Hindawi
Complexity
Volume 2018, Article ID 9598307, 11 pages
https://doi.org/10.1155/2018/9598307

Research Article

Adaptive Gradient-Based Iterative Algorithm for Multivariable Controlled Autoregressive Moving Average Systems Using the Data Filtering Technique

Jian Pan,[1] **Hao Ma**,[1] **Xiao Jiang**,[1] **Wenfang Ding**,[1] **and Feng Ding**[1,2,3]

[1]*Hubei Collaborative Innovation Center for High-Efficiency Utilization of Solar Energy, Hubei University of Technology, Wuhan 430068, China*
[2]*College of Automation and Electronic Engineering, Qingdao University of Science and Technology, Qingdao 266042, China*
[3]*School of Internet of Things Engineering, Jiangnan University, Wuxi 214122, China*

Correspondence should be addressed to Jian Pan; jpan@163.com

Received 31 March 2018; Accepted 3 June 2018; Published 24 July 2018

Academic Editor: Jing Na

The identification problem of multivariable controlled autoregressive systems with measurement noise in the form of the moving average process is considered in this paper. The key is to filter the input–output data using the data filtering technique and to decompose the identification model into two subidentification models. By using the negative gradient search, an adaptive data filtering-based gradient iterative (F-GI) algorithm and an F-GI with finite measurement data are proposed for identifying the parameters of multivariable controlled autoregressive moving average systems. In the numerical example, we illustrate the effectiveness of the proposed identification methods.

1. Introduction

Parameter estimation plays an important role in system control [1–4], system analysis [5–8], and signal processing [9–13]. Parameter estimation is significant in system modeling [14, 15]. Multi-input multi-output systems widely exist in industrial control areas, which are also called multivariate systems or multivariable systems [16–18]. They are more complex in model structures than single-input single-output systems and always have high dimensions and numerous parameters, which make the parameter estimation more difficult. In this literature, Ding et al. proposed a filtering decomposition-based least squares iterative algorithm for multivariate pseudolinear ARMA systems [19]. Ma et al. studied the parameter estimation problem of multivariate Hammerstein systems and presented a modified Kalman filter-based recursive least squares algorithm to give the parameter estimates [20]. Pan et al. proposed a filtering-based multi-innovation extended stochastic gradient algorithm for multivariable systems [21].

The data filtering technique is an important approach in system identification [22] and state estimation. Chen and Ding applied the data filtering technique to identify the multi-input and single-output system based on the maximum likelihood recursive least squares algorithm [23]. Mao et al. derived an adaptive filtering-based multi-innovation stochastic gradient algorithm for the input nonlinear system with autoregressive noise [24]. They introduced a linear filter to filter the input and output signals and decomposed the identification model into two subidentification models (i.e., a noise model and a system filtered model), which can improve the convergence rate and computation efficiency [25]. The identification methods can be applied to many areas [26–29].

The gradient search is useful for identification as an optimization method [30, 31]. Many gradient-based algorithms, including the stochastic gradient algorithms [32–34] and the gradient-based iterative algorithms, have been developed using the multi-innovation identification theory, the maximum likelihood estimation methods [35, 36], the key-term separation principle [37, 38], and the data filtering theory.

For example, Ma et al. presented an iterative variational Bayesian method to identify the Hammerstein varying systems with parameter uncertainties. Chen et al. studied the identification problem of bilinear-in-parameter systems and presented a gradient-based iterative algorithm by using the hierarchical identification principle and the gradient search [39]. Deng and Ding developed a Newton iterative identification method for an input nonlinear finite impulse response system with moving average noise [40]. Other methods can be referred as to the transfer function identification [41–45], linear system identification [46–51], and nonlinear system identification [52–59].

This paper uses the hierarchical identification principle to study the data filtering-based iterative identification methods for a multivariable controlled autoregressive moving average (M-CARMA) system. The basic idea is to introduce a linear filter to decompose the original identification model into two subidentification models and then obtain the parameter estimates using the negative gradient search. The main contributions are as follows:

(i) A filtering-based gradient iterative (F-GI) algorithm is proposed using the data filtering technique and the gradient search.

(ii) A filtering-based gradient iterative algorithm with finite measurement data is developed to obtain the parameter estimates.

The layout of the remainder of this paper is as follows. Section 2 derives the identification model for the M-CARMA system. In Section 3, we derive a data filtering-based gradient iterative algorithm based on the data filtering technique. A filtering-based gradient iterative algorithm with finite measurement data is developed to estimate the unknown parameters in Section 4. A numerical example is shown in Section 5 to illustrate the benefits of the proposed methods in this paper. Finally, some concluding remarks are given in Section 6.

2. The Problem Formulation

Some notation is introduced for convenience: $\widehat{\boldsymbol{\theta}}(t)$ denotes the estimate of $\boldsymbol{\theta}$ at time t; "$A =: X$" or "$X := A$" stands for" A is defined as X"; the symbol $\mathbf{I}$ ($\mathbf{I}_n$) represents an identity matrix of appropriate size ($n \times n$); the symbol $\mathbf{1}_n$ represents an n-dimensional column vector whose elements are 1; z denotes a unit forward shift operator like $z\mathbf{x}(t) = \mathbf{x}(t+1)$ and $z^{-1}\mathbf{x}(t) = \mathbf{x}(t-1)$; the superscript T symbolizes the vector/matrix transpose; and the norm of a matrix $\mathbf{X}$ is defined by $\|\mathbf{X}\|^2 := tr[\mathbf{X}\mathbf{X}^{\mathrm{T}}]$.

The following multivariable controlled autoregressive moving average system in Figure 1 is considered,

$$\mathbf{A}(z)\mathbf{y}(t) = \mathbf{B}(z)\mathbf{u}(t) + D(z)\mathbf{v}(t), \tag{1}$$

where $\mathbf{u}(t) \in \mathbb{R}^r$ is the system input vector, $\mathbf{y}(t) \in \mathbb{R}^m$ is the system output vector, $\mathbf{v}(t) \in \mathbb{R}^m$ is a white noise vector with zero mean, $\mathbf{A}(z)$ and $\mathbf{B}(z)$ are the matrix polynomials

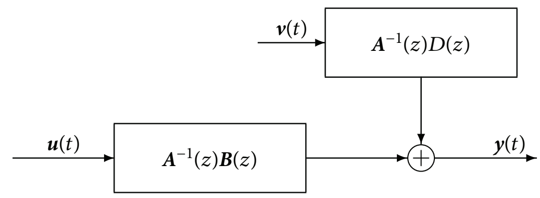

Figure 1: A multivariable controlled autoregressive moving average system.

in the unit backward shift operator z^{-1}, and $D(z)$ is the polynomial in z^{-1}.

$$\begin{aligned}
\mathbf{A}(z) &:= \mathbf{I} + \mathbf{A}_1 z^{-1} + \mathbf{A}_2 z^{-2} + \cdots + \mathbf{A}_{n_a} z^{-n_a}, \quad \mathbf{A}_i \in \mathbb{R}^{m\times m},\\
\mathbf{B}(z) &:= \mathbf{B}_1 z^{-1} + \mathbf{B}_2 z^{-2} + \cdots + \mathbf{B}_{n_b} z^{-n_b}, \quad \mathbf{B}_i \in \mathbb{R}^{m\times r},\\
D(z) &:= 1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_{n_d} z^{-n_d}, \quad d_i \in \mathbb{R}.
\end{aligned} \tag{2}$$

Assume that the orders n_a, n_b, and n_d are known, and $\mathbf{u}(t) = 0$, $\mathbf{y}(t) = 0$, and $\mathbf{v}(t) = 0$ for $t \le 0$. The intermediate variable is defined as

$$\mathbf{w}(t) := D(z)\mathbf{v}(t) \in \mathbb{R}^m. \tag{3}$$

The system information vector $\boldsymbol{\varphi}_{\mathrm{s}}(t)$, the noise information vector $\boldsymbol{\psi}(t)$, the system parameter matrix $\boldsymbol{\theta}_{\mathrm{s}}$, and the noise parameter vector $\boldsymbol{\theta}_{\mathrm{n}}$ are defined as

$$\begin{aligned}
\boldsymbol{\varphi}_{\mathrm{s}}(t) &:= \big[-\mathbf{y}^{\mathrm{T}}(t-1), -\mathbf{y}^{\mathrm{T}}(t-2), \ldots, -\mathbf{y}^{\mathrm{T}}(t-n_a),\\
&\qquad \mathbf{u}^{T}(t-1), \mathbf{u}^{\mathrm{T}}(t-2), \ldots, \mathbf{u}^{\mathrm{T}}(t-n_b)\big]^{\mathrm{T}}\\
&\quad \in \mathbb{R}^{mn_a+rn_b},\\
\boldsymbol{\psi}(t) &:= [\mathbf{v}(t-1), \mathbf{v}(t-2), \ldots, \mathbf{v}(t-n_d)] \in \mathbb{R}^{m\times n_d},\\
\boldsymbol{\theta}_{\mathrm{s}}^{\mathrm{T}} &:= \big[\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_{n_a}, \mathbf{B}_1, \mathbf{B}_2, \ldots, \mathbf{B}_{n_b}\big]\\
&\quad \in \mathbb{R}^{m\times(mn_a+rn_b)},\\
\boldsymbol{\theta}_{\mathrm{n}} &:= \big[d_1, d_2, \ldots, d_{n_d}\big]^{\mathrm{T}} \in \mathbb{R}^{n_d}.
\end{aligned} \tag{4}$$

Equations (3) and (1) can be written as

$$\begin{aligned}
\mathbf{w}(t) &= \left(1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_{nd} z^{-n_d}\right)\mathbf{v}(t)\\
&= d_1\mathbf{v}(t-1) + d_2\mathbf{v}(t-2) + \cdots + d_{n_d}\mathbf{v}(t-n_d) + \mathbf{v}(t)\\
&= \boldsymbol{\psi}(t)\boldsymbol{\theta}_n + \mathbf{v}(t),
\end{aligned} \tag{5}$$

$$\begin{aligned}
\mathbf{y}(t) &= [\mathbf{I} - \mathbf{A}(z)]\mathbf{y}(t) + \mathbf{B}(z)\mathbf{u}(t) + D(z)\mathbf{v}(t)\\
&= \boldsymbol{\theta}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{\varphi}_{\mathrm{s}}(t) + \mathbf{w}(t) = \boldsymbol{\theta}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{\varphi}_{\mathrm{s}}(t) + \boldsymbol{\psi}(t)\boldsymbol{\theta}_{\mathrm{n}} + \mathbf{v}(t).
\end{aligned} \tag{6}$$

Equation (5) is the noise identification model. For the M-CARMA system in (1), choose the polynomial $L(z) := 1/D(z)$ as a filter. Define the filtered input vector $\mathbf{u}_{\mathrm{f}}(t)$, the filtered output vector $\mathbf{y}_{\mathrm{f}}(t)$, and the filtered information vector $\boldsymbol{\varphi}_{\mathrm{f}}(t)$ as

$$\begin{aligned}
\mathbf{u}_{\mathrm{f}}(t) &:= L(z)\mathbf{u}(t) = \frac{1}{D(z)}\mathbf{u}(t) \in \mathbb{R}^{r}, \\
\mathbf{y}_{\mathrm{f}}(t) &:= L(z)\mathbf{y}(t) = \frac{1}{D(z)}\mathbf{y}(t) \in \mathbb{R}^{m}, \\
\boldsymbol{\varphi}_{\mathrm{f}}(t) &:= L(z)\boldsymbol{\varphi}_{\mathrm{s}}(t) = \frac{1}{D(z)}\boldsymbol{\varphi}_{s}(t) \\
&= \left[-\mathbf{y}_{\mathrm{f}}^{\mathrm{T}}(t-1), -\mathbf{y}_{\mathrm{f}}^{\mathrm{T}}(t-2), \ldots, -\mathbf{y}_{\mathrm{f}}^{\mathrm{T}}(t-n_a),\right. \\
&\qquad \left.\mathbf{u}_{\mathrm{f}}^{\mathrm{T}}(t-1), \mathbf{u}_{\mathrm{f}}^{\mathrm{T}}(t-2), \ldots, \mathbf{u}_{\mathrm{f}}^{\mathrm{T}}(t-n_b)\right]^{\mathrm{T}} \\
&\in \mathbb{R}^{mn_a+rn_b}.
\end{aligned} \tag{7}$$

Multiplying both sides of (1) by $L(z)$ obtains

$$\mathbf{A}(z)L(z)\mathbf{y}(t) = \mathbf{B}(z)L(z)\mathbf{u}(t) + \mathbf{v}(t), \tag{8}$$

or

$$\mathbf{A}(z)\mathbf{y}_{\mathrm{f}}(t) = \mathbf{B}(z)\mathbf{u}_{\mathrm{f}}(t) + \mathbf{v}(t). \tag{9}$$

Then we have

$$\mathbf{y}_{\mathrm{f}}(t) = [\mathbf{I} - \mathbf{A}(z)]\mathbf{y}_{\mathrm{f}}(t) + \mathbf{B}(z)\mathbf{u}_{\mathrm{f}}(t) + \mathbf{v}(t) = \boldsymbol{\theta}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{\varphi}_{\mathrm{f}}(t) + \mathbf{v}(t). \tag{10}$$

Equations (10) and (5) form the filtered identification models of the M-CARMA system.

3. The F-GI Algorithm

In this section, a linear filter $L(z)$ is applied to deal with the moving average noise. A gradient-based iterative identification algorithm is proposed for M-CARMA systems by using the data filtering technique [60–63].

Considering the newest p data from $i = t - p + 1$ to $i = t$, the stacked filtered output matrix $\mathbf{Y}_{\mathrm{f}}(p, t)$, the stacked filtered information matrix $\boldsymbol{\Phi}_{\mathrm{f}}(p, t)$, the stacked noise vector $\mathbf{W}(p, t)$, and the stacked noise information matrix $\boldsymbol{\Phi}_{\mathrm{n}}(p, t)$ are defined as

$$\begin{aligned}
\mathbf{Y}_{\mathrm{f}}(p, t) &:= [\mathbf{y}_{\mathrm{f}}(t), \mathbf{y}_{\mathrm{f}}(t-1), \ldots, \mathbf{y}_{\mathrm{f}}(t-p+1)] \in \mathbb{R}^{m\times p}, \\
\boldsymbol{\Phi}_{\mathrm{f}}(p, t) &:= [\boldsymbol{\varphi}_{\mathrm{f}}(t), \boldsymbol{\varphi}_{\mathrm{f}}(t-1), \ldots, \boldsymbol{\varphi}_{\mathrm{f}}(t-p+1)] \in \mathbb{R}^{(mn_a+rn_b)\times p}, \\
\mathbf{W}(p, t) &:= [\mathbf{w}(t), \mathbf{w}(t-1), \ldots, \mathbf{w}(t-p+1)]^{\mathrm{T}} \in \mathbb{R}^{pm}, \\
\boldsymbol{\Phi}_{\mathrm{n}}(p, t) &:= [\boldsymbol{\psi}(t), \boldsymbol{\psi}(t-1), \ldots, \boldsymbol{\psi}(t-p+1)]^{\mathrm{T}} \in \mathbb{R}^{(pm)\times n_d}.
\end{aligned} \tag{11}$$

Define a quadratic criterion function:

$$\begin{aligned}
J_1(\boldsymbol{\theta}_{\mathrm{s}}) &= \left\|\mathbf{Y}_{\mathrm{f}}(p, t) - \boldsymbol{\theta}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{\Phi}_{\mathrm{f}}(p, t)\right\|^2, \\
J_2(\boldsymbol{\theta}_{\mathrm{n}}) &= \|\mathbf{W}(p, t) - \boldsymbol{\Phi}_{\mathrm{n}}(p, t)\boldsymbol{\theta}_{\mathrm{n}}\|^2.
\end{aligned} \tag{12}$$

Let $k = 1, 2, 3, \ldots$ be an iterative variable. Let $\widehat{\boldsymbol{\theta}}_{\mathrm{s},k}(t)$ and $\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t)$ be the estimates of $\boldsymbol{\theta}_{\mathrm{s}}$ and $\boldsymbol{\theta}_{\mathrm{n}}$ at iteration k. Minimizing $J_1(\boldsymbol{\theta}_{\mathrm{s}})$ and $J_2(\boldsymbol{\theta}_{\mathrm{n}})$ and using the negative gradient search will give the following iterative relations for obtaining the parameter estimates of $\boldsymbol{\theta}_{\mathrm{s}}$ and $\boldsymbol{\theta}_{\mathrm{n}}$:

$$\begin{aligned}
\widehat{\boldsymbol{\theta}}_{\mathrm{s},k}(t) &= \widehat{\boldsymbol{\theta}}_{\mathrm{s},k-1}(t) - \frac{\mu_{1,k}(t)}{2}\operatorname{grad}[J_1(\boldsymbol{\theta}_{\mathrm{s}})] \\
&= \widehat{\boldsymbol{\theta}}_{\mathrm{s},k-1}(t) + \mu_{1,k}(t)\boldsymbol{\Phi}_{\mathrm{f}}(p, t) \\
&\quad \cdot \left[\mathbf{Y}_{\mathrm{f}}(t) - \widehat{\boldsymbol{\theta}}_{\mathrm{s},k-1}^{\mathrm{T}}(t)\boldsymbol{\Phi}_{\mathrm{f}}(p, t)\right]^{\mathrm{T}},
\end{aligned} \tag{13}$$

$$\begin{aligned}
\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t) &= \widehat{\boldsymbol{\theta}}_{\mathrm{n},k-1}(t) - \frac{\mu_{2,k}(t)}{2}\operatorname{grad}[J_2(\boldsymbol{\theta}_{\mathrm{n}})] \\
&= \widehat{\boldsymbol{\theta}}_{\mathrm{n},k-1}(t) + \mu_{2,k}(t)\boldsymbol{\Phi}_{\mathrm{n}}^{\mathrm{T}}(p, t) \\
&\quad \cdot \left[\mathbf{W}(t) - \boldsymbol{\Phi}_{\mathrm{n}}(p, t)\widehat{\boldsymbol{\theta}}_{\mathrm{n},k-1}(t)\right],
\end{aligned} \tag{14}$$

where $\mu_{1,k}(t) \geq 0$ and $\mu_{2,k}(t) \geq 0$ are the iterative step size or the convergence factor. However, the difficulty is that the noise information matrix $\boldsymbol{\Phi}_{\mathrm{n}}(p, t)$ (i.e., $\boldsymbol{\psi}(t)$) contains the unmeasured vector $\mathbf{v}(t-i)$. So the gradient-based iterative algorithm in (13) and (14) cannot give the parameter estimate $\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t)$ directly. The solution is to use the hierarchical identification principle and to replace the unknown variable $\mathbf{v}(t-i)$ with its corresponding estimates $\widehat{\mathbf{v}}_{k-1}(t-i)$ at iteration $k-1$, and to define the estimate of $\boldsymbol{\psi}(t)$ as

$$\widehat{\boldsymbol{\psi}}_k(t) := [\widehat{\mathbf{v}}_{k-1}(t-1), \widehat{\mathbf{v}}_{k-1}(t-2), \ldots, \widehat{\mathbf{v}}_{k-1}(t-n_d)] \in \mathbb{R}^{m\times n_d}. \tag{15}$$

Using $\widehat{\boldsymbol{\psi}}_k(t)$ to construct the estimate of $\boldsymbol{\Phi}_{\mathrm{n},k}(p, t)$ obtains

$$\widehat{\boldsymbol{\Phi}}_{\mathrm{n},k}(p, t) := \left[\widehat{\boldsymbol{\psi}}_k(t), \widehat{\boldsymbol{\psi}}_k(t-1), \ldots, \widehat{\boldsymbol{\psi}}_k(t-p+1)\right]^{\mathrm{T}} \in \mathbb{R}^{(pm)\times n_d}. \tag{16}$$

Replacing t in (6) with $t - i$ gives

$$\mathbf{w}(t-i) = \mathbf{y}(t-i) - \boldsymbol{\theta}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{\varphi}_{\mathrm{s}}(t-i). \tag{17}$$

Replacing $\boldsymbol{\theta}_{\mathrm{s}}$ in (17) with $\widehat{\boldsymbol{\theta}}_{\mathrm{s},k-1}(t)$ obtains the estimate of $\mathbf{w}(t-i)$ at iteration k:

$$\widehat{\mathbf{w}}_k(t-i) = \mathbf{y}(t-i) - \widehat{\boldsymbol{\theta}}_{\mathrm{s},k-1}^{\mathrm{T}}(t)\boldsymbol{\varphi}_{\mathrm{s}}(t-i). \tag{18}$$

From (6), we have

$$\mathbf{v}(t-i) = \mathbf{y}(t-i) - \widehat{\boldsymbol{\theta}}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{\varphi}_{\mathrm{s}}(t-i) - \boldsymbol{\psi}(t-i)\boldsymbol{\theta}_{\mathrm{n}}. \tag{19}$$

Replacing $\boldsymbol{\theta}_{\mathrm{s}}$, $\boldsymbol{\theta}_{\mathrm{n}}$, and $\boldsymbol{\psi}(t-i)$ with $\widehat{\boldsymbol{\theta}}_{\mathrm{s},k}(t)$, $\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t)$, and $\widehat{\boldsymbol{\psi}}_k(t-i)$ obtains the iterative estimate of $\mathbf{v}(t-i)$ at iteration k:

$$\widehat{\mathbf{v}}_k(t-i) = \mathbf{y}(t-i) - \widehat{\boldsymbol{\theta}}_{\mathrm{s},k}^{\mathrm{T}}(t)\boldsymbol{\varphi}_{\mathrm{s}}(t-i) - \widehat{\boldsymbol{\psi}}_k(t-i)\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t). \tag{20}$$

Then, using $\widehat{\mathbf{w}}_k(t)$ to construct the iterative estimate of $\mathbf{W}(p, t)$ at iteration k gives

$$\widehat{\mathbf{W}}_k(p, t) = [\widehat{\mathbf{w}}_k(t), \widehat{\mathbf{w}}_k(t-1), \ldots, \widehat{\mathbf{w}}_k(t-p+1)]^{\mathrm{T}} \in \mathbb{R}^{pm}. \tag{21}$$

Use $\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t) = [\widehat{d}_{1,k}(t), \widehat{d}_{2,k}(t), \ldots, \widehat{d}_{n_d,k}(t)]^{\mathrm{T}}$ to construct the estimate of $D(t, z)$ as

$$\widehat{D}_k(t,z) = 1 + \widehat{d}_{1,k}(t)z^{-1} + \widehat{d}_{2,k}(t)z^{-2} + \cdots + \widehat{d}_{n_d,k}(t)z^{-n_d}. \tag{22}$$

Using $\widehat{D}_k(t,z)$ to filter $\mathbf{y}(t)$ and $\mathbf{u}(t)$ gives the filtered estimates $\widehat{\mathbf{y}}_{\mathrm{f},k}(t)$ and $\widehat{\mathbf{u}}_{\mathrm{f},k}(t)$ of $\mathbf{y}_\mathrm{f}(t)$ and $\mathbf{u}_\mathrm{f}(t)$:

$$\begin{aligned}\widehat{\mathbf{u}}_{\mathrm{f},k}(t) &= \mathbf{u}(t) - \widehat{d}_{1,k}(t)\widehat{\mathbf{u}}_{\mathrm{f},k}(t-1) - \widehat{\mathbf{d}}_{2,k}(t)\widehat{\mathbf{u}}_{\mathrm{f},k}(t-2) \\ &\quad - \cdots - \widehat{d}_{n_d,k}(t)\widehat{\mathbf{u}}_{\mathrm{f},k}(t-n_d), \\ \widehat{\mathbf{y}}_{\mathrm{f},k}(t) &= \mathbf{y}(t) - \widehat{d}_{1,k}(t)\widehat{y}_{\mathrm{f},k}(t-1) - \widehat{d}_{2,k}(t)\widehat{\mathbf{y}}_{\mathrm{f},k}(t-2) \\ &\quad - \cdots - \widehat{d}_{n_d,k}(t)\widehat{\mathbf{y}}_{\mathrm{f},k}(t-n_d).\end{aligned} \tag{23}$$

Furthermore, we use $\widehat{\mathbf{y}}_{\mathrm{f},k}(t)$ to construct the estimate of $\mathbf{Y}_\mathrm{f}(p,t)$, use $\widehat{\mathbf{y}}_{\mathrm{f},k}(t)$ and $\widehat{\mathbf{u}}_{\mathrm{f},k}(t)$ to construct the estimate of $\boldsymbol{\varphi}_\mathrm{f}(t)$, and use $\widehat{\boldsymbol{\varphi}}_{\mathrm{f},k}(t)$ to construct the estimate of $\boldsymbol{\Phi}_\mathrm{f}(p,t)$ at iteration k:

$$\begin{aligned}\widehat{\mathbf{Y}}_{\mathrm{f},k}(p,t) &:= \left[\widehat{\mathbf{y}}_{\mathrm{f},k}(t), \widehat{\mathbf{y}}_{\mathrm{f},k}(t-1), \ldots, \widehat{\mathbf{y}}_{\mathrm{f},k}(t-p+1)\right] \in \mathbb{R}^{m\times p}, \\ \widehat{\boldsymbol{\varphi}}_{\mathrm{f},k}(t) &:= \left[-\widehat{\mathbf{y}}^{\mathrm{T}}_{\mathrm{f},k}(t-1), \ldots, -\widehat{\mathbf{y}}^{\mathrm{T}}_{\mathrm{f},k}(t-n_a), \right. \\ &\quad \left. \widehat{\mathbf{u}}^{\mathrm{T}}_{\mathrm{f},k}(t-1), \ldots, \widehat{\mathbf{u}}^{\mathrm{T}}_{\mathrm{f},k}(t-n_b)\right]^{\mathrm{T}} \in \mathbb{R}^{mn_a+rn_b}, \\ \widehat{\boldsymbol{\Phi}}_{\mathrm{f},k}(p,k) &:= \left[\widehat{\boldsymbol{\varphi}}_{\mathrm{f},k}(t), \widehat{\boldsymbol{\varphi}}_{\mathrm{f},k}(t-1), \ldots, \widehat{\boldsymbol{\varphi}}_{\mathrm{f},k}(t-p+1)\right] \\ &\quad \in \mathbb{R}^{(mn_a+rn_b)\times p}.\end{aligned} \tag{24}$$

From the above derivation, we can summarize a filtering-based multi-innovation gradient iterative identification algorithm:

$$\begin{aligned}\widehat{\boldsymbol{\theta}}_{\mathrm{s},k}(t) &= \widehat{\boldsymbol{\theta}}_{\mathrm{s},k-1}(t) + \mu_{1,k}(t)\widehat{\boldsymbol{\Phi}}_{\mathrm{f},k}(t) \\ &\quad \cdot \left[\widehat{\mathbf{Y}}_{\mathrm{f},k}(t) - \widehat{\boldsymbol{\theta}}^{\mathrm{T}}_{\mathrm{s},k-1}(t)\widehat{\boldsymbol{\Phi}}_{\mathrm{f},k}(t)\right]^{\mathrm{T}},\end{aligned} \tag{25}$$

$$\widehat{\mathbf{Y}}_{\mathrm{f},k}(p,t) = \left[\widehat{\mathbf{y}}_{\mathrm{f},k}(t), \widehat{\mathbf{y}}_{\mathrm{f},k}(t-1), \ldots, \widehat{\mathbf{y}}_{\mathrm{f},k}(t-p+1)\right], \tag{26}$$

$$\boldsymbol{\Phi}_{\mathrm{f},k}(p,t) = \left[\widehat{\boldsymbol{\varphi}}_{\mathrm{f},k}(t), \widehat{\boldsymbol{\varphi}}_{\mathrm{f},k}(t-1), \ldots, \widehat{\boldsymbol{\varphi}}_{\mathrm{f},k}(t-p+1)\right], \tag{27}$$

$$\begin{aligned}\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t) &= \widehat{\boldsymbol{\theta}}_{\mathrm{n},k-1}(t) + \mu_{2,k}(t)\widehat{\boldsymbol{\Phi}}^{\mathrm{T}}_{\mathrm{n},k}(t) \\ &\quad \cdot \left[\widehat{\mathbf{W}}_k(t) - \widehat{\boldsymbol{\Phi}}_{\mathrm{n},k}(t)\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t)\right],\end{aligned} \tag{28}$$

$$\widehat{\mathbf{W}}_k(p,t) = \left[\widehat{\mathbf{w}}_k(t), \widehat{\mathbf{w}}_k(t-1), \ldots, \widehat{\mathbf{w}}_k(t-p+1)\right]^{\mathrm{T}}, \tag{29}$$

$$\widehat{\boldsymbol{\Phi}}_{\mathrm{n},k}(p,t) = \left[\widehat{\boldsymbol{\psi}}_k(t), \widehat{\boldsymbol{\psi}}_k(t-1), \ldots, \widehat{\boldsymbol{\psi}}_k(t-p+1)\right]^{\mathrm{T}}, \tag{30}$$

$$\begin{aligned}\widehat{\boldsymbol{\varphi}}_{\mathrm{f},k}(t) &= \left[-\widehat{\mathbf{y}}^{\mathrm{T}}_{\mathrm{f},k}(t-1), \ldots, -\widehat{\mathbf{y}}^{\mathrm{T}}_{\mathrm{f},k}(t-n_a), \right. \\ &\quad \left. \widehat{\mathbf{u}}^{\mathrm{T}}_{\mathrm{f},k}(t-1), \ldots, \widehat{\mathbf{u}}^{\mathrm{T}}_{\mathrm{f},k}(t-n_b)\right]^{\mathrm{T}},\end{aligned} \tag{31}$$

$$\widehat{\boldsymbol{\psi}}_k(t) = \left[\widehat{\mathbf{v}}_{k-1}(t-1), \widehat{\mathbf{v}}_{k-1}(t-2), \ldots, \widehat{\mathbf{v}}_{k-1}(t-n_d)\right], \tag{32}$$

$$\begin{aligned}\boldsymbol{\varphi}_\mathrm{s}(t) &= \left[-\mathbf{y}^{\mathrm{T}}(t-1), \ldots, -\mathbf{y}^{\mathrm{T}}(t-n_a), \right. \\ &\quad \left. \mathbf{u}^{\mathrm{T}}(t-1), \ldots, \mathbf{u}^{\mathrm{T}}(t-n_b)\right]^{\mathrm{T}},\end{aligned} \tag{33}$$

$$\begin{aligned}\widehat{\mathbf{u}}_{\mathrm{f},k}(t) &= \mathbf{u}(t) - \widehat{d}_{1,k}(t)\widehat{\mathbf{u}}_{\mathrm{f},k}(t-1) - \widehat{d}_{2,k}(t)\widehat{\mathbf{u}}_{\mathrm{f},k}(t-2), \\ &\quad - \cdots - \widehat{d}_{nd,k}(t)\widehat{\mathbf{u}}_{\mathrm{f},k}(t-n_d),\end{aligned} \tag{34}$$

$$\begin{aligned}\widehat{\mathbf{y}}_{\mathrm{f},k}(t) &= \mathbf{y}(t) - \widehat{d}_{1,k}(t)\widehat{\mathbf{y}}_{\mathrm{f},k}(t-1) - \widehat{d}_{2,k}(t)\widehat{\mathbf{y}}_{\mathrm{f},k}(t-2), \\ &\quad - \cdots - \widehat{d}_{n_d,k}(t)\widehat{\mathbf{y}}_{\mathrm{f},\mathrm{k}}(t-n_d),\end{aligned} \tag{35}$$

$$\widehat{\mathbf{w}}_k(t-i) = \mathbf{y}(t-i) - \widehat{\boldsymbol{\theta}}^{\mathrm{T}}_{\mathrm{s},k-1}(t)\boldsymbol{\varphi}_\mathrm{s}(t-i), \tag{36}$$

$$\widehat{\mathbf{v}}_k(t-i) = \mathbf{y}(t-i) - \widehat{\boldsymbol{\theta}}^{\mathrm{T}}_{\mathrm{s},k}(t)\boldsymbol{\varphi}_\mathrm{s}(t-i) - \widehat{\boldsymbol{\psi}}_k(t-i)\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t), \tag{37}$$

$$\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t) = \left[\widehat{d}_{1,k}(t), \widehat{d}_{2,k}(t), \ldots, \widehat{d}_{n_d,k}(t)\right]^{\mathrm{T}}, \tag{38}$$

$$\mu_{1,k}(t) \le 2\left(\left\|\widehat{\boldsymbol{\Phi}}_{\mathrm{f},k}(t)\right\|^2\right)^{-1}, \tag{39}$$

$$\mu_{2,k}(t) \le 2\left(\left\|\widehat{\boldsymbol{\Phi}}_{\mathrm{n},k}(t)\right\|^2\right)^{-1}. \tag{40}$$

The identification steps of the algorithm in (25), (26), (27), (28), (29), (30), (31), (32), (33), (34), (35), (36), (37), (38), (39), and (40) to compute $\widehat{\boldsymbol{\theta}}_{\mathrm{s},k}(t)$ and $\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t)$ are listed as follows:

(1) Set the initial values: let $t = 1$, give the data length p, and give a small positive number ε. Set the initial values $\widehat{\boldsymbol{\theta}}_\mathrm{s}(0) = \mathbf{1}_{(mn_a+rn_b)\times m}/p_0$, $\widehat{\boldsymbol{\theta}}_\mathrm{n}(0) = \mathbf{1}_{n_\mathrm{d}}/p_0$, $p_0 = 10^6$.

(2) Collect the input–output data $\mathbf{u}(t)$ and $\mathbf{y}(t)$ and construct $\boldsymbol{\varphi}_\mathrm{s}(t)$ using (33).

(3) Let $k = 1$ and set the initial values $\widehat{w}_0(t-i) = 1_m/po, \widehat{v}_0(t-i) = 1_m/p_0, \widehat{y}_{\mathrm{f},0}(t-i) = 1_m/p_0, \widehat{u}_{f,0}(t-i) = 1_r/p_0$, $i = 1, 2, \ldots, \max[n_a, n_b, n_d]$.

(4) Construct $\widehat{\boldsymbol{\psi}}_k(t)$ and $\widehat{\boldsymbol{\varphi}}_{\mathrm{f},k}(t)$ using (31) and (32).

(5) Compute $\widehat{\mathbf{w}}_k(t)$ using (36) and form the stacked noise vector $\widehat{\mathbf{W}}_k(p,t)$ using (29) and the stacked noise matrix $\boldsymbol{\Phi}_{\mathrm{n},k}(p,t)$ using (30).

(6) Choose $\mu_{2,k}(t)$ using (40) and update the noise parameter estimates $\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t)$ using (28).

(7) Read $\widehat{d}_{i,k}(t)$ from $\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t)$ in (38). Compute $\widehat{\mathbf{u}}_{\mathrm{f},k}(t)$ and $\widehat{\mathbf{y}}_{\mathrm{f},k}(t)$ using (34) and (35).

(8) Construct $\boldsymbol{\Phi}_{\mathrm{f},k}(p,t)$ using (27) and $\widehat{\mathbf{Y}}_{\mathrm{f},k}(p,t)$ using (26).

(9) Choose $\mu_{1,k}(t)$ using (39) and update the system parameter estimates $\widehat{\boldsymbol{\theta}}_{\mathrm{s},k}(t)$ using (25).

(10) Compute $\widehat{\mathbf{v}}_k(t-i)$ using (37).

(11) Compare $\widehat{\boldsymbol{\theta}}_{\mathrm{s},k}(t)$ with $\widehat{\boldsymbol{\theta}}_{\mathrm{s},k-1}(t)$ and compare $\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t)$ with $\widehat{\boldsymbol{\theta}}_{\mathrm{n},k-1}(t)$: if $\|\widehat{\boldsymbol{\theta}}_{\mathrm{s},k}(t) - \widehat{\boldsymbol{\theta}}_{\mathrm{s},k-1}(t)\| > \varepsilon$ and $\|\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t) - \widehat{\boldsymbol{\theta}}_{\mathrm{n},k-1}(t)\| > \varepsilon$, increase k by 1 and turn to Step 4;

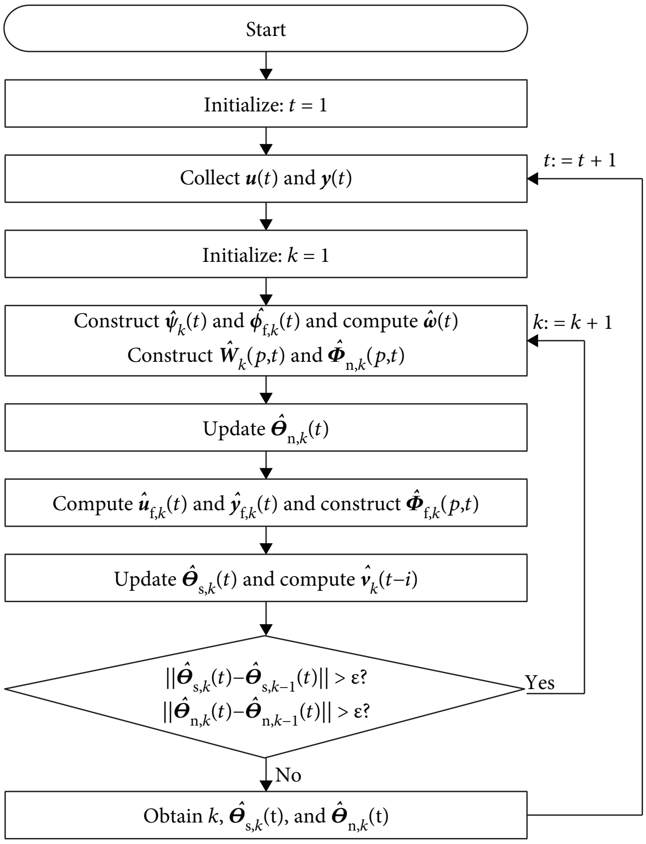

FIGURE 2: The flowchart of computing the F-GI parameter estimates.

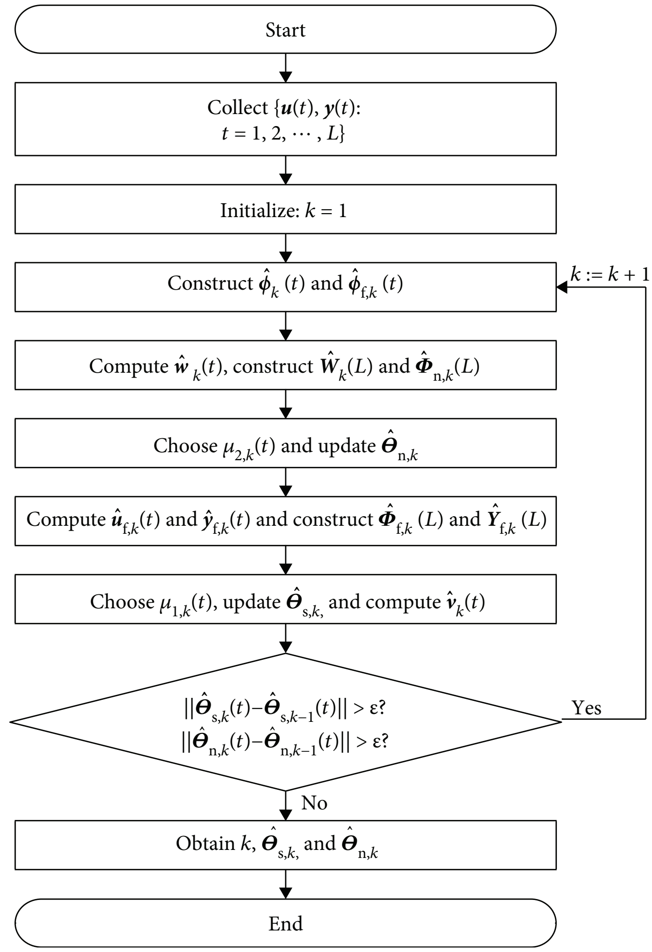

FIGURE 3: The flowchart of computing the F-GI parameter estimates with finite measurement data.

otherwise, obtain k and the parameter estimate vector $\widehat{\boldsymbol{\theta}}_{\mathrm{s},k}(t)$ and $\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t)$, let $\widehat{\boldsymbol{\theta}}_{\mathrm{s},0}(t+1) := \widehat{\boldsymbol{\theta}}_{\mathrm{s},k}(t)$, $\widehat{\boldsymbol{\theta}}_{\mathrm{n},0}(t+1) := \widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t)$, increase t by 1 and turn to Step 2.

The flowchart of computing $\widehat{\boldsymbol{\theta}}_{\mathrm{s},k}(t)$ and $\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}(t)$ from the F-GI algorithm is shown in Figure 2.

4. The F-GI Algorithm with Finite Measurement Data

Consider the data from $t=1$ to $t=L$ and define the stacked filtered output matrix $\mathbf{Y}_{\mathrm{f}}(L)$, the stacked filtered information matrix $\boldsymbol{\Phi}_{\mathrm{f}}(L)$, the stacked noise vector $\mathbf{W}(L)$, and the stacked noise information matrix $\boldsymbol{\Phi}_{\mathrm{n}}(L)$ as

$$\begin{aligned}
\mathbf{Y}_{\mathrm{f}}(L) &:= [\mathbf{y}_{\mathrm{f}}(L), \mathbf{y}_{\mathrm{f}}(L-1), \ldots, \mathbf{y}_{\mathrm{f}}(1)] \in \mathbb{R}^{m\times L},\\
\boldsymbol{\Phi}_{\mathrm{f}}(L) &:= [\boldsymbol{\varphi}_{\mathrm{f}}(L), \boldsymbol{\varphi}_{\mathrm{f}}(L-1), \ldots, \boldsymbol{\varphi}_{\mathrm{f}}(1)] \in \mathbb{R}^{(mn_a+rn_b)\times L},\\
\mathbf{W}(L) &:= [\mathbf{w}(L), \mathbf{w}(L-1), \ldots, \mathbf{w}(1)]^{\mathrm{T}} \in \mathbb{R}^{Lm},\\
\boldsymbol{\Phi}_{\mathrm{n}}(L) &:= [\boldsymbol{\psi}(L), \boldsymbol{\psi}(L-1), \ldots, \boldsymbol{\psi}(1)]^{\mathrm{T}} \in \mathbb{R}^{(Lm)\times n_d}.
\end{aligned} \tag{41}$$

Note that $\mathbf{Y}_{\mathrm{f}}(L)$ and $\boldsymbol{\Phi}_{\mathrm{f}}(L)$ contain all the measured data $\{u(t), y(t)\colon t = 1, 2, \ldots, L\}$.

The two gradient criterion functions are defined as

$$\begin{aligned}
J_3(\boldsymbol{\theta}_{\mathrm{s}}) &:= \left\|\mathbf{Y}_{\mathrm{f}}(L) - \boldsymbol{\theta}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{\Phi}_{\mathrm{f}}(L)\right\|^2,\\
J_4(\boldsymbol{\theta}_{\mathrm{n}}) &:= \|\mathbf{W}(L) - \boldsymbol{\Phi}_{\mathrm{n}}(L)\boldsymbol{\theta}_{\mathrm{n}}\|^2.
\end{aligned} \tag{42}$$

Similarly, minimizing $J_3(\boldsymbol{\theta}_{\mathrm{s}})$ and $J_4(\boldsymbol{\theta}_{\mathrm{n}})$, we can derive a filtering-based gradient iterative (F-GI) algorithm with the data length L for the M-CARMA system:

$$\begin{aligned}
\widehat{\boldsymbol{\theta}}_{\mathrm{s},k} &= \widehat{\boldsymbol{\theta}}_{\mathrm{s},k-1} + \mu_{1,k}(t)\widehat{\boldsymbol{\Phi}}_{\mathrm{f},k}(L)\\
&\quad \cdot \left[\widehat{\mathbf{Y}}_{\mathrm{f},k}(L) - \widehat{\boldsymbol{\theta}}_{\mathrm{s},k-1}^{\mathrm{T}}\widehat{\boldsymbol{\Phi}}_{\mathrm{f},k}(L)\right]^{\mathrm{T}},
\end{aligned} \tag{43}$$

$$\widehat{\mathbf{Y}}_{\mathrm{f},k}(L) = \left[\widehat{\mathbf{y}}_{\mathrm{f},k}(L), \widehat{\mathbf{y}}_{\mathrm{f},k}(L-1), \ldots, \widehat{\mathbf{y}}_{\mathrm{f},k}(1)\right], \tag{44}$$

$$\widehat{\boldsymbol{\Phi}}_{\mathrm{f},k}(L) = \left[\widehat{\boldsymbol{\varphi}}_{\mathrm{f},k}(L), \widehat{\boldsymbol{\varphi}}_{\mathrm{f},k}(L-1), \ldots, \widehat{\boldsymbol{\varphi}}_{\mathrm{f},k}(1)\right], \tag{45}$$

$$\begin{aligned}
\widehat{\boldsymbol{\theta}}_{\mathrm{n},k} &= \widehat{\boldsymbol{\theta}}_{\mathrm{n},k-1} + \mu_{2,k}(t)\widehat{\boldsymbol{\Phi}}_{\mathrm{n},k}^{\mathrm{T}}(L)\\
&\quad \cdot \left[\widehat{\mathbf{W}}_k(L) - \widehat{\boldsymbol{\Phi}}_{\mathrm{n},k}(L)\widehat{\boldsymbol{\theta}}_{\mathrm{n},k-1}\right],
\end{aligned} \tag{46}$$

$$\widehat{\mathbf{W}}_k(L) = [\widehat{\mathbf{w}}_k(L), \widehat{\mathbf{w}}_k(L-1), \ldots, \widehat{\mathbf{w}}_k(1)]^{\mathrm{T}}, \tag{47}$$

Table 1: The F-GI parameter estimates and errors ($L = 1000$).

σ^2	k	a_{11}	a_{12}	b_{11}	b_{12}	a_{21}	a_{22}	b_{21}	b_{22}	d_1	δ (%)
0.5^2	1	0.14294	0.09843	0.00868	0.00915	0.00296	0.08637	0.00937	−0.00661	−0.27864	86.50364
	10	0.42310	0.34081	0.15604	0.12072	−0.02498	0.29864	0.20824	−0.15361	−0.70856	56.91872
	50	0.63701	0.59049	0.44414	0.30015	−0.21622	0.83432	0.36331	−0.32901	−0.77686	16.06327
	200	0.65476	0.60674	0.46465	0.31821	−0.31707	1.02684	0.38855	−0.39407	−0.56444	3.58517
	500	0.65477	0.60673	0.46466	0.31824	−0.31710	1.02689	0.38857	−0.39418	−0.56441	3.58655
1.2^2	1	0.14514	0.09935	0.00228	0.00199	0.00410	0.08912	0.00243	−0.00127	−0.28184	86.71589
	10	0.52198	0.41436	0.13005	0.04549	−0.05221	0.40777	0.16182	−0.15682	−0.91038	54.85474
	50	0.65005	0.60460	0.37591	0.17468	−0.29039	0.97368	0.33485	−0.29105	−0.60764	13.34916
	200	0.65652	0.60385	0.46751	0.25681	−0.31942	1.02762	0.38992	−0.36323	−0.56348	6.73257
	500	0.65706	0.60351	0.48224	0.27468	−0.31970	1.02832	0.39775	−0.37791	−0.56279	5.95620
True values		0.65000	0.60000	0.45000	0.35000	−0.30000	1.00000	0.38000	−0.40000	−0.60000	

$$\mathbf{\Phi}_{\text{n},k}(L) = \left[\widehat{\boldsymbol{\psi}}_k(L), \widehat{\boldsymbol{\psi}}_k(L-1), \dots, \widehat{\boldsymbol{\psi}}_k(1)\right]^{\text{T}}, \tag{48}$$

$$\begin{aligned}\widehat{\boldsymbol{\varphi}}_{\text{f},k}(t) = \big[&-\widehat{\mathbf{y}}_{\text{f},k}^{\text{T}}(t-1), \dots, -\widehat{\mathbf{y}}_{\text{f},k}^{\text{T}}(t-n_a), \\ &\widehat{\mathbf{u}}_{\text{f},k}^{\text{T}}(t-1), \dots, \widehat{\mathbf{u}}_{\text{f},k}^{T}(t-n_b)\big]^{\text{T}},\end{aligned} \tag{49}$$

$$\widehat{\boldsymbol{\psi}}_k(t) = [\widehat{\boldsymbol{v}}_{k-1}(t-1), \widehat{\boldsymbol{v}}_{k-1}(t-2), \dots, \widehat{\boldsymbol{v}}_{k-1}(t-n_d)], \tag{50}$$

$$\begin{aligned}\boldsymbol{\varphi}_{\text{s}}(t) = \big[&-\mathbf{y}^{\text{T}}(t-1), \dots, -\mathbf{y}^{\text{T}}(t-n_a), \\ &\mathbf{u}^{\text{T}}(t-1), \dots, \mathbf{u}^{\text{T}}(t-n_b)\big]^{\text{T}},\end{aligned} \tag{51}$$

$$\begin{aligned}\widehat{\mathbf{u}}_{\text{f},k}(t) = \mathbf{u}(t) &- \widehat{d}_{1,k}(t)\widehat{\mathbf{u}}_{\text{f},k}(t-1) - \widehat{d}_{2,k}(t)\widehat{\mathbf{u}}_{\text{f},k}(t-2) \\ &- \cdots - \widehat{d}_{n_d,k}(t)\widehat{\mathbf{u}}_{\text{f},k}(t-n_d),\end{aligned} \tag{52}$$

$$\begin{aligned}\widehat{\mathbf{y}}_{\text{f},k}(t) = \mathbf{y}(t) &- \widehat{d}_{1,k}(t)\widehat{\mathbf{y}}_{\text{f},k}(t-1) - \widehat{d}_{2,k}(t)\widehat{\mathbf{y}}_{\text{f},k}(t-2) \\ &- \cdots - \widehat{d}_{n_d,k}(t)\widehat{\mathbf{y}}_{\text{f},k}(t-n_d),\end{aligned} \tag{53}$$

$$\widehat{\mathbf{w}}_k(t) = \mathbf{y}(t) - \widehat{\boldsymbol{\theta}}_{\text{s},k-1}^{\text{T}}\boldsymbol{\varphi}_{\text{s}}(t), \tag{54}$$

$$\widehat{\boldsymbol{v}}_k(t) = \mathbf{y}(t) - \widehat{\boldsymbol{\theta}}_{\text{s},k}^{\text{T}}\boldsymbol{\varphi}_{\text{s}}(t) - \widehat{\boldsymbol{\psi}}_k(t)\widehat{\boldsymbol{\theta}}_{\text{n},k}, \tag{55}$$

$$\widehat{\boldsymbol{\theta}}_{\text{n},k} = \left[\widehat{d}_{1,k}(t), \widehat{d}_{2,k}(t), \dots, \widehat{d}_{n_d,k}(t)\right]^{\text{T}}, \tag{56}$$

$$\mu_{1,k}(t) \le 2\left(\left\|\mathbf{\Phi}_{\text{f},k}(L)\right\|^2\right)^{-1}, \tag{57}$$

$$\mu_{2,k}(t) \le 2\left(\left\|\mathbf{\Phi}_{\text{n},k}(L)\right\|^2\right)^{-1}. \tag{58}$$

The identification steps of the F-GI algorithm with finite measurement data in (43), (44), (45), (46), (47), (48), (49), (50), (51), (52), (53), (54), (55), (56), (57), and (58) to compute $\widehat{\boldsymbol{\theta}}_{\text{s},k}$ and $\widehat{\boldsymbol{\theta}}_{\text{n},k}$ are listed as follows.

(1) Collect the input–output data $\{\mathbf{u}(t), \mathbf{y}(t), t = 1, 2, \dots, L\}$and give a small positive number ε. Construct $\boldsymbol{\varphi}_{\text{s}}(t)$ using (51).

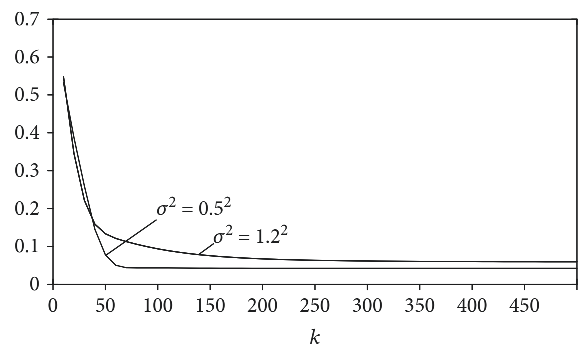

Figure 4: The CARMA-FGI estimation errors δ with different noise variances ($L = 1000$).

(2) Let $k = 1$ and set the initial values $\widehat{\boldsymbol{\theta}}_{\text{s},0} = \mathbf{1}_{(mn_a+rn_b)\times m}/p_0, \widehat{\boldsymbol{\theta}}_{\text{n},0} = \mathbf{1}_{n_d}/p_0, \widehat{\mathbf{w}}_0(t) = \mathbf{1}_m/p_0, \widehat{\boldsymbol{v}}_0(t) = \mathbf{1}_m/p_0, \widehat{\mathbf{y}}_{\text{f},0}(t) = \mathbf{1}_m/p_0, \widehat{\mathbf{u}}_{\text{f},0}(t) = \mathbf{1}_r/p_0, p_0 = 10^6$.

(3) Construct $\widehat{\boldsymbol{\psi}}_k(t)$ and $\widehat{\boldsymbol{\varphi}}_{\text{f},k}(t)$ using (49) and (50).

(4) Compute $\widehat{\mathbf{w}}_k(t)$ using (54), construct $\widehat{\mathbf{W}}_k(L)$ using (47), and construct $\widehat{\mathbf{\Phi}}_{\text{n},k}(L)$ using (48).

(5) Choose $\mu_{2,k}(t)$ using (58) and update $\widehat{\boldsymbol{\theta}}_{\text{n},k}$ using (46).

(6) Read $\widehat{d}_{i,k}(t)$ from $\widehat{\boldsymbol{\theta}}_{\text{n},k}$ in (56) and compute $\widehat{\mathbf{u}}_{\text{f},k}(t)$ and $\widehat{\mathbf{y}}_{\text{f},k}(t)$ using (52) and (53).

(7) Construct $\widehat{\mathbf{\Phi}}_{\text{f},k}(L)$ using (45) and construct $\widehat{\mathbf{Y}}_{\text{f},k}(L)$ using (44).

(8) Choose $\mu_{1,k}(t)$ using (57) and update $\widehat{\boldsymbol{\theta}}_{\text{s},k}$ using (43).

(9) Compute $\widehat{\boldsymbol{v}}_k(t)$ using (55).

(10) Compare $\widehat{\boldsymbol{\theta}}_{\text{s},k}$ with $\widehat{\boldsymbol{\theta}}_{\text{s},k-1}$ and compare $\widehat{\boldsymbol{\theta}}_{\text{n},k}$ with $\widehat{\boldsymbol{\theta}}_{\text{n},k-1}$: if $\|\widehat{\boldsymbol{\theta}}_{\text{s},k} - \widehat{\boldsymbol{\theta}}_{\text{s},k-1}\| > \varepsilon$ and $\|\widehat{\boldsymbol{\theta}}_{\text{n},k} - \widehat{\boldsymbol{\theta}}_{\text{n},k-1}\| > \varepsilon$, increase k by 1 and turn to Step 3; otherwise, obtain k and the parameter estimate vector $\widehat{\boldsymbol{\theta}}_{\text{s},k}$ and $\widehat{\boldsymbol{\theta}}_{\text{n},k}$.

Table 2: The F-GI parameter estimates and errors ($L = 2000$).

σ^2	k	a_{11}	a_{12}	b_{11}	b_{12}	a_{21}	a_{22}	b_{21}	b_{22}	d_1	δ (%)
0.5^2	1	0.14346	0.09808	0.01067	0.00762	0.00511	0.08406	0.01113	−0.00704	−0.27896	86.56006
	10	0.39890	0.32288	0.22390	0.15247	−0.01745	0.27349	0.21267	−0.15857	−0.68297	57.02995
	50	0.63521	0.59550	0.44755	0.31132	−0.21207	0.82865	0.34932	−0.32757	−0.77919	16.41676
	200	0.65416	0.60726	0.45998	0.33687	−0.31265	1.01818	0.37903	−0.40383	−0.58336	1.98279
	500	0.65417	0.60725	0.45998	0.33691	−0.31269	1.01824	0.37904	−0.40396	−0.58332	1.98657
1.2^2	1	0.14560	0.09899	0.00315	0.00157	0.00466	0.08794	0.00329	−0.00156	−0.28174	86.73735
	10	0.51092	0.41373	0.17192	0.06708	−0.05673	0.42188	0.14417	−0.12991	−0.91458	53.92889
	50	0.64851	0.60875	0.38838	0.20796	−0.28513	0.96729	0.30507	−0.28172	−0.62099	12.59343
	200	0.65613	0.60516	0.46036	0.30206	−0.31371	1.01919	0.36807	−0.38499	−0.58216	3.60147
	500	0.65684	0.60452	0.47158	0.31970	−0.31423	1.02001	0.37658	−0.40343	−0.58126	2.91324
True values		0.65000	0.60000	0.45000	0.35000	−0.30000	1.00000	0.38000	−0.40000	−0.60000	

The flowchart of computing $\widehat{\boldsymbol{\theta}}_{\mathrm{s},k}$ and $\widehat{\boldsymbol{\theta}}_{\mathrm{n},k}$ from the F-GI algorithm with finite measurement data is shown in Figure 3.

The algorithm in (43), (44), (45), (46), (47), (48), (49), (50), (51), (52), (53), (54), (55), (56), (57), and (58) for multivariable CARMA systems is based on the filtering technique and the gradient search and can be extended to more complex multivariable systems with cooled noises.

5. Numerical Example

Consider a two-input two-output CARMA system:

$$\mathbf{A}(z)\mathbf{y}(t) = \mathbf{B}(z)\mathbf{u}(t) + D(z)\mathbf{v}(t), \tag{59}$$

where

$$\begin{aligned}
\mathbf{y}(t) &= \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, \\
\mathbf{u}(t) &= \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}, \\
\mathbf{v}(t) &= \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}, \\
\mathbf{A}(z) = \mathbf{I} + \mathbf{A}_1 z^{-1} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} z^{-1} \\
&= \begin{bmatrix} 1 + 0.65z^{-1} & 0.60z^{-1} \\ -0.30z^{-1} & 1 + 1.00z^{-1} \end{bmatrix}, \\
\mathbf{B}(z) = \mathbf{B}_1 z^{-1} &= \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} z^{-1} = \begin{bmatrix} 0.45z^{-1} & 0.35z^{-1} \\ 0.38z^{-1} & -0.40z^{-1} \end{bmatrix}, \\
D(z) &= 1 + d_1 z^{-1} = 1 - 0.60z^{-1}, \\
\boldsymbol{\theta}_{\mathrm{s}}^{\mathrm{T}} &= [\mathbf{A}_1, \mathbf{B}_1], \\
\boldsymbol{\theta}_{\mathrm{n}} &= d_1.
\end{aligned} \tag{60}$$

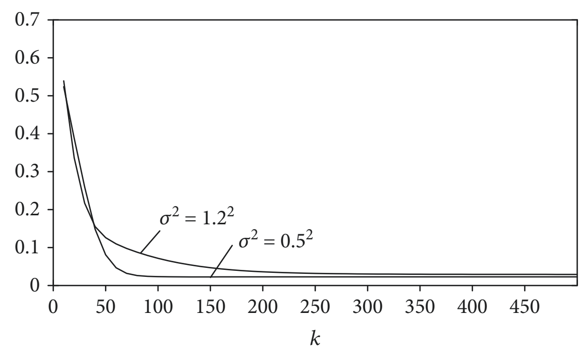

Figure 5: The CARMA-FGI estimation errors δ with different noise variances ($L = 2000$).

In simulation, $\{u_1(t)\}$ and $\{u_2(t)\}$ are taken as two persistent excitation signal sequences with zero mean and unit variance, and $\{v_1(t)\}$ and $\{v_2(t)\}$ as two white noise sequences with $\sigma_1^2 = \sigma_2^2$. Take the data length $L = 1000$ and $L = 2000$ and apply the F-GI algorithm with finite measurement data in (43), (44), (45), (46), (47), (48), (49), (50), (51), (52), (53), (54), (55), (56), (57), and (58) to estimate the parameters of this M-CARMA system. The parameter estimates and errors are shown in Table 1 with $L = 1000$, $\sigma_1^2 = \sigma_2^2 = 0.5^2$, and $\sigma_1^2 = \sigma_2^2 = 1.2^2$, and the parameter estimation errors versus t are shown in Figure 4. For comparison with the different data length $L = 2000$, the simulation results are shown in Table 2 and Figure 5.

From Tables 1–3 and Figures 4 and 5, we can draw the following conclusions.

(1) The parameter estimation errors obtained by the presented algorithms gradually become smaller with the iterative variable k increasing. Thus, the proposed algorithms for M-CARMA systems are effective.

(2) The system parameter estimates converge to their true values with the increasing of the data length.

(3) Under the same data length, a smaller noise variance leads to higher parameter estimation accuracy and a faster convergence rate.

Table 3: The F-GI parameter estimates and errors ($k = 500$).

σ^2	L	a_{11}	a_{12}	b_{11}	b_{12}	a_{21}	a_{22}	b_{21}	b_{22}	d_1	δ (%)
0.5^2	1000	0.65477	0.60673	0.46466	0.31824	−0.31710	1.02689	0.38857	−0.39418	−0.56441	3.58655
	2000	0.65417	0.60725	0.45998	0.33691	−0.31269	1.01824	0.37904	−0.40396	−0.58332	1.98657
1.2^2	1000	0.65706	0.60351	0.48224	0.27468	−0.31970	1.02832	0.39775	−0.37791	−0.56279	5.95620
	2000	0.65684	0.60452	0.47158	0.31970	−0.31423	1.02001	0.37658	−0.40343	−0.58126	2.91324
True values		0.65000	0.60000	0.45000	0.35000	−0.30000	1.00000	0.38000	−0.40000	−0.60000	

(4) A longer data length L leads to a smaller estimation error under the same noise level.

6. Conclusions

An F-GI algorithm and an F-GI algorithm with finite measurement data are proposed for identifying the multivariable controlled autoregressive system with measurement noise in this paper. The linear filter is introduced to filter the input–output data, and the hierarchical identification principle is applied to decompose the identification model into two subidentification models. The simulation results show that the proposed algorithms can generate accurate estimates. The proposed approaches in the paper can combine other mathematical tools [64–69] and statistical strategies [70–75] to study the performances of some parameter estimation algorithms and can be applied to other multivariable systems with different structures and disturbance noises and other literature [76–86] such as system identification [87–92].

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (no. 61471162) and the National First-Class Discipline Program of Light Industry Technology and Engineering (LITE2018-26).

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