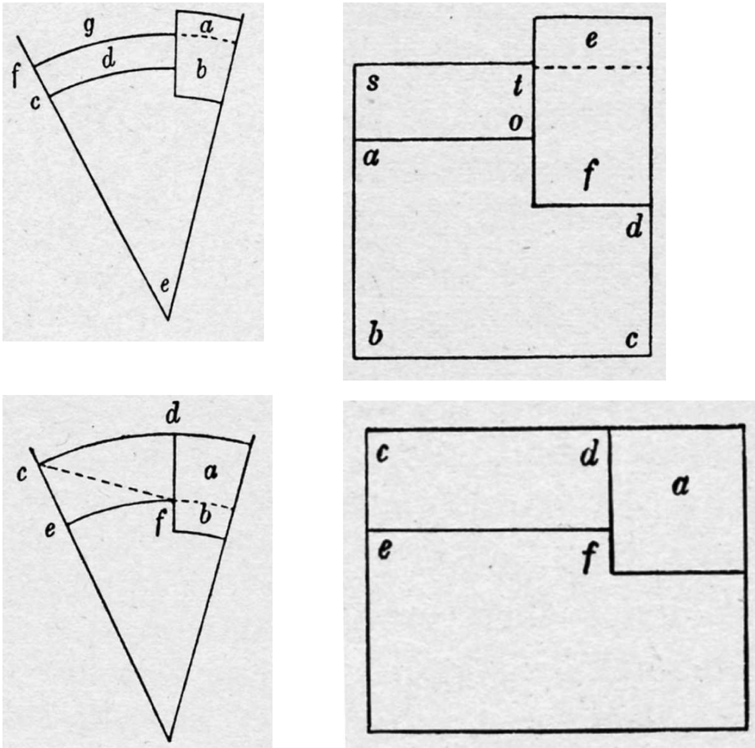

FIG. 2. (*left*) Diagrams from the first draft of *De motu* (Galilei 1890–1909, i, 381–4); (*right*) diagrams from the subsequent drafts of *De motu* (Galilei 1890–1909, i, 255–6).

reality. Galileo came close to an answer. In the final draft of *De motu*, after discussing the proportions of motions along inclined planes, he raises the objection that his demonstrations are based on the assumption that the lines of action of weights are parallel, while in reality they converge to the centre of the Earth. His answer is that Archimedes himself makes the same assumption in his work on the quadrature of the parabola. Thus we may assume that Galileo might have answered our question following a similar line of argument.[21]

Galileo's explanatory mechanism of buoyancy is basically a balance mechanism, the equality of the weights [*gravitates*] of the floating body and of a volume of water equal to the volume of the body's submerged portion. It was still inspired by Archimedes's analysis of buoyancy on the spherical shell of water. It is by means of a 'reductio' argument based on the equilibrium of portions of water that Archimedes proves that the surface of any water mass at rest is spherical, and that the centre of the spherical surface coincides with that of the Earth. Let us consider the diagram in Figure 3 (upper part). If the water surface were *abc*, instead of the spherical surface,

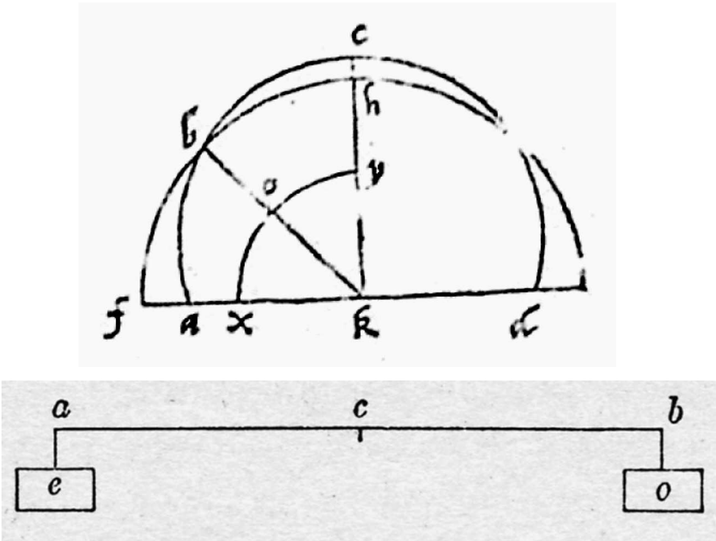

FIG. 3. (*above*) The equilibrium condition of the spherical shell of water in Archimedes, *De iis quae vehuntur in aqua libri duo, a Federico Commandino Urbinate in pristinum nitorem restituti et commentariis illustrati* (ref. 19), 2 recto; (*below*) the balance of equal arms in *De motu* (Galilei 1890–1909, i, 257).

fbh, with centre *k*, then different masses of water, *xabo* and *obcy*, would press differently the underlying layer of water, *xop* (*xoy*, in the figure, mistakenly). Thus the whole water could not be at rest. The diagram and the argument suggest that the equilibrium of the water masses could be modelled in analogy with a balance of equal arms. Let us now consider Galileo's representation of a balance of equal arms in Figure 3 (part below).

According to Galileo, three events might occur in relation to weight *e*. It might remain at rest, move upwards, or move downwards. If it is heavier than weight *o* it will move downwards; if it is less heavy than *o* it will move upwards, not because it does not have gravity, but because *o* is heavier. From this, Galileo argues, it is evident that in the balance both upward and downward motions originate from gravity, but in a different way. For *e*'s upward motion occurs because of *o*'s gravity, whereas *e*'s downward motion occurs because of its own gravity.[22]

The balance of equal arms perfectly models the motions of a body within fluid media. The body is represented by one weight. The other weight will represent a portion of the medium having the same volume as the body's volume. The explanatory mechanism of buoyancy is the same as the more general explanatory mechanism of motion and rest. The body's motion or rest will follow according as the body is heavier than, lighter than, or as heavy as a volume of the medium equal to the body's own volume.[23]

Once the Archimedean framework is in place, Galileo goes on to reject Aristotle's claim that speed of motion is caused by the subtility [*subtilitas*] of the medium while slowness [*tarditas*] of motion by the thickness [*crassities*] of the medium.[24]

By the following train of thought Galileo closes in on what will eventually become the "restricted theory" of falling bodies in the void. According to the restricted theory, all bodies of the same matter will fall with the same speed in the same medium. His starting point is a theory of free fall that he attributes to Aristotle. All bodies of the same kind fall with ratios of speed that are the same as the ratios of their bulk. Thus, a piece of lead twice as large as another piece of lead will fall twice as fast.[25]

A heavy body *b* moves along line *ce* (Figure 4). Let the line be divided at point *d*. If mobile *b* is divided in the same ratio as that according to which the line is divided by point *d*, then in the same time in which body *b* moves along the whole line, *ce*, its part will move along line *cd*.[26] According to this theory then, all bodies of the same kind, i.e., of the same matter, such as wood, or lead, will fall with ratios of speeds that are the same as the ratios of their magnitudes.

In contrast to Aristotle, Galileo now claims that mobiles of the same kind, although different in volume, will move with the same speed.

> Let us thus say that mobiles of the same kind [things of the same kind are said to be those mobiles that are made of the same matter, such as lead, wood, etc.], though different in volume, will move with same speed, so that a greater stone will not fall faster than a smaller one.[27]

In my view, the realization at the root of Galileo's entire process of idealization in the 1590s is nested in the following observational analogy. The reason why mobiles of the same kind, although different in volume, fall with the same speed is the same as

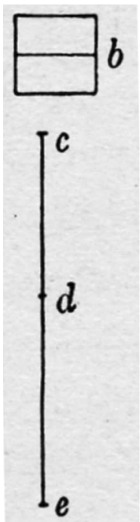

FIG. 4. Aristotle's view, according to Galileo: bodies of the same matter fall with ratios of speeds that are the same as the ratios of their magnitudes. Galilei 1890–1909, i, 263.

why both a chip of wood and a large wooden beam float.[28] After the passage where Galileo makes this astonishing claim, we find a sequence in which four distinct stages of thinking can be discerned. In the first two, Galileo begins by saying "... si mente concipieremus", i.e., if we conceive with mind, or in mind. Then, in the third, he proposes a mechanical analogy (but does not use this term). Finally, the fourth he calls an "argument".

First stage. Let us consider a wooden beam and a chip of the same wood floating on water. Then imagine the specific weight of water decreasing so that at one point water will become specifically lighter than wood. Who could claim, Galileo asks, that the beam will begin to descend before or faster than the chip? The reason why the beam's behaviour will be the same as the chip's is that while descending both will have to raise an amount of water equal to their volumes. Thus the volumes of water raised will have the same ratio as that of the chip's and the beam's volumes. In consequence, the ratio of the weight of the beam to the weight of its displaced volume of water is the same as that of the weight of the chip to the weight of its displaced volume of water. With the same ease, both the chip and the beam will overcome the resistance of the water that has to be displaced.[29]

Second stage. Consider a volume of wax floating on water (wax is specifically lighter than water). Now imagine mixing with the wax a modicum of material specifically heavier than water, such as sand, so that the mixed body will start descending in water, but very, very slowly. There is no reason, Galileo claims, why a chip of the mixed body will descend slower than the whole mixed body itself, or even not descend at all.[30] Here Galileo has, so to say, reversed the strategy of the first stage. Instead of imagining the specific weight of the medium varying, he imagines the specific weight of the body varying.

Third stage. "And the same one can experience in the balance...", exclaims Galileo. In his words,

> ... for if very large, equal weights are placed on each side, and then to one of them something heavy, but only modestly so, is added, the heavier will then go down, but not any more swiftly than if the weights had been small. And the same reasoning holds in water: for the beam corresponds to one of the weights of the balance, while the other weight is represented by an amount of water as great in size as the size of the beam: if this amount of water weighs the same as the beam, then the beam will not go down; if the beam is made slightly heavier in such a way that it goes down, it will not go down more swiftly than a small piece of the same wood, which weighed the same as an [equally] small part of the water, and then was made slightly heavier.[31]

Fourth stage. At this point Galileo claims that the entire reasoning can be confirmed by the following argument. He assumes that if one of two mobiles moves faster than the other, the composite of both will move faster than the slower body yet slower than the faster one. Then he goes on to state that mobiles of the same kind having different volumes will move with the same speed (Figure 5). The strategy is as follows.

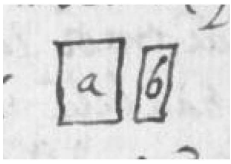

FIG. 5. Fourth stage. What happens when *a* and *b* join together? G. Galilei, *De motu*, folio 70 verso; Galilei 1890–1909, i, 265.

Let *a*, *b* be the two bodies, and let *a* be larger than *b*. If possible, let *a* move faster than *b*. The composite will move faster than *b* yet slower than *a*. But the composite is larger than *a*, therefore a larger body will move slower than a smaller body, which is awkward ["quod quidem est inconveniens"].[32] It is a 'reductio' argument, as the language employed by Galileo clearly suggests ("inconveniens" is virtually another way of saying "absurd"[33]).

Briefly, all bodies of the same specific gravity will fall at the same speed in the same medium. A volume of matter (lead, for example) in different media (air, water, etc.) will fall with ratios of speeds that are the same as the ratios of the differences between the weight of the volume and the weight of an equal volume of medium ["idem mobile in diversis mediis descendens, eam is suorum motuum celeritate, servare proportione..."]. By the same token, equal volumes of different matter (wood, lead, etc.) will fall in the same medium (air, for example) with ratios of speed that are the same as the ratios of the difference between the weights of the volumes and the weights of equal volumes of the medium. Here it must be stressed again that in Galileo's language "proportion" is to be interpreted as "sameness" of two ratios formed by two couples of magnitudes, or numbers.

The next level of the idealization process concerns the medium and the void. Is motion possible in the void? Galileo argues against the reasons given by Aristotle to prove the impossibility of the void. The strongest argument brought by Aristotle, Galileo claims, is to be found at *Physics*, IV, 215b (*cf.* Figure 6, which Galileo draws to illustrate Aristotle's argument).[34]

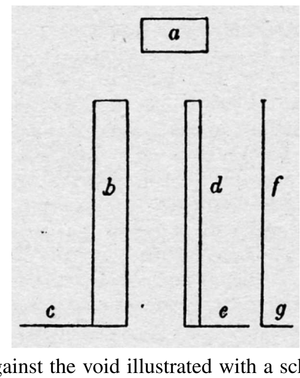

FIG. 6. Aristotle's argument against the void illustrated with a schema by Galileo (Galilei 1890–1909, i, 396).

> He [Aristotle] then argued as follows: Let mobile *a* cross medium *b* in time *c*; but let it cross a more subtle medium, namely *d*, in time *e*: it is manifest that, as the thickness of *b* is to the thickness of *d*, thus time *c* is to time *e*. Then let *f* be a void; and let mobile *a*, if that can happen, cross this *f*, not in an instant, but in time *g*; and let the thickness of medium *d* be to the thickness of another medium as time *e* is to time *g*. Now, from the things that have been established, mobile *a* will be moved through this medium that has just been found in time *g*, since medium *d* has the same ratio to this medium that has just been found as time *e* to time *g*; but, in this same time *g*, *a* is also moved through void *f*: hence *a*, in the same time, will be moved through two equal distances, one of which is a plenum, but the other a void; which assuredly is impossible. Therefore the mobile will not be moved through the void in time; therefore [the motion will take place] in an instant.[35]

To which Galileo, fortified by the conclusions already established, replies as follows:

> This is Aristotle's demonstration: to be sure it would have concluded very much to the point and from necessity, if Aristotle had demonstrated the things he took for granted, or, if they had not been demonstrated, if they had at least been true; but he has been deceived in this, since the things which he took as well known axioms, are not only not manifest to the senses, but have never been demonstrated, and are further not demonstrable, because they are totally false. For he has assumed that the motions of the same mobile in different media observe the same ratio to one another, in swiftness, as that which the subtilities of the media have: that this is assuredly false, has been abundantly demonstrated above.[36]

Thus Galileo sprightly jumps to the final level of idealization allowed by the restricted theory. He jumps from the shoulders of a mathematical giant, none other than Archimedes. He characterizes his own achievement with the following summary words:

> ... to put it briefly, my whole intent is the following: if there is a heavy thing *a*, whose proper and natural heaviness is 1000, in any plenum whatever its heaviness will be less than a thousand, and, consequently, the swiftness of its motion in any plenum whatever will be less than a thousand. And if we take a medium, such that the heaviness of a volume of it equal to the volume of *a* is only 1, the heaviness of *a* in this medium will be 999; thus its swiftness also will be 999: and the swiftness of this same *a* will only be a thousand in the medium where its heaviness is one thousand; and that will be nowhere but in the void.[37]

In sum, in the void all bodies of the same matter fall with the same speed, and the ratios of the speeds are the same as those of the specific weights. In a plenum bodies fall with speed ratios that are the same as the ratios of the differences between their specific weight and that of the medium. According to Galileo, however, these *proportiones motuum* are not confirmed by experience. Before expanding on this discrepancy, Galileo argues, the reason must be considered why natural motion downwards is slower at the beginning.[38] So, first of all, how is this progression from slowness to swiftness to be conceived? We find an answer in *De motu accelerato*.

3. PARADOX AND IDEALIZATION

In *De motu accelerato*, Galilean idealization begins with a powerfully simple question. When a stone initially at rest falls from a height and I realize that new increments of speeds are acquired, why should I not believe that these increments manifest themselves according to the simplest relationship?[39] The stone remains the same stone, the moving principle remains the same moving principle, why not all the other characteristics? Thus speed, too, will remain the same ... but no, not speed. Why, Galileo insists? Because increments of speed manifest themselves. Identity, uniformity, and simplicity, therefore, must be sought elsewhere, not in the stone's speed, but in the increments of speed, namely, in acceleration [*acceleratio*].[40] The simplest increment is that occurring successively while always remaining equal to itself. The mode of this special occurrence, Galileo claims, can be grasped by reflecting on the affinity between motion and time. Uniformity of motion is defined through equality of spaces and times. By the same token, uniformity of acceleration must be definable through the equality of speed increments and times. Thus, mind conceives uniformly accelerated motion when it grasps the uniform increments of speed occurring in equal particles of time [*particula temporis*].[41]

There is, however, an unfortunate consequence. Since no time short enough can be assigned such that a shorter interval cannot be conceived, then, after the beginning of motion downwards, no degree of speed will be so small that an even smaller degree of speed, or greater degree of slowness, cannot be conceived. But how might it be possible that a huge mobile is possessed of such a degree of slowness as not to traverse an inch of space in one hour, in one day, not even in one year?[42] "Quod profecto mirum, seu potius absurdum videtur ...", which seems paradoxical, or rather absurd, Galileo protests. This is the great paradox that Galileo will have to come to terms with over the next few decades.

For the time being, it is observational analogy, by which a new path towards idealization is opened, that makes this paradox appear less paradoxical, and the absurd consequence almost but evaporate.

Consider a big iron or lead weight placed on top of a pole. The pole will sink underneath the ground only up to a point. But if the weight strikes the pole after falling from on high it will make it sink deeper and deeper according as the height of fall increases. This additional compression can only be due to the impulse of percussion, which is ultimately caused by the weight's terminal speed of fall. Thus we may deduce a direct relation between the pole's sinking and its speed of fall. But who does not see that a fall from one or two inches will not make the pole sink sensibly? Then the weight may indeed acquire such insignificant degrees of speed as not to cause an appreciable sinking of the pole.[43]

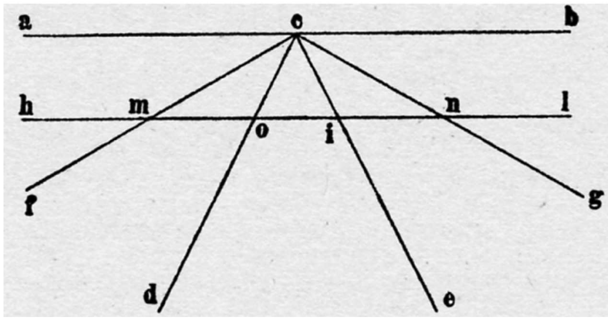

FIG. 7. Another example illustrates a possible solution to the paradox (Galilei 1890–1909, ii, 264).

Analogical thinking is of the essence here. Another way of explaining the paradox away analogically, an example [*exemplum*], suggests that Galileo is working within the framework of the restricted theory (Figure 7).

Imagine line *hl* uniformly descending according to the flowing of time. Lines *ab*, *fc*, *dc*, *ec*, and *gc* are fixed. Now imagine line *oi*'s length increasing as line *hl* descends. Imagine also line *mn* increasing as line *hl* descends. Surely no line *oi* is so small that at one previous instant another line *mn* cannot have been equal to it. Yet you can imagine angle *fcg* so obtuse that the rate of increase of line *mn* is so higher than that of line *oi* as to make you almost doubt that this may be possible. Note that Galileo's diagram almost conjures an optical illusion effect. For all lines, *oi*'s must be contained within lines *mn*'s![44] Still the argument is impeccable, geometrically. Thus if lines *oi* and *mn* represent degrees of speed it is not absurd that they may increase much quicker than each other yet both decrease till they evanesce at point *c*.[45] So a lead ball may descend much quicker than a wood ball, yet both of them transit through all the degrees of the evanescing speed.

Degree of speed [*gradus velocitatis*] and particle of time [*particula temporis*], as the reader will have noticed, are at the heart of the idealization process in *De motu accelerato*. Galilean idealization, as I have suggested, gradually divests phenomena of their perceptible properties. Galileo's descriptive language, accordingly, has to be read with respect for its nuances. With this interpretive sensitivity we may illuminate the fundamental question which, in my view, is at the root of the received assumption that *De motu accelerato* was written sometime between 1604 and 1630, after the discovery of the "correct" sameness of ratios of degrees of speed to time.[46] The question has to do with Galileo's views on samenesses of ratios and on the structure of the continuum. I will consider them in turn.

I believe that since in *De motu accelerato* a definition is furnished of accelerated motion in terms of increments of speed and particles of time, Galileo scholars have taken for granted that *De motu accelerato* was the logical outcome of the discovery of the sameness of ratios between speeds and times.[47] This belief has probably been framed by the retrospective illusion created by Galileo's recasting a portion of *De motu* *accelerato* in *Two new sciences*, almost verbatim, at the beginning of the so-called Third Day, where the mathematical treatment of accelerated motion is introduced.[48] In fact, neither *De motu accelerato* nor its recast in *Two new sciences* revolve around the samenesses of ratios. Why so?

First of all, no mention of sameness of ratios is to be found in either text. In both of them, Galileo says, "it is by no means contrary to reason, if we assume that the intension of speed occurs according to the extension of time [*a recta ratione absonum nequaquam esse videtur, si accipiamus, intensionem velocitatis fieri iuxta temporis extensionem*]".[49] Galileo's mature approach to *sameness of ratios* is that expounded by Euclid in the fifth book of the *Elements*.[50] It has nothing to do with the much later representation of ratios and equalities of ratios in algebraic symbolism. Further, and most importantly, it has nothing to do with our modern linear functions between variables.[51] It is based on the construction and comparison of all possible equal multiples of the four magnitudes forming the two ratios of a *sameness of ratios*. The Euclidean criterion for sameness of ratios is as follows. Two ratios are said to be the *same* when all possible equal multiples of the four magnitudes forming the two ratios are to one another in a certain relation.[52] More specifically, four given magnitudes

> are said to be in the same ratio, the first to second, and the third to the fourth, when the equimultiples of the first and the third, both alike equal, alike exceed, or alike fall short of, the equimultiples of the second and the fourth — whatever this multiplication may be — and those equimultiples are considered that correspond to one other.[53]

As is clear from this definition, two magnitudes are not enough to form a sameness of ratios. Four magnitudes are required. Neither the medievals, nor Galileo, for instance, would have thought in terms of a single speed proportional to a single space. This kind of thinking always involved at least four quantities (actually two pairs of homogeneous quantities), as the ratio of the first space to the second, so the ratio of the first speed to the second. In Section 5 we shall see an example of how Galileo applied the definition of sameness of ratios to weights (from the 1590s *De motu*).

The *De motu accelerato* and its partial recast in *Two new sciences* have been read misleadingly as a virtually identical prelude to the sameness of ratios between speeds and times. This has been an unfortunate effect of the modern habit of thinking samenesses of ratios in terms of linear functions of variables, so that the definition of uniformly accelerated motion in *De motu accelerato* has been equated with an implicit statement of the sameness of ratios between speeds and times. The fact is that it took Galileo many years and the 1604 detour in order to bring together in a coherent whole the Euclidean samenesses of ratios on uniformly accelerated motions developed during the Padua period and the earlier analysis of uniform acceleration of *De motu accelerato*.

In fact, in the document in which Galileo derives the times-squared law from the "erroneous" sameness of ratios between speeds and spaces, he does not start from the Euclidean definition of sameness of ratios, whose applicative complexity was

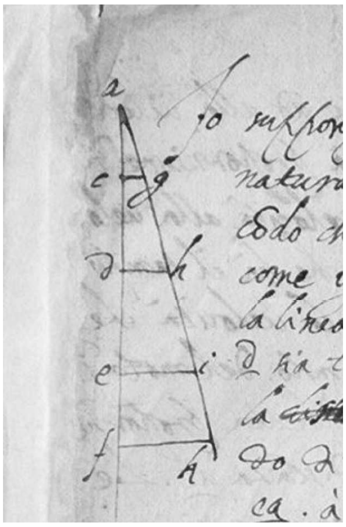

FIG. 8. The folio where Galileo derives the times-squared law from the "erroneous" principle communicated to Sarpi in 1604. *Cf.* Galilei 1600–38, folio 128 recto.

almost insurmountable, but from a geometrical diagram. He represents the spaces fallen through and the degrees of speed as the sides of right-angled triangles, which he knows from the sixth book of Euclid's *Elements* to be proportional magnitudes (Figure 8).

The underlying assumptions as to the composition of the continuum in *De motu accelerato*, though not explicitly stated, explain why in 1604 Galileo regarded the sameness of ratios between spaces and speeds as fully consistent with the earlier analysis of acceleration of *De motu accelerato*. The paradox Galileo sees in the passing of heavy bodies through "infinitely" small degrees of speed reveals that, in his view, the continuum is infinitely divisible. On the other hand, the language of *particles of time* [*particula temporis*] suggests that Galileo also conceives of the structure of the temporal continuum as "granular", that is, he thinks of time as being made up of particles. Can these two apparently discordant views on the composition of continua be reconciled? Surprisingly, they can.

In the *Postils to Rocco*, with which we shall be concerned later on, Galileo explicitly asserts that his apparently contradictory theses on the composition of continua are in fact both true. Continua are endlessly divisible yet structurally granular. They are composed of an infinite number of indivisibles which have no extension. Further, Galileo claims, to assert that they are endlessly divisible is exactly the same as to assert that they are made of an infinite number of indivisibles.[54] There is a structural affinity between the continua of space and time and all continua in general. Thus the regularity of uniform acceleration with respect to time may naturally be mathematized as a Euclidean sameness of ratios, by assuming the sameness of ratios between degrees of speed and spaces traversed on the vertical trajectory. Galileo did so by imagining a correspondence between the infinite points discernable in the vertical trajectory and the infinite *particula* discernable in the elapsed time. On this view of the continuum, endlessly divisible yet ultimately granular, there is no difference between "imagining in space" and "thinking in time". The "error" that Koyré attributed to an early modern tendency to spatialization, was no more than a direct consequence of Galileo's unorthodox views on the fine structure of continua.

Furthermore, we have another key document, a folio in *Manuscript 72*, possibly supporting my claim that Galileo's idealization process in the *De motu accelerato* preceded his attempt to *mathematize*, i.e., express in terms of Euclidean samenesses of ratios, his definition of uniform accelerated motion.[55] In this document, Galileo wishes to prove the following: "In motion from rest the moment of speed [*momentum velocitatis*] and the time of motion increase in the same ratio."[56] This statement is interesting in two ways. First of all, Galileo intends to prove a *sameness of ratios*. In the 1604 "erroneous" attempt to prove the times-squared law he *started* from the sameness of ratios between speeds and spaces but did not try to prove it. Secondly, the proof contains an assumption concerning the geometrical representation of time which can be illuminated by considering Galileo's views on the structure of continua. Galileo represents space and time of fall with a geometrical line (Figure 9). He supposes, for instance, that *ac* represents both the distance fallen through, from rest at *a*, and the time elapsed from *a* to *c*.

Since he knows that motion accelerates during fall he cannot represent the time elapsed while the body reaches the bottom, at point *b*, with the same distance *ab*. This prohibition, so to say, is *visually* conveyed by the diagram itself, not by a prejudgement concerning samenesses of ratios. Since, as can be seen in the manuscript diagram, the length of *ab* is virtually twice the length of *ac*, the time of fall from *a* to *b* cannot be twice the time from *a* to *c*. To represent the time from *a* to *b* Galileo chooses a shorter distance, *as*, equal to the geometrical mean between *ba* and *ac*. Now chronology becomes important. If the 1604 attempt preceeded this proof, then

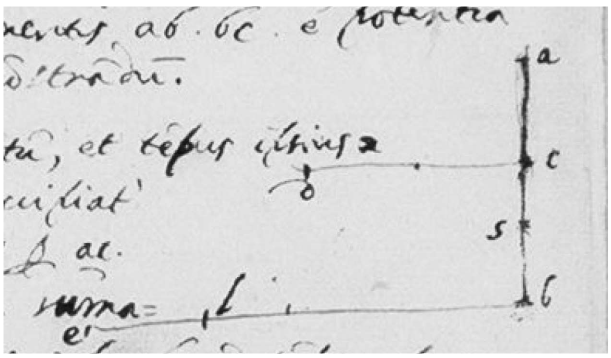

FIG. 9. Galileo represents space and time of fall from *a* to *c* with the same vertical distance, *ac* (Galilei 1600–38, folio 91 verso).

this proof may indeed have suggested to Galileo that the 1604 sameness of ratios assumed between degrees of speed and spaces traversed was incorrect.[57] If vice versa this proof predates the 1604 attempt, as I am inclined to think, then we may read it as a mathematization, in terms of samenesses of ratios, of the correspondence between the infinite points of the vertical trajectory and the infinite *particula* of the elapsed time. This mathematization now leads to the conclusion that a Euclidean sameness of ratios occurs between degrees of speed and times elapsed. On this interpretation, the subsequent episode of 1604 would simply be an inverse attempt at deciphering the correspondence between the degrees of speed and the points of the vertical trajectory, by assuming the direct correspondence of points and *particula* of time. Both strategies of mathematization must have been seen as viable ones, though only for a short time.

For we know that Galileo did not abandon the "wrong" sameness of ratios between speeds and spaces because he had discovered the "right" sameness of ratios between speeds and times, but because he had found a counterargument, in his eyes powerful and conclusive. In *Two new sciences*, Galileo has his mouthpiece, Salviati, admit that he [Galileo] had been of the erroneous opinion for some time, but then changed his mind when he found a counterargument, which is fully related in the subsequent paragraph.[58] This is also proved by a letter written to Galileo in 1611 by one of his former pupils in Padua, who says:

> I have thought about that proposition of yours: *A mobile acquiring speed according to the proportion of distances from the terminus from which it started will move in an instant*. And since this proposition more and more seems to me to be true and demonstrable, I have been thinking whether a motion such as this might be possible....[59]

The proposition attributed to Galileo in the letter is exactly the claim for the counterargument that Galileo himself has to offer in *Two new sciences*. The claim is that if motion were uniformly accelerated according to the sameness of ratios between speeds and spaces, then it would have to occur in an instant, which, Galileo argues, is absurd.[60] In addition, we note that in order to introduce the question of the relation between speeds and spaces, Galileo has Sagredo, the interlocutor who here voices Galileo's early opinion, say that the definition of uniformly accelerated motion given by Salviati-Galileo might have been stated *equally well, and perhaps more clearly*, "without changing the concept", in terms of the relation of speeds and spaces.[61] This confirms that Galileo's early opinion, though short-lived, was originally seen not *as alternative to*, but rather *as compatible with* the idealization in terms of degrees of speed uniformly accruing in time, furnished in the early *De motu accelerato*.

4. PARADOX AND EXPERIMENT

But how small is the degree of speed acquired after a very modest fall? Can it be measured? Or even only thought of? Is it really all that small, converging to nil according as the height of fall diminishes to zero?

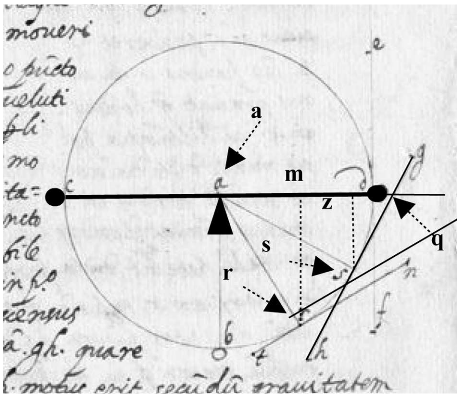

FIG. 10. Here I have emphasized the fundamental elements of Galileo's manuscript diagram accompanying the derivation of the inclined plane rule, the balance and two tangents at points *r*, *s*. From *Manuscript 71* (ref. 31), folio 95 recto.

Galileo found a way to measure that evanescing degree of speed on inclined planes. In effect one might say that he found a way to put a figure on the power of idealization. To understand how he measured evanescence we need to go back to the 1590s *De motu*, once again. There Galileo started out not by experimenting with inclined planes but by thinking with inclined planes. The question that Galileo poses is the same as that concerning vertically falling bodies: to find the proportions of motion [*proportiones motuum*]; in this case, those of the same mobile on differently inclined planes.[62]

Let us refer to Figure 10. Galileo resolved this puzzle with a diagram, the principle of the balance of equal arms, and the mathematical technique of proportional reasoning. Consider a balance of equal arms whose fulcrum is *a*, and *c*, *d* are two equal weights at the extremities of the arms. To know the proportions of motions along inclines we need to know how the slope decreases the weight of the body. If the balance starts rotating clockwise, weight *d* starts descending and its weight will be in the first point as that acting along vertical line *ef*. Thus at *s* and *r* the weight's incipient motion will be along the inclines *sh*, *rt*. The weight of the body at *d* is equal to that of *c* because the arms are equal and therefore the balance is in equilibrium. But at *s* and *r* the distances of the body from the fulcrum, *am*, *az*, respectively, are smaller than *ad*, and therefore the equilibrium will no longer be preserved, which, in effect, means that the body's gravity is decreased. And it is decreased according to the ratio of *am*, *az*, to *ad*, respectively. Thus both weight and speed of the body at points *s*, *r*, i.e., on the inclines *sg*, *tr*, are in the same ratio as distances *za*, *ma*. But as *qa* is to *az* so *qs* is to *sz*. In conclusion, weight and speed on an inclined plane are to weight and speed along the vertical elevation of the inclined plane as the length of the vertical elevation is to the length of the incline.

Along differently inclined planes the weights and speeds of a body are reciprocally

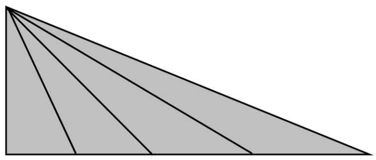

FIG. 11. On different inclines weights and speeds are to one another reciprocally as the lengths of the inclines.

as the lengths of the inclines. Therefore, on a inclined plane with fixed vertical elevation the longer the incline the slower the descending body according to the inverse ratio of the incline to the elevation. This theory, though still not distinguishing between 'speed', as a global quantity, and 'degree of speed', as a local quantity, would easily have guided Galileo in the search for a measure of the degree of speed once he had refined his thinking enough in terms of degrees of speed. What was needed was one step further. At some point, perhaps on the basis of experiments with pendula, Galileo convinced himself that the degree of speed acquired after a fall is always the same, providing that the height from which the fall begins remains the same.[63] This means that along any inclined plane whose elevation is constant, the degree of speed at the foot of the plane is the same regardless of the length of the incline, i.e., regardless of the inclination of the plane (Figure 11). The so-called "double distance" rule was the calculation technique that allowed Galileo to measure the degree of speed (Figure 12).

On folio 163 verso of *Manuscript 72*, Galileo gave a derivation of the double-distance rule. We need not concern ourselves with the details of the proof. All we need to know is what it states: in the absence of external impediments, after a fall along an incline a body will continue to move on the horizontal plane with constant degree of speed, covering a distance *double the length of the incline* in a time equal to the time of the descent along the inclined plane (Figures 12 and 13).[64] With this

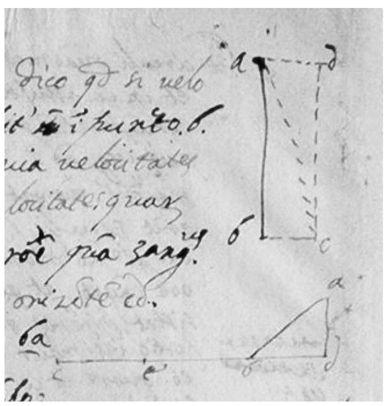

FIG. 12. Galileo's manuscript diagram accompanying the derivation of the *double distance rule* (Galilei 1600–38, folio 163 verso).

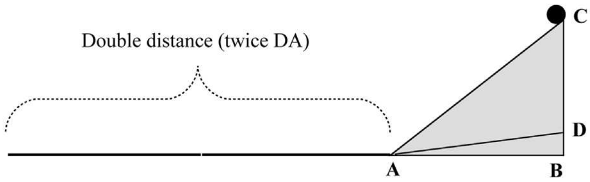

FIG. 13. A body passes through all the infinitely small degrees of speed.

rule Galileo could measure the degree of speed at the foot of an inclined plane. He presented this technique in the *Dialogue concerning the two chief world systems* (1632), where he published a solution to the paradox of the mobile passing through all the infinitely small degrees of speed (Figure 13).[65]

A body starting from rest at *D* and falling along the short vertical, *DB*, will acquire a degree of speed at *B* equal to the degree acquired by a body starting from rest at *D* and falling along incline *DA*. This degree is of course smaller than that acquired after falling from rest at *C*, regardless of the path, and no doubt, Galileo claims, one can imagine the degrees of speed becoming smaller and smaller as *D* approaches *B*. Therefore the body falling along the perpendicular *DB* may well start from a point so close to *B* that the degree of speed aquired, measurable by the time needed to cover the double distance (twice *DA*), is evanescently small (the time needed, in this case, becoming longer and longer).

Galileo's celebrated account of the inclined plane experiment, in *Two new sciences*, has puzzled historians in more ways than one, even though ever since Thomas Settle repeated the experiment according to Galileo's indications, there has seemed to be little doubt that the experiment was really carried out.[66] It has never been satisfactorily explained, however, why Galileo would have done experiments with such a modestly inclined plane as that described, if the only purpose of his investigations had been to measure the progression of space according to time. The traditional explanation has been that only with modestly inclined planes could Galileo have slowed down the phenomenon enough to take accurate measurements. Of course one might wonder why Galileo should have set out to take those measurements in the first place, given that he had no theoretical framework for the times-squared law. What he had was a framework of questions concerning degrees of speed. So, while not denying that the slowing down of the phenomenon may ultimately have been an important factor in practice, I wish to suggest that very modest inclines are exactly what was needed for Galileo to investigate the evanescent degrees of speed.

Here experimentation serves the purpose of idealization. The fact that in the course of those investigations with inclined planes Galileo also found the times-squared law may just have been a fortunate incident.

5. WEIGHTLESS BODIES: COGNITIVE AUTOBIOGRAPHY

Is weight a body's property independent of its volume? Or does it depend on volume? If this is the case, according to what relation? Remember that until the mid-1630s

Galileo thinks within the framework of the restricted theory of falling bodies in the void, in which ratios of speed are the same as the ratios of the specific weights. Already in the 1590s, however, Galilean idealization tends to progress further. If weight turned out to be related to volume then both weight and volume might be causal factors of fall. Weight might no longer be seen as the sole cause of fall.

In *De motu*, Galileo furnishes a proof that, for bodies of the same type of matter, the weights are to one another as the volumes. He does so by directly having recourse to Euclid's criterion of sameness of ratios (Figure 14).

Let *a* and *b* be two unequal volumes [*moles*]. Let *c* and *d* be their weights [*gravitates*]. Galileo sets out to prove that *c* has to *d* the same ratio that *a* has to *b*. In order to show that two weights of different volumes of bodies having the same specific weight are in the same ratio as their volumes, all that is needed is to construct equimultiples of the two weights and two volumes and prove that they satisfy a kind of "perpetual accord", as given by Euclid in the definition sameness of ratios. Let *efg* and *hk* be the multiples of volumes *a*, *b*, respectively. Let *nop* and *lm* be the multiples of weights *c* and *d*, respectively. Here, by way of example, Galileo has chosen to represent triple and double multiples. If, for any choice whatever of the multiples, when *efg* greater than / equal to / less than *hk* is true, it follows that *nop* greater than / equal to / less than *lm* is also true, then one has proved that the two weights are in the same ratio as their volumes, in the sense of Euclid.[67] No geometry, however, could lead Galileo to the general theory. Further idealization was required. The decisive step only came many years later.

In Galileo's *Postils to Rocco*, we find a fascinating autobiographical analysis concerning the train of thought that a few decades earlier led Galileo to the restricted theory. Not surprisingly, Galileo's *ex post facto* reconstruction tells an enriched story, not exactly tallying with what we recounted above on the basis of the 1590s *De motu*.

Galileo begins by claiming that it was reason, not experience, that initially persuaded him that all bodies of the same specific gravity fall at the same speed.

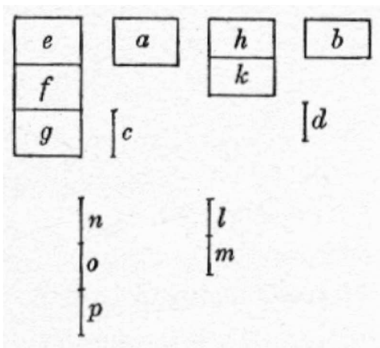

FIG. 14. The mathematization of weight according to equal multiple sameness of ratios (Galilei, 1890–1909, i, 349). Note that weights are represented by lines separately from magnitudes.

> ... I formed an axiom such that nobody could ever object to. I hypothesized that any heavy body whatever that descends has in its motion degrees of speed so limited by nature and fixed, that it would be impossible to alter them, by increasing or decreasing its speed, without using violence in order to speed up, or slow down, its natural course. I then figured in my mind two bodies equal in volume and weight, such as, for instance, two bricks, which depart from the same height at the same time. These will doubtless descend with the same speed assigned to them by nature. If this speed has to be increased by another mobile, it is necessary that this mobile move more swiftly. Yet, if one imagines the two bricks joining together while descending, which one will double the other's speed by adding impetus to it, given that speed cannot be increased by an arriving mobile if it does not move more swiftly?[68]

Galileo then asserts that "... from this first discourse I moved on to another, more convincing proof", which is more or less the same argument, already given in *De motu*, restricted to bodies of the same specific gravity. The only difference (highly significant, however) is that in the *Postils* Galileo keeps referring to "degrees of speed", a clear indication that he now knows very well that speed of fall increases according to a certain progression. In 1634 Galileo has of course long been in possession of the times-squared law.

In Galileo's view, it is one thing to talk of weight in relation to the effects on a balance, so that a brick put on top of another equal brick will double the latter's weight, but it is quite another to generalize this model to free-falling bricks. Weight, Galileo claims, is but to feel burdened.[69] Put your hand below a 100-pound cannon ball hanging by a rope, you will not feel burdened. But if the rope is severed then you will feel burdened when trying to keep the ball from descending. However, when you make your hand swiftly recede with the same speed below the free-falling ball, you will hardly feel burdened, because no resistance is opposed to the ball. Thus when two equal bricks join together upon each other, since they fall with the same speed neither of them will increase the weight of the other.[70]

Note that in 1634 Galileo has somehow moved away from the *De motu* image of two free-falling bricks *adjacent* to each other and joining together (*cf*. Figure 5). His thought focuses on the image of two free-falling bricks joining *upon* each other, clearly motivated by the analogy of the hand and the cannon ball. Thus for falling bodies Galileo is actually claiming that if brick 1 (W1) and brick 2 (W2) fall at the same speed, then W1 = W1 + W2 (that is, brick W2 cannot add weight to the other brick, W1, which is equally fast), and analogously, W2 = W2 + W1 (that is, brick W1 cannot add weight to the other brick, W2, which is equally fast). Therefore it follows that W1 = W2 = 0. In the final analysis, for the mature Galileo, a falling body must in some sense be regarded as weightless. A speck of dust and a cannon ball, light and heavy, it makes no difference. Weight is somehow "lost" during fall. No surprise that the cannon ball may indeed go through all the degrees of speed like a tiny speck of dust.

In a discussion intended to be added to future editions of *Two new sciences*, Galileo

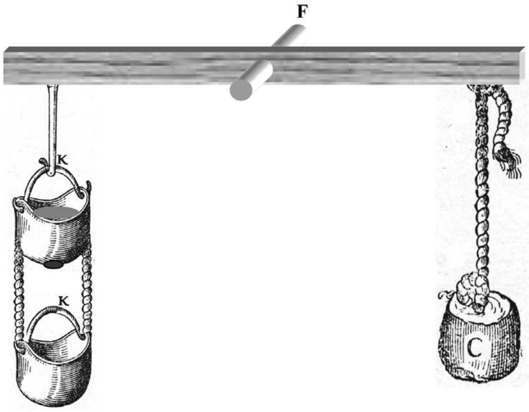

FIG. 15. My reconstruction of the balance excogitated by Galileo to test the force of percussion . A balance of equal arms turns around fulcrum *F*. Weight *C* counterbalances the total weight of the two buckets, *K*, and the water inside the upper bucket. What happens when the water is let fall from the upper bucket into the bucket below?

introduces again the idea of the *weightless* condition of falling bodies. He relates a fascinating experiment devised to test the force of percussion. Consider a wooden beam turning around fulcrum *F*. On the left two buckets hang from the beam. The upper one is filled with water, the one below is empty. On the right a weight exactly counterbalances the buckets and the water. Galileo's expectation was that once water was let flow from a hole in the upper bucket into the bucket below, the balance would turn anti-clockwise because of the added force due to percussion, the impact of water on the bucket below (Figure 15).

However, contrary to expectation, Galileo says, as soon as the hole was open and water began to flow downwards the balance turned the other way round, so that weight *C* started descending. Amazingly, he continues, no sooner had water impacted the bucket below than weight *C* stopped descending and inverted its motion, returning to the original equilibrium position, though very slowly. Since the column of descending water loses its weight while accelerating downwards, and since the balance goes back to equilibrium (after the transient initial dip), then the loss of weight must be compensated by the speed of the descending water. Thus the effect of percussion of the water impacting the bucket is due solely to the water's speed, and can be measured by the equivalent weight of the column of descending water (if it were at rest).[71]

Galileo has realized that weight might not be the cause of speed after all, if a body's speed of fall can only be increased by an arriving body already possessed of a greater speed. The Aristotelian relation between speed and weight, in his eyes, relies on the hidden (and fallacious) assumption that the joining together of the bricks descending equally fast must increase their weight, or rather the weight of the newly-joined body. The analogy of the hand below the cannon ball "rewrites" Galileo's memory creatively.

He relates processes occurring decades earlier but the new thinking of the two bricks upon each other takes the place of the old thinking of the two bricks adjacent to each other. Galileo lived in a culture still heavily influenced by orally-shaped modes of thinking. It comes as no surprise that his memory works differently from later modes of recollecting information verbatim associated with written texts.

We have another example of the intriguing phenomena associated with memory, recollection, and memory-rewriting in Galileo. In his traditional *Treatise on the sphere*, used as a basis for lectures at the University of Padua, we find a section entitled "That the Earth is immobile", in which Galileo suggests that he is closely following chap. 7 of Ptolemy's *Almagest* (where a rebuttal of the Earth's diurnal rotation is furnished). However, even though Galileo's text is presented as a quasi-paraphrase of Ptolemy's text, the series of arguments attributed to Ptolemy by Galileo does not match *Almagest*'s chap. 7. We find in Galileo's paraphrase traces of texts by Copernicus and Christoph Clavius. Galileo's assertion that "essendo il moto circolare e veloce accommodato non all' unione, ma più tosto alla divisione e dissipazione" is strongly reminiscent of Copernicus's assertion in *De revolutionibus* that "[q]uae vero repentina vertigine concitantur, videntur ad collectionem prorsum inepta, magisque unita dispergi". In his *Commentary on the sphere* Clavius presents an argument against the diurnal rotation of the Earth. If the Earth rotated around the axis of the world in twenty-four hours, "all edifices would be destroyed, and in no way could they remain firm". In Galileo's *Treatise* we find the same argument given by Clavius. Indeed the section "That the Earth is immobile" of Galileo's *Treatise* was (unconsciously?) modelled on Clavius's discussion. This conclusion is further supported by the list of arguments not matching *Almagest*'s chap. 7 that are summarized by Galileo, and which appear in Clavius's *Commentary*. In particular, the argument involving an arrow thrown upwards vertically, which would not fall back in the same place, and the image of a stone falling from the mast of a moving ship, are discussed by Clavius immediately following the catastrophic picture of collapsing buildings. Thus, Ptolemy's, Copernicus's, and Clavius's texts coalesced in Galileo's memory, forming a converging framework of ideas. He reorganized, so to speak, the intricate network of arguments and mental images related to *Almagest*'s chap. 7, which he found in relevant contemporary works.[72] Galileo's culture was still influenced by a style of approach to texts typical of oral cultures. Memorizing content rather than checking for the verbatim exactness of quotations was often a scholar's more urgent mode of interaction with books. As Walter Ong has masterfully taught us, oral cultures are aggregative rather than analytic.[73] Content is constantly re-shaped by oral memory. Memory-rewriting is just an effect that orality has on the workings of recollection.

What about bodies of different matters, such as wood, lead, cork, and metals? This became for Galileo the crux of the matter. "Spuntar lo scoglio più duro... [overcoming the hardest obstacle...]", Galileo tells us.[74] What, in his words, ultimately convinced him that all bodies in a vacuum fall with the same innate speed (always increasing as the square of time,[75] however) are no more than "conjectures", since, he candidly admits, experience might in this case turn out to be impossible.[76]

> My conjecture was founded on a certain effect that can be observed concerning speeds of mobiles of different gravity in fluid media. Speeds become more and more different according as the media become heavier and heavier. Only gold, the heaviest matter known to us, descends within quicksilver, in which all other metals float. It is clear that a mixed body of gold and silver can be made so that it will descend within mercury very slowly. Thus gold would sink one braccio within mercury in, say, one pulse beat, whereas the mixed body would take no less than 50 or 100 beats. If we let the bodies fall to the bottom of four braccia of water, pure gold will not precede the mixed body one-tenth of the time required of the latter. But in air, from a height of 100 braccia, no difference in time of fall would be discernible.[77]

Other examples by Galileo include a sphere made of beeswax mixed with lead, a marble sphere, a cork sphere, and a gold sphere. The line of reasoning is always the same. The medium is merely responsible for the divergence in the speeds of fall. There was, however, a serious difficulty in this argument. Galileo had known since the 1590s that the divergence of speeds could also be predicted by his restricted theory, on the assumption that *ratios* of speeds are taken "geometrically", i.e., according as *ratio* is treated in Euclid's theory of samenesses of ratios (*cf.* Table 1, for an example).[78]

In the early *De motu*, in order not to face the complications of forming a non-Archimedean ratio between the specific weight of a medium and that of the vacuum (equal to zero), Galileo had had recourse to the theory of "arithmetical ratio".[79] In this case, as Galileo explains, ratios are not defined directly, but a specific criterion of "sameness" for arithmetical ratios is given, according to which two ratios are the *same* ratio when the differences between the quantities forming the ratios are the same.[80] On the assumption that ratios are taken arithmetically, divergence of speed does not follow in the early theory (*cf.* Table 1, where the differences are in fact always equal to 20). The trick worked in *De motu*, but we know from early preparatory material on mechanics and motion, especially *Manuscript 72*, as well as the later *Two new sciences*, that Galileo soon realized that toying with arithmetical ratios was a blind

TABLE 1. Two bodies of equal volume, *A*, *B*, weighing 10 and 30, have speeds as 9, 8, 6, 2, and 29, 28, 26, 22, when falling in media (*M*) of increasing specific weights, such as 1, 2, 4, 8. Speeds diverge as the specific weight of the medium increases. According to Euclid's theory of samenesses of ratios, the ratio of 22 to 2 is much greater than the ratio of 29 to 9, but if ratios are considered arithmetically, the ratio of 22 to 2 is the same as the ratio of 29 to 9 (the differences between their quantities being equal).

	Speeds of Body *A* = 10	Speeds of Body *B* = 30
M = 1	9 (= 10 – 1)	29 (= 30 – 1)
M = 2	8 (= 10 – 2)	28 (= 30 – 2)
M = 4	6 (= 10 – 4)	26 (= 30 – 4)
M = 8	2 (= 10 – 8)	22 (= 30 – 8)

alley. Indeed he abandoned arithmetical ratios for good, as their complete disappearance from his subsequent writings makes clear.

At one point both theories of motion in the void, the restricted and the general, must have competed in Galileo's mind. Both theories predicted the divergence of speeds in media of increasing specific weight, on the assumption that ratios are considered geometrically according to Euclid's *Elements*. So, how did Galileo go about discriminating between the two theories?

6. THE GENERAL THEORY: PENDULA EXPERIMENTS

The question still remained open of whether the divergence of speeds in fluid media could be interpreted *inversely*, as either leading to exact equality of speeds in a vacuum for bodies of all types of matter, or simply leading to a convergence of speeds in a vacuum, still to be considered different, according to the restricted theory. The diagrams in Figure 16 should clarify what I mean by *inverse* interpretation of the phenomenon of divergence.

We know, Galileo asserts, that if we let two spheres of gold and cork (having the same volume) fall from a height of 100 braccia in air, the golden sphere will precede the cork one by, say, two or three braccia. However, this is due to resistance caused by the medium. For when the spheres are let fall from a height of one or two braccia the difference will disappear altogether. Thus if we could remove all the impediments caused by the medium the difference would disappear even when the spheres fall from a great height. That the difference must stem from the impediments of the medium, Galileo suggests, is indicated by the following considerations. If gold were faster solely in virtue of gravity then it would be reasonable to expect that once all the impediments due to the medium have been removed, the golden sphere's speed would exceed the cork sphere's speed with the same ratio as that of their gravities, even when the spheres fall from very small heights. Therefore, when these experiments are conducted from very small heights, so that all the impediments due to the medium are kept to a minimum, if we observe that the speeds of the two spheres tend to become equal in media of decreasing heaviness, so that even in a very light medium, such as air, differences of speeds all but disappear, then, Galileo claims, we

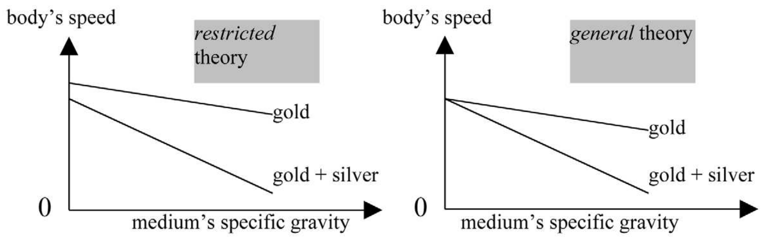

FIG. 16. Two possible inverse interpretations of the phenomenon of the divergence of speeds. In both cases speeds converge as the medium's specific gravity tends to zero (left areas of the diagrams), but in the restricted theory speeds remain different while in the general theory they become equal.

are entitled to hypothesize that in a vacuum their speeds would be identical.[81]

The argumentative strategy hinges on a theory of the resistance to motion caused by the fluid medium, which Galileo has expounded in the discussion leading up to this final point.[82] Briefly, he has argued that four forms of resistance are exerted by fluid media. First, the Archimedean thrust (let's call it R1), which simply makes the body lighter. Second, the resistance due to the viscous property of the medium (let's call it R2). Third, the resistance due to the body's speed (let's call it R3), so that the faster the body moves the greater the resistance opposed by the medium. Fourth, the resistance due to friction between a body's surface and the fluid (let's call it R4). The latter is caused by the sticking of the medium particles to the asperities of the body's surface.

It should now be clear why the experimental conditions proposed by Galileo reduce resistance to a minimum. Since he considers two equal spheres of different materials, both the Archimedean thrust (R1, due to volume) and surface friction (R4, due to surface) are the same for both spheres. Since speeds must be compared within the same medium, resistance caused by viscosity is cancelled in the comparison (R2 is the same for both spheres). Finally, and most importantly, from very small heights bodies do not acquire great speeds, so that resistance due to speed is almost negligible (R3 tends to become zero). However, as Galileo is well aware, this resistance cannot be eliminated altogether. Also we may note that, in comparing the resistance due to friction between the surfaces of the two spheres and the fluid, Galileo is assuming that the effect is the same regardless of the corrugation of the materials the spheres are made of. In sum, Galilean idealization here aims at abstracting from type of matter, weight, friction due to surface corrugation, and speed/resistance effects due to the medium.

If we accept his autobiographical account, this is the tortuous path that Galileo followed to satisfy himself that the theory of the equality of speeds for all bodies in the void holds true. Did Galileo really conduct the crucial experiments from very small heights that he mentions in his account? We do not know; I doubt it. For in *Two new sciences*, Galileo ambiguously says that the experience of two bodies largely different in weight falling from on high "is subject to some difficulty [*patisce qualche difficoltà*]".[83] Did he mean that he had tried, but unsuccesfully? Or that he never did?

Be that as it may, he goes on to explain how he designed and performed experiments with pendula, by which, he claims, he had found a way of accumulating the effects of falls from very small heights, so as to eliminate interference due to resistance yet be able to observe differences, if there were any. He thus convinced himself that the general theory holds true. Since he does not mention these pendula experiments in the *Postils* we have reason to believe that they might have been excogitated by Galileo in his very late years, precisely in preparation for the publication of *Two new sciences*. Here is Galileo's account in his own words. It is worth reading in its entirety since he adopts the autobiographical style, once again, and also because there is a revealing reference to the choice of modestly inclined planes.

> So I fell to thinking how one might many times repeat descents from small heights, and accumulate many of those minimal differences of time that might intervene between the arrival of the heavy body at the terminus and that of the light one, so that added together in this way they would make up a time not only observable, but easily observable. In order to make use of motions as slow as possible, in which resistance by the medium does less to alter the effect dependent upon simple heaviness, I also thought of making the movables descend along an inclined plane not much raised above the horizontal. On this, no less than along the vertical, one may observe what is done by heavy bodies differing in weight. Going further, I wanted to be free of any hindrance that might arise from contact of these movables with the said inclined plane. Ultimately, I took two balls, one of lead and one of cork, the former being at least a hundred times as heavy as the latter, and I attached them to equal thin strings four or five braccia long, tied high above. Removed from the vertical, these were set going at the same moment, and falling along the circumferences of the circles described by the equal strings that were the radii, they passed the vertical and returned by the same path. Repeating the goings and comings a good hundred times by themselves, they showed by sense perception that the heavy one kept time with the light one so well that not in a hundred oscillations, nor in a thousand, does it get ahead in time even by a moment, but the two travel with equal pace.[84]

Thus Galileo claims he has found a way to measure the possible accumulation of even tiny discrepancies of periods of oscillation between two bodies of a different type of matter. The two pendula keep perfect pace, i.e., they remain synchronous and no accumulation of time differences can be observed. The pendula experiments also showed Galileo the effects of air resistance. For while oscillating, the pendula part company. As Galileo further explains, the cork bob is slowed down more quickly than the lead bob, because of the greater net damping effect due to the difference between air resistance and the driving force of weight. Crucially, however, Galileo tells us that the slowing down does not alter the synchronism of the bobs (they keep crossing the lowest point at the same instant).

A tantalizing question arises, though. In 1994 David Hill argued that Galileo knew a great deal more about pendula than he was willing to publish. In particular, some manuscript folios have been interpreted by Hill as evidence of experiments and calculations performed by Galileo with pendula. Hill's challenging conclusions are that Galileo was well aware of the non-isochronic behaviour of pendula.[85] I believe that Hill's conclusions are widely exaggerated. His whole argument hinges on the crucial assumption that by calculating times of descent through rectilinear chords of a circle and extrapolating the results to the arcs of the circle, Galileo must have realized the non-isochronism of swinging bobs. There is no evidence that Galileo might have allowed such an arbitrary extrapolation. However, we might legitimately ask whether Galileo really *observed* the perfect synchronism of the cork and lead bobs alluded to in *Two new sciences*. What if the bobs do not keep perfect pace? Might Galileo have "sanitized" results from only partially successful experiments?

How about the role of the air resistance unequally slowing down the swinging bobs? Intriguing as these questions are, we must leave them open at the present time. Further investigations are much needed, ultimately, I think, by replicating the experiments Galileo claimed to have performed.

In sum, Galileo crowned his idealization process by directly observing, or perhaps partially constructing, or even only imagining the beguiling synchronism of swinging bobs. Be that as it may, he finally succeeded in cognizing the separation of the slowing-down effect of air resistance from the naturally constant rhythm of the two pendula. All bodies fall with the same uniformly accelerated speed in the void. The idealized phenomenon of fall in the void was achieved. The validity of the general theory was sealed.

7. CONCLUSION: A LOOK AT SOME RESPONSES TO GALILEO'S CLAIMS

Galileo had started out in *De motu* seeking ratios of speeds for bodies of different matters moving in different fluid media. He concluded five decades later, with the publication of *Two new sciences*, by realizing that all bodies fall with the same degrees of speed in the void. The initial search terminated at the ultimate level of idealization, namely, in the imagination of a void space populated by heavy bodies whose material constitution was no longer a factor in determining the rhythm of their fall. In such a space of undifferentiated falls no paradox undermines the belief, already infusing the early *De motu accelerato*, that the simplicity of the order of nature endows all bodies with the same uniform acceleration. On that level of abstraction Galileo founded the sequence of mathematical proofs which, in his eyes, would constitute the beginning of a "new science" of motion.

Most modern commentators have argued about that perplexing and celebrated passage in *Two new sciences*, where Galileo dismisses the search for a causal explanation of acceleration as not pertinent to his project.[86] Some have seen in that dismissal the sign of a nascent, modern scientific methodology unfettered by metaphysical concerns. I believe that it was simply the logical outcome of Galileo's life-long search for the idealized phenomenon of fall in the void. The most likely candidate for the cause of acceleration, gravity itself, became puzzling. Heavy bodies accelerate downwards. We feel burdened by weight yet a body's weight seems to disappear when the body is freely falling in the void. Bodies weigh differently yet fall in the void with the same degrees of speed. We might somehow measure the constant rhythm of speed increment. We can perceive the synchronicity of pendula unaffected by the damping effect of air resistance. But nothing in the perceptual properties of falling bodies, or swinging bobs, reveals the effect of gravity as a cause. Galilean idealization had, so to speak, overrun its course. What did Galileo's contemporaries think of his claims about falling bodies? Can some light be shed on Galilean idealization by looking at their reactions?

Regrettably we still know too little about the responses by seventeenth-century natural philosophers to Galileo's claims concerning falling bodies, and more

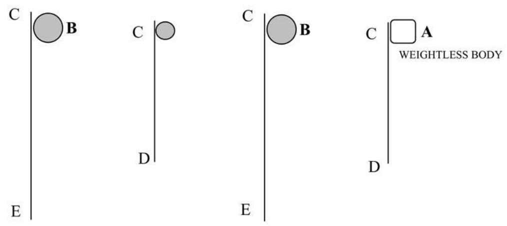

FIG. 17. Coresio's reconstruction of Aristotle's argument. To help the reader I have tentatively added the figures, which are not in Coresio's text.

specifically concerning the law of free fall. Recent research, however, has drawn attention to some of those responses, especially by Jesuit philosophers around the middle of the seventeenth century.[87] I will discuss some relevant points emerging from these investigations in a moment. First I wish to turn to a much earlier example of the Aristotelian reception of Galileo's mode of argumentation on falling bodies, which has so far not been studied, but which is a precious clue to the early impact of Galileo's claims.

I am referring to the case of a professor of Greek at the University of Pisa who in 1612 discussed an aspect of Galileo's early critique of Aristotle. The professor was Giorgio Coresio, who refers to Aristotle's *De caelo*, at 301 a–b.[88] Coresio criticized not Galileo's *De motu* — then still unpublished — but the view of another Pisa professor, Jacopo Mazzoni (1548–98). That view, Coresio claimed, had been shared by Galileo himself in the late 1580s. Coresio asserted that Mazzoni, in referring to *De caelo*, 301 a–b, had argued that experience contradicts Aristotle's contention according to which one could divide a heavy body as the ratio of line *CE* to line *CD*, so that if the whole body moved along line *CE* then the part would have to move along *CD* in the same time (Figure 17).

But, Coresio insists, to understand correctly this argument against the idea that a light body could possibly descend, we need to attribute to Aristotle's notion of 'heavy' in the context of *De caelo*, 301 a–b, the meaning of an idealized minimum of heaviness, i.e., of a heaviness that is less than any possible heaviness. If we do not interpret Aristotle's passage in this way, Coresio argues, then it is possible to derive a contradiction, which is this. Suppose that the chosen part of the heavy body is such that another part can be chosen which is less heavy. Now, since it has been concluded that both the part and the weightless move in the same time, then it follows that the weightless will have to move faster than any portion smaller than the part, and therefore that the non-heavy will move faster than the heavy, which is against the initial stipulation that the heavy moves faster than the non-heavy.[89] Coresio's implication seems to be that Aristotle's original argument (according to which, let us remember, one could divide a heavy body as the ratio of line *CE* to line *CD*, so that if the whole body moved along line *CE* then the part would move in the same time along *CD*) turns out to be correct if we assume that Aristotle intended a body whose heaviness is less than any possible heaviness. In this case the smallest possible speed corresponds to the smallest possible heaviness, and no light body will ever descend. Although his argument may at first sound confusing, Coresio seems implicitly willing to accept that there is a sense of 'heavy' that is both compatible with Aristotle's proportionality at *De caelo*, 301 a–b (regardless of what Coresio might take Aristotle to mean by 'proportionality'), and according to which all heavy bodies might in the limit be said to fall (or rather cease to fall) at the same rate, a proposition that would not violate Aristotle's fundamental dictum that no light body can descend.[90] It is clear that in Coresio's view, Aristotle's line of reasoning at *De caelo*, 301 a–b, must be understood under idealized conditions, if truly absurd consequences such as the descent of light bodies are not to follow.

I now wish to discuss the relevance of the present research to the debate on Galileo's point-atomism, the law of free fall, and idealization.[91] Let us first recall very briefly what Galileo's theory of point-atoms [*atomi non quanti*] consisted of. Galileo's theory appears to have been developed in connection with his solution to the so-called paradox of the Rota Aristotelis. The latter was one of the problems discussed in the pseudo-Aristotelian *Mechanical questions*. It basically reduces to the question "as to how it is that a greater circle when it revolves traces out a path of the same length as a smaller circle, if the two are concentric".[92] In *Two new sciences*, Galileo tackled the Rota Aristotelis by first considering two concentric hexagons and then reasoning that circles are but polygons of infinitely-many sides. He thus concluded that both the geometrical continuum and physical bodies are made up of infinite point-atoms and infinite point-vacua, or void interstices, interposed between the point-atoms. Galileo claimed that this solution allowed him to solve not only the paradox of the Rota Aristotelis, but also a number of natural-philosophical *vexatae quaestiones*, such as that of the condensation and rarefaction of physical bodies.[93] Two scholars, A. Mark Smith and Carla Rita Palmerino, have hypothesized a relationship between the physical-mathematical point-atomism put forward by Galileo in *Two new sciences* and his application of geometry to the study of the natural world. I will briefly review these contributions before pointing out how my analysis suggests a new direction of research in relation to the question raised by these two scholars.

In Mark Smith's view, Galileo's theory of point-atoms was perfectly compatible with Aristotelian metaphysics. According to Mark Smith, it was Galileo's reliance on Euclidean geometry that "permitted him to preserve, essentially intact, the Aristotelian view of the continuum", because geometry allowed Galileo to neutralize the "problem of possible material discontinuity within the traditional world plenum".[94] It had been none other than Aristotle, Mark Smith claims, who influenced the formation of Euclid's *Elements*, especially as far as their formal structure based on definition and deduction is concerned. But the *Elements*, Mark Smith continues, reflect Aristotle's view of the real world in terms of extension and process. Points, lines, and planes are all abstractions of reality so that the "world of Euclidean geometry can be regarded as a disembodied replica retaining bounds without the bounded". However, in Mark Smith's view, Aristotle did not share the conception of the Euclidean continuum. His conception of the continuum was instead based on his theory of prime matter, which represents the unvarying substratum of qualitative change. Mark Smith suggests that this Aristotelian view was passed on virtually intact to the Middle Ages and the Renaissance, even though challenges to Aristotle's ideas on the composition of the continuum were issued every now and then during the scholastic period. Galileo, like Aristotle, was fully certain that "the real world consisted in a material plenum", so that he clung to that certitude in spite of innumerable difficulties. Thus, Mark Smith claims that since Galileo treated the world as fully geometrical he needed to meet the "Euclidean criterion of absolute continuity", while accepting the basic Aristotelian position. But, Mark Smith concludes, since Galileo was able to avail himself of the current of mathematical thought brought to light by the Renaissance, he could attempt the supreme compromise of reconciling the Euclidean vision of the mathematical continuum with the possibility of discontinuity in natural philosophy. In Mark Smith's words, Galileo's "notion of growing temporal increments of speeds in accelerated motion may have led him to seek a valid compromise, demonstrable by means of Euclidean geometry, that would paradoxically permit of a substantial discontinuity". In sum, for Mark Smith, the mathematization of the law of free fall, based on the notion of *gradus velocitatis*, must have suggested to Galileo the possibility of developing a general theory of both the mathematical and the physical continua that was in fundamental agreement with the Aristotelian conception of prime matter as the substratum of all qualitative change.[95]

Along not very dissimilar lines, Carla Rita Palmerino has pointed out that in the seventeenth century the physical-mathematical atomism of *Two new sciences* could be seen (though not necessarily accepted) as an "infinitist foundation" of the mathematization of nature, and more specifically of the law of free fall. She has noted, especially in relation to Pierre Gassendi, who rejected Galileo's analysis of the Rota Aristotelis and offered an alternative theory, that both authors' solutions to that paradox constituted conceptual bridges between their theories of matter and their theories of falling bodies. However, for Palmerino, Galileo's point-atoms and Gassendi's extended atoms were totally incompatible. Thus, according to her, when Gassendi tried to incorporate Galileo's theory within his own atomistic philosophy he ran into a series of foundational problems that, vice versa, are not present in Galileo's theory.[96] Indeed, Palmerino argues, Galileo's point-atomism was intended to "establish a mathematical foundation for the theory of acceleration which he [i.e., Galileo] discusses in the second part of the work [i.e., *Two new sciences*], by demonstrating that space, time, and motion are composed of extensionless indivisibles".[97] Palmerino's basic argument is as follows. Galileo hypothesized that the passage from rest to motion cannot occur with a jump but must be the result of a continuous process of change. The falling body must pass through infinite degrees of speed. In response to the objections to this hypothesis raised by Salviati's interlocutors in *Two new sciences*, the latter argues that the falling body descends without remaining in any degree of speed for more than an instant. In the explanation of the Rota Aristotelis paradox, Galileo imagines that when the hexagons rotate, each of their sides stays fixed for a finite period of time, whereas he argues that "in circles the delays of the ends of their infinitely many sides are momentary, because an instant in a finite time is a point in a line that contains infinitely many [points]".[98] Thus both the rotating wheel and the falling stone pass infinitely many points without remaining there for more than one instant. It is this connection between the structure of matter, time, and motion that, in Palmerino's view, would constitute the conceptual bridge between Galileo's point-atomism and his mathematical theory of motion. The former would thus afford a justification of the legitimacy of the latter, and more generally of a mathematical natural philosophy.

In sum, while Mark Smith sees Galileo's point-atomism as grown out of the latter's concerns with the reconciliation of the Aristotelian physical continuum with the Euclidean mathematical continuum, Palmerino thinks that Galileo's solution to the Rota Aristotelis paradox was intended to lay the philosophical foundation of the mathematization of the laws of falling bodies.

In my opinion, both arguments are unconvincing. Mark Smith's analysis flies in the face of all Galileo's anti-Aristotelianism, and indeed of the lack of any evidence supporting the view that Galileo would have shared a conception of the physical continuum based on the Aristotelian notion of prime matter. As to Palmerino's analysis, it must be stressed that Galileo's point-atomism is expounded in the First Day of *Two new sciences*, where Galileo adopted the Italian vernacular instead of the Latin in which he chose to present the theory of motion (Third and Fourth Days). This suggests to me that he did not intend to offer a rigorous foundation for the theory of motion, which he would have prefaced in Latin to the Third Day of *Two new sciences*, but at most a free discussion of the problems connected with the paradox of the Rota Aristotelis. This interpretation would be consistent with Shea's conclusion that Galileo's interest in the constitution of matter was in fact derivative.[99]

I would argue that, around the middle of the seventeenth century, the interpretation of Galileo's mathematization of nature, specifically the law of free fall, was open to different possibilities. Indeed, it could be interpreted in almost opposite ways. According to Palmerino's analysis, Pierre Gassendi, for example, discussed Galileo's point-atomism as a possible foundation of the theory of falling bodies, though eventually rejecting the notion of point-atom.[100] On the other hand, Mark Smith has argued that Galileo's point-atomism may have been driven by Aristotelian preoccupations. I suggest that there was a third interpretative possibility, open to Galileo's contemporaries, much more concerned with the mathematical foundations of Galileo's claims about falling bodies. Indeed, as we shall see in a moment, we may discern its presence, for instance, in Evangelista Torricelli (1608–47) and Pierre Fermat (1601–65). We have seen that Galileo's idealization was to a large extent based on the possibility of applying Euclidean samenesses of ratios to natural philosophy. From a mathematical point of view, the technical problems connected with Galileo's mathematization mostly concerned the definition of equimultiple sameness of ratios. In a sense, we might argue that, since Galileo never articulated any foundational concerns with the applicability of samenesses of ratios to natural philosophy, his entire enterprise might have been interpreted either as mathematization of nature (and as such in need of some form of justification) or as naturalization of mathematics. In other words, it is not unreasonable to expect that while some of his contemporaries would have stressed the need for a foundational bridge between theory of motion and theory of matter, others might have looked at the possibility of a mathematization in which nature itself was regarded as essentially mathematical, and therefore in no need of further justification.

I will now turn to two examples, further research on which might substantiate my claim. The first regards Fermat's proof of an assertion concerning free fall that Galileo made in *Two new sciences*. The second regards Torricelli's re-formulation of Galileo's theory of accelerated motion, and his expunction from it of the equimultiple definition of sameness of ratios.

At the beginning of the Third Day of *Two new sciences*, Simplicio raises the question of natural acceleration by arguing that falling bodies descend with a speed which increases in relation to space ["a ragion dello spazio"]. Salviati answers claiming that such a view is as impossible and false as it is that "motion should be made instantaneously". Stillman Drake has discussed some aspects of the reception of the free fall law during the period 1632–49, especially amongst French mathematicians and natural philosophers.[101] One principal point at issue amongst those who became interested in Galileo's times-squared law was precisely his assertion on instantaneous motion left unproven in *Two new sciences*. It was, one might say, the perfect terrain on which different attitudes towards the mathematization of nature could become more explicit. In a sense, the hypothesis that falling bodies accelerate in relation to space instead of time lends itself to being seen in direct correlation with the theory of point-atoms. Indeed, if there is a connection between varying degrees of speed and points along the descent trajectory, then a theory of both the physical and geometrical continua might well constitute a foundation of the mathematization of motion. Yet Pierre Fermat furnished a proof of Galileo's assertion that was not motivated by the perception of a lack of a physico-geometrical foundation, but was entirely developed within the confines of a purely Archimedean approach. Writing to Gassendi, Fermat told him that instead of wasting time in long replies to the Jesuit opponents of the times-squared law, with whom Gassendi had entered into a controversy, it was necessary to find a rigorous proof of Galileo's assertion. In the letter to Gassendi, Fermat presented what he claimed to be a fully Archimedean demonstration. We need not dwell on the details of Fermat's long and complex proof.[102] Briefly, Fermat imagines a point uniformly accelerating along a line following the ratio of the traversed spaces. He then shows that if one considers a series of spaces in a continuous proportion along that line, then they will be traversed in the same interval of time. This part of the proof is the most complex because in its development Fermat follows an Archimedean method based on a double 'reductio ad absurdum'. He then goes on to prove Galileo's assertion by constructing another 'reductio ad absurdum'. In addition, the whole proof takes into account Galileo's results on uniform motion expounded in *Two new sciences*. To reiterate, in Fermat's view, what was most urgent was to find the mathematical proof of Galileo's assertion.

It thus seems that for Fermat Galileo's mathematization of nature might have been further developed along purely Archimedean lines, unconstrained by concerns with its physico-geometric foundations.

Evangelista Torricelli was the most talented of Galileo's pupils. In 1642, after Galileo's death, he became mathematician to the Grand Duke Ferdinand II. Torricelli was in Arcetri during Galileo's terminal illness and wrote down the tract on proportions which Galileo dictated to him. In 1644, he published the *Opera geometrica*, which amongst other purely mathematical works contains a tract entitled *De motu*.[103] Torricelli's *De motu* is basically a re-construction of the Third and Fourth Days of Galileo's *Two new sciences*, in which Torricelli furnishes new, more concise and elegant proofs based on the introduction of some geometrical properties of the parabolic line. The *De motu*'s complete title announced that in the treatise Torricelli would demonstrate the ingenuity of nature which plays with the parabolic line. This assertion should, in my view, be taken as a serious statement of Torricelli's programme intended to develop a mathematical science *de motu*, i.e., a mathematization of nature, in the sense indicated above of an extension of the language of Euclidean samenesses of ratios. Even though it is true that Torricelli introduces the parabolic line, and a few results concerning conic sections from Apollonius, his re-construction of Galileo's theory of naturally accelerated motion remains substantially faithful to Euclid's language of samenesses of ratios. However, a feature immediately strikes the reader of Torricelli's *De motu* who is also acquainted with Galileo's *Two new sciences*. The analysis of uniform motion based on equimultiple sameness of ratios, presented by Galileo at the beginning of the Third Day of *Two new sciences*, was replaced by Torricelli with a series of geometrical theorems aimed at demonstrating a postulate left unproven by Galileo.[104] Further, Torricelli proposed a 'reform' of Euclid's theory of samenesses of ratios, in which the equimultiple definition was abandoned for good.[105] Torricelli's attitudes towards the mathematization of nature became all too apparent in 1646, when he answered a letter from the French mathematician Gilles Personne de Roberval (1602–75). Roberval had objected in his letter to Torricelli that Galileo's law of fall was incorrect, except for very short heights and for very heavy objects. In Roberval's view, all falling bodies reached a final uniform speed so that Galileo's law appeared to contradict experience. Torricelli pointed out that Archimedes himself had hypothesized that projectiles moved in spiral lines and subsequently composed his tract on spirals. Afterwards, according to Torricelli, Archimedes realized that projectiles do not follow spiral lines. What, Torricelli asks, should Archimedes have done? Should he have abandoned his mathematical tract on spirals altogether? No, Torricelli answers, what was needed was simply to remove all references to physical projectiles and only consider generic points.[106] By the same token, Torricelli suggests to Roberval, we simply need to remove from Galileo's treatise on motion all references to physical objects and all the physical terms such as projectiles, so that in essence all that remains is pure geometry — a set of abstract propositions. The rest, Torricelli concludes, is fable.[107] Lanfranco Belloni, the editor of Italian edition of Torricelli's *De motu*, has argued that such comments by Torricelli are a blatant negation of the physical value of Galileo's mathematization of nature.[108] However, I wish to suggest that we need not interpret Torricelli's view as a lack of interest in the legitimate search for a foundation of the relationship between mathematics and physics. Such a search was by no means necessarily inscribed — so to speak — in Galileo's whole project of idealization. It was just one possibility way to interpret its meaning.

In conclusion, Galileo's point-atomism has been considered both as an attempt to reconcile the Aristotelian metaphysics of the physical continuum with the Euclidean continuum, and as an implicit foundation of Galileo's project on falling bodies. However, Torricelli's re-formulation of Galileo's *de motu* science, and perhaps Fermat's invitation to Gassendi not to waste time in physico-mathematical disputes, are two examples of a radically alternative interpretation of Galileo's idealized claims about falling bodies. They emphasize a *naturalization* of mathematics for which no metaphysical or meta-mathematical justifications seem required of the sort relished by modern historians and philosophers of science.

REFERENCES

1. In effect, the document in which Galileo derived the law from the "erroneous" principle communicated to Sarpi, though traditionally associated with the 1604 letter, cannot be dated with certainty. It was first published in *Le opere di Galileo Galilei*, ed. by Antonio Favaro (Edizione Nazionale, 20 vols, Florence, 1890–1909; cited below as Galilei 1890–1909, followed by the Roman numeral of the volume and the page numbers in Arabic numerals). *Cf.* Galilei 1890–1909, viii, 373–4.
2. Alexandre Koyré, *Études galiléennes* (Paris, 1966), 86ff., 96.
3. William Shea, *Galileo's intellectual revolution* (New York, 1972), pp. vii–viii.
4. Shea, *Galileo's intellectual revolution* (ref. 3), 3.
5. P. Damerow, G. Freudenthal, P. McLaughlin and J. Renn, *Exploring the limits of preclassical mechanics* (New York and Berlin, 1992; but *cf.* also the second edition, New York and Berlin, 2004, with a few relevant revisions, especially in chap. 3, pp. 135–41); Stillman Drake, "Galileo's 1604 fragment on falling bodies", *The British journal for the history of science*, iv (1969), 340–58, reprinted in Stillman Drake, *Essays on Galileo and the history and philosophy of science* (3 vols, Toronto, Buffalo and London, 1999; hereafter *Essays*), ii, 187–207; *idem*, "Uniform acceleration, space, and time", *The British journal for the history of science*, xvii (1970), 21–43, reprinted in Drake, *Essays*, ii, 208–32; *idem*, *Galileo studies: Personality, tradition and revolution* (Ann Arbor, 1970); *idem*, "The uniform motion equivalent to a uniformly accelerated motion from rest", *Isis*, lxiii (1972), 28–38, reprinted in Drake, *Essays*, ii, 233–47; *idem*, "Galileo's experimental confirmation of horizontal inertia: Unpublished manuscripts", *Isis*, lxiv (1973), 291–305, reprinted in Drake, *Essays*, ii, 147–59; *idem*, "Galileo's discovery of the law of free fall", *Scientific American*, ccxxviii (1973), 84–92, reprinted in Drake, *Essays*, ii, 248–64; *idem*, "Galileo's work on free fall in 1604", *Physis*, xvi (1974), 309–22, reprinted in Drake, *Essays*, ii, 281–91; *idem*, "Free fall from Albert of Saxony to Honoré Fabri", *Studies in history and philosophy of science*, v (1975), 347–66, reprinted in Drake, *Essays*, iii, 239–57; *idem*, "Galileo's accuracy in measuring horizontal projections", *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, x (1985), 3–14 , reprinted in Drake, *Essays*, ii, 321–31; *idem*, *History of free fall: Aristotle to Galileo* (Toronto, 1989); and *idem*, *Galileo at work: His scientific biography* (New York, 1995; 1st edn, Chicago, 1978).

6. *Cf.* Antonio Favaro, *Galileo Galilei e lo studio di Padova* (2 vols, Padua, 1966; 1st edn, Florence, 1883); *idem*, *Galileo Galilei a Padova* (Padua, 1968); Drake, *Galileo at work* (ref. 5); and the interesting essays in G. Santinello (ed.), *Galileo e la cultura padovana* (Padua, 1992), on various aspects of the Padua period. See also D. Hill, "Galileo's work on 116v: A new analysis", *Isis*, lxxvii (1986), 283–91; *idem*, "Dissecting trajectories: Galileo's early experiments on projectile motion and the law of fall", *Isis*, lxxix (1988), 646–68; *idem*, "Pendulums and planes: What Galileo didn't publish", *Nuncius*, ix (1994), 499–515; Ronald Naylor, "The evolution of an experiment: Guidobaldo del Monte and Galileo's *Discorsi* demonstration of the parabolic trajectory", *Physis*, xvi (1974), 323–46; *idem*, "Galileo and the problem of free fall", *The British journal for the history of science*, vii (1974), 105–34; *idem*, "Galileo's simple pendulum", *Physis*, xvi (1974), 23–46; *idem*, "Galileo: Real experiment and didactic demonstration", *Isis*, lxvii (1976), 398–419; *idem*, "Galileo: The search for the parabolic trajectory", *Annals of science*, xxxiii (1976), 153–72; *idem*, "Galileo's need for precision: The point of the fourth day pendulum experiment", *Isis*, lxviii (1977), 97–103; *idem*, "Galileo's theory of motion: Processes of conceptual change in the period 1604–1610", *Annals of science*, xxxiv (1977), 365–92; *idem*, "Mathematics and experiment in Galileo's new sciences", *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, iv (1979), 55–63; *idem*, "Galileo's theory of projectile motion", *Isis*, lxxi (1980), 550–70; *idem*, "Galileo's method of analysis and synthesis", *Isis*, lxxxi (1990), 695–707; J. Renn, "Galileo's manuscripts on mechanics: The project of an edition with full critical apparatus of MSS. GAL. Codex 72", *Nuncius*, iii (1988), 193–241; J. Renn, P. Damerow and S. Rieger, "Hunting the white elephant: When and how did Galileo discover the law of fall?", *Science in context*, xiii (2000), 299–419: Thomas Settle, "Galilean science: Essays in the mechanics and dynamics of the 'Discorsi'", Ph.D. Thesis, Cornell University, 1966, copy consulted on line at: http://www.mpiwg-berlin.mpg.de/litserv/diss/settle/html/Page001.htm; W. Wisan, "A new science of motion: A study of Galileo's *De motu locali*", *Archive for history of exact sciences*, xiii (1974), 103–306; and *idem*, "Galileo and the process of scientific creation", *Isis*, lxxv (1984), 269–286.
7. For the sake of brevity, I have quoted *Manuscript 72* with arbitrary dates as "Galilei 1600–38", followed by the arabic numeral of the folio. The manuscript is available on-line, at the ECHO, European Cultural Heritage Online, website: http://xserve02.mpiwg-berlin.mpg.de:18880/echo_nav/echo_pages/content/scientific_revolution/galileo. *Cf.* Antonio Favaro, "Documenti inediti per la storia dei manoscritti Galileiani nella Biblioteca Nazionale di Firenze", *Bullettino di bibliografia e di storia della scienze matematiche e fisiche*, xviii (1885), 1–112 and 151–230, for the intriguing story of the formation of the entire manuscript collection at Florence. Most of the content of *Manuscript 72*, however, has been known since it was first published in Galilei 1890–1909, viii, 363–448, *cf*. Paolo Galluzzi, *Momento: Studi galileiani* (Rome, 1979), 276–83. Ronald Naylor claims that in experiments aimed at studying the motion of projectiles Galileo found results conflicting with the sameness of ratios hypothesized in 1604; *cf*. his "Galileo's theory of projectile motion" (ref. 6), 562ff. Damerow *et al*. argue that this cannot be the case, and that the process of "discovery of the proportionality between the degree of velocity and time" was mostly a mathematical process; *cf.* Damerow, Freudenthal, McLaughlin and Renn, *Exploring the limits of preclassical mechanics* (ref. 5), 177ff. In W. Wisan's view, the discovery was based on a crucial experiment concerning projection, documented in Galilei 1600–38, folio 116 verso. See Wisan, "Galileo and the process of scientific creation" (ref. 6), 227–9. Hill, "Galileo's work on 116v" (ref. 6), rejects Wisan's reconstruction, and suggests that the experiment was tried to test a mathematical connection between degree of speed and the square root of the distance fallen through. All things considered I believe that these interpretations of single folios of *Manuscript 72*, though interesting, raise mostly unresolvable questions since they are based on reconstructions which are so underdetermined that a verdict cannot be reached. Finally, *cf.* E. Sylla, "Galileo and the Oxford calculatores: Analytical languages and the mean-speed theorem for accelerated motion", in W. Wallace (ed.), *Reinterpreting Galileo* (Washington, DC, 1986),

53–110, on some technical aspects of the mathematics underlying the 1604 reasoning, and P. Palmieri, "Galileo's mathematical natural philosophy", Ph.D. Thesis, London University, 2002, 43–45, for a criticism of Sylla's argument.

8. *Cf.* Galilei 1890–1909, ii, 261–6. The editor of the National Edition of Galileo's works, Antonio Favaro (Galilei 1890–1909, ii, 259–60), believed it to have been written about 1604, in connection with the 1604 letter to Sarpi. Koyré, *Études galiléennes* (ref. 2), 138, thought it to have been written in 1609, because in it Galileo claims that it took him a long time to come up with the "correct" definition. Wisan, "A new science of motion" (ref. 6), 277–8, argues that the *De motu accelerato* should be dated even later, to about 1630, on the basis of scant evidence concerning the quality of paper of the folios. Damerow, Freudenthal, McLaughlin and Renn, *Exploring the limits of preclassical mechanics* (ref. 5), 226, follow Wisan. As was already noted by Koyré, on the other hand, the nineteenth-century editor of a collection of Galileo's works, Eugenio Albèri, printed it among the Pisa period writings, *De motu antiquiora*, in G. Galilei, *Le opere di Galileo Galilei*, ed. by Eugenio Albèri (17 vols, Florence, 1842–56), xi. For the date and the internal chronology of *De motu antiquiora*, *cf.* I. Drabkin, "A note on Galileo's *De motu*", *Isis*, li (1960), 271–7; Raymond Fredette, "Les *De motu 'plus anciens'* de Galileo Galilei: Prolegomenes", Ph.D. Thesis, Montreal University, 1969, copy consulted on line at: http://www.mpiwg-berlin.mpg.de/litserv/diss/fred_1/ HTML/Page001.htm; Stillman Drake, "The evolution of *De motu*", *Isis*, lxvii (1976), 239–50, reprinted in Drake, *Essays* (ref. 5), i, 201–14; *idem*, "Galileo's pre-Paduan writings: Years, sources, motivations", *Studies in history and philsophy of science*, xvii (1986), 429–48, reprinted in Drake, *Essays* (ref. 5), i, 215–35; W. Hooper, "Galileo and the problem of motion", Ph.D. Thesis, Indiana University, 1992, 62–138; M. Camerota, *Gli scritti 'De motu antiquiora' di Galileo Galilei: Il Ms Gal 71* (Cagliari, 1992); and E. Giusti, "Elements for the relative chronology of Galilei's *De motu antiquiora*", *Nuncius*, xiii (1998), 427–60. The only exception to the early date is A. Carugo and A. C. Crombie, "The Jesuits and Galileo's ideas of science and nature", *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, viii (1983), 3–68, convincingly rejected, I believe, by Camerota in his *Gli scritti 'De motu antiquiora' di Galileo Galilei* cited above in this endnote.

9. P. Palmieri, "Mental models in Galileo's early mathematization of nature", *Studies in history and philosophy of science*, xxxiv (2003), 229–64, and Nenad Mišcevic, "Mental models and thought experiments", *International studies in the philosophy of science*, vi (1992), 215–16. Mental models are mental representations analogical with the states of affairs they are intended to represent.

10. *Cf.* Carmen Armijo, "Un nuevo rol para las definiciones", in J. Montesinos and C. Solís (eds), *Largo campo di filosofare: Eurosymposium Galileo 2001* (La Orotava, Tenerife, 2001), 85–99; Stillman Drake, "Velocity and Eudoxan proportion theory", *Physis*, xv (1973), 49–64, reprinted in Drake, *Essays* (ref. 5), ii, 265–80; *idem*, "Mathematics and discovery in Galileo's physics", *Historia mathematica*, i (1974), 129–50, reprinted in Drake, *Essays* (ref. 5), ii, 292–306; *idem*, "Euclid Book V from Eudoxus to Dedekind", *Cahiers d'histoire et de philosophie des sciences*, n.s., xxi (1987), 52–64, reprinted in Drake, *Essays* (ref. 5), iii, 61–75; and A. Frajese, *Galileo matematico* (Rome, 1964). For a general treatment of the various aspects of the Euclidean theory of samenesses of ratios, I have relied on: I. Grattan-Guinness, "Numbers, magnitudes, ratios, and proportions in Euclid's *Elements*: How did he handle them?", *Historia mathematica*, xxiii (1996), 355–75; C. Sasaki, "The acceptance of the theory of proportions in the sixteenth and seventeenth centuries", *Historia scientiarum*, xxix (1985), 83–116; K. Saito, "Compounded ratio in Euclid and Apollonius", *Historia scientiarum*, xxxi (1986), 25–59; and *idem*, "Duplicate ratio in Book VI of Euclid's *Elements*", *Historia scientiarum*, l (1993), 115–35. P. Rose, *The Italian renaissance of mathematics: Studies on humanists and mathematicians from Petrarch to Galileo* (Geneva, 1975), is an extensive, immensely erudite survey of Renaissance mathematics in Italy from a non-technical point of view. *Cf.* also E. Sylla, "Compounding ratios: Bradwardine, Oresme, and

the first edition of Newton's *Principia*", in E. Mendelsohn (ed.), *Transformation and tradition in the sciences: Essays in honor of I. Bernard Cohen* (Cambridge, 1984), 11–43.

11. E. Giusti, "Aspetti matematici della cinematica Galileiana", *Bollettino di storia delle scienze matematiche*, i (1981), 3–42; *idem*, "Ricerche galileiane: Il trattato 'De motu equabili' come modello della teoria delle proporzioni", *Bollettino di storia delle scienze matematiche*, vi (1986), 89–108; *idem*, "Galilei e le leggi del moto", in G. Galilei, *Discorsi e dimostrazioni matematiche intorno a due nuove scienze attinenti alla meccanica ed i movimenti locali*, ed. by Enrico Giusti (Turin, 1990), pp. ix–lx; *idem*, "La teoria galileiana delle proporzioni", in L. Conti (ed.), *La matematizzazione dell' universo: Momenti della cultura matematica tra '500 e '600* (Perugia, 1992), 207–22; *idem*, *Euclides reformatus: La teoria delle proporzioni nella scuola galileiana* (Turin, 1993); *idem*, "Il filosofo geometra: Matematica e filosofia naturale in Galileo", *Nuncius*, ix (1994), 485–98; *idem*, "Il ruolo dela matematica nella meccanica di Galileo", in A. Tenenti *et al*., *Galileo Galilei e la cultura veneziana* (Venice, 1995), 321–38; S. Maracchia, "Galileo e Archimede", in Montesinos and Solís (eds), *Largo campo di filosofare* (ref. 10), 119–30; F. Palladino, "La teoria delle proporzioni nel Seicento", *Nuncius*, vi (1991), 33–81; P. Palmieri, "The obscurity of the equimultiples: Clavius' and Galileo's foundational studies of Euclid's theory of proportions", *Archive for history of exact sciences*, lv (2001), 555–97; and *idem*, "Galileo's mathematical natural philosophy" (ref. 7).
12. The *Postils to Rocco* is a set of comments and notes that Galileo wrote in response to a book published in 1633 by an Aristotelian philosopher, Antonio Rocco, and attacking the *Dialogue concerning the two chief world systems* (published by Galileo the year before). The *Postils to Rocco* has been published in Galilei 1890–1909, vii, 569–750. On Rocco, *cf*. Antonio Favaro, "A. Rocco", *Atti del Reale Istituto Veneto di Scienze Lettere e Arti*, 7th series, iii (1892), 615–36.
13. Galilei 1890–1909, i, 307.
14. In general, on the possible influence on Galileo of Francesco Buonamici, a professor at Pisa in the 1590s, *cf*. Koyré, *Études galiléennes* (ref. 2), 24–47. M. O. Helbing has studied in detail Buonamici's natural philosophy and concluded that there seems to be some affinity between the two authors' philosophical attitudes towards mathematics; *cf*. his *La filosofia di Francesco Buonamici* (Pisa, 1989), 352–71. On Galileo and Girolamo Borri (1512–92), a professor at Pisa when Galileo was a student there, see Anna de Pace, "Galileo lettore di Girolamo Borri nel *De motu*", in Enrico Rambaldi (ed.), *De motu: Studi di storia del pensiero su Galileo, Hegel, Huygens e Gilbert* (Milan, 1990), 3–70. *Cf.* also M. Camerota and M. O. Helbing, "Galileo and Pisan Aristotelianism: Galileo's *De motu antiquiora* and the *Quaestiones de motu elementorum* of the Pisan professors", *Early science and medicine*, v (2000), 319–65. This paper "tries to show that some of the main topics discussed in *De motu antiquiora* (the famous experiments on falling bodies and the questions of acceleration and Archimedean extrusion) are *connected with the debate* [emphasis added] on the 'motion of the elements' (*motus elementorum*) which took place in the second half of the sixteenth century at the 'Studio Pisano' among the local professors of philosophy" (*ibid*., 323). However, to my mind, aside from emphasizing vague similarities between scant passages in the *De motu antiquiora* and texts by two Aristotelian professors at Pisa, the authors fail to explain what the 'connections' between Galileo and the debate on the motion of the elements would really have consisted in. Camerota and Helbing also claim that Galileo's view on *Archimedean extrusion* (another vague expression that the paper's authors never clarify) was influenced by Girolamo Borri, and exactly the opposite of Buonamici's. For a radically different analysis of Galileo's views on Archimedean buoyancy in *De motu antiquiora*, which were firmly grounded in Archimedes's mathematical treatment of floating bodies, *cf*. P. Palmieri, "The cognitive development of Galileo's theory of buoyancy", *Archive for history of exact sciences*, lix (2005), 189–222. A fundamental work is C. Lewis, *The Merton tradition and kinematics in late sixteenth and early seventeenth century Italy* (Padua, 1980), especially 127–70, with further bibliography. See also C. Schmitt, "The faculty of arts at Pisa at the time of Galileo",

Physis, xiv (1972), 243–72, "Towards a reassessment of Renaissance Aristotelianism", *History of science*, xi (1973), 159–93, "The University of Pisa in the Renaissance", *History of education*, iii (1974), 3–17, and *Aristotle and the Renaissance* (Cambridge, MA, 1983).

15. Galilei 1890–1909, i, 251.
16. Only Latin versions of *On floating bodies* had circulated in Western Europe until Heiberg's discovery of the Constantinople palimpsest, in the early twentieth century. See Archimedes, *Archimedis opera omnia*, ed. and transl. by J. L. Heiberg, ii (Leipzig, 1913).
17. *Cf.* Galilei 1890–1909, i, 215ff., and 379 (on the problem of Hiero's crown), and *ibid*., 233ff. (postils to *On the sphere and the cylinder*). *Cf.* also Shea, *Galileo's intellectual revolution* (ref. 3), 1–11, and C. Dollo, *Galileo Galilei e la cultura della tradizione* (Soveria Mannelli, Catanzaro, 2003), 63–86, in regard to the general influence of Archimedes on Galileo.
18. *Cf.* Galilei 1890–1909, i, 187ff., and G. Di Girolamo, "L' influenza archimedea nei 'Theoremata' di Galilei", *Physis*, xxxvi (1999), 21–54.
19. Archimedes, *Opera Archimedis Syracusani philosophi et mathematici ingeniosissimi* (Venice, 1543) and *De iis quae vehuntur in aqua libri duo, a Federico Commandino urbinate in pristinum nitorem restituti et commentariis illustrati* (Bologna, 1565).
20. "... sit aquae status, ante quam magnitudo in ipsam demittatur, *abcd*.... Necessarium itaque est, ut, dum magnitudo *f* demergitur, aqua attollatur." *Cf.* Galilei 1890–1909, i, 255–6.
21. *Cf.* Galilei 1890–1909, i, 300. Galileo used to teach a course at Padua on traditional astronomy, preparing notes for a *Trattato della sfera*, which is published in Galilei 1890–1909, ii, 211–55. Unfortunately he does not comment on the Earth's size.
22. Galilei 1890–1909, i, 258. It is worth noting that the diagram of the watery spheres was repeated by Galileo in the *Trattato della sfera* (ref. 21), 219–20. Here Galileo wished to prove that the surface of water is spherical.
23. "... ita ut, nempe, mobile naturale unius ponderis in lancem vicem gerat; tanta autem moles medii, quanta est mobilis moles, alterum in lance pondus repraesentet" (Galilei 1890–1909, i, 259).
24. Galilei 1890–1909, i, 260–2. Galileo explicitly refers to Aristotle's passage in *Physics*, IV, at 215b. *Cf.* Aristotle, *The complete works of Aristotle*, ed. by J. Barnes (2 vols, Princeton, 1984), i, 366.
25. "De illis mobilibus quae sunt eiusdem speciei dixit Aristoteles, illud velocius moveri quod maius est", Galilei 1890–1909, i, 262–3.
26. Galilei 1890–1909, i, 263. *Cf.* Aristotle's *De caelo*, at 301b. The passage is rather difficult. See, for example, a Renaissance edition in Latin, commented by Simplicius, in *In quatuor libros* De coelo *Aristotelis* (Venice, 1563), 206–7, and the modern English translations by W. K. C. Guthrie, in Aristotle, *De caelo* (Cambridge, MA, 1939), 277 (with the Greek original text on the facing page), and by Barnes, in Aristotle, *The complete works of Aristotle* (ref. 24), i, 494. *Cf.* also Palmieri, "Mental models in Galileo's early mathematization of nature" (ref. 9), 258ff., for a discussion of an interesting interpretation of this passage by a defender of Aristotle, a contemporary of Galileo and professor of Greek at Pisa University.
27. Galilei 1890–1909, i, 263.
28. It is worth quoting the original passage of this startling analogy. "Qua conclusione [i.e., that all mobiles of the same kind fall at the same speed] qui mirantur, mirabuntur etiam, tam maximam trabem quam parvum lignum aquae supernatare posse: eadem enim est ratio." Galilei 1890–1909, i, 263–4.
29. Galilei 1890–1909, i, 264. Galileo also specifies that it must be proven that volumes of the same matter have the same ratio as their weights. He proves this claim further on in *De motu*, *cf.* Galilei 1890–1909, i, 348–50.
30. Galilei 1890–1909, i, 264.
31. G. Galilei, *De motu*, trans. by R. Fredette (ECHO, European Cultural Heritage Online website, at http://xserve02.mpiwg-berlin.mpg.de:18880/echo_nav/echo_pages/content/scientific_revolution/

galileo), 17. On this website, there are available both the translation and images of the manuscript folios, the so-called *Manuscript 71*. *Cf.* the original, in Galilei 1890–1909, i, 264.

32. Galilei 1890–1909, i, 264.
33. This can be gathered by looking at the Latin versions of commentaries on Aristotle circulating in the Renaissance. *Cf.*, for example, Simplicius's commentary on *De caelo*, 301b, in Simplicius, *In quatuor libros* De coelo *Aristotelis* (ref. 26), 207, in which he asserts that Aristotle's demonstration [*ostensio*] is "by reductio ad absurdum [*per deductionem ad impossibilem*]", and then concludes his paraphrase of Aristotle's text with the same formula used by Galileo, "quod est inconveniens".
34. Aristotle, *The complete works of Aristotle* (ref. 24), i, 366.
35. Galilei, *De motu* (ref. 31), 28. *Cf.* the original in Galilei 1890–1909, i, 277.
36. Galilei, *De motu* (ref. 31), 28–29, with a few changes. Original in Galilei 1890–1909, i, 277–8.
37. Galilei, *De motu* (ref. 31), 33, with a few changes. Original in Galilei 1890–1909, i, 282.
38. Galilei 1890–1909, i, 273.
39. Galilei 1890–1909, ii, 262.
40. *Ibid*.
41. *Ibid.*
42. Galilei 1890–1909, ii, 263.
43. *Ibid*.
44. If you have difficulty, as I had at first, in focusing on the diagram because of the disturbing effects of optical illusionism, imagine drawing vertical lines passing through *o* and *i*.
45. Galilei 1890–1909, ii, 265.
46. In effect the origin of the tradition to date *De motu accelerato* to 1604 or later may have been Antonio Favaro's decision to publish the short writing in the second volume of Galilei 1890–1909 (the so-called National Edition), according to his conviction that the piece should have been written in connection with the letter to Sarpi (Galilei 1890–1909, ii, 259–60). No other reason, however, was brought by Favaro to support his decision, though he knew that Albèri had published it together with the *De motu* manuscript material, with which it is still bound today.
47. The *De motu accelerato* ends with the following definition of uniformly accelerated motion: "Motum uniformiter, seu aequabiliter, acceleratum dico illum, cuius momenta, seu gradus, celeritatis a discessu ex quiete augentur iuxta ipsiusmet temporis incrementum a primo instanti lationis" (Galilei 1890–1909, ii, 266). In *Two new sciences* Galileo rephrased this definition, as follows: "Motum aequabiliter, seu uniformiter, acceleratum disco illum, qui, a quiete recedens, temporibus aequalibus aequalia celeritatis momenta sibi superaddit" (Galilei 1890–1909, viii, 198).
48. Galilei 1890–1909, viii, 197–8.
49. Galilei 1890–1909, ii, 263 (*De motu accelerato* version), and *ibid*., viii, 198 (*Two new sciences* version, with only a slight grammatical difference due to sentence construction).
50. Euclid, *The thirteen books of the Elements*, translated with commentary by Thomas Heath (2nd edn, 3 vols, New York, 1956), ii, 112–86. See Armijo, "Un nuevo rol para las definiciones" (ref. 10); Drake, "Velocity and Eudoxan proportion theory" (ref. 10); Drake, "Mathematics and discovery in Galileo's physics" (ref. 10); Drake, "Euclid Book V from Eudoxus to Dedekind" (ref. 10); Frajese, *Galileo matematico* (ref. 10); Giusti, "Aspetti matematici della cinematica Galileiana" (ref. 11); Giusti, "Ricerche galileiane: Il trattato 'De motu equabili' come modello della teoria delle proporzioni" (ref. 11); Giusti, "Galilei e le leggi del moto" (ref. 11); Giusti, "La teoria galileiana delle proporzioni" (ref. 11); Giusti, *Euclides reformatus* (ref. 11); Giusti, "Il filosofo geometra" (ref. 11); and Giusti, "Il ruolo dela matematica nella meccanica di Galileo" (ref. 11).
51. On the non-algebraic character of Galileo's approach to sameness of ratios, *cf.* Drake, "Velocity and Eudoxan proportion theory" (ref. 10); Drake, "Mathematics and discovery in Galileo's physics"

(ref. 10); Drake, "Euclid Book V from Eudoxus to Dedekind" (ref. 10); Palmieri, "The obscurity of the equimultiples" (ref. 10); Palmieri, "Galileo's mathematical natural philosophy" (ref. 7); and Palmieri, "Mental models in Galileo's early mathematization of nature" (ref. 10).

52. *Cf.* the details of the definition of "sameness" of ratios, in Giusti, "Aspetti matematici della cinematica Galileiana" (ref. 11); Giusti, "Ricerche galileiane" (ref. 11); *idem*, "Galilei e le leggi del moto" (ref. 11); *idem*, "La teoria galileiana delle proporzioni" (ref. 11); *idem*, *Euclides reformatus* (ref. 11); and Palmieri, "The obscurity of the equimultiples" (ref. 10). Euclid's definition of "sameness" of ratios is complex. Galileo considered it to be obscure, and tried to replace it with a new one, in the very final months of his life. On Galileo and the equal multiples definition, *cf.* especially Giusti, *Euclides reformatus* (ref. 11), 57–82, and Palmieri, "The obscurity of the equimultiples" (ref. 11).

53. I have translated this definition from a common Renaissance edition of Euclid's *Elements*, that edited and commented upon by Christoph Clavius. The original is as follows: "In eadem ratione magnitudines dicuntur esse, prima ad secundam, et tertiam ad quartam, cum primae et tertiae aeque multiplicia, a secundae et quartae aeque multiplicibus, qualiscunque sit haec multiplicatio, utrumque ab utroque vel una deficiunt, vel una equalia sunt, vel una excedunt; si ea sumantur quae inter se respondent." Christoph Clavius, *Commentaria in Euclidis Elementa geometrica* (Hildesheim, 1999; facsimile edn of the first volume of *Christophori Clauii Bambergensis e Societate Iesu Opera mathematica V tomis distributa* (Mainz, 1611–12)), 209.

54. "Io ... stimo vera l' una e l' altra proposizione: essendo certo che il continuo costa di parti sempre divisibili, dico che è verissimo e necessario che la linea sia composta di punti, e il continuo d' indivisibili; e cosa forse più inopinata vi aggiungo, cioè che, essendo il vero uno solo, conviene che il dire che il continuo costa di parti sempre divisibili, col dire che il continuo costa d' indivisibili, siano una medesima cosa..." (Galilei 1890–1909, vii, 745). In addition, we know that Galileo had planned to write a tract on the composition of the continuum well before 1610 (Galilei 1890–1909, x, 352). All studies of Galileo's views concerning the continuum that I am aware of, traditionally based on *Two new sciences*, miss the fundamental unorthodoxy of Galileo with respect to the then long-standing dichotomy between endless divisibility and indivisibility. In his last work, Galileo's discussion of atomism and the continuum meanders through paradoxes and questions that tend to hide his conviction of the equivalence between infinite indivisibles and endless divisibility, explicitly stated in his postils to Rocco. *Cf.* K. Lasswitz, *Geschichte der Atomistik* (2 vols, Hamburg and Leipzig, 1890), ii, 37–54; E. J. Dijksterhuis, *The mechanization of the world picture* (London, 1961), 419–24; W. Shea, "Galileo's atomic hypothesis", *Ambix*, xviii (1970), 13–27; A. Mark Smith, "Galileo's theory of indivisibles: Revolution or compromise?", *Journal of the history of ideas*, xxxvii (1976), 571–88; S. Quan, "Galileo and the problem of infinity: A refutation and a solution. Part 1: The geometrical demonstrations", *Annals of science*, xxvi (1970), 115–51, and "Part 2: The dialectical arguments, and the solution", *Annals of science*, xxviii (1972), 237–84; P. Redondi, *Galileo eretico* (Turin, 1988), 11–31; *idem*, "I problemi dell' atomismo", in Montesinos and Solís (eds), *Largo campo di filosofare* (ref. 9), 661–76; C. R. Palmerino, "Una nuova scienza della materia per la scienza nova del moto: La discussione dei paradossi dell'infinito nella prima giornata dei Discorsi galileiani", in R. Gatto and E. Festa (eds), *Atomismo e continuo nel XVII secolo* (Naples, 2000), 275–319; and *idem*, "Galileo's and Gassendi's solutions to the rota Aristotelis paradox: A bridge between matter and motion theories", in J. E. Murdoch, W. R. Newman and C. H. Lüthy (eds), *Medieval and early modern corpuscular matter theories* (Leiden, 2001), 381–422.

55. Unfortunately, this folio of *Manuscript 72* (Galilei 1600–38, viii, folio 91 verso), much as the folio containing the "erroneous" proof of the times-squared law, cannot be dated with certainty. Damerow, Freudenthal, McLaughlin and Renn, *Exploring the limits of preclassical mechanics* (ref. 5), 178ff., interpret this document as a proof of the correct sameness of ratios between speeds and times, and thus assume that it was written after the 1604 erroneous attempt (*ibid.*,

181). Their argument, however, is based on the questionable assumption that truth must have followed error. Date is not a problem for my purposes here, on the other hand, since all I wish to show is that more or less at the same time as the 1604 "error", Galileo was trying to express his early analysis of *De motu accelerato* in terms of samenesses of ratios.

56. "In motu ex quiete eadem ratione intenditur velocitatis momentum, et tempus ipsius motus" (Galilei 1600–38, viii, folio 91 verso). On the meanings of "momento" in Galileo, *cf*. Galluzzi, *Momento* (ref. 7).
57. Damerow, Freudenthal, McLaughlin and Renn, *Exploring the limits of preclassical mechanics* (ref. 5), 181, claim that this result amounts to a contradiction because Galileo had previously used the sameness of ratios between speeds and spaces.
58. Galilei 1890–1909, viii, 203–4.
59. Galilei 1890–1909, xi, 85.
60. Galilei 1890–1909, viii, 203–4. The details of the counterargument are irrelevant for our purposes. *Cf.* Drake, *History of free fall* (ref. 5), 74–75.
61. Galilei 1890–1909, viii, 203.
62. Galilei 1890–1909, i, 296–302.
63. In *Two new sciences*, this is assumed as the fundamental postulate of the entire axiomatic structure of the treatise on motion. Galileo only furnished a corroborative argument from the motion of pendula. *Cf.* Galilei 1890–1909, viii, 205–8. Eventually Galileo found a "mechanical" proof of this postulate in the final years of his life, *cf*. Galilei 1890–1909, viii, 442–5.
64. Galilei 1890–1909, viii, 383–4.
65. Galilei 1890–1909, vii, 51–52.
66. *Cf.* Galilei 1890–1909, viii, 212–13, for Galileo's account, and Thomas Settle, "An experiment in the history of science: with a simple but ingenious device Galileo could obtain relatively precise time measurements", *Science*, cxxxiii (1961), 19–23.
67. Galilei 1890–1909, i, 348–50. *Cf.* Euclid, *The thirteen books of the Elements* (ref. 50), ii, 114.
68. Galilei 1890–1909, vii, 731.
69. In *De motu*, Galileo had already given this definition of weight, but had not recognized its relevance for the case of free falling bodies. "We say that we feel burdened [*gravari*] when some weight is placed upon us which tends downwards because of its gravity, in which case we have to oppose a force so that the weight no longer descends; that opposing is what we call to feel burdened [... *tunc dicimur gravari, quando super nos incumbit aliquod pondus quod sua gravitate deorsum tendit, nobis autem opus est nostra vi resistere ne amplius descendat; illud autem resistere est quod gravari appellamus*]". See Galilei 1890–1909, i, 288, 388. Since 'gravari' is a passive infinitive we might render it more literally as "be burdened". However, let me explain why I prefer "feel burdened". The question has to do with meaning reconstruction. We cannot simply attach the dictionary or grammatical value to a cognate. The context of the passage suggests that Galileo has in mind a human behaviour (that of opposing a force). It goes as follows: "... tunc dicimur gravari, quando super nos incumbit aliquod pondus quod sua gravitate deorsum tendit, nobis autem opus est nostra vi resistere ne aplius descendat; illud autem resistere est quod gravari appellamus." So "We say that we feel burdened [*gravari*] when some weight is placed upon us which tends downwards because of its gravity, in which case we have to oppose a force so that the weight no longer descends; that opposing is what we call to feel burdened [*gravari*]". It is significant that Galileo uses "gravari", the passive infinitive, not "gravare", the active infinitive. Why? Precisely because Galileo has in mind a behaviour typical of human beings. When burdened with a weight we need to exert an opposing force ["... quando super nos incumbit aliquod pondus ... nobis autem opus est nostra vi resistere"]. So by translating "feel burdened" I wish to convey this idea that weight is for human beings to experience. Stones too support weights, but do not

feel burdened (how awkward it would have been if Galileo had said "lapidibus autem opus est sua vi resistere" ?!). In humans action and feeling are interconnected.

70. "... l' unire e soprapporre l' uno all' altro de' sopranominati mattoni" (Galilei 1890–1909, vii, 733).
71. Galilei 1890–1909, viii, 324–5.
72. *Cf.* Galileo's treatise, in Galilei 1890–1909, ii, 223–4, and the relevant passages by Ptolemy, Copernicus, and Clavius, in, respectively, Ptolemy, *Almagestum Cl. Ptolemei* (Venice, 1515), 4 (this edition of the *Almagest* was in Galileo's personal library), Copernicus, *De revolutionibus orbium coelestium, Libri VI* (Nuremberg, 1543), 5r–v, and Christoph Clavius, *In sphaeram Ioannis de Sacro Bosco commentarius* (Rome, 1585), 196.
73. Walter Ong, *Orality and literacy: The technologizing of the word* (London, 1988), 38.
74. Galilei 1890–1909, vii, 743.
75. There is, I think, a phrasing problem in this passage. Galileo has by now long realized that it is space that increases as the square of time. Here he claims that it is a body's "innate speed" that increases as the square of time. This may have been a slip of the pen, or he may have been thinking of "innate speed" as a global quantity, measurable by the space traversed, the latter being the quantity which in effect increases as the square of time.
76. Clearly Galileo believed that to make experiments in the void might turn out to be impossible. *Cf.* Galilei 1890–1909, vii, 743ff. Here I need to point out that in a short piece of writing, which cannot be dated, but presumably is part of a family of similar works concerning hydraulics problems written about the very early 1630s, before the *Postils to Rocco*, Galileo already asserted that he believed that bodies of all types of matter would move in the void with the same speed. He supported the claim with the argument that will be discussed in Section 6, based on the *divergence* of speeds of bodies in different media. At the beginning of this document, Galileo addresses one Bertizzolo, who apparently had made some objections to Galileo, as if his critic were still alive. Antonio Favaro identified the "Bertizzolo" of the writing with the Mantua engineer, Emanuele Bertazzolo (1570–1626), whom Galileo may have met in 1604 on a visit to that city. Thus this piece might have been written earlier than 1630. If this is case then by the mid-1630s Galileo had for some time been convinced that all bodies fall at the same speed in the void. What he still had not realized is the weightless condition of fall. *Cf.* A. Favaro, *Scampoli galileiani*, ed. by L. Rossetti and M. L. Soppelsa (2 vols, Trieste, 1992), ii, 510–15.
77. Galilei 1890–1909, vii, 743.
78. Euclid, *The thirteen books of the Elements* (ref. 50), ii, 112–86.
79. Galilei 1890–1909, i, 276ff.
80. Thus, for example, the arithmetical ratio of 5 to 8 is the same as that of 20 to 23, since 8 – 5 = 3, which is equal to 23 – 20. Galilei 1890–1909, i, 278ff.
81. Galilei 1890–1909, vii, 743–4.
82. Galilei 1890–1909, vii, 734–42.
83. Galilei 1890–1909, viii, 128.
84. G. Galilei, *Two new sciences. Including centres of gravity and force of percussion*, ed. and transl. by Stillman Drake (Madison, 1974), 87, with small changes. *Cf.* the original, in Galilei 1890–1909, viii, 128–9.
85. David Hill, "Pendulums and planes: What Galileo didn't publish", *Nuncius*, ix (1994), 499–515. *Cf.* also S. Drake, *Galileo: Pioneer scientist* (Toronto, 1990), 9–31, who proposes a different interpretation of the manuscripts studied by Hill.
86. Galilei 1890–1909, viii, 202–3.
87. C. R. Palmerino, "Galileo's and Gassendi's solutions to the Rota Aristotelis paradox: A bridge between matter and motion theories", in J. E. Murdoch, W. R. Newman and C. H. Lüthy (eds),

Medieval and early modern corpuscular matter theories (Leiden, 2001), 381–422; *idem*, "Infinite degress of speed: Marin Mersenne and the debate over Galileo's law of free fall", *Early science and medicine*, v (1999), 269–328; *idem*, "Two Jesuit responses to Galileo's science of motion: Honoré Fabri and Pierre La Cazre", in M. Feingold (ed.), *The new science and Jesuit science: Seventeenth century perspectives* (Dordrecht, Boston and London, 2003), 187–227; and *idem*, "Galileo's theories of free fall and projectile motion as interpreted by Pierre Gassendi", in C. R. Palmerino and J. M. M. H. Thijssen (eds), *The reception of the Galilean science of motion in seventeenth-century Europe* (Dordrecht, Boston and London, 2004), 137–64. Palmerino's articles have further interesting bibliography. *Cf.* also E. Giusti, "A master and his pupils: Theories of motion in the Galilean school", in C. R. Palmerino and J. M. M. H. Thijssen (eds), *The reception of the Galilean science of motion in seventeenth-century Europe* (Dordrecht, Boston and London, 2004), 119–36; and M. T. Borgato, "Riccioli e la caduta dei gravi", in M. T. Borgato (ed.), *Giambattista Riccioli e il merito scientifico dei Gesuiti nell' età Barocca* (Forence, 2002), 79–118, on Riccioli's experiments on falling bodies in Bologna about the middle of the seventeenth century.

88. Little is known of Coresio, see F. P. De Ceglia, "Giorgio Coresio: Note in merito a un difensore dell' opinione di Aristotele", *Physis*, xxxvii (2000), 393–437. The passage by Aristotle discussed by Coresio was rendered into English by W. K. C. Guthrie as follows: "Suppose a body A to be weightless, and another body B to have weight, and let the weightless body move a distance CD and the body B move in an equal time CE. (The heavy body will move farther.) Now if the heavy body be divided in the proportion in which CE stands to CD (and it can quite well bear such a relationship to one of its parts), then if the whole traverses the whole distance CE, the part must traverse CD in an equal time. Thus that which has weight will traverse the same distance as that which has none, and this is impossible." See Aristotle, *On the heavens*, transl. by W. K. C. Guthrie (Cambridge, MA, and London, 1986; 1st edn, 1939), 277–8.
89. Galilei 1890–1909, iv, 240–1. Coresio's argument is in his *Operetta intorno al galleggiare de' corpi solidi* (Galilei 1890–1909, iv, 197–244).
90. Galileo's pupil Benedetto Castelli compiled a list of 'errors' contained in Coresio's *Operetta*, but, significantly, he had nothing to say about the latter's counter-argument to Mazzoni-Galileo's critique of Aristotle (Galilei 1890–1909, iv, 246–85).
91. As for Galileo's atomism in general, I have relied on: K. Lasswitz, *Geschichte der Atomistik* (2 vols, Hamburg and Leipzig, 1890), ii, 37–54; E. J. Dijksterhuis, *The mechanization of the world picture* (London, 1961), 419–24; W. Shea, "Galileo's atomic hypothesis", *Ambix*, xvii (1970), 13–27; A. Mark Smith, "Galileo's theory of indivisibles: Revolution or compromise?", *Journal of the history of ideas*, xxxvii (1976), 571–88; and P. Redondi, *Galileo eretico* (Turin, 1988; 1st edn, Turin, 1983), 11–31.
92. Aristotle, *Minor works*, transl. by W. H. Hett (Cambridge, MA, 1993; 1st edn, 1936), 387ff. *Cf.* also J. E. Drabkin, "Aristotle's wheel", *Osiris*, ix (1950), 346–59, still the best discussion on this question.
93. Galilei, *Two new sciences* (ref. 84), 29ff.
94. Smith, "Galileo's theory of indivisibles" (ref. 91), 571–2.
95. *Ibid*., 583–4.
96. Palmerino, "Galileo's and Gassendi's solutions to the Rota Aristotelis paradox" (ref. 87), 384–5.
97. *Ibid*., 398.
98. Galilei, *Two new sciences* (ref. 84), 56, and Palmerino, "Galileo's and Gassendi's solutions to the Rota Aristotelis paradox" (ref. 87), 398–400.
99. Shea, "Galileo's atomic hypothesis" (ref. 91), 13.
100. Palmerino, "Galileo's and Gassendi's solutions to the Rota Aristotelis paradox" (ref. 87), 405ff.
101. Drake, *History of free fall* (ref. 5).

102. P. Fermat, *Oeuvres*, ed. by P. Tannery and C. Henry (5 vols, Paris, 1894–1922), ii, 267–76, and iii, 302–9.
103. E. Torricelli , *Opera geometrica* (Florence, 1644). I have based my analysis on E. Torricelli, *Opere scelte*, ed. by L. Belloni (Turin, 1975), and E. Torricelli, *Opere*, ed. by G. Loria and G. Vassura (5 vols, Faenza, 1919–44).
104. 'Geometrical foundation' has to be taken in the sense that Torricelli furnishes a geometrical proof of Galileo's postulate, which in *Two new sciences* had been justified on the basis of an analogy with mechanical principles. *Cf.* Torricelli, *Opere scelte* (ref. 103), 156ff.
105. Torricelli's proposal of reform of Euclid's theory of samenesses of ratios was probably inspired by Galileo. *Cf.* Giusti, *Euclides reformatus* (ref. 9), 83–114. Torricelli attempted to reproduce all the basic results of the Fifth and Sixth Books of the *Elements* without having recourse to the notion of equimultiple. To do so he introduced nine definitions and six postulates.
106. It goes without saying that such a reconstruction of the genesis of Archimedes's tract on spiral lines is purely the product of Torricelli's fantasy.
107. Torricelli, *Opere scelte* (ref. 103), 173, and Torricelli, *Opere* (ref. 103), iii, 384.
108. Torricelli, *Opere scelte* (ref. 103), 173.